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A General Overview of Risk Theory and its Application to Agriculture

Burton Pflueger
South Dakota State University

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A General Overview of Risk Theory
and its Application to Agriculture

by
Burton Pflueger*

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* Dr. Burton Pflueger is an Assistant Professor in the Economics Department at South Dakota State University.

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dynamic process occurring over a long period of time and covering a wide range of events. The process of risk management concerns designing strategies to cope with risks, tactics for implementing these strategies and controlling risks when they occur, and restoring a firm's ability to implement these strategies after periods of distress have passed. The different types of risk responses discussed below can have one of three effects: (a) risk responses can reduce the likelihood of risk occurrence by absorbing the risks within the business; (b) risk responses can reduce the likelihood of risk occurrence by transferring the risk outside the business; and, (c) risk responses can better enable the firm to bear the risks when they do occur.

Each sector of the farm business has a variety of ways to respond to risk. These risk responses are generally collected into three groups: production, marketing, and finance. Before any tactic or strategy is undertaken, its risk reducing effect must be compared to the costs of potentially lower income from its adoption.

Marketing responses to risk are used to minimize the risks of fluctuating prices. The types of market responses include: inventory management, forward commitments, pooling arrangements, and information and learning. Inventory management can be used to reduce risk by changing production and storage policies in order to affect the timing and magnitude of market transactions. One method of implementing inventory management is through spreading sales over time. As Boehlje and Trede state, "the primary advantage of spreading sales is that a producer is not tied to a specific marketing date and retains flexibility in marketing."

This seems contrary to the ideas of forward commitments which involve hedging, contract sales, and forward pricing of inputs. By hedging, a

producer shifts price risk to a speculator by use of the futures market. Such action allows a producer to establish a "set" price for his commodity, and thus "lock in" a profit margin. With such profits assured, the farmer knows that funds will be available to meet cash flow obligations. However, hedging subjects the farmer to basis risk (changes in the difference between cash and futures prices) and to additional financial risk from possible margin calls. The farmer must also avoid hedging into a loss position.

Forward contracts can also guarantee a price but do not experience margin calls. A sales contract is less flexible than a hedge as it requires delivery of the commodity. The producer should assure that the commodity will meet contract specifications at sale time. Forward pricing could also fix input costs. While reducing price risk, forward contracting may increase production risk through the fixed delivery commitment.

Other marketing responses to risk include pooling arrangements where a farmer could transfer storage, sale, or pricing to a larger organization. The larger organization would presumably be in a more competitive position than the individual farmer. The larger organization could sell greater quantities of the good at one time and perhaps command a higher price. Also the larger organization might have greater storage capabilities. Thus, the larger organization could pass the benefits of these pooling arrangements on to the farmer.

The other marketing response is to increase information and learning. If a producer can increase his education or collect additional information, he could formulate more realistic expectations about the future and better adjust his operations in response to this information.

A General Overview of Risk Theory and its Application to Agriculture

1 Risk in Agriculture

Before discussing the risks of agriculture it is useful to consider why risk and risk analysis have commanded so much time and effort of academicians. A successful farm business achieves an "optimal" organization of activities in production, marketing, investment, financing, and consumption. A common thread among these activities is the influence of uncertainty. For the farm business, operating in a competitive environment, it is impossible to eliminate uncertainty. The nature of the production process makes the producer a speculator against weather, disease, market fluctuations, and even the actions of other producers.

In 1972 Scott and Baker stated, "Much has been written about risk in farming, including major causes of risk such as plant and animal diseases, variations in the weather and other environmental changes, price fluctuations, variations in human ability and judgement, etc.." Robison and Brake state, "randomness of commodity output and prices are well-known phenomena that have plagued both farmers and their lenders as they develop plans and financial programs for the coming year." The Federal Crop Insurance Corporation may have best characterized the situation when they wrote, "In farming, risk comes with the business. It always has and always will. Fortunately though, farmers today have more and better tools to help them manage risks, or, at least, to manage certain kinds of risk."

Before considering the concept of risk management, it is important to identify the sources of risk in agriculture which Boehlje and Trede clas-

sified into six categories: price risk, governmental policy and regulations, business and legal risk, management discontinuity risk, risks of production, and technological and obsolescence risk. Price risk arises from fluctuating product and input prices, both affecting the profits of the farm business. Requiring farmers to comply with changes in governmental policy leaves the farmer uncertain about acceptable chemical usage in production operations. Changing governmental export policy, and the subsequent effect on commodity prices, is another type of government risk. Business and legal risks are increasing in importance as farmers make more use of contractual arrangements and are more subject to potential lawsuits. Management discontinuity risk arises from the need to have continuous management for efficient operation. The death of a manager could jeopardize the operation without provision for continuity of management. Weather, disease, and insects can affect production performance and constitute the major components of production risk. Technological changes can make current production methods obsolete in a short time, and being too early or late adopting new technology could be costly to the agricultural producer.

2 Risk Management

Risk or uncertainty (the terms are used interchangeably) is not necessarily a detriment. The presence of risk enables the producer to obtain profits, and compels careful business judgement. Since it is impossible to eliminate risk, business judgement is needed to reduce the uncertainty concerning the time, place, or extent of this risk.

Good business judgement involves making decisions concerning many aspects, not the least of which is risk management. Risk management is

As with marketing responses, there are different categories of production and financial responses to risk. In production these categories include: enterprise choice, enterprise diversification, flexibility, informal insurance, and cost control. Enterprise choice may reduce production risk by selecting those enterprises that exhibit low variability of income. Using one definition of risk (discussed in the following section), low variability of outcome means low risk. Similarly, enterprise diversification can reduce production risk depending on the correlation conditions for different enterprises. Correlation of incomes indicates how the income of one enterprise changes when the income of another enterprise changes. If the correlation is positive, the incomes of the two enterprises move together; the higher the correlation, the more closely the two move together. A low positive correlation or a negative correlation (resulting when incomes from the enterprises move in opposite directions) would be desired. When the returns from one enterprise are unfavorable, the returns from the other may be favorable. By diversifying, a producer could invest part of his resources in a low profit enterprise because of its favorable correlation with other enterprises. A diversified producer will sacrifice the prospects of very high expected earnings in return for a lower but more stable level of expected income.

Flexibility in operations enables the producer to better adjust to changing conditions in agriculture. Flexibility could be product flexibility - changing from one product to another (feeder cattle to feeder pigs or feeder cattle to slaughter cattle) - or cost flexibility. Cost flexibility is achieved by maintaining a high proportion of total costs as variable costs.

Producers may use informal insurance to reduce risks. Selection of disease or insect-resistant varieties of crops and livestock, health care activities, irrigation, and machinery sizing decisions are examples. Cost control results in lower costs and thus wider profit and safety margins.

Financial responses to risk include: formal insurance, holding credit reserves, managing leverage, pacing of investment and disinvestment, hedging interest rates, and use of leasing programs. Formal insurance provides a source of liquidity that may be a viable alternative for some farmers. Insurance allows an individual an amount of protection for the cost of the premium. This concept enters the decision to use public programs. A farmer can use crop insurance to guarantee a minimum level of yield for the price of the premium. These guarantees provided by public programs can affect more than the price and yield outcome expectations of farmers; they could influence lenders' credit decisions as well.

Past studies have shown that the size, structure, cost, and other attributes of credit are sensitive to the actions of farm borrowers. Credit can provide a source of liquidity from lenders. Borrowing from credit reserves prevents liquidation of assets to meet debt obligations and then reacquiring them after the stress period has passed. The amount of credit reserve is the difference between a borrower's credit limit and the funds already borrowed. The credit limit is determined by the lender based on evidence supplied by the borrower concerning his ability to meet debt obligations. The lenders' credit decisions affect a firm's performance, and this effect increases as firms experience greater reliance on leverage and liquidity. Lenders may respond to farmers' use of public programs by stabilizing or lowering the costs of borrowing. If a lender must foreclose on a farmer, there is a drain on the supply of

loanable funds. Therefore if a farmer maintains liquidity and thus reduces the probability of bankruptcy, he may be "rewarded" by the lender in the form of lower borrowing costs. Knowing the relationship between lenders' responses and the actions of farm borrowers may indicate possible modifications in financial programs and debt repayment plans used by farmers as responses to risk.

Leverage management involves structuring liabilities so that current debt obligations do not exceed the farm business' capacity to meet those obligations. Adjusting the pace of business growth or disinvestment attempts to better match debt obligations with the pattern of debt servicing capacity. A pace that is too slow or fast can cause cash flow problems.

The hedging of interest rates would allow a farmer to stabilize the costs of borrowed capital. Leasing land on a share-rent basis allows farmers to share their business risk with land owners, and enables farmers to control an asset's use without having to pay the full acquisition price. Thus leasing can alter a firm's liquidity position.

3 Risk Definitions and Measurement of Risk

The definitions of risk differ with no apparent consensus on the "correct" definition. The risk definition chosen is based on the conditions of a particular study. Lowrance has defined risk as the probability and severity of adverse effects. This definition relates the causes and effects of risk in determining who is exposed to what risks, in what way, and for how long. In this approach, analysts identify the adverse effects of risk in order to determine the producer's risk bearing capacity.

Frank Knight was one of the first to examine the distinction between risk and uncertainty. He defined risk as a situation in which the future can be predicted with a specified degree of probability. Uncertainty was the situation where no probability could be assigned to future events. In recent literature no distinction is made between risk and uncertainty, due to the emergence of subjective concepts of probability. Other definitions of risk include: uncertainty in regard to cost, loss, or damage [Hardy]; the chance of loss [Webster's Dictionary]; the probability of disaster, and the dispersion of possible outcomes (Roumasset).

