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# Production Uncertainty and Factor Price Disparity in the Slaughter Cattle Market: Theory and Evidence

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**PRODUCTION UNCERTAINTY AND FACTOR  
PRICE DISPARITY IN THE SLAUGHTER  
CATTLE MARKET: THEORY AND EVIDENCE<sup>1</sup>**

BY

SCOTT FAUSTI AND DILLON FEUZ<sup>2</sup>

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**PRODUCTION UNCERTAINTY AND FACTOR PRICE DISPARITY  
IN THE SLAUGHTER CATTLE MARKET: THEORY AND EVIDENCE**

**Introduction:**

The theoretical analysis of competitive firm behavior under economic uncertainty has been explored in the areas of input and output price uncertainty in the papers by Baron (1970), Sandmo (1971), Batra and Ullah (1974), and Blair (1974) among others. The issue of the competitive firm facing production uncertainty generated by input quality variability was addressed in a paper by Ratti and Ullah (1976). Other papers applied their approach to specific areas, such as, wage discrimination being explained by labor quality variability.<sup>1</sup>

A simple model of a competitive firm confronting production uncertainty, generated by the variability in the flow of factor service (input), is presented below. The authors believe that the assumptions of the model developed in this paper provides a realistic description of the short run behavior of firms engaged in meat packing operations in the upper midwest and purchasing cattle in the slaughter cattle market.

This paper follows the approach used to analyze production uncertainty developed by Ratti and Ullah. The purpose of our study is to analyze firm behavior when it must purchase its input (steers) in an auction market under two different informational conditions. This in essence, creates two submarkets for the purchasing of the input. The firm has either complete information or incomplete information concerning the "contribution to production" of the input it is purchasing.<sup>2</sup> Incomplete information implies that there is uncertainty over the "contribution to production" of the input when purchased. The term "contribution to production", will be denoted CTP throughout the rest of the paper.<sup>3</sup>

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<sup>1</sup> For example see the paper by Baldwin (1991).

<sup>2</sup> "Contribution to production", refers to that part of total product attributable to a particular input (slaughter cattle).

<sup>3</sup> The issue of uncertainty over the "contribution to production", is an issue of quantity and quality uncertainty. The yield of a carcass effects the total supply of the final output (beef), i.e., uncertainty over dressing percentage. The quality of an animal effects the quality of the final product

It is assumed that sellers of the input (cattle producers) have the choice of selling their product under either market alternative. It is further assumed that the firm (packer) is forced to compete in both markets when it purchases its input requirements. This is a reasonable assumption for firms operating in the meatpacking industry. If sellers choose method (A) to sell their product, then there is full information on the input's CTP when the firm makes its input purchase at the auction determined price. This is the profit maximization under certainty case for the firm. If sellers choose method (B) to sell their product, then there is uncertainty over the input's CTP when the firm makes its input purchase at the auction determined price.<sup>4</sup> Thus, sellers are separating the market for their product into two distinct submarkets.<sup>5</sup>

The model analyzes the firm's input purchasing decisions within this market structure. The theoretical results derived in the model are then empirically tested using data from the U.S. slaughter cattle market to determine if there is statistical evidence to support the conclusions of the model.

In section 1, the model is presented. In section 2, the relationship between a firm's preference toward risk and production uncertainty is analyzed. In section 3, the issue of firm input pricing behavior across market alternatives is addressed. In section 4, the effect of a change in the variability of the input's contribution to production on firm input purchasing behavior is examined. In section 5, the results derived in sections 3 and 4 are empirically tested.

The theoretical analysis demonstrates that a risk neutral firm will pay less for an input with uncertainty over its CTP in a auction market, than for an input when its contribution is known with certainty. The implication of this

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(beef), i.e., uncertainty over the grade.

<sup>4</sup> Marketing method (A) represents the grade and yield method of marketing and method (B) is the live weight method of marketing. Ward (1987) provides a detailed description of these two marketing methods.

<sup>5</sup> It is assumed that cattle producers sell all of their herd in one market. They do not sell part of their herd in one market and the rest in the other market.

result is that firm purchasing behavior is generating price disparity.<sup>6</sup> As uncertainty over an input's contribution increases, the price paid for the input declines as compared to the certainty case. The implication of this result is that as uncertainty increases, the degree of price disparity increases. The empirical study in section 5 provides strong evidence that there does exist price differentials between the full information marketing method and incomplete information marketing method for slaughter cattle.

