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Vibration Analysis of a Boat and Dock

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VIBRATION ANALYSIS OF A BOAT AND DOCK

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ABSTRACT

A structure is designed to withstand the maximum force that will be applied. In this case a boat dock is examined as a spring system. This exercise uses a typical design for a lake side boat dock, tires for buoys, and an average fishing boat. Vibration analysis can predict when the highest forces will occur. A solution can be found to graph the motion of the dock. In addition to the analytical solution, the MATLAB program is used to visualize the vibrations. With these tools the magnitude and timing of forces can be verified.

INTRODUCTION

To understand the forces on a dock when a boat hits it, it is possible to apply vibration analysis. If it is known how these forces act, better docks can be designed. Safety and strength can be improved by knowing the magnitude and timing of the force. Significant efforts have been put forward for vibration analysis of boats and docks by individuals whose interests lay in the various fields [1, 2, 3]. This paper goes through the process of vibration analysis and animation to discuss motion behaviors of the boat under the given initial conditions and its influence to the dock.

Figure 1 is a simple drawing of a boat and a dock. The left side of the dock is a fixed section while the rest is an attached floating dock. In this real world situation there are forces, transitional and rotational, in all three dimensions. That makes a very complicated six degree of freedom system.

To focus on the spring motion of the dock the system is reduced to a one degree of freedom system.

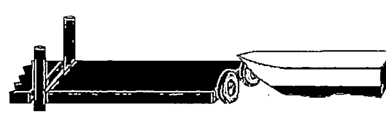

Figure 1. Real world system

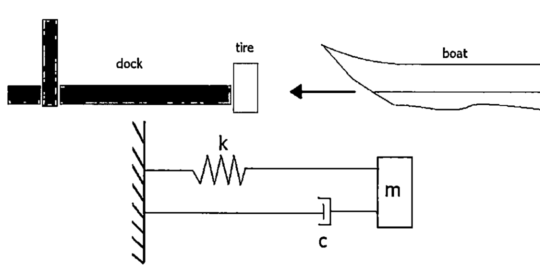

Figure 2. Reducing to a DOF system

The primary forces that will be analyzed act in a single direction, so the depth of the system can be ignored. Next the parts of the dock and boat system will be simplified into components of a Free Vibration Damped system: k-spring constant, c-damper, and m-mass.

The dock is made of wood that will absorb the impact of the boat and act as a spring for the system. A buoy, usually a tire or some rubber material, that protects the boat and dock from damage, plays the role of a damper. Water also acts as a damper on the boat as it approaches the dock. When the boat contacts the dock, it is assumed to be an elastic collision (boat and dock 'stick' together). This assumption means that none of the force is lost to the surroundings. In this way, the boat's mass is the mass of the system.

Other interactions, such as waves and currents, are considered negligible. This reaction is analyzed as a free vibration, where the initial velocity of the boat drives the motion. It is assumed that the boat is moving at a sufficiently slow speed and will not cause a forced vibration.

METHODS

From the diagram of the system as shown in figure 2., the equation of motion is set up based on Newton's second law of motion $\sum F = ma$ as follows:

$$m\ddot{x}(t) = -c\dot{x}(t) - kx(t) \qquad (1)$$

Manipulating equation (1), we have the following result

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0 \qquad (2)$$

In equation (1) and (2), $\ddot{x}(t)$, $\dot{x}(t)$, and $x(t)$ are the acceleration, velocity, and displacement of the vibration system under consideration. t is time.

To find the solution, and to simulate and visualize the results using MATLAB program, three basic parts *m, c,* and *k* of the vibrating system must be determined. The mass of the system is the mass of the boat where m=500kg. The dock is formed by parallel wood beams. The axial directions of these beams are in the motion direction of the boat. Then the following equation is used for calculation of the spring constant k [4]

$$k = \frac{AE}{L} \qquad (3)$$

where A is cross-sectional area , E young's modulus, and L is the original length of wood beams. The area is 2.5m by 0.5 m, and the length is 10 m. There is a large range for the Young's Modulus of wood. These are highly dependent on the type and direction of the wood beams. Softer types of wood will adsorb more force. Since pine is both relatively soft and cheep it often used for docks. Its modulus is about 11GPa [5]. Calculating the spring constant from equation (3) will be:

$$k = \frac{(1.5m * 0.5m)11x10^9 \frac{N}{m^2}}{10m} = 825\frac{MN}{m}$$

This is a large spring constant but consistent for this system. The dock acts as a very stiff spring, deflecting only a small amount. Damping constant is the last to be found. Since it is known that a tire on a car will damp the vibration 233 N/(m/s). Tires in this system

do not have inner tubes deflecting in a horizontal direction. The damping constant is taken to be c=100N/(m/s) for this calculation.
Now that the constants are known, the system can be examined as a whole. The spring constant is very large and the damping force is small. After the initial deflection of the spring, the large spring constant will overcome the force. This allows a prediction of an under-damped system, as shown in figure 3.

