

3-21-2005

# Livestock Risk Protection: An Application of Black's Option Pricing Model

Matthew A. Diersen

South Dakota State University, [Matthew.Diersen@SDSTATE.EDU](mailto:Matthew.Diersen@SDSTATE.EDU)

Follow this and additional works at: [http://openprairie.sdstate.edu/econ\\_comm](http://openprairie.sdstate.edu/econ_comm)

 Part of the [Agricultural and Resource Economics Commons](#), and the [Regional Economics Commons](#)

---

## Recommended Citation

Diersen, Matthew A., "Livestock Risk Protection: An Application of Black's Option Pricing Model" (2005). *Economics Commentator*. Paper 451.

[http://openprairie.sdstate.edu/econ\\_comm/451](http://openprairie.sdstate.edu/econ_comm/451)

This Newsletter is brought to you for free and open access by the Economics at Open PRAIRIE: Open Public Research Access Institutional Repository and Information Exchange. It has been accepted for inclusion in Economics Commentator by an authorized administrator of Open PRAIRIE: Open Public Research Access Institutional Repository and Information Exchange. For more information, please contact [michael.biondo@sdstate.edu](mailto:michael.biondo@sdstate.edu).



# ECONOMICS COMMENTATOR

South Dakota State University

No. 460

March 21, 2005



## Livestock Risk Protection: An Application of Black's Option Pricing Model

by

Matthew A. Diersen\*

Associate Professor/ Extension Specialist

### Background

The Agricultural Risk Protection Act of 2000 authorized federal insurance for livestock. The Risk Management Agency (RMA) administers Livestock Risk Protection (LRP), one of the initial products available under the Act. LRP is price insurance sold through crop/livestock insurance agents. LRP is a pilot program with risk protection features similar to, but distinct from put options on futures. LRP shares features similar to serial contracts, cash-index options, and e-mini contracts. RMA has experimented with subsidizing put options directly, for example through the Dairy Options Pilot Program. LRP premiums carry a small subsidy, which has sparked some interest among producers. For details on the mechanics of the program and how it fits into a marketing plan see Diersen (2004).

A literature search revealed a single related study, Hart, Babcock, and Hayes (2001). They examined livestock insurance, but primarily from the perspective of covering the feeding margin. Under their assumptions, risk-averse producers would benefit from insurance and other risk management tools. They have scenarios without feed cost coverage and with contract sizes adjusted below the standard sizes. The relative merits of insurance strategies, futures strategies, and options strategies depend on risk aversion and contract size.

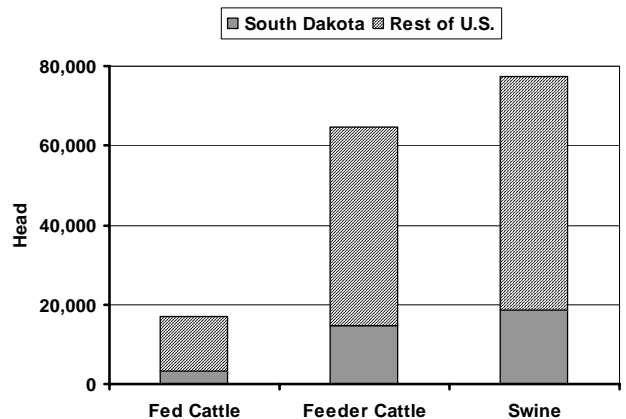
South Dakota producers have already purchased a relatively large number of LRP policies (figure 1). Producers can cover fed cattle, feeder cattle (including heifers and calves in 2005), and swine. Exposure to

information about the coverage in 2004, some features that can reduce basis risk and the ability to cover a small number of head make LRP desirable in South Dakota. However, producers have covered only a small percent of total marketings. Cattle producers have expressed interest in the product and increased use is likely in late spring and early summer as they look to protect this year's calf crop.

LRP is new and has some desirable features to manage against risk from low livestock prices. However, when producers are initially exposed to the product the array of prices and coverage levels can be difficult to sort out. Likewise, producers can replicate LRP coverage using put options. With more than one product to choose from, understanding what determines the prices will help producers pick the most cost-effective tool.

The purpose of this *Commentator* is to discuss a conceptual framework for valuing LRP coverage. The framework, Black's option-pricing model, allows for an accurate cost comparison between LRP and put options. The framework is also useful for uncovering volatility patterns, which can affect the choice between tools.

Figure 1. Head covered under LRP as of March 2, 2005



Source: USDA-RMA

\* Participants at a departmental seminar on this topic provided helpful comments.

## Pricing Options on Futures

Black (1976) has a theoretical model for pricing or valuing options on futures contracts. Livestock options are written on underlying futures contracts. One can also use Black's model to value LRP, which functions in a way similar to that of put options. Black's model shows how different aspects of options or LRP coverage influence premiums.