#### **I. Assumptions and the Model:**

The analysis assumes a short run time frame for the firm (packer). The firm operates in a competitive setting in both the output and factor markets.<sup>7</sup> All inputs are assumed to be fixed except one. The one variable input X (one unit of cattle) must be purchased in an auction market, where sellers have the choice of selling their product via a full information marketing method (A) or an incomplete information marketing method (B).<sup>8</sup> The firm is forced to compete in both markets and the information that is available concerning X's CTP when purchased in market (B) is assumed symmetric.<sup>9</sup> It is assumed that the firm's purchasing decisions with respect to the two marketing methods are independent, and they are determined by demand and supply conditions in the output market. Once the firm has purchased the slaughter cattle, X, via method (A) or (B), X's CTP becomes known to the firm. Hence, the method employed to purchase X has no effect on the production process. Therefore, it is assumed that the price paid for cattle by the firm is based on profit maximization criteria and is dependent

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<sup>6</sup> Price disparity, unlike price discrimination, is not generated by market failure, e.g., the packer does not exert monopsony power.

<sup>7</sup> The authors recognize that the number of firms engaged in the meatpacking industry do not meet the criteria for a perfectly competitive market. However, our paper will show that price discrimination can occur in the absence of market power, and if there is market power present then it would most likely affect the mean level of prices being paid in the market and not alter the price differentials found in the submarkets.

<sup>8</sup> One unit of cattle refers to per head, per pen, etc.

<sup>9</sup> We are assuming that the slaughter cattle market does not have an asymmetric information problem.

on how they purchase X. However, the pricing decision arrived at for method (A) is independent of the pricing decision arrived at for method (B).<sup>10</sup> This type of market environment generates a firm pricing strategy that produces price disparity. This set of assumptions, in the authors' opinion, provides a realistic description of the slaughter cattle market in the United States.<sup>11</sup>

Assume a short run production process that uses only one variable input X, slaughter cattle. Define X as unit of slaughter cattle acquired for current use in the production of beef products. If slaughter cattle are purchased in market (A), then the firm knows the amount of output that can be generated by input X.<sup>12</sup> If slaughter cattle are purchased in market (B), then  $X_1$  represents the unit of slaughter cattle available for the production of beef. Given that actual quality of slaughter cattle is not known if purchased in market (B), its CTP is uncertain at the time of purchase. In order for the firm to maximize total profit the ratio of the marginal physical products (MPP) of X to  $X_1$  must be equal to the ratio of the prices paid for X and  $X_1$ .

Following the modeling procedure developed by Ratti and Ullah, X and  $X_1$  are linked in the following way:

$$X_1 = vX \quad (1)$$

where v is a strictly positive random variable with the variable's density function defined as  $f(v)$  with a unit mean.<sup>13</sup>

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<sup>10</sup> Independence is only assumed for individual lots of cattle purchased via method (A) or (B) and not for the mean price level found in the two markets. That is, the mean price level in both submarkets are dependent on the output price.

<sup>11</sup> Feuz, Fausti, and Wagner (1993), discuss the structure of the slaughter cattle market and provide empirical evidence that average profits to cattle feeders vary among marketing methods.

<sup>12</sup> Cattle are not homogenous inputs, however, their contribution to production is known with certainty when they are purchased via the G&Y market.

<sup>13</sup> In the following analysis, the model developed in this paper is a modified version of the model developed by Ratti and Ullah. Ratti and Ullah give credit to Walters (1960), and Roodman (1972) for the method of specification of the input variables. In this paper, as in the paper by Ratti and Ullah, the issue is uncertainty over the flow of factor services to production.

The firm's short run production function when it purchases in market A or B is defined as,

$$Q = h(X_1) = h(vX), \quad h'(X_1) > 0, \quad h''(X_1) < 0. \quad (2)$$

The third derivative of the production function is assumed to exist, and the marginal product of the input is positive but declining. The marginal product of the input is the first derivative of the input's CTP.