Gabriel and Baker separate total risk into the components of business risk and financial risk. They define business risk as the variability of net operating income or net cash flows. Business risk is independent of the firm's financial organization; it can be further classed into the production and marketing risks discussed above. Financial risk also has two components: the risk of cash insolvency and the risk accruing to equity holders from a firm's fixed financial obligations associated with debt financing and leasing.

The discussion of measuring risk is as varied as the definitions of risk. If risk is defined as the dispersion of possible outcomes, the appropriate risk measure is one that describes this dispersion such as the variance of the probability distribution. If risk is defined as the chance of loss or probability of disaster, the measure of risk is given by the probability of obtaining an outcome below a specified level; this risk constraint can be stated in terms of outcome levels or probabilities.

Roumasset reports Joseph Stiglitz as defining risk similar to love; a term familiar to all and defineable by none. This definition places risk as a general term and never tries to quantify it. Generally those

striving to find the appropriate measure of risk are seeking some number that will represent risk in the most valid way. Rothschild and Stiglitz attempted to define risk from the concept of risk aversion. Dillon suggests that risk definitions should be avoided but believes risk aversion can be discussed without confusion (Roumasset). Stiglitz has provided evidence for avoiding classifications of risk attitudes (e.g. risk neutrality, risk aversion, relative risk aversion). However, as Arrow has pointed out, since the writings of Bernoulli it has been common to assume that individuals tend to display aversion to the taking of risks, and that this concept of risk aversion is an explanation for many observed phenomena in the economic world.

4 Risk Theory

To explain these economic observations, a theory has been sought to predict how individuals react in uncertain situations. This theory would contain a set of propositions about choice rules to indicate which action, from a set of available actions, would be followed. "Generally it is assumed that certain consistency relations hold among the choices from different sets of possible actions and that these sets belong to some restricted class" (Arrow).

Arrow states that the basic need for a theory of behavior under uncertainty arises from two considerations. The first involves the decision maker's subjective feelings of imperfect knowledge in choice decisions. The second consideration is that some observed phenomena, like insurance, cannot be explained on the assumption that individuals act with subjective certainty. A general approach that includes the decision maker's attitudes towards additional wealth is sought. The need to in-

clude this attitude has precluded using a criterion like maximizing expected money value in favor of the more general expected utility theory. Dillon suggests that expected utility is the only approach that is normatively coherent and logical as a basis for choice decisions under uncertainty.

The expected utility model gives a single-value index for ordering choices, while considering the preferences of the decision maker. It separates the decision maker's attitude toward additional wealth from his perception of the amount of uncertainty involved. Dillon points out that

on the basis of three simple and reasonable postulates about rational choice, it (the expected utility model) implies: (a) the existence of a subjective probability distribution for the uncertain outcomes associated with any risky alternative; (b) a utility function that reflects the decision maker's preferences between alternative risky choices; and (c) that risky chance is optimized by choosing the alternative with the highest expected utility.

The three postulates about rational choice form the basis of Bernoulli's principle. These axioms have been given a variety of names and some works contain elegant proofs of the theorem. The first axiom is that of ordering and transitivity. This axiom states that a decision maker either prefers one of two risky prospects or is indifferent between them. A presumption of ordering is not trivial and becomes less so as we extend the concept to transitivity by adding more than two prospects. The concept of transitivity states that if prospect A is preferred to prospect B and prospect B is preferred to prospect C, then prospect A is preferred to prospect C.

The second axiom, called the continuity axiom, states that if the decision maker prefers A to B to C, then there exists some probability $P(A)$ other than zero or one such that the decision maker is indifferent

between prospect B and a lottery that yields prospect A with the probability $P(A)$ and prospect C with the probability $1 - P(A)$. This axiom implies that, faced with a risky situation involving both a good and a bad outcome, the decision maker will take the risk if the probability of getting the bad outcome is low enough.

The third axiom is the independence axiom. The concept of independence is illustrated as follows: if prospect A is preferred to prospect B and prospect C is any other risky prospect, then a lottery with outcomes of A and C will be preferred to a lottery with outcomes B and C provided the probability of obtaining outcome A is equal to the probability of outcome B. The preference of outcome A over outcome B is unaffected by, or independent of, the presence of C.

Based on these axioms, the expected utility theorem is stated as follows: for the decision maker whose preferences do not violate the axioms of choice, there exists a utility function, U , by which preference values can be assigned to possible outcomes and the expected value of which is in terms of the decision maker's probability distribution for outcomes under each choice alternative. Some authors (Arrow) refer to utility in terms of actions and states of the world. Actions are alternative ways of getting a job done. Each act has an outcome or range of outcomes which are affected by states of the world. The consequence (c_{ij}) of an action (a_j) is affected by the state of the world $(P(\theta_i))$, which in turn affects the utility derived from that action. Thus the utility of a risky prospect is obtained by finding the expected value of the utility function in terms of the consequences of the action taken. If there are discrete outcomes then the utility of action a_j is found by: $U(a_j) = \sum_j U(a_j | \theta_i) P(\theta_i)$. If the outcomes are continuous, the utility formula is:

$U(a_j) = \int U(a_j | \theta) f(\theta) d(\theta)$. The scale on which utility is defined is arbitrary. While some speak of cardinal utility measures, it is generally believed that utility measures can only provide an ordinal classification of risky prospects. While tempting, it is meaningless to say one prospect yields twice as much utility as another or to compare utility values between individuals.

A theory of choice based on these axioms has both merits and limitations. According to Dreze, the merits and limitations of a theory can be examined from the viewpoints of relevance, usefulness, generality, and integration with other theories. From the viewpoint of relevance, expected utility, (EU), has strong normative appeal. The axioms of EU theory are considered acceptable, but the strength of conclusions derived from them may limit the descriptive realism of the theory. The actions of some individuals may fail to satisfy the axioms. Also the formalization of even a simple decision problem proves amazingly intricate when all variables facing the decision maker are considered. It would be surprising that any theory could correctly portray the decision situation for such complex problems. However, decision makers usually simplify the decision situation for an orderly, manageable approach that may not utilize all available information in the best possible way. Thus, modeling approaches that appear abstract due to sparse detail may correspond more closely with actual decision situations than is actually believed.

The viewpoint of usefulness considers the theory's capacity to resolve logical difficulties in stating and solving problems. In this regard expected utility theory is considered more general and flexible than other alternatives. Expected utility theory also applies the mathematical theory of probability to decision problems. Its usefulness has

been questioned on such points as the calibration of probabilities, reliability in eliciting utility functions, and the discovery of an optimal act from a given set (Dreze).

Dreze suggests that expected utility theory has almost complete formal and logical generality. Formal generality is earned by the ability to encompass problems that before were handled by pure mathematical theory; that is, problems where the relative frequencies in repeated situations could only be handled by the formalism of probability. Logical generality involves expected utility as the decision criterion. Thus a single answer is given to various decision situations.

A major virtue of expected utility theory is its integrated treatment of statistics, economic, and decision theory. In economics the concept of risk is described through events and consequences, valued in utility terms. The probabilistic measure on events provides the bridge with statistical theory. The theory applies to a single decision maker, but can easily be extended for group decision analysis.

5 Risk Attitudes

5.1 Risk Attitudes and Risk Aversion

The preference of a decision maker for one risky prospect over another depends on his risk attitude. Generally the attitude toward risk is evidenced by the shape of the utility function, where wealth is the object of utility. As depicted in Figure 1, the decision maker's utility function will have one of three general forms: linear, concave or convex with respect to the origin. A linear function would imply a risk neutral attitude. A concave function shows aversion to risk and a decision maker with a convex utility function would be classified as risk preferring.

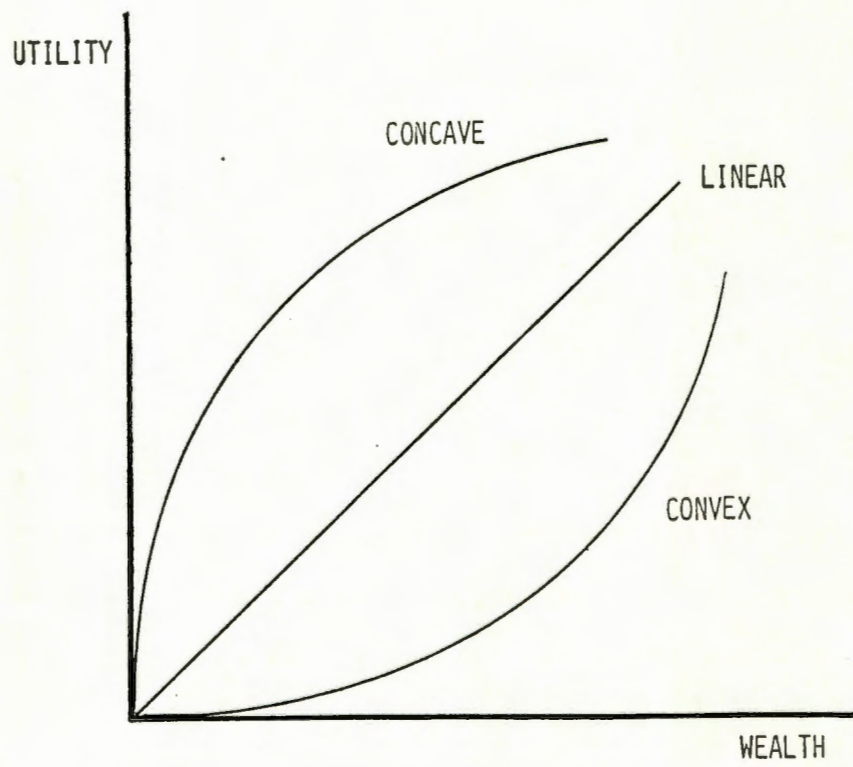


Figure 1. Shapes of Utility Functions

However, the Friedman-Savage approach and work by Kahneman and Tversky suggest that an individual's utility function could have both concave and convex segments.