Black's model has a formula for put option prices,  $p$ , defined as:

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)]$$

where

$$d_1 = [\ln(F/K) + \sigma^2 T/2] / \sigma T^{0.5},$$

$$d_2 = [\ln(F/K) - \sigma^2 T/2] / \sigma T^{0.5},$$

$e$  is the number 2.71828,

$r$  is the known risk-free interest rate,

$T$  is the known time until expiration,

$K$  is the known strike price,

$\ln$  is the natural log,

$F$  is the known futures or forward price,

$\sigma$  is the unknown price volatility, and

$N(x)$  is cumulative probability or area under the normal curve.

The first term in Black's put formula,  $e^{-rT}$ , is a discount factor that negatively affects price. The term in brackets makes some intuitive sense at expiration. When time reaches zero, the put value reduces to the difference between the strike price and futures price. The interaction between time and volatility is what makes up the "time value" of the option. Both components positively influence put prices and either could be the driving force behind option values. In general, the volatility of cattle prices has increased in recent years, which directly affects the price paid for options and for LRP. Note that all the parameters are known except the volatility level. When the put price is unknown one needs to estimate the volatility. However, with observed prices volatility can be backed out of the formula.

LRP and put options are similar, but not identical. LRP premiums receive a federal price subsidy. A brokerage commission adds to the quoted cost of a put option. LRP contracts are written on underlying forward prices, called the "expected ending value" in the policy endorsement. For feeder cattle and swine LRP endorsements, the forward and futures prices settle to the same cash prices. For the fed cattle LRP endorsement, the forward prices are similar to the cash price used to settle live cattle futures contracts. LRP coverage is available daily with ending dates a fixed number of weeks in the future (the shortest

period being 13 weeks). Put options have set expiration dates and set strike prices. LRP coverage is available at fixed percentages of the expected ending value, in essence becoming strike prices. Thus, the known aspects of Black's model may differ between puts and LRP.

## Comparing Costs of Puts and LRP

One use of Black's formula is to accurately assess any cost difference in LRP and put option coverage. With different tools to choose from, producers will want to buy the lowest cost coverage (after subsidies and commissions). If the expiration dates and strike prices are the same, one can compare the costs without using a pricing formula. When the known aspects differ, one should compare LRP coverage to the put option with the maturity date closest to, but after the ending date of LRP coverage.

Here is where one can use the fact that all of the parameters in Black's model are known except for the volatility. The formula has one equation and one unknown. Beginning with an observed put premium one can back out or derive the implied volatility. Enter the known parameters and adjust the volatility until the formula price matches the observed price.<sup>1</sup> Then, by holding volatility constant at the implied level, adjust the days until expiration, the strike price, and the futures price to match a given level of LRP coverage. The resulting formula value can then be compared directly to the cost of LRP coverage.

For example, consider the situation on October 15, 2004 and compare a feeder cattle put option on the March 2005 futures contract to LRP-feeder cattle coverage. The prices are per cwt. The March futures and options contracts expire on March 24, 2005 giving  $T=159/360$  days. The risk-free interest rate was 2 percent per year giving  $r=0.02$ . The March futures settled that day at  $F=\$104.08$ . At strike price  $K=\$88$  the put premium settled at  $p=\$0.75$ , before commission. Using a spreadsheet I found the corresponding implied volatility was 21 percent (annually) or  $\sigma=0.21$ . Similarly, LRP coverage on October 15, 2004 was available with a March 11, 2005 ending date. The expected ending value was \$104.34

---

<sup>1</sup> Obtain the implied volatility by either entering the formulas into a spreadsheet or using an on-line calculator (for example, the options calculator at <http://www.ace.uiuc.edu/ofor/>).

and the coverage price of \$88.26 would cost \$0.78 before the subsidy.

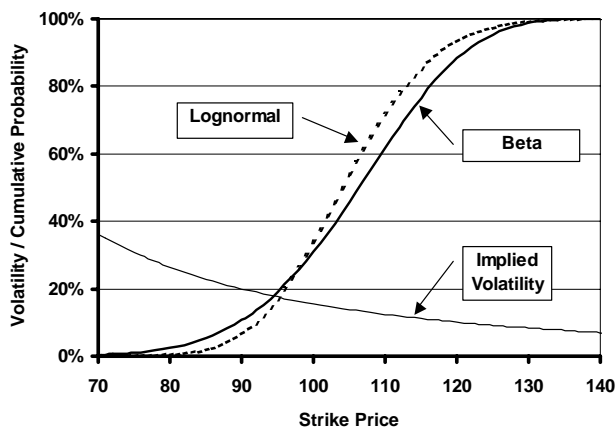
How did cost compare across products? Adjusting the put option parameters to match the LRP features, while holding the volatility constant, gives  $p = \$0.64$ . The lower price implies the expected cost of replicating LRP coverage using a put option was cheaper than buying LRP coverage outright. However, the likely commission of \$0.12 brought the total option cost to \$0.76 and the subsidy of \$0.10 reduced the LRP cost to \$0.68. Thus, transaction costs and subsidies were large enough in this example to reverse the decision in favor of LRP coverage.