If  $X$  is purchased in market (A), then  $v$  is assumed to be a constant, with a value of one. If  $X$  is purchased in market (B), then  $v$  is defined as a random variable, which implies output ( $Q$ ) is also a random variable. The random variable  $v$  is assumed to be positive with the variable's density function defined as  $f(v)$ , and the expected value of  $v$  is defined as  $E[v] = 1$ . Given the nature of the live market (market B) for slaughter cattle, it is reasonable to assume that on average firms are correct about the quality of the cattle purchased. Thus, the unit mean assumption imposed on  $v$  is reasonable.<sup>14</sup>

Beginning with firm behavior under certainty (market A), it is assumed the firm's goal is to maximize profits  $\Pi$ . The variables  $p$ ,  $r$ , and  $C$  are defined respectively as the output price of beef and the input price of slaughter cattle and the fixed cost. The firm's profit function is defined as

$$\Pi = p \cdot h(X) - r \cdot X - C. \quad (3)$$

The first order condition for profit maximization is

$$d\Pi/dX = p \cdot h'(X) - r = 0. \quad (4)$$

The second order condition for profit maximization is

$$d^2\Pi/dX^2 = p \cdot h''(X) < 0. \quad (5)$$

Rearranging the equation 4, the following equilibrium condition is arrived at

$$p \cdot h'(X) = r \text{ or } p = r/h'(X). \quad (6)$$

Equilibrium condition (6) is the standard result. The firm will purchase the input for a price equal to its marginal value product (MVP), i.e., the value of its marginal contribution to the production of beef.

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<sup>14</sup> Cattle buyers make a living purchasing cattle in the live market. Therefore, one would not expect systematic errors in their bidding over time.



If firms purchase a unit of cattle in market (B), then there is uncertainty over the CTP of that unit purchased. In other words, the CTP of the unit purchased becomes a random variable. Profit is now defined in terms of expected utility. Assuming that the firm's utility function conforms to characteristics of a Von Neumann-Morgenstern utility function and its third derivative exists, the firm's expected utility from profit can be written as

$$E[U(\Pi)] = E[U(p \cdot h(X_1) - r \cdot X - C)]. \quad (7)$$

It is assumed that the marginal utility of profit is positive  $U'(\Pi) > 0$ , and the value of  $U''(\Pi)$  being negative if the firm is risk averse, 0 if the firm is risk neutral, and positive if the firm is risk preferring.

The first order condition for maximizing expected utility of profit is

$$dE[U(\Pi)]/dX = E[U'(\Pi) \cdot (p \cdot v \cdot h'(X_1) - r)] = 0. \quad (8)$$

The second order condition is

$$d^2E[U(\Pi)]/dX^2 = E[U''(\Pi) \cdot (p \cdot v \cdot h'(X_1) - r)^2 + p \cdot v^2 \cdot h''(X_1) \cdot U'(\Pi)] < 0. \quad (9)$$

## II. The Effect of Uncertainty on Firm Behavior:

The issue in this section is, how uncertainty over the input's CTP, in conjunction with the firm's attitude toward risk, affect the rate of return paid to the cattle producer by the firm (packer). The analysis begins with rewriting equation (8) in the following manner

$$E[U'(\Pi) \cdot (p \cdot v \cdot h'(X_1))] = E[U'(\Pi)] \cdot r. \quad (10)$$

Adopting Horowitz's (1970) alternative way of expressing equation (10),

$$p \cdot E[v \cdot h'(X_1)] = r - \{p \cdot \text{Cov}(U', v \cdot h'(X_1)) / E[U'(\Pi)]\}. \quad (11)$$

From above it is clear that the MPP and MVP of slaughter cattle are now random variables given by  $v \cdot h'(X_1)$  and  $p \cdot v \cdot h'(X_1)$  respectively. Examining the covariance term in equation (11), it is clear that when  $U''(\Pi) = 0$ , the covariance term is also equal to zero. The implication of equation (11) is that the risk neutral firm purchases cattle in the live market for  $r = E[MVP]$ . However, when  $U''(\Pi) \neq 0$ , the sign of the covariance term can not be ascertained. However, it can be demonstrated that when it is assumed that the elasticity of the marginal

product curve of the input has an absolute value of less than one, then  $\text{sign Cov} = \text{sign } U''(\Pi)$ :

$$\mathcal{E} = dh'(X_1)/dX_1 \cdot X_1/h'(X_1) = X_1 \cdot h''(X_1)/h'(X_1) > -1. \quad (12)$$

