The Journal of Undergraduate Research

Volume 6 *Journal of Undergraduate Research, Volume 6: 2008* | Article 11

2008

# Analysis of Wind Turbine Transverse Vibration

Matt Hein
*South Dakota State University*

Whitney Karpen
*South Dakota State University*

# Analysis of Wind Turbine Transverse Vibration

Authors: Matt Hein, Whitney Karpen
Faculty Sponsor: Shawn Duan
Department: Mechanical Engineering

## ABSTRACT

Large wind turbines have become an appealing choice for renewable electrical energy production throughout the world. The United States Midwest contains massive potential for wind energy development and is therefore being primed for large wind turbines to harvest this resource and bring it to market. Because of this, safety concerns have arisen in regard to their ability to withstand the harsh climate of the region. Analysis of the transverse vibration caused by severe weather will be preformed. A simplified model will allow for an analytical solution and displacement graphs produced with MATLAB code. This information will allow for an unbiased conclusion about large wind turbine safety.

## INTRODUCTION

Wind power is the process through which wind is used to generate electricity. A wind turbine as shown in Figure 1 plays a key role for this kind of energy conversion. The wind pushes against the surface of the slanted blades that move as the air molecules slide against it. The kinetic energy of the wind is converted into mechanical power, and subsequently electricity. Though a wind turbine is an effective device for wind energy conversion, vibration as a critical issue is born with it.

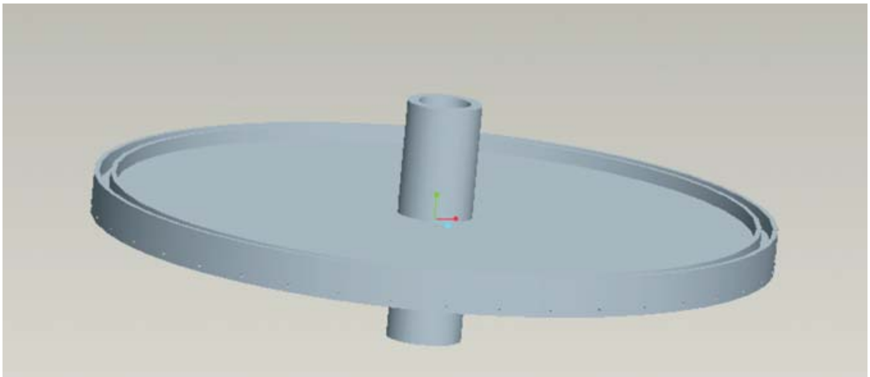


**Figure 1.** A wind turbine on prairie

In this paper, the wind turbine will be approximated as a cantilever beam that is fixed at the ground. The nacelle and blades will be combined and approximated as one mass. The supporting beam will be modeled as a spring. The model will be undamped with one degree of freedom, accounting for translational motion. The model of the system (wind turbine) can be simplified because it is a large machine and small variables will have little effect on the vibrational analysis as a whole as shown in Figure 2. The flow induced vibration of the wind turbine tower will be neglected from this analysis because it is a marginally small effect compared to the translational wind force transferred from the blades to the towers maximum height.

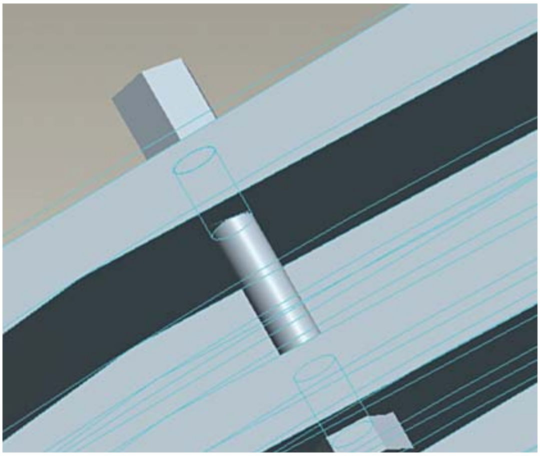


**Figure 2.** Simplification of the wind turbine to a one-degree of freedom system [1]

Equations of Motion will be developed for the model to determine if the structure can withstand severe weather of the northern United States. According to the National Weather Service website record storms in South Dakota have had wind speeds as high as 40 to 50mph. If these wind speeds were present the turbine would break its rotor at a wind speed of 25 mph a pitch its blades to reduce drag and lift. For our scenario we will consider the possibility of the pitch motors failing so that a maximum cross-sectional area is directly exposed to the highest wind speed of 50 mph.

The data being used is for a GE 1.5MW wind turbine which is one of the most widely used models in the United States.