The concept of risk aversion states that a risk averse individual is one who, starting from a position of certainty, is unwilling to take a fair bet. Alternatively, one can say that a risk averter will require compensation for taking risks. In contrast, the risk neutral individual would make his decision without requiring compensation for risk bearing. The risk preferring decision maker would pay for the chance to take risks. Empirical evidence has shown most individuals display risk aversion, although the degree of risk aversion will vary among individuals.

Arrow and Pratt individually related the concept of risk aversion to the concept of utility in order to derive a measure of risk aversion that would be useful for interpersonal comparisons and other purposes. They specified utility as a function of wealth, as shown in Figure 1. The first derivative of the utility function is the marginal utility of wealth; the second derivative quantifies the rate of change of marginal utility with respect to changes in wealth. Arrow and Pratt sought a measure of risk aversion based on the second derivative of the utility function, but modified so as to remain unchanged under positive linear transformations of the utility function. A measure with these properties is the ratio of the second derivative of the utility function to the first derivative. Two such measures are:

$$\text{absolute risk aversion} = R_a(W) = -U''(W) / U'(W)$$

$$\text{relative risk aversion} = R_r(W) = -WU''(W) / U'(W)$$

The absolute risk aversion measure appraises risk in absolute terms, whereas the relative risk aversion measure is in proportion to the wealth

position. The measure of relative risk aversion is a measure of the elasticity of the marginal utility of wealth. It changes not only with changes in the units of utility, but also with respect to changes in wealth.

5.2 Determining Risk Attitudes

Where the theory of risk aversion is fairly straight-forward, the determination of an individual's risk attitude is more complex. Obviously, the problem would be less complex if the individual's utility function were known. Since this is generally not the case, an individual's utility function is estimated by fitting a specific function to utility values elicited from that individual. More will be said later on the functional forms of the utility function, but first the derivation of individual utility values is examined.

According to Robison, Barry, Kliebenstein, and Patrick the approaches to measuring risk attitudes include: (a) direct elicitation of utility functions (DEU); (b) interval measures of risk aversion; (c) experimental methods; and, (d) observed economic behavior (OEB).

The DEU method involves, as the name would imply, direct contact with the decision maker. Generally the direct elicitation method involves the expected utility approach. Most elicitation procedures use hypothetical gambles - generally 50/50 gambles - where each outcome has the same probability of occurrence. These outcomes may be expressed as monetary gains or losses. The various elicitation procedures yield a series of points in utility-monetary outcome space that can then be fitted with a utility function.

Three well known varieties of the DEU method are the Von Neumann-Morgenstern (VN) method, the modified Von Neumann-Morgenstern method, and the Ramsey method. The Von Neumann-Morgenstern method requires the decision maker to identify the probability of occurrence for the favorable outcome that would leave him indifferent between the risky prospect and a certain prospect whose value is the average of the favorable and unfavorable outcomes. This method is also called the equally-likely-risky-outcome method (ELRO).

For this method the solicitor starts by defining a reference interval over two money outcomes. This interval is then broken into smaller intervals on which the individual will be tested about his utility preferences. Next the solicitor establishes an arbitrary origin and scale of measurement. Then the decision maker is presented with hypothetical lotteries, generally with equally likely (50/50) probabilities; the value of reward on one lottery is changed until the decision maker is indifferent between that lottery and one that has a specified utility value. A combination of such lotteries yields a series of money-utility pairings that is plotted, and a functional relationship established.

The modified Von Neumann-Morgenstern method uses the 50/50 lotteries and elicits these utility values or preferences considering equally likely risky prospects, and then determines the certainty equivalent (ELCE). A certainty equivalent is the amount that a decision maker would exchange for certain that leaves him indifferent between the risky prospect and the certain prospect. As an example, consider a risk averse decision maker who is faced with the following prospect that has two equally likely outcomes. Outcome A is the possible payoff of \$11,000 and outcome B has the possible payoff of \$9,000. The expected monetary value of this

situation is \$10,000. The expected utility of this payoff is shown by point B in Figure 2. The purchase price, or cost of the prospect, if equal to the expected monetary value, is represented by point F which exceeds point B. Thus the investor will not undertake this prospect because the zero monetary gain translates into a utility loss for the decision maker. The concave utility function portrays decreasing marginal utility of wealth such that the \$1,000 between \$9,000 and \$10,000 is more valuable to the decision maker than is the \$1,000 between \$10,000 and \$11,000. Thus, for the risk averse decision maker, a fair bet, one where the cost to play is equal to the expected monetary payoff, translates to a utility loss and the decision maker would not play. He would, however, be willing to play if the price he had to pay would yield the same utility as the expected utility of the investment. For the example above, the expected utility of the prospect is point D (see Figure 2) yielding a certainty equivalent of \$9,700.

For a risk averse decision maker the certainty price, or certainty equivalent, is always less than the expected monetary value of the prospect. The difference is the risk premium; in the above example the risk premium is \$300 ($\$10,000 - \$9,700 = \300). A series of such 50/50 "lotteries" would yield a series of certainty equivalent points that are plotted on a graph where utility and wealth are the axes. Thus the series of lotteries and the certainty equivalents are used to estimate the decision maker's utility curve.

The Ramsey method is similar to the modified Von Neumann-Morgenstern method. It elicits certainty equivalents for a series of risky alternatives. This modification is intended to overcome decision maker's possible biases towards gambling or toward selected probability levels.

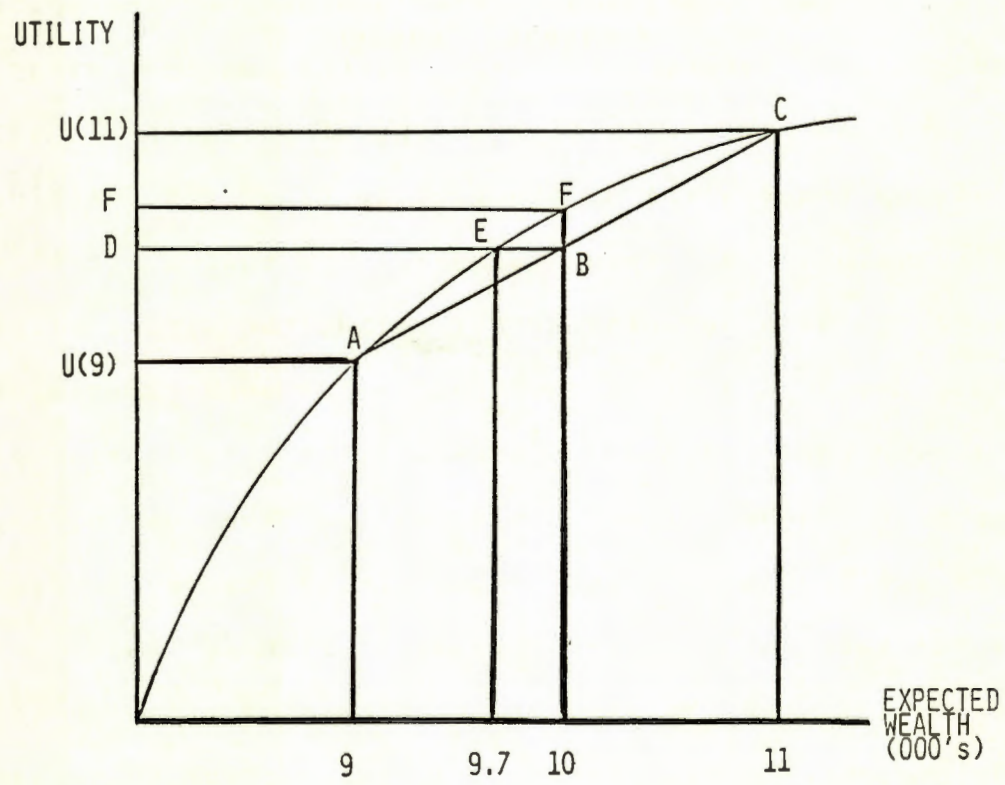


Figure 2. Certainty Equivalents

The direct elicitation method of determining decision maker's utility has been criticized because, as some (Young, Robison et al.) proclaim, it is subject to bias. This bias could arise from different interviewers administering the test, decision makers' preferences for special probability conditions or aversion to gambling, the non-inclusion of extraneous variables, time and experience limitations of participants as well as their beliefs on the realism of the game setting, the functional form of the fitted function, and compounding errors in the elicitation process. While these concerns about bias appear valid, the direct elicitation method should not be heavily discounted as few other approaches can provide such a rich empirical base. Moreover, the elicitation procedure for the DEU method has been refined considerably. The cost of the elicitation procedure is still high, however, which limits its use in micro decision situations and research concerning farmers' risk attitudes.