If equation (12) is true, then examining the derivatives of the two components of the covariance term with respect to  $v$ ,

$$d[v \cdot h'(X_1)]/dv = h'(X_1) \cdot [1 + \mathcal{E}] > 0, \quad (13)$$

and

$$dU'(\Pi)/dv = U''(\Pi) \cdot p \cdot X \cdot h'(X_1), \quad (14)$$

verifies that  $\text{sign Cov} = \text{sign } U''(\Pi)$ . That is, since the sign of equation (14) is dependent on  $U''(\Pi)$ , and equation (13) is positive,  $\text{sign Cov}$  must equal  $\text{sign } U''(\Pi)$ .

Applying this result to equation (11), the following condition is arrived at

$$p \cdot E[v \cdot h'(X_1)] \begin{matrix} \geq \\ < \end{matrix} r, \quad (15)$$

depending on whether  $U''(\Pi) \begin{matrix} \leq \\ > \end{matrix} 0$ .

The above result can be interpreted as follows; at the margin: 1) the risk neutral firm will purchase a unit of cattle at a price equal to its  $E[\text{MVP}]$  in the live market; 2) the risk averse firm will purchase a unit of cattle at a price less than its  $E[\text{MVP}]$  in the live market; and 3) the risk preferring firm will purchase a unit of cattle at a price greater than its  $E[\text{MVP}]$  in the live market. The implications of these results are that firm's input demand for  $X$  is dependent on its attitude toward risk.<sup>15</sup>

### III. Market Separation and Price Disparity:

In this section the analysis will begin with the assumption that the firm is risk neutral. As stated above, sellers of slaughter cattle choose one of two methods to sell their cattle and firms purchase cattle under both systems. This market structure implies that there are actually two submarkets for slaughter cattle. This paper assumes that the firm's purchasing decisions in each market

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<sup>15</sup> These results are consistent with the results derived in the paper by Ratti and Ullah.

are made independently because of the informational disparities between the two submarkets with respect to the input's CTP. It follows that, independent profit maximization decisions with respect to purchases of  $X$  are made for each submarket. The implication is that the meat packing firm will maximize profit by setting  $MVP=MC$  in each market. Rearranging equations (6) and (11), yields equations 16 and 17 respectively;

$$p = r/h'(X), \quad (16)$$

and

$$p = [r - \{p \cdot \text{Cov}(U', v \cdot h'(X_1)) / E[U'(\cdot)]\}] / E[v \cdot h'(X_1)]. \quad (17)$$

To simplify the analysis, replace  $r$  in equation (17) with  $r^*$ . Given that output price  $p$  is the same regardless of the input market the firm purchases in, the following equilibrium condition is derived from equations (16) and (17),

$$r/h'(X) = [r^* - \{p \cdot \text{Cov}(U', v \cdot h'(X_1)) / E[U'(\cdot)]\}] / E[v \cdot h'(X_1)]. \quad (18)$$

Equation (18) leads to the first proposition in the paper:

**PROPOSITION I.** *If a risk neutral firm purchases its inputs from two (seller separated) submarkets, where the two subgroups have equal average productivity and differ only in the amount of information available on an input's CTP, then the firm will purchase the input from the group with the uncertainty over its CTP at a lower price than from the group where the CTP is known with perfect information.*

To establish the above proposition, it is assumed that the third derivative of the production function is negative. This implies that the marginal product function  $h'(X_1)$  is itself a concave function. This assumption is consistent with equation (12), and implies that  $d^2/dX_1 < 0$ . The implications of  $h''(X_1) < 0$  is that the MPP of  $X_1$  is a non-increasing function of  $X_1$ .<sup>16</sup> Under the assumption that  $h''(X_1) < 0$ , and employing Jensen's inequality the following result is attained,

$$E[h'(vX)] < h'(X).^{17} \quad (19)$$


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<sup>16</sup> As noted by Ratti and Ullah, this assumption is consistent with many of the common forms of production functions used in economic analysis. For example, the Cobb-Douglas and CES production functions have this property.