## METHODS

### *Total Translational Force Determination*

A blade of GE 1.5 MW wind turbine is represented in Figure 3. The total translational force acting on the blade can be expressed by the following equations:

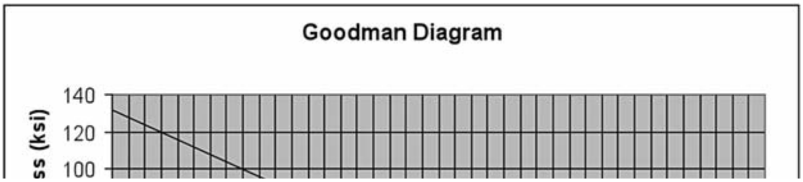



**Figure 3.** Picture of largest acting forces on blade due to wind.

wind speed: $v = 50mph = 22.35m/s$ (1)
blade weight: $W = 7tons = 6.35kN$ (2)
blade length: $l = 37.8m$ (3)
drag coefficient: $C_D = .015$ (4)
lift coefficient: $C_L = 1.3$ (5)
**Note:** *drag and lift coefficients from the reference [2]*
average air density: $\rho = 1.024kg/m^3$ (6)
average blade diameter: $D = 1.8m$ (7)

The area of the blade tangential to the wind is approximated by taking the product of the blade diameter and length.

blade area: $A = D * l = 1.8m * 37.8m = 68.0m^2$ (8)

drag force: $$F_D = C_D \rho A \frac{v^2}{2} = .015(1.024\frac{kg}{m^3})(68.0m^2)\frac{(22.35m/s)^2}{2} = 260.9N \quad (9)$$

lift force: $$F_L = C_L \rho A \frac{v^2}{2} = 1.3(1.024\frac{kg}{m^3})(68.0m^2)\frac{(22.35m/s)^2}{2} = 22.6kN \quad (10)$$

total translational force:
$$F_T = \sqrt{(F_L)^2 + (F_D)^2} = \sqrt{(22600N)^2 + (260.9N)^2} = 22.602kN \quad (11)$$

## *Equivalent Tower Stiffness Determination*

Tower height to hub: $l = 64.7m$
Tower material: assuming tower is constructed of Structural A36 Steel Alloy and the Modulus of Elasticity
Outside diameter, thickness of tower: 4.5m outside diameter with a 15mm thickness

$$k = \frac{3EI}{l^3} = \frac{3E \cdot [\pi/64 (d_o^4 - d_i^4)]}{l^3} = \frac{3 \cdot 200GPa \cdot [\pi/64 (4.5^4 - 4.47^4)]}{64.7^3} = 7.849 \times 10^5 Ns/m \quad (12)$$

## *Equivalent Mass Determination*

mass of individual blade: $m_{blade} = 647.3kg$ [3] (13)
mass of nacelle and internal components: $m_{nacelle} = 91{,}060\,kg$ (14)
$$m_{eq} = 3m_{blade} + m_{nacelle} = 3(647.3) + 91060 = 96{,}886\,kg \quad (15)$$

## *Vibrational Equation of Motion of 1-DOF Simplified Model*

System equilibrium is selected as the reference for modeling.

$$m_{eq}\ddot{x} + kx = F_T\cos(\omega t) \quad (16)$$

where: $m_{eq}$= total equivalent mass of blades, rotor, and nacelle.
$k$= stiffness of the simplified tower.
$F_T$= total translational force caused by wind.
$\omega$= frequency of harmonic wind force.

Further we use the following assumed relationship:

$$\omega = 2m_{eq}\omega_n \quad (17)$$

where

$$\omega_n = \sqrt{\frac{k}{m_{eq}}} = \sqrt{\frac{7.844\times10^5}{96886}} = 2.85rad \quad (18)$$

and

$$\omega = 2\cdot 96886\cdot 2.85 = 552250 rad/s \quad (19)$$

The total solution of the system is the sum of the transient and steady-state solutions.

$$x(t) = x_h(t) + x_p(t) \quad (20)$$

where

$$x_h(t) = C_1\cos(\omega_n t) + C_2\sin(\omega_n t) \quad (21)$$

and

$$x_p(t) = \mathrm{X}\cos(\omega t) \quad (22)$$

In equation (22), the magnitude X can be expressed as follows:

$$\mathrm{X} = \frac{F_T}{k - m_{eq}\omega^2} \quad (23)$$

If the initial conditions are represented by $x_0(mm)$ and $\dot{x}_0(m/s)$, then we have

$$C_1 = x_0 - \frac{F_T}{k - m_{eq}\omega^2} \quad (24)$$

and

$$C_2 = \frac{\dot{x}_0}{\omega_n} \quad (25)$$

Substituting (24) and (25) into (23), we have

$$x(t) = (x_0 - \frac{F_T}{k - m_{eq}\omega^2})\cos(\omega_n t) + (\frac{\dot{x}_0}{\omega_n})\sin(\omega_n t) + (\frac{F_T}{k - m_{eq}\omega^2})\cos(\omega t) \quad (26)$$