In response to the problems of the direct elicitation approach, King and Robison propose the risk interval approach. For this approach decision makers are asked to order pair-wise comparisons of probability density functions so that a confidence interval for a risk attitude measure can be identified. They propose the Arrow-Pratt absolute risk aversion measure on which to identify the interval. The first step in this procedure is to assume that a constant risk aversion measure, symbolized as λ , is a good approximation of the true risk aversion function, at least over a small range. Next the risk aversion measure is calculated such that the calculated expected utilities for two different probability density functions are approximately equal. The decision maker then states a preference for one of the two density functions. By stating his preference the decision maker has bisected the range of the constant risk aver-

sion measure. A series of such questions would narrow the range of measure until it converges on a single risk aversion value. This method affords the analyst greater flexibility in measuring risk attitudes. It also allows greater generality in stating the relationship between risk aversion and monetary outcomes as perceived by the decision maker. This generality would allow for changes in a decision maker's risk attitude whereas a specified functional relationship would have an associated pattern of risk attitude.

Binswanger used a different approach to determine risk attitudes. His approach, called experimentation, is based on gaming situations. This approach resembles the direct elicitation approach but reduces some of its bias. Actual financial compensation was included to add realism; the experiment was conducted over time to permit the respondents to reflect on their decisions, and respondents were taught about the gaming process prior to the analysis.

Similar to the DEU method is an approach proposed by Patrick, Blake, and Whitaker. Their approach to measuring risk attitudes, labeled "magnitude estimation", is in the class of "ratio-scaling" techniques. The decision maker is asked to judge the importance of one item relative to a base item. In this case farmers could be interviewed about the importance of different goals. More will be said about multiple goals later.

The magnitude estimation procedure has several advantages. It is a well-documented procedure that has proved reliable in a variety of contexts. It is easy to use. Individuals are asked to assign values to certain items according to their relative importance. Also, the judgments obtained from farmers should be comparable across individuals and thus allow for aggregation of results to analyze macro decisions.

The observed economic behavior method draws inferences about risk attitudes from comparisons between decision makers' actual behavior and their behavior as predicted by empirical models. If, for example, two decision makers were stating preferences about risky alternatives, one having a lower return than the other, the decision maker choosing the lower risk alternative is considered more risk averse.

Young summarizes the advantages of the observed economic behavior method. Like the direct elicitation approach, it can provide empirical measures of risk aversion. It can handle large amounts of data and is less costly than interview methods. It determines risk attitudes from actual situations and not hypothetical gaming situations. One major drawback of this method, however, is that any difference between actual performance and predicted behavior is attributed entirely to risk aversion. The actual behavior exhibited by a decision maker is probably influenced by other variables as well.

6 Multidimensional Utilities

Most decision makers realize that money and risk are not the only variables worth considering. To capture the effects of these other variables some analysts deal with multidimensional utilities [Rausser and Yassour, Herath, Hardaker, Anderson]. Assessment of multidimensional utilities involves the benchmark, "quasi-separable" utility function, or the additive utility function approach (Anderson, Dillon, Hardaker).

For the benchmark approach it is essential that for every multidimensional consequence another consequence can be found that is indifferent to it and has constant values in all dimensions, but one whose preference is independent of the others. Using an example to illustrate this concept,

consider a consequence consisting of possible outcomes x , y , and z . If a decision maker states a preference for outcome (x_1, y_1, z_j) over outcome (x_2, y_1, z_j) , then for him values of x are preferentially independent of constant values in the other attributes. To further illustrate the benchmark approach consider a simple case using only two attributes x and y , x being preferentially independent of y . A benchmark level of y is selected, y^* , and a value for x , x' , is sought such that when paired with y^* , the decision maker is indifferent between the hypothetical consequence (x', y^*) and the particular consequence (x, y) . This consequence pairing is then associated with a single utility value and a set of such associations would give a plot of points for which a functional relationship could be established. For a more detailed discussion of this approach the reader is referred to Raiffa, 1968 or von Winterfeldt and Fischer, 1973 as referenced by Anderson, Dillon and Hardaker.

Keeney recognized that the above approach could become tedious if there were several consequences with many attributes. He developed the "quasi-separable" utility function approach to aid in this problem. For this approach we need to develop the concepts of joint preferentially independence and utility independence. For a decision maker, attributes are jointly preferentially independent if the location and shape of the indifference curve for a combination of two are independent of the level of other attributes. An attribute is utility-independent if the decision maker's preference for consequences involving that attribute do not depend on the level of other attributes.

If we have a case of n attributes and can assume that the requirements of joint preference and utility independence hold, then the utility function can be written as a function such that $U(x_1, x_2, \dots, x_n) =$

$f(U_1(x_1), U_2(x_2), \dots, U_n(x_n))$ where the utility for a particular attribute is scaled from zero to one. If the utility is scaled, the function will either be additive or multiplicative form as shown.

$$\text{Additive: } U(x_1, x_2, \dots, x_n) = \sum_{i=1}^n k_i U_i(x_i)$$

$$\text{Multiplicative: } U(x_1, x_2, \dots, x_n) = \left\{ \prod_{i=1}^n [Kk_i U_i(x_i) + 1] - 1 \right\} / K$$

where k_j is the scaling factor for the utility to be between zero and one and K is another scaling constant.

In this manner one now only need assess n one-dimensional utility functions and the n scaling factors of k_i instead of an n -dimensional utility function directly. The value of k is based on the probability, p_i , such that the decision maker is indifferent between (a) the consequence where one attribute is at its most preferred stage and all other attributes are at their least preferred stage and (b) a lottery of probability p_i that yields the consequence where all attributes are most preferred and the chance of $1 - p_i$ that all consequences will be at their least preferred state. Thus in a two attribute case, $U(x_1^+, x_2^-) = p_i U(x_1^+, x_2^+) + (1 - p_i) U(x_1^-, x_2^-)$ and hence $U(x_1^+, x_2^+) = p_i$. Again, a more detailed description of this approach is found in Keeney.

The additive utility function approach is the simplest and most widely used to evaluate multidimensional utilities. Only determination of the one dimensional utility function for each attribute and scaling factor is involved. Each attribute is scaled as in the quasi-separable approach with the most preferred set of attributes given a scale factor of one. The other attribute consequences are scaled between zero and one against

the most preferred set. Even though the assumptions of the additive utility function may rarely be met, the approach will yield results similar to, but without the complications of, a non-additive function.

A lexicographic utility ordering exists if the decision maker does not allow tradeoffs between attributes. The goals are sequentially ordered, but overachievement has no effect on total utility whereas underachievement yields infinite disutility. In terms of the axioms of expected utility theory, this ordering implies the continuity axiom no longer holds. Utility is no longer expressed as a single real number, but as a priority vector reflecting the expected utility at each attribute dimension.

Since underachievement of a goal results in disutility, the most preferred attribute must be achieved before considering the next most preferred, and so on. Most writings concerning lexicographic utility preferences use the term "safety-first" rule. Here, the decision maker satisfies the safety preference first and then seeks to meet other goals. Pyle and Turnovsky discuss three safety-first rules. The first, put forth by Tesler, assumes that the decision maker maximizes his expected returns after assuring that the chance of receiving a return less than the specified safety level (E_{\min}) is not greater than a specified probability (P). This risk can be expressed as Maximize E subject to Probability ($E \leq E_{\min}$) $\leq P$. The values of E_{\min} and P indicate the degree and direction of the risk attitude of the decision maker.

Kataoka introduced a second safety-first rule. Under this rule the decision maker chooses the plan that maximizes returns at a lower confidence limit (L) subject to a constraint that the chance of receiving less than the lower limit is not greater than the specific value of P .

The second rule could be written: maximize L subject to Probability ($E < L$) $\leq P$. This rule maximizes the return along a lower confidence limit \bar{P} .

The third safety-first rule chooses a plan that has the smallest probability of yielding a return below the specified minimum. Roy developed this plan that could be written: minimize Probability ($E < E_{\min}$).

7 Functional Forms of Utility Functions

A functional relationship for the plotted preference points is needed to derive the single attribute utility function which becomes the basis for future analysis. (Single attribute utility functions are the most commonly used). Several forms are possible: linear, polynomial, logarithmic, power, and exponential. The linear function characterizes a risk neutral decision maker as shown in Figure 1. However, empirical evidence shows most decision makers are risk averse; thus the linear function is not appropriate.

A polynomial utility function can be justified as it is a Taylor series approximation of the unknown true utility function. This equation could be written as $U(w) = w + bw^2 + cw^3 + \dots$. The number of terms to include is based on a priori judgements and on the best statistical fit of the elicited points in an empirical analysis. This will usually be done by matching the number of terms in the equation to the number that the decision maker considers when stating his preferences. Empirically, good fits have been obtained by using two (quadratic) or three (cubic) terms (moments of the probability distribution).

The quadratic utility function could be specified as $U(w) = w + bw^2$ where the restriction $dU/dW > 0$ necessitates $W < -1/2 b$, if $b < 0$, to

display decreasing marginal utility of wealth. If X were a risky prospect under consideration, its utility could be found by $U(W) = E(W) + b[E(W)]^2 + b[M_2(W)]$ where $M_2(W)$ is the variance of W . Since $M_2(W)$ must be positive and $U / M_2(W) = b$, then the requirement for a risk averse decision maker, $b < 0$, implies that variability hinders utility. The utility of the expected value of this prospect can be shown by $U[E(W)] = E(W) + b[E(W)]^2$. Comparing this function with the utility function stated earlier, it can be shown that $U(W) - U[E(W)] = b M_2(W)$. Since $M_2(W)$ is positive, the utility of a risky prospect is smaller than the utility of the expected value of a certain prospect for the risk averse decision maker, as shown earlier in Figure 2.