<sup>17</sup> The Jensen inequality states that if a function is concave the following is true:  $E[h(X)] < h[E(X)]$ . The implication for our model is that the MPP of  $X$  in an uncertain environment is less than the MPP of  $X$  if production had taken place with the expected value of the random variable  $X$ , i.e., a certain environment. See Rao (1973, p.58) for an explanation of Jensen's inequality.

Equation (19) implies that the risk neutral firm's expected MPP generated by  $X_1$  is less than the MPP that would be achieved under conditions of certainty given the same factor combination. Certainty implies a situation where the random variable  $v$  is replaced by its expected value. Applying the result derived in equation (19) to equation (18) implies that for the risk neutral firm,  $r$  must be greater than  $r^*$ . This establishes proposition I.

Proposition I demonstrates that a perfectly competitive firm facing a competitive but segregated market structure for factor inputs, and the two submarkets varying only on the information available on the input's CTP, will engage in a pricing strategy that generates price disparity. That is, although all cattle are paid their expected marginal value product. However, firms in a sense are discriminating between the two groups because they have equal average productivity, but sellers receive unequal average returns for their cattle. This proposition presents an interesting case of a firm's pricing strategy producing a form of price discrimination (disparity) in its factor market without market power.

If it is assumed that the firm is risk averse, then equation (18) demonstrates that the degree of price disparity will increase. This last statement leads to the second proposition of the paper;

**PROPOSITION II.** *The degree of price disparity that is generated between market alternatives will vary positively with the degree of firm risk aversion.*

To establish proposition II, proposition I is reasserted. Proposition I established that  $r$  is greater than  $r^*$  for the risk neutral firm. Then by equations (15 & 18),  $r^*$  must be greater than say any  $r^{**}$ , the price that a risk averse firm would pay for a unit of cattle in the live market. Thus, proposition II is established.

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**Note:** risk neutrality implies that the covariance terms in equations 17 & 18 are zero.

#### IV. Comparative Statics: A Decline In Uncertainty:

In this section the effect of an change in the amount of information available to the firm on the CTP of a unit of cattle purchased in market (B) is examined. A change in the amount of information available implies a change in the amount of uncertainty associated with X marketed via marketing method (B). To capture this effect of a marginal change in uncertainty, the distribution of v will under-go a mean preserving change in the dispersion of the distribution. The results developed below are only determinant in the risk neutral case. A modification of equation (8) is now undertaken by replacing v with  $v^* = (\alpha \cdot v + \beta)$ , where  $\alpha$  is a shift parameter and  $\beta$  is a function of  $\alpha$  with the following properties:

1)  $\beta' = -E[v] = -1$ , and 2)  $\beta(\alpha=1) = 0$ . This transformation implies that  $X_1 = (\alpha \cdot v + \beta) \cdot X$ . Assuming the firm is risk neutral equation (8) is now

$$dE[\pi]/dX = E[p \cdot v^* \cdot h'(X_1) - r] = 0. \quad (20)$$

Replacing  $v^*$  with  $(\alpha \cdot v + \beta)$ , and renaming equation (20)  $E[Z]$ ,

$$E[Z] = E[p \cdot (\alpha \cdot v + \beta) \cdot h'(X_1) - r] = 0, \quad (21)$$

the comparative static analysis can begin. Invoking the implicit function theorem around the equilibrium value of X and  $\alpha=1$ , then taking the total differential of  $E[Z]$  and setting all of the differentials to zero except dX and  $d\alpha$ , the partial derivative  $\partial X/\partial \alpha$  is derived

$$\partial X/\partial \alpha = -E[\{p \cdot (v-1) \cdot h'(X_1) \cdot (1 + \epsilon)\} / \{p \cdot v^2 \cdot h''(X_1)\}]. \quad (22)$$

The sign of the partial derivative derived above can be determined by examining the following relationship:

$$p \cdot E[(v-1) \cdot h'(X_1) \cdot (1 + \epsilon)] = \text{Cov}((v-1), h'(X_1) \cdot (1 + \epsilon)). \quad (23)$$

By ascertaining the sign of  $\text{Cov}((v-1), h'(X_1) \cdot (1 + \epsilon))$ , the sign of the numerator of equation (23) can be determined. Examining the derivatives of the two components of the covariance term with respect to v,

$$d[h'(X_1) \cdot (1 + \epsilon)]/dv < 0, \quad (24)$$

and

$$d(v-1)/dv = 1 > 0, \quad (25)$$

verifies that the sign of the covariance is negative and thus the sign of the partial derivative  $\partial X/\partial \alpha < 0$ .