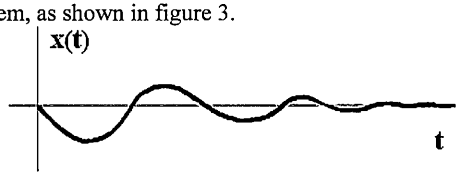

Figure 3: Sketch of the waveform

The graph tends toward zero due to the damping effect. However because the damping in this case is so small, the fluctuations will occur. After a short time the fluctuations will die out. It also should be noted that the graph is negative at the start. This shows that the initial reaction is to have a compressed spring.

To continue toward the solution of the system, the type of the system has to be mathematically determined. From definition of natural frequency and damping ratio, we have the following equations:

$$\omega_n = \sqrt{\frac{k}{m}}, \qquad (4)$$

and

$$\zeta = \frac{c}{c_c} \qquad (5)$$

where ω_n is natural frequency and ζ is damping ratio. In equation (5) the critical damping constant is c_c can be expressed as follows:

$$c_c = 2m\omega_n \qquad (6)$$

From equations (4), (5), and (6), numerical value of the damping ratio can be calculated as follows:

$$\zeta = \frac{c}{2m\sqrt{k/m}} = \frac{100}{2*500*\sqrt{\frac{825x10^6}{500}}} = 2.46x10^{-6}$$

The value of the damping ratio indicates that the system is an underdamped system.

Next the initial conditions will be applied to the system to obtaining the vibration response of the system. The initial conditions of the system are x_o=0 and $\dot{x}_0 = 5$ m/s. Since $\zeta < 1$, the vibration response of this system can be represented by the following equation:

$$x(t) = e^{-\zeta\omega_n t}\left(x_0 \cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right) + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\sqrt{1-\zeta^2}\,\omega_n} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right)\right) \quad (7)$$

Substituting numerical values of each term into equation (7), the vibration response of the system can be represented by the following equation:

$$x(t) = -e^{-.00316t}\left(0.0389 \sin(1284.5t)\right) \quad (8)$$

Next MATLAB software will be utilized to verify the mathematical derivation and dynamical analysis we had above. MATLAB [7] is a software package from MathWorks. With basic coding, it is possible to graph and animate a equation in MATLAB. The file extension is ".m" and is only used by MATLAB.

Two programs that were written by Dr. Nordenholz [6] have been used to make animation. They are as follows:

```
function freesmd_sim(t,x)
%Animation function for a horizontal spring/mass/damper
%Written by T. Nordenholz, Fall 05
%To use, type free_sim(t,x) where t and x are the time (sec) and
%displacement (m) arrays
%Geometrical and plotting parameters can be set within this program
% set geometric parameters
W=.05; %width of mass
H=.1; %height of mass
L0=1;%unstretched spring length
Wsd=.5*H; %spring width
xrect=[-W/2,-W/2,W/2,W/2,-W/2]; % plotting coordinates of mass
yrect=[0,H,H,0,0];
%set up and initialize plots
%x vs t plot
Hf=figure('Units','normalized','Position',[.1,.1,.8,.8]);
Ha_f2a1=subplot(2,1,1);
Hls_f2plot1=plot(t(1),x(1));axis([0,t(end),-L0,L0]),grid on,...
 xlabel('t (sec)'),ylabel('x (m)')
% animation plot
Ha_f2a2=subplot(2,1,2);
% create mass
Hp_f2rect=fill(xrect+x(1),yrect,'b');axis([-L0,.5,-H,2*H]),grid on
Hl_cm=line(x(1),H/2,'Marker','O','MarkerSize',8,'MarkerFaceColor','k');
% create spring/damper
Hgt_springdamp=hgtransform;
Hl_Lend=line([0,.1],[0,0],'Color','k','Parent',Hgt_springdamp);
Hl_Rend=line([.9,1],[0,0],'Color','k','Parent',Hgt_springdamp);
Hl_Lbar=line([.1,.1],Wsd*[-1,1],'Color','k','Parent',Hgt_springdamp);
Hl_Rbar=line([.9,.9],Wsd*[-1,1],'Color','k','Parent',Hgt_springdamp);
```

```
Hl_spring=line(linspace(.1,.9,9),Wsd*[1,2,1,0,1,2,1,0,1],'Color','k','Parent',Hgt_springd
amp);
Hl_dampL=line([.1,.4],Wsd*[-1,-1],'Color','k','Parent',Hgt_springdamp);
Hl_dampLpist=line([.4,.4],Wsd*[-1.3,-.7],'Color','k','Parent',Hgt_springdamp);
Hl_dampR=line([.6,.9],Wsd*[-1,-1],'Color','k','Parent',Hgt_springdamp);
Hl_dampRcyl=line([.55,.6,.6,.55],Wsd*[-.5,-.5,-1.5,-
1.5],'Color','k','Parent',Hgt_springdamp);
% set initial length
L=L0+x(1)-W/2;
set(Hgt_springdamp,'Matrix',[L,0,0,-L0;0,1,0,H/2;0,0,1,0;0,0,0,1]);
text(0,1.5*H,'|-> x');
% draw and hold for 1 second
drawnow
tic;while toc<1,end
tic

% Animate by looping through time and x arrays
% and redrawing at each value
for n=1:length(t)
 L=L0+x(n)-W/2;
 set(Hls_f2plot1,'XData',t(1:n),'YData',x(1:n));
 set(Hp_f2rect,'XData',xrect+x(n));
 set(Hl_cm,'XData',x(n));
 set(Hgt_springdamp,'Matrix',[L,0,0,-L0;0,1,0,H/2;0,0,1,0;0,0,0,1]);
 while toc<t(n),end;
 drawnow;
end
```