A similar statement applies to the cubic function, $U(W) = w + bw^2 + cw^3$. The condition, $3c - b^2 > 0$, must hold if the first derivative of the utility function with respect to wealth is to be everywhere positive. The cubic function shows an initial stage of decreasing marginal utility, followed by a stage of increasing marginal utility. This could occur for a particular decision maker, but generally there is a restricted range in which the cubic function will be relevant.

While useful in empirical application, polynomial utility functions have been faulted on theoretical grounds. One criticism is that polynomial functions are not everywhere monotonically increasing. This may not be a problem for estimated functions used over a specific range of values.

Another criticism of polynomial functions is that an n dimensional function implies that only n moments of the probability distribution affect the decision. Depending on the importance of higher moments of the probability distribution, this could lead to a wrong assessment of the

true function. If the derivative of the utility function with respect to the moment in question indicates that moment's importance, then the smaller is the derivative, the less its importance.

Perhaps the most critical shortcoming of polynomial functions is their failure to exhibit decreasing risk aversion with increases in wealth. Again this may pose no problem if the polynomial function represents a local approximation of a decreasing risk-averse function for wealth. A new function could be fit for the decision maker if the change in wealth was significant.

To overcome the faults of polynomial functions, other functional forms have been used. These functions may be more theoretically appealing, but are harder to work with. In general, the function chosen should give the best representation of the true utility function.

A logarithmic utility function would be specified as $U = \log_e W$. A power function is expressed as $U = W^c$. An exponential function is represented as $U = 1 - e^{-cW}$. If the analyst is working with normal probability distributions, the exponential function is equivalent to $EU(V) = E(V) - b\sigma_v^2$. This expression, in expected utility terms, contains the expected value, variance, and a risk aversion measure.

Assuming that an individual bases his decision on two parameters, mean and variance, a quadratic utility function could be inferred and portfolio theory can be used as our analysis tool. The decision maker's preferences could be expressed by a field of indifference curves as in Figure 3. In the figure indifference is expressed between M_R (mean of returns from an activity) and σ_r (standard deviation of those returns) on the respective axes. A decision maker is indifferent between all points

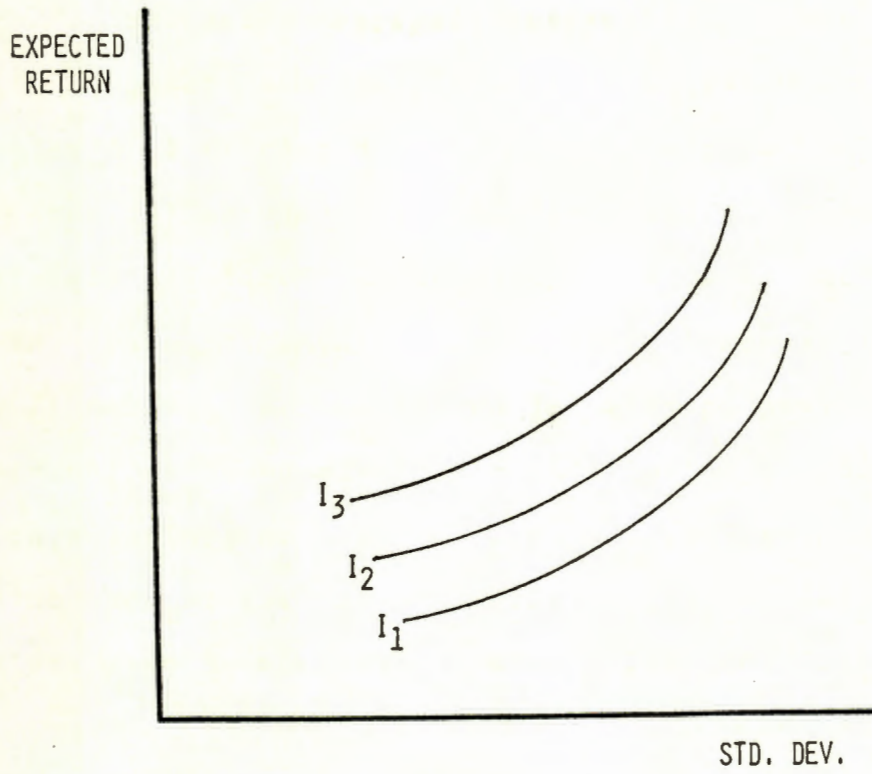


Figure 3. Indifference Curves

that lie on a curve such as I_1 and indifference curves that lie above and to the right of another are preferred (I_2 preferred to I_1).

Portfolio theory is used to explain a decision maker's choice of the "best" portfolio, where a portfolio is defined as any combination of assets. "Best" means that for a specified rate of return the portfolio has the lowest risk, or for a specified risk level the portfolio has the highest return. The expected return for a portfolio is a weighted average return over all assets in the portfolio; the returns for each asset are weighted by its proportion in the portfolio. Portfolio risk is measured by the variance or standard deviation of returns (Figure 3 then graphs the indifference curves in risk-return space). A fundamental concept of portfolio theory is the belief that the riskiness of an individual asset differs depending on whether the asset is held in isolation or as part of a portfolio. Portfolio analysis will be discussed in more detail when other models for decision making under conditions of risk are examined.

8 Risk Models

8.1 General Risk Models

Different models have been developed to study decision making under conditions of uncertainty and the effects of risk responses on the farm business. These models fall into three main categories: (a) maximizing methods; (b) efficiency analysis methods; and, (c) satisficing methods (Anderson). Each group has several subclassifications. The categories and subclassifications are discussed as follows.

Maximizing methods are the most commonly used by analysts, perhaps because they provide orderly solutions and single answers. These methods are often based on portfolio concepts involving the mean and variance of

returns. Portfolio analysis will be examined in detail, but first other methods are briefly discussed.

As Table 1 shows, the first type of maximizing model is the expected utility model. In essence, this method elicits preferences for hypothetical prospects which are then encoded into a function which is used as a choice indicator. Difficulties with this method involve the need to empirically estimate utility functions and subjective probabilities.

The next method in this category is moment expected utility. Since probability distributions can be described by moments of those distributions, the expected utility function could represent a weighted sum of the series of moments (each moment weighted by its importance to the decision choice where the total weight is one). Here the difficulty is in estimating the number of relevant moments. The expected profit method, third in this category, considers only one moment of the distribution. Such a model is applicable only for risk-neutral decision makers.

The second category of maximizing methods is labeled "security". Security methods focus on the lower ends of probability distributions and a critical level of outcomes to be met. Although not based on utility functions, Markowitz and Masson state that it is possible to relate these two categories.

First in the security category is the safety principle. The concept here is to minimize the probability that the profit goal will not fall below a specified critical level. This method usually bases the decision rule on the mean and standard deviation of the probability distribution.

Next in this category is the safety-first rule which is operationally equivalent to chance constrained programming. Here the principle is to

Table 1. Summary of Methods for Modeling Single Attribute Risky Decisions (reproduced from Anderson, page 45)

Category	Expository Reference	Exemplary Agricultural Application Reference
A. Maximizing methods		
1. Utility function		
a. Expected utility	Borch, 1968	Dillon, 1971
b. Moment expected utility	Hadar, 1971	McArthur and Dillon, 1971
c. Expected profit	Hart, 1951	Heady and Dillon, 1961
2. Security		
a. Safety principle	Roy, 1952	Boussard, 1969
b. Safety first	Charnes and Cooper, 1959; Shackle, 1952; Tesler, 1955	Boussard and Petit, 1967; Kennedy and Francisco, 1974
c. Safety fixed	Kataoka, 1963	Roumasset, 1974
d. Maximin	Wald, 1950 --	Dillon, 1962
3. Lexicography		
a. LSF	Encarnacion, 1964	Roumasset, 1976
4. Elimination by aspects	Tversky, 1972	Gladwin, 1975
B. Efficiency analysis methods		
1. Stochastic dominance		
a. FSD	Quirk and Saposnik, 1962	O'Mara, 1971
b. SSD	Hadar and Russell, 1969	Anderson, 1974
c. TSD	Whitmore, 1970	Hardaker and Tanago, 1973
d. DSD	Vickson, 1975	--
e. n-th order SD	Hammond, 1974	--
f. convex SD	Fishburn, 1974b	Drynan, 1977
2. Utility-family-specific orderings		
a. Polynomial	Hanoch and Levy, 1970	Anderson, et al., 1977
b. Exponential	Hammond, 1974	Drynan, 1977
3. Others		
a. E-V	Markowitz, 1959	Freund, 1956
b. E-A	Markowitz, 1959	Hazell, 1971
c. E-SV	Markowitz, 1959	Hazell, 1971
d. E-H	Philippatos and Gressios, 1975	--
e. Partial FSD	Anderson, 1974	Anderson, 1974
f. E-Safety	Baumol, 1963	Webster and Kennedy, 1975
C. Satisficing methods		
a. Arbitrary and unclassifiable	Simon, 1957	--

maximize an objective function which is constrained by a specified critical probability. The specified probability constrains the "disaster" in that the probability of meeting the minimum acceptable level of the objective function is greater than or equal to the specified probability. Mathematically this concept is presented as: Maximize profit (π) subject to the probability (P) of profit being below the critical level (π_c) is less than or equal to the specified probability (P^*) $\text{Max } \pi \text{ s.t. } P(\pi < \pi_c) \leq P^*$.