The above result leads to the third proposition of the paper:

**PROPOSITION III.** *For risk neutral firms, as uncertainty over CTP declines, the degree of price disparity between marketing alternatives will decline.*

To establish the above proposition the implications of  $\partial X/\partial \alpha$  being negative are analyzed. The negative sign indicates that as the uncertainty over CTP declines, demand for cattle via marketing method (B) increases. The implication is that for a fixed unit amount of cattle purchased, a decrease in uncertainty increases the expected MPP of a unit of cattle. This means that  $E[v \cdot h'(X_1)] < E[v^* \cdot h'(X_1)]$  when  $\alpha < 1$ . Examining this result in the context of equation (18), an increase in the expected MPP of a unit of cattle purchased via marketing method (B) will increase  $r^*$  relative to  $r$ . Thus, the degree of price disparity declines as uncertainty declines and thus proposition III is established.

#### **V. Empirical Findings: The Slaughter Cattle Market.**

In this section two hypothesis tests are constructed to test if there is evidence to support propositions I and III. The data for the empirical analysis were collected from the South Dakota Retained Ownership Demonstration Project.<sup>18</sup> Over a three year period beginning in April of 1991 218 pens of cattle were marketed via the grade and yield marketing method.<sup>19</sup> Market price data for the live and dressed weight markets were collected for the same type of cattle in the same marketing area<sup>20</sup>. This enabled us to construct average revenue per pen for

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<sup>18</sup> The Dept. of Animal and Range Science at South Dakota State University is administering the project. A description of the project can be found in Wagner, et al. (1991, 1992 and 1993).

<sup>19</sup> Marketing cattle in pens is a common practice in this market. Each pen in the demonstration project contained five steers.

<sup>20</sup> Hartman indicated the cattle from the project were representative of the cattle being purchased in the general market area. The Nebraska Direct Dressed and Live weight market prices were obtained from Data Transmission Network and the USDA, Livestock and Wool Statistics, and were then adjusted down for the local basis by \$1/cwt and \$0.64/cwt for dressed and live weight, respectively.

the cattle in the project for all three markets. Thus, it is possible to make a comparison of average revenue received per head for each pen of cattle in the project under three different marketing methods; 1) live weight, 2) dressed weight, and 3) grade and yield.<sup>21</sup> This unique data set allowed the following empirical testing procedures to be used to test if there is a difference in average revenue paid for a particular pen of cattle across the three marketing methods. The empirical testing of proposition I below will employ the "Difference between Population Means: Matched Pairs Test".<sup>22</sup>

To test proposition I, the null hypothesis is, average revenue per pen paid to sellers in the live and grade & yield marketing methods are equal. Against the alternative that average revenue per pen paid is higher in the grade & yield marketing method.

#### HYPOTHESES TEST I

$H_0: U_x - U_y = 0$   
 $H_1: U_x - U_y > 0$   
 Decision rule: reject  $H_0$  if the following is true  
 $\hat{u} / (s_u/n^{1/2}) > t_{n-1, \alpha}$   
 where  $\hat{u} = \Sigma(x_i - y_i) / n$  and  $s_u = [1/(n-1) \cdot \{\Sigma u_i^2 - n\hat{u}^2\}]$

$U_x$  is the population mean value of revenue paid per pen in the grade & yield marketing method.  $U_y$  is the population mean value of the revenue paid per pen in the live marketing method. The variable  $x_i$  is the average revenue paid for the  $i^{\text{th}}$  pen in the grade & yield market. The variable  $y_i$  is the average revenue paid for the  $i^{\text{th}}$  pen in the live market. The variable  $s_u$  is the observed sample standard deviation for the  $n$  differences:  $u_i = (x_i - y_i)$ .