## Volatility Patterns

It is common to compute the implied volatility of options using quotes from at-the-money options. Such options have strike prices closest to the futures price and tend to have greater trading liquidity, with resulting prices that more accurately reflect the volatility. When I first compared prices from October 15, 2004, I used at-the-money options and found the implied volatility was 14.4 percent. Using 14.4 percent volatility gave a formula price of \$0.12 (before commission) for the \$88 strike put option, leading me to initially conclude that LRP coverage (of \$0.68) was very expensive.

However, a distinct volatility pattern, a relationship between strike prices and implied volatility, existed on that date. Using Black's formula, I derived the implied volatility for all March feeder cattle options trading across different strike prices. The volatility decreased as the strike prices increased (figure 2). The resulting pattern was not the commonly known "volatility smile", where the lowest volatility is observed for at-the-money options. The pattern was a "volatility skew", consistent with the

**Figure 2. Implied Volatility and Price Distributions of March Feeder Cattle Futures**



volatility pattern on equities (Hull, 2000). To account for the skew one should measure volatility for the option with a strike price closest to the LRP coverage level when comparing costs. The adjustment is necessary because the volatility pattern suggested an atypical underlying futures price distribution was expected at that time.

An underlying assumption of Black's model is that prices follow a lognormal distribution. Converting the implied volatility from the at-the-money option to the variance of a lognormal distribution with a mean equal to the futures price gave the lognormal distribution shown in figure 2. The lognormal distribution understated the probability of the futures price falling below the lower strike prices and understated prices for out-of-the-money put options.

One can use option price information to derive the entire implied underlying futures price distribution. Sherrick, Garcia, and Tirupattur (1996) provide an empirical example of fitting different distributions to option prices. They use a risk-neutral valuation method that assumes volatility is time-additive, but not necessarily constant. I fit a beta distribution to the full span of prices at different strike prices.<sup>2</sup> The best-fitting beta distribution has fatter tails than the lognormal distribution and more accurately explains the observed option prices (figure 2). For example, using the beta distribution gave a put price of \$0.52 for the \$88 strike price option.

## Importance of Option Types

A producer may reach the end of the coverage period and still own the livestock. Regardless of what has happened to prices and any indemnity levels, the producer now faces price risk until the livestock are sold. Hence, underestimating the coverage period or end date is not a prudent strategy if prices were profitable when coverage is purchased, the producer is substantially risk-averse, or events may cause large jumps in prices. If prices remain steady or increase after the purchase of LRP, producers can market livestock early and not be concerned with the coverage.

If prices decline after the purchase of LRP and the producer sells the cattle within 30 days of the end date of coverage, then the producer faces basis risk. Prices

<sup>2</sup> I used Excel to change the Beta distribution parameters to minimize the squared differences between implied and actual option values.

may rise between the time the livestock are sold and the indemnity is settled. Producers cannot collect LRP indemnity payments until the ending date; meaning LRP is similar to European-style options. The comparable put options are American-style; the holder can exercise or sell them at any time.

Black's model is for European-style options. American-style in-the-money put options are worth more than European-style options because they can be exercised and not incur the discount factor shown in Black's formula. When a producer with LRP has to sell livestock before the 30-day window is in effect, it is possible to sell the coverage. Because LRP is European-style coverage, it will be worth the discounted intrinsic value when the livestock are sold.

### Conclusion

Option pricing theory provides a useful conceptual framework for valuing LRP. Put options can be adjusted to make a cost comparison against LRP. A volatility skew can affect the option prices, but making comparisons at similar strike prices can mitigate the bias. Finally, the European style will influence the price of any secondary sales of LRP coverage.

### References

- Black, F. "The Pricing of Commodity Contracts." *Journal of Financial Economics* 3(1976): 167-179.
- Dietersen, M.A. 2004. *Livestock Risk Protection: Feeder Cattle, Fed Cattle, and Swine*. FS927, Cooperative Extension Service, South Dakota State University, December.
- Hart, C.E., B.A. Babcock, and D.J. Hayes. "Livestock Revenue Insurance." *The Journal of Futures Markets* 21(2001): 553-580.
- Hull, J.C. *Options, Futures, & Other Derivatives*. Upper Saddle River, NJ: Prentice Hall, 2000.
- Sherrick, B.J., P. Garcia, and V. Tirupattur. "Recovering Probabilistic Information From Option Markets: Tests of Distributional Assumptions." *The Journal of Futures Markets* 16(1996): 545-560.

\*\*\*\*\*

### ECONOMICS COMMENTATOR

\*\*\*\*\*

Economics Department  
South Dakota State University  
Box 504 Scobey Hall  
Brookings, SD 57007-0895

Phone: 605-688-4141  
Fax: 605-688-6386  
E-Mail: Penny\_Stover@sdstate.edu

300 copies of this newsletter were produced at a cost of less than \$100



SOUTH DAKOTA STATE UNIVERSITY  
Economics Department  
Box 504  
Brookings SD 57006-0895

**Change Service Requested**

Non-Profit Org.  
U.S. Postage  
  
PAID  
  
Brookings, S.D.  
Permit 24