The safety-fixed rule is used when the decision maker is seeking to maximize his minimum return with a probability at least equal to a minimum specified probability. Mathematically the method could be specified as (using the notation above): $\text{Max } \pi_c \text{ s.t. } P(\pi \leq \pi_c) \leq P^*$. As can be seen, this rule is an alternative version of the safety-first rule. The maximum method is just a special case where P^* is zero.

Third in the class of maximizing models is lexicographic ordering. This method is usually applied to multiple goal planning, and may include the safety-first rules. These concepts were discussed earlier in this chapter. The "elimination by aspects" method is similar to lexicographic ordering. While probably better classified as a satisficing method, it is associated with the maximizing methods because of the possible application to single attribute decisions and for ease of cross reference. In this method prospects are compared, one aspect at a time, and those not meeting predetermined standards are eliminated.

Differing from the maximizing methods are the "efficiency analysis" methods. Maximizing methods result in a complete ordering of risky prospects while efficiency methods achieve a partial ordering. The first subclass of efficiency analysis methods is stochastic dominance.

Different degrees of dominance have been established, where increasing the number of degrees weakens the dominance condition. In general, the dominance rules are based on comparisons between cumulative distribution functions (CDF) for risky prospects. First degree dominance (FSD), the weakest rule having the least ordering power, occurs when one risky prospect is preferred over another by all expected utility maximizers. If this condition was shown graphically, the dominant CDF would lie to the right of the other CDF, and nowhere to the left. If the distributions were normal, this could only happen if the variables were the same but the means differed.

Second degree stochastic dominance (SSD) applies when the area encompassed by one CDF remains smaller than another, as the area is summed sequentially over larger values of stochastic outcomes. Again looking to two prospects, if A dominates B by SSD, then A would be preferred over B by all risk averse utility maximizers.

Third degree stochastic dominance is applied to utility maximizers with concave utility functions that have a positive third derivative. Third degree stochastic dominance (TSD) discriminates among prospects based on the skewness of the distribution. DSD, decreasing stochastic dominance, is weaker than third degree stochastic dominance; it applies to all risk averse decision makers with decreasing absolute risk aversion.

Stochastic dominance with respect to a function was used by King and Robison in connection with their interval method explained previously. Convex stochastic dominance and n-th order stochastic dominance have been explored by some authors. The analytical burden and questionable gains from using these more complex methods make them inappropriate for use in many studies.

The next set of methods is "utility family specific orderings". These methods build on the algebraic forms of the utility function. Anderson reports the two most prominent types of utility functions used in research are the polynomial utility function [Anderson et al. 1977] and the negative exponential utility function [Dryan 1977] (Anderson). The generalities of these functions were examined earlier.

The EV method, mean variance rule, is a special case combining both efficiency and maximizing methods. EV methods have had long and extensive use in portfolio analysis and quadratic programming. The popularity of this method warrants further discussion below.

Mean-absolute deviation analysis (EA) is a linear alternative to EV analysis. EA analysis defines risk as the mean absolute income deviation instead of the variance of income for EV analysis. Some argue that this definition is less satisfactory than variance, but EA analysis is readily adaptable to linear programming and thus has a pragmatic appeal.

Another alternative to EV analysis is ESV analysis. ESV, mean-semivariance analysis, is used when the researcher believes that semi-variance is a more satisfactory measure of risk than variance. Here only the deviations below the mean, or below some disaster level, are considered. EH, mean-entropy, analysis is another alternative that uses entropy, or disorder, as a distribution-free measure of risk. The results from EH analysis are similar to EV analysis; therefore EH analysis is not used often in decision analysis.

Anderson lists models that modify the FSD rule. These modifications are based on the belief that the analysis of overlapping CDF's are unimportant and can be ignored. Intersections in the lower tail of the CDF's occur so infrequently that they can be ignored. Those intersections that

occur above the mean or in the upper tails are also regarded as unimportant to the decision maker and can be ignored. Anderson states that there are no theoretical foundations for these beliefs; nonetheless they still have practical appeal. E-Safety methods involve the safety-first attitudes of decision makers, as discussed above.

The satisficing methods complete the classification of models. These methods use satisficing rules, as opposed to maximizing rules involving predetermined levels of attributes. These methods are vague and unconstructed; any such rules could be formulated to explain decision makers' behavior. While they may show significant explanatory power, their lack of theoretical basis limits their usefulness for formal study.

8.2 Portfolio Analysis

Because of its popularity with researchers, portfolio analysis warrants a more detailed discussion. Portfolios will differ regarding the assets that comprise the portfolio and the amount of investment in each asset. The expected return of a portfolio is the weighted average of the returns from individual assets; the asset's return weighted by the proportion of investment committed to that asset. Numerically this is shown as $R_p = \sum_{i=1}^n r_i x_i$ where R_p = return on the portfolio, r_i = return on individual asset i , x_i = proportion of investment in asset i and n = number of assets.

The "risk" of the portfolio is measured by the standard deviation of returns. The numerical calculation of risk is

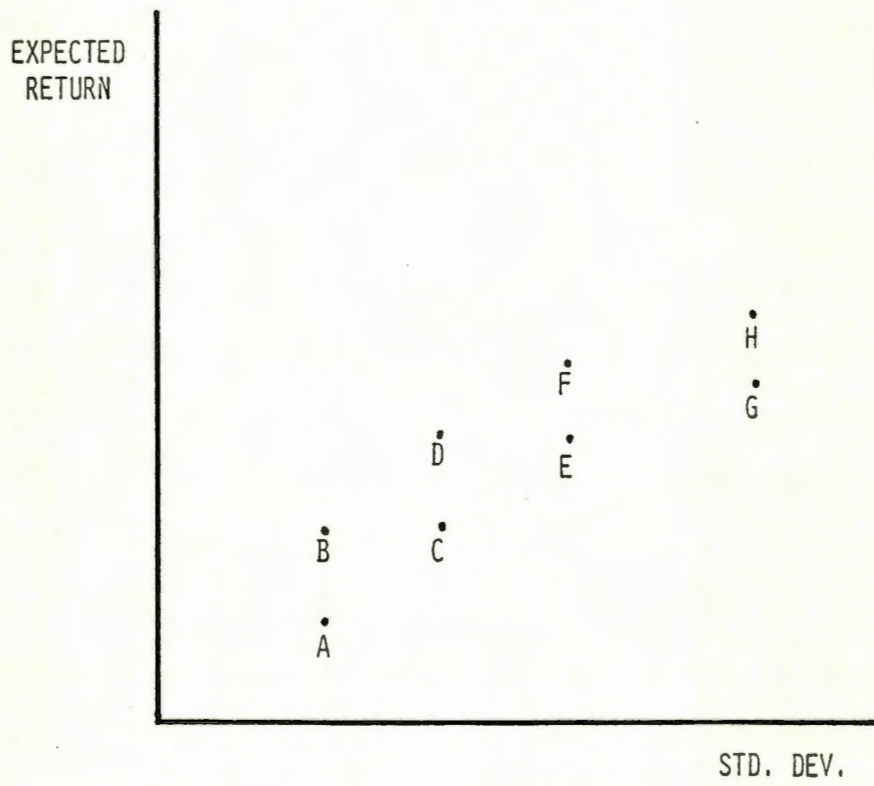
$$\sigma_p = \sqrt{\sum_{i=1}^n x_i^2 \sigma_i^2 + 2 \sum_{\substack{i=1 \\ i \neq j}}^n \sum_{j=1}^n x_i x_j c_{ij} \sigma_i \sigma_j}$$

where σ_p = standard deviation of portfolio return, $x_{i(j)}$ = proportion of investment in asset $i(j)$, $\sigma_{i(j)}$ = standard deviation of returns of asset $i(j)$, and c_{ij} = correlation of returns for assets i and j . Portfolio risk is a function of the proportions of investment, the variances of asset returns (first term under the radical), and the covariances of asset returns (second term under the radical).

The expected return and the standard deviation of a set of portfolios are plotted on a graph as shown in Figure 4.

Based on the relationships in the figure, portfolio B is superior to portfolios A and C for a risk averse decision maker. Portfolios B and A experience the same risk, but B has the higher return. Similarly portfolios B and C have the same expected returns, but portfolio C has the higher risk. The set of "efficient" portfolios are those that maximize return for a given risk level or minimize risk for a given level of return. Connecting the points representing these efficient portfolios yields an efficient frontier. This efficient frontier (EV set), shown in Figure 5, is positively sloped, indicating increasing returns are achieved only with added risk.

Some writings include a risk-free asset to show the possible gains in risk efficiency, and the implications for the financial structure of the portfolio. In Figure 6 a risk-free asset is shown as R_F , yielding return R_F and zero risk. Point M represents a portfolio on the efficient frontier. Combining the risk-free asset with those assets in portfolio M yields an extended efficient set MR_F . Points to the right of M on line MR represent portfolios that can be obtained if the investor borrows money at the risk-free rate R_F and invests in portfolio M. If no borrowing is allowed, the efficient frontier is the line R_FME .