The next program is called Freevib.m and is written as follows:

```
%free spring/mass/damper
clear,clc,close all
%set parameters
%all dimensions in m, kg, s
k=825*1066;m=500;c=100;
x0=0;v0=5;
%calculate Wn(natural frequency) and z(damping ratio)
Wn=sqrt(k/m);
z=c/2/sqrt(k*m);
%define time array
t=0:.02:5;
%generate x array (depends on z)
if z<1
 %underdamped case
 Wd=Wn*sqrt(1-z^2);
 A=sqrt(x0^2+((v0+z*Wn*x0)/Wd)^2);
```

```
 phi=atan2(x0,(v0+z*Wn*x0)/Wd);
 x=A*exp(-z*Wn*t).*sin(Wd*t+phi);
else
     x=0
end
%plot and animate
freesmd_sim(t,x);
```

The first highlighted section sets up the constants that have already been determined. For this case it is already known that our system is an underdamped case. If the program finds this to be true it will graph the solution. Other wise it will return a x=0. Other programs can be written for critically damped or overdamped systems.

RESULTS

Figure 4 is the resultant graph produced by two MATLAB programs. The lower section of figure 4 oscillates with the graph to show the range of motion. These results illustrate with what is known and assumed about the system.

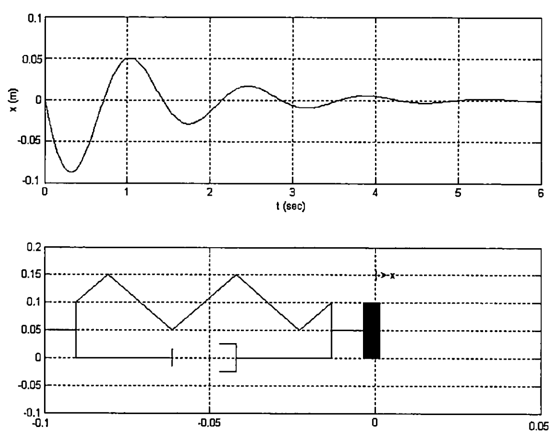

Figure 4. MATLAAB results

The damping is small creating an underdamped system. The boat impacts the dock, creating a compression in the system. The wood makes a very stiff spring, so its deflection is small. That deflection quickly tends goes to zero.

Now that this work is completed, the effect this has on design can be examined. The initial force and deflection is the largest in the reactions of the spring. So when designing a dock these are the forces that should be analyzed for strength based design. The motion and vibrations can be further used in fatigue based design.

LIMITATIONS

This paper is a preliminary theoretical derivation and analysis. It could be improved through a forced vibration analysis, accounting for more degrees of freedom, and performing experiments on actual docks. This project could also be extended to include larger ships, forces, etc. This author intends to apply what has been learned to a specific problem in practice and provide preliminary results for further analysis and research.

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