Entering the necessary information with assuming zero initial conditions, we have:

$$x(t) = (0 - \frac{22600}{7.844\times10^5 - 96886\cdot(552250)^2})\cos(2.85\cdot t) + (\frac{0}{2.85})\sin(2.85\cdot t) + (\frac{22600}{7.844\times10^5 - 96886\cdot(552250)^2})\cos(552250\cdot t) \quad (27)$$

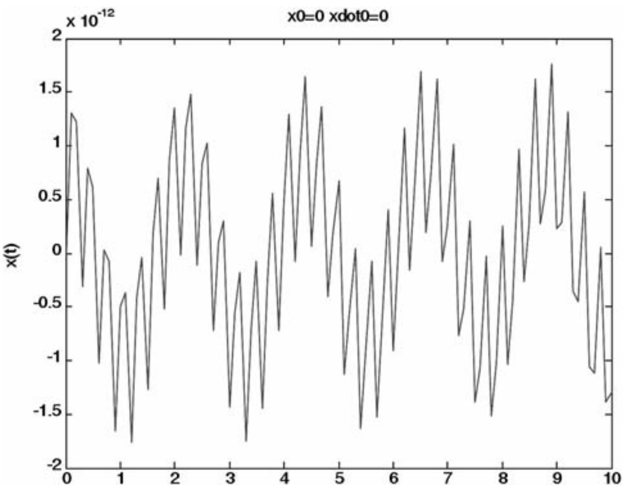
Simplifying equation (27), we have the following equation:

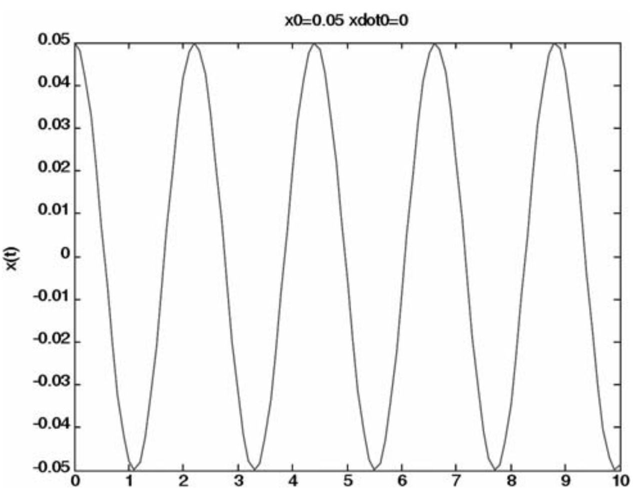
$$x(t) = 7.648\times10^{-13}\cos(2.58\cdot t) - 7.648\times10^{-13}\cos(552250\cdot t) \quad (28)$$

The resulting equation of motion is graphed in figure 4 using MATLAB

## RESULTS

Figure 4 shows the vibration responses of equation (26) with four different initial conditions for and in time domain. The associated Matlab codes are given in the appendix.