4. Plot of Possible Portfolios

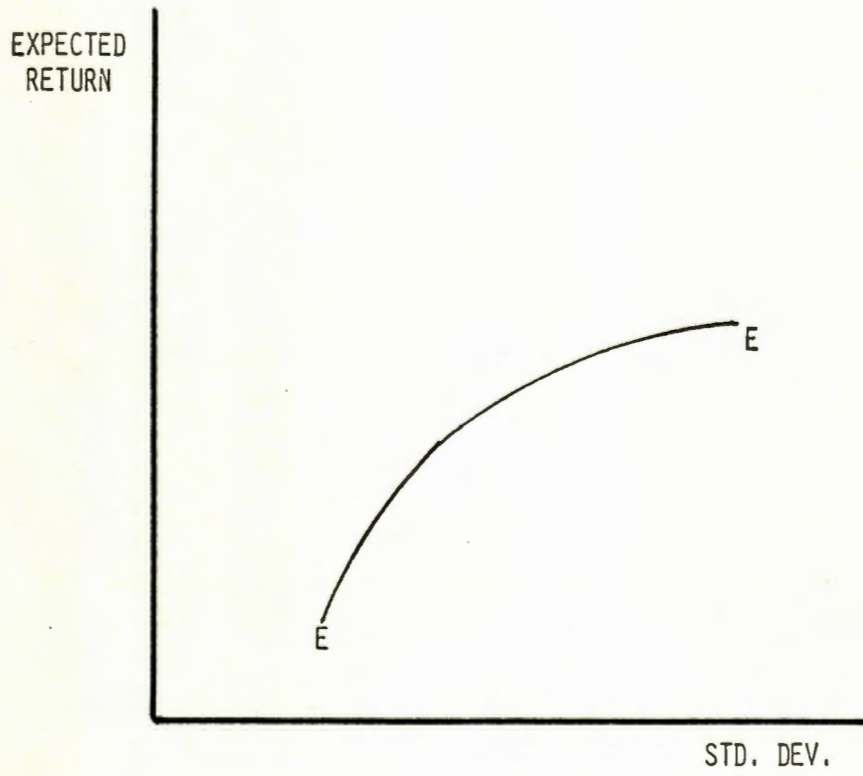


Figure 5. Efficient Frontier

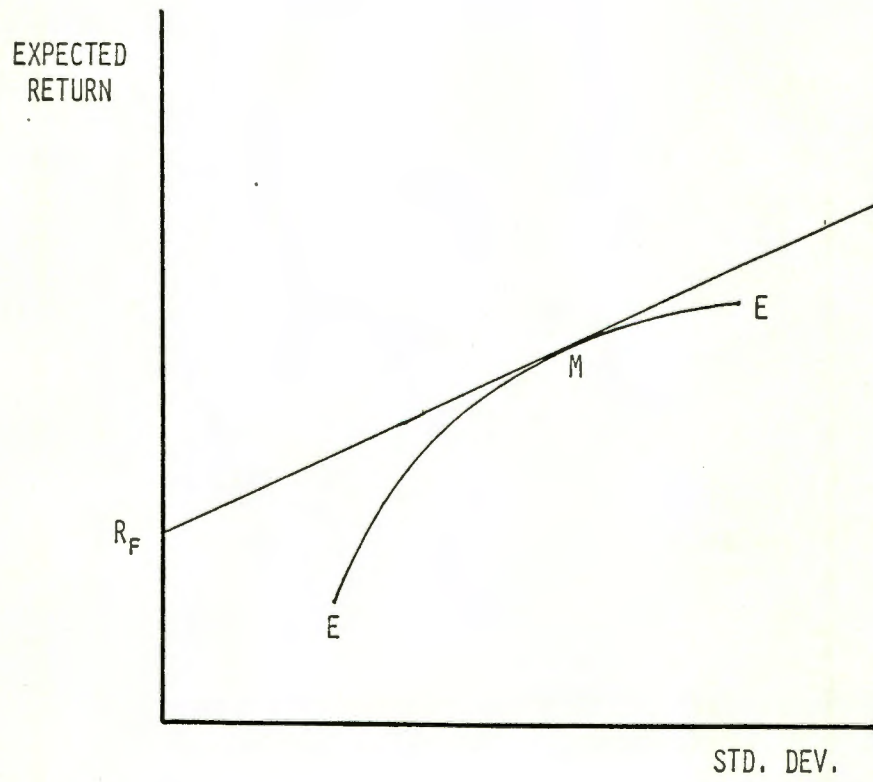


Figure 6. Efficiency Frontier Including a Risk-Free Asset

Suppose borrowing and lending were both allowed, but at differing rates. Figure 7 shows lending at rate R_F and borrowing at rate R_F' , both risk free. Lending activities involve positive holdings of the risk-free asset and borrowing represents negative holdings of the risk-free asset, both in connection with the holdings of portfolios M and N, respectively, on the efficient frontier. In this case the efficient frontier becomes $R_F M N R_F'$.

Once the efficient frontier is determined, the analysis expands to account for the decision maker's indifference curves. These curves (Figure 3) depict the decision maker's preferences. All points along an indifference curve are equal in preference and a more preferred point lies above and to the right of another (I_2 preferred to I_1).

The indifference curves of Figure 3 are combined with the efficient frontier to indicate the optimal portfolio. The optimal portfolio is represented by the tangency point between an indifference curve and the efficient frontier, as shown by point M in Figure 8. Including a risk-free asset enables the investor to reach a higher indifference curve through lending or borrowing. This is shown in Figure 9.

9 Shifting the EV Frontier

Robison and Barry examined how an optimal portfolio will change as decision makers alter their level of risk aversion or are subject to circumstances which may cause a shift in the efficient frontier. Shifts in the efficient set are caused by changes in the return of a risk-free asset, changes in the means or variances of returns on risky assets, and changes in beginning wealth position. Shifts in the EV set can be paral-

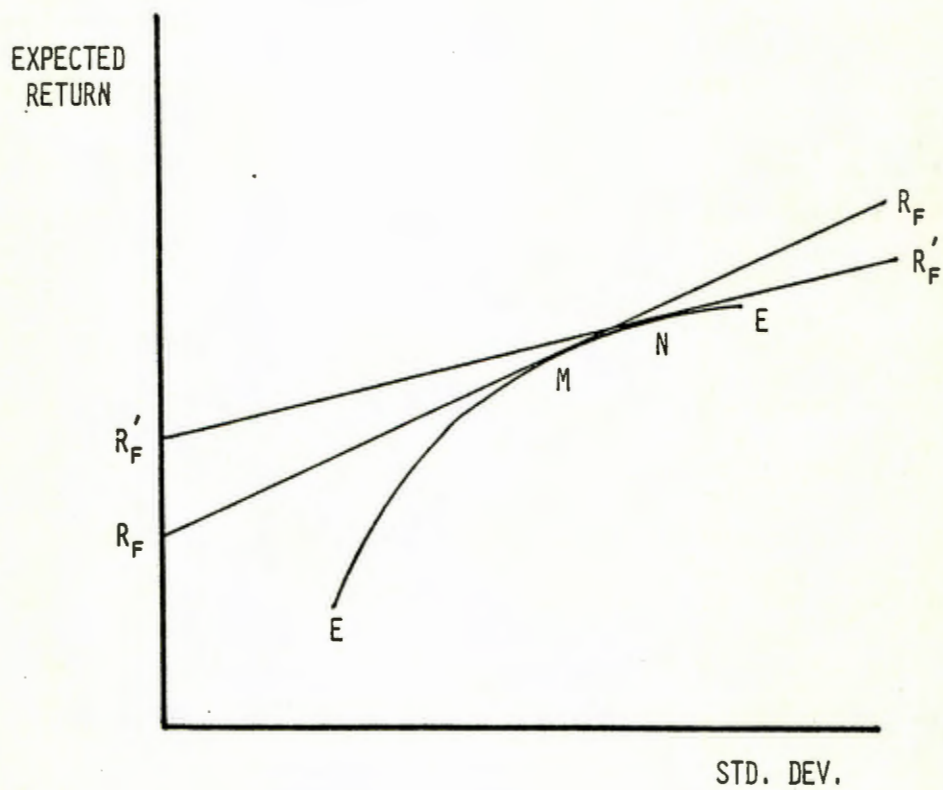


Figure 7. Efficiency Frontier for Different Risk-Free Borrowing and Lending Rates