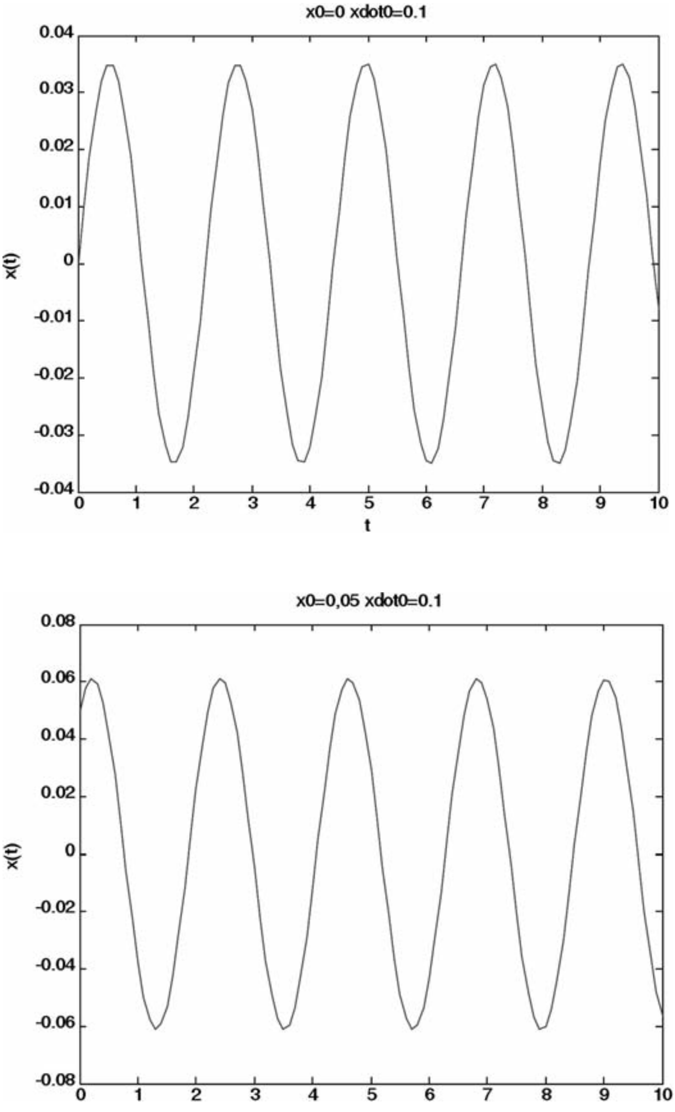



**Figure 4.** MATLAB graphical output of vibration responses.

## DISCUSSION

The solution for this vibrational system gives some preliminary data in the design of large wind turbine towers. With the wind being modeled as strong (50 mph) and gusting the movement was limited to less than 8x10-13 meters. For a system this large that amount of movement would be nearly impossible to detect. It may be possible for the induced movement to increase if the wind is "optimized" so that the frequency nearly matches the natural frequency of the wind turbine but the purpose of this discussion does not include that analysis.

## LIMITATIONS

The simplified model of the wind turbine and weather conditions account for the greatest forces in the system but do not accurately represent the directional and intensity changes for wind of severe weather. Thickness of the tower wall is a critical dimension but can only be approximated because of the competitive advantage this value represents in reducing production costs.

## ACKNOWLEDGEMENTS

Thank members of wind energy industry for providing information on physical characteristics of wind turbine. Thank Dr. Duan for guidance in analysis of the system and the production of this article. Thank Mechanical Engineering Department for providing the educational curriculum that makes this research possible

## REFERENCES

[1] Rao, S. S. Mechanical Vibrations, 4th Edition Pearson/Prentice-Hall, 2004
[2] Dixon, S.L. Fluid Mechanics and Thermodynamics of Turbomachinery. Elsevier Butterworth-Heinemann. 5th edition. 2005
[3] GE spec sheet

## APPENDIX

```
% Matlab Codes for Wind Turbine Tower Vibration
p=1.204;         %average air density (kg/m^3)
v=22.35;                 %Wind Speed (m/s)
dblade=1.8;                     %average blade diameter
lblade=37.8;                                   %length (m)
ablade=lblade*dblade;          %blade area
cdrag=0.015;                                   %drag coefficient
```

```
clift=1.3;                        %lift coefficient

%Force Calculations
fdrag=cdrag*p*ablade*v^2/2;             %drag force (N)
flift=clift*p*ablade*v^2/2;             %lift force (N)
F0=((flift)^2+(fdrag)^2)^.5;            %total force (N)

wn = 2.85;          %rad/s
m = 96886;          %kg
w = 552250;         %rad/s

xa0 = 0;
xa0_dot = 0;
xb0 = .05;
xb0_dot = 0;
xc0 = 0;
xc0_dot = 0.1;
xd0 = .05;
xd0_dot = 0.1;

f_0 = F0/m;
for i = 1: 101
 t(i) = 10 * (i-1)/100;
 x1(i) = xa0_dot*sin(wn*t(i))/wn + (xa0 - f_0/(wn^2-w^2))*cos(wn*t(i))...
 + f_0/(wn^2-w^2)*cos(w*t(i));
 x2(i) = xb0_dot*sin(wn*t(i))/wn + (xb0 - f_0/(wn^2-w^2))*cos(wn*t(i))...
 + f_0/(wn^2-w^2)*cos(w*t(i));
 x3(i) = xc0_dot*sin(wn*t(i))/wn + (xc0 - f_0/(wn^2-w^2))*cos(wn*t(i))...
 + f_0/(wn^2-w^2)*cos(w*t(i));
 x4(i) = xd0_dot*sin(wn*t(i))/wn + (xd0 - f_0/(wn^2-w^2))*cos(wn*t(i))...
 + f_0/(wn^2-w^2)*cos(w*t(i));

xend

figure
plot(t,x1);
ylabel('x(t)');
title('x0=0 xdot0=0');

figure
plot(t,x2);
ylabel('x(t)');
title('x0=0.05 xdot0=0');
```

```
figure
plot(t,x3);
xlabel('t');
ylabel('x(t)');
title('x0=0 xdot0=0.1');

figure
plot(t,x4);
ylabel('x(t)');
title('x0=0,05 xdot0=0.1');
```