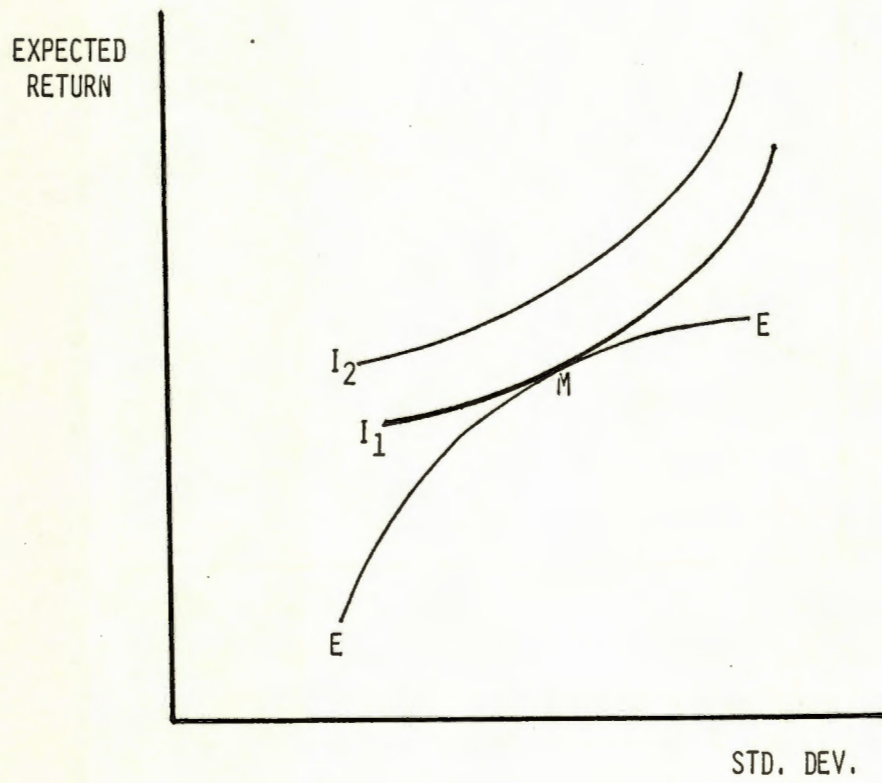


Figure 8. Optimal Portfolio of Risky Assets

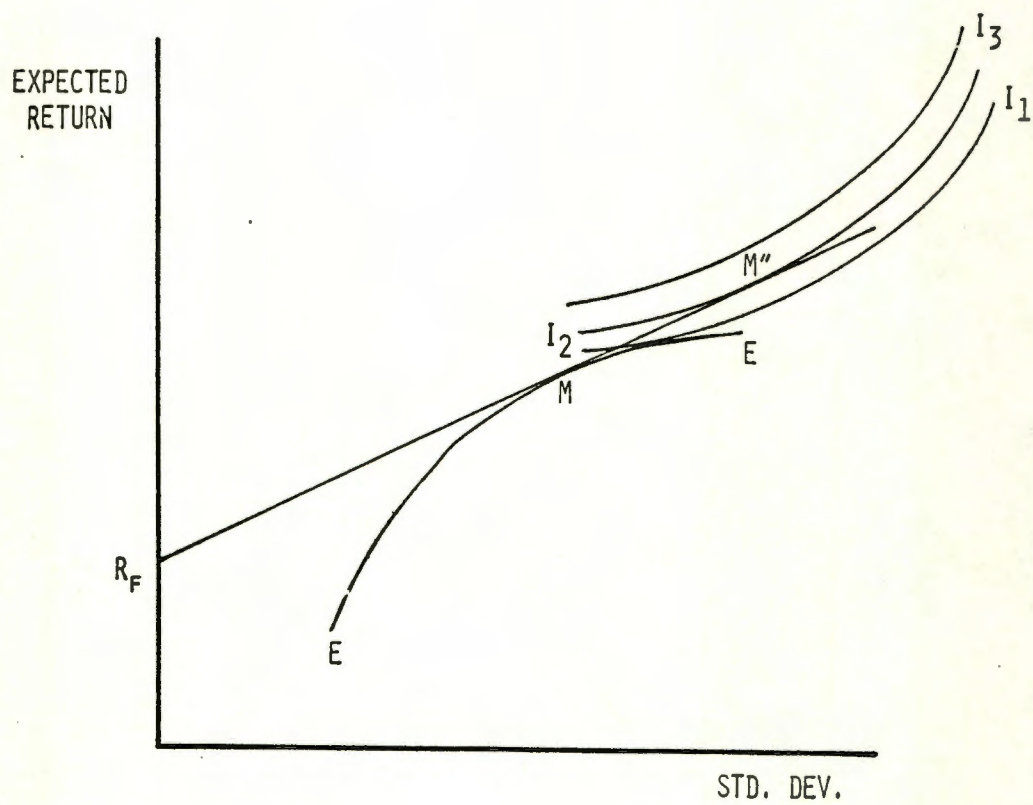


Figure 9. Increased Utility Attainment From Borrowing or Lending

lel or nonparallel, and nonparallel shifts can be either positive or negative.

Robison and Barry state, "Parallel shifts of the EV set are caused by changes in the decision maker's initial wealth." By adding wealth, the expected returns for each asset in the EV set are increased by equal amounts while the variances remain unchanged. For a parallel shift, the slope of the new EV set is the same as the slope of the old set at the same level of variance.

Robison and Barry describe positive and negative nonparallel shifts as, "A negative (positive), nonparallel shift in the EV set occurs if lines drawn tangent to the EV sets at points of equal variance form a smaller (larger) angle of inclination with the abscissa, the axis measuring expected wealth." (In their work they sketch the EV set on a graph where variance, used as the risk indicator, is measured on the vertical axis and wealth is measured on the horizontal axis.) Changes in the distribution of returns on risky assets, either expected return or variance of returns, will cause these nonparallel shifts. A nonparallel shift could result from a change in the return on the risk-free asset. These shifts will differ in that the change affecting the risk-free asset will change the point at which the EV set intersects the wealth axis whereas a change in the distribution of risky assets will leave this point of intersection unchanged. If the expected return of a risky asset were increased with no change in the variance, then any portfolio containing this asset would experience increasing returns, the same variance, and the EV set would shift to the right. A similar shift would occur if the expected return was held constant but the variance was reduced.

Changes in the portfolio holdings and therefore changes in the EV frontier can be divided conceptually into two components: income effect and substitution effect. The parallel shift of the EV frontier described earlier is considered the income effect of a change in demand for a given portfolio. In this analysis, prices of assets are considered random variables with their distribution remaining constant as risk-free wealth increases. Therefore the income effect results from increasing the risk-free wealth position of the decision maker. Cass and Stiglitz have stated a theorem relating the income effect and absolute risk aversion measures: if there are two assets, one risky and one safe, total purchases of the risky asset increase, remain constant, or decrease with increases in initial as absolute risk aversion decreases, remains constant, or increases. Robison and Barry define the substitution effect as "the change in quantity demanded resulting from a change in the probability distribution of prices after compensating the decision maker for a change in risk-free income." They summarize this concept as follows: "Any change in the quantity of risky assets that occurs while holding the equilibrium slope of the EV frontier constant is due entirely to the substitution effect. Any change in the quantity of risky assets that occurs when the slope changes is due entirely to the income effect."

Robison and Barry also presented this discussion graphically. In Figure 10 the original EV frontier is shown as curve 1. Let the initial equilibrium be at point A. The frontier then undergoes a negative, non-parallel shift to the right; the new equilibrium is point D. To determine how much of the change is caused by the substitution effect, the new frontier, curve 2, is shifted to the left until tangent with line BC, which approximates the decision maker's indifference curve. Curve 3 shows

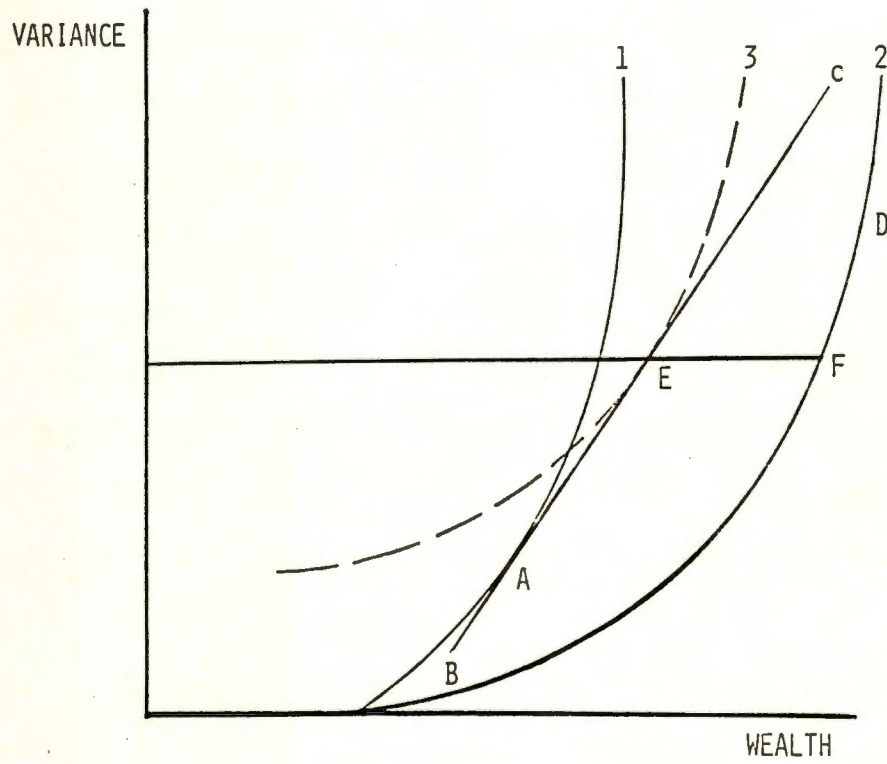


Figure 10. Income and Substitution Effects

the position of curve 2 after the shift; the tangency point is at E. The substitution effect is determined by the difference in quantity of assets in the portfolios A and E. The income effect shows the change in assets as the portfolio moves from E to D.

Shifts in the EV frontier and changes in equilibrium slopes have the following effects on the return and variance of the portfolio held. Parallel shifts on the frontier will cause the equilibrium variance to increase, remain constant, or decrease if the decision maker exhibits decreasing, constant, or increasing absolute risk aversion. If the shift of the frontier is a negative nonparallel shift to the right, then the portfolio wealth and variance increase for constant or decreasing absolute risk aversion. Positive nonparallel shifts to the right result in decreasing portfolio variance if there is constant or increasing absolute risk aversion. Similar statements can be made if the shift is to the left. It is possible to determine the effects of shifts in the EV frontier on the holdings of individual assets. This determination requires more rigorous mathematical derivation and therefore is not included here. If such is of interest, the reader is referred to Robison.

This paper has not attempted to provide a comprehensive review of all studies of risk and risk management in agriculture. Rather, the object was to present the basic ideas and theoretical foundations on which most risk studies have been constructed.

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