If the null hypothesis is rejected, it can be concluded that there is evidence to suggest that average revenue paid per pen in the grade & yield market

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<sup>21</sup> Average revenue per pen implies that the total revenue per pen received is divided by the number of cattle in that pen.

<sup>22</sup> A description of the "Difference between Population Means: Matched Pairs Test" can be found in Newbold (1991, pp. 377-80). The robustness of the test is dependent on the distribution of the differences following a normal distribution. The Bowman/Shelton test for normality was conducted and a test statistic of .9314 was derived. This result strongly supports the assumption that the distribution of differences is normal for the data used to test proposition I. A description of the normality test can be found in Newbold (1991, pp. 442-44).

is higher than in the live market. The results of the hypothesis test produced a mean difference of average revenue per pen paid of \$9.22, and a t statistic of 5.33. The null hypothesis is rejected with a p-value of .0001. The conclusion is that there is strong evidence in favor of proposition I, which implies that there is a price differential between the two marketing systems.

An increase in information on an input's contribution to production implies a decrease in uncertainty. The dressed weight marketing method represents an intermediate marketing method between live and grade & yield with respect to the amount of information available to buyers in the slaughter cattle market. The testing procedure used above is not the appropriate testing procedure because the distribution of the differences between live and dressed weight is not normal.<sup>23</sup> To test proposition III, the null hypothesis becomes, the distribution of differences for revenue per pen paid to sellers in the live and dressed weight marketing methods is centered on zero. Against the alternative that the center of the distribution is greater than zero.

#### HYPOTHESIS TEST II

Ho: Distribution is centered on zero  
H1: Center of distribution is greater than zero  
Decision rule: reject Ho if the following is true  
 $(T - \mu_T) / \sigma_T < -Z_\alpha$ ,  
where T is the observed value of the Wilcoxon statistic  
and  $\mu_T$ ,  $\sigma_T$  are the E(T) and Var(T) respectively.

If the null hypothesis is rejected, it can be concluded that there is evidence to suggest that average revenue paid per pen in the dressed weight market is higher than in the live market. The implication of a rejection of the null hypothesis is that price disparity declines as uncertainty declines.

The mean difference of average revenue per pen paid for dressed weight minus live weight is \$6.74, and a Z test statistic derived from the Wilcoxon test is -2.3655. The null hypothesis is clearly rejected with a p-value of .01. The

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<sup>23</sup> The Bowman-Shelton test statistic derived from this data is 10.51. The assumption that the distribution of the differences is normal is rejected with a p-value of less than .05. Therefore, the non-parametric Wilcoxon Test (large sample), will be employed to test if there is evidence to support proposition III. A description of the Wilcoxon test can be found in Newbold (1991, pp. 419-423).



conclusion is that there is strong evidence in favor of proposition III, which implies that the price differential (disparity) between the two marketing systems declines as uncertainty declines.

#### VI. SUMMARY:

A short run model of a competitive firm facing uncertainty over a factor's CTP was presented in this paper. The assumptions of the model reflect, in the authors' opinion, a realistic characterization of the U.S. slaughter cattle market. The paper provides a theoretical explanation and empirical evidence of packer purchasing behavior, in the absence of market failure, generating price disparity,.

Under the assumptions of the model, it was demonstrated first that a competitive, risk neutral firm will engage in a pricing strategy that will produce price disparity when it purchases a single input in two submarkets which differ only in the amount of information available on the CTP of the single factor. Next, it was shown that the degree of price disparity is positively related to the degree of firm risk aversion. The final theoretical result established that the degree of price disparity is inversely related to the amount of information available on an input's CTP.

The empirical section of the paper provides strong support for the existence of factor price disparity due to uncertainty over an input's CTP. Evidence also was provided in support of an inverse relationship between the degree of price disparity and the amount of information available on a factor's CTP.

The findings presented in this paper raise new questions on how uncertainty affects competitive factor markets. A natural extension of this research is to relax the assumption of competitive factor markets. Another possible extension would be to relax the assumption of symmetric information.

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# FACTOR QUALITY UNCERTAINTY, FACTOR PRICE DISCRIMINATION

## AND THE COMPETITIVE FIRM: THEORY AND EVIDENCE

### MATHEMATICAL APPENDIX

$$EQ. 8: \frac{dE[U(\Pi)]}{dx} = E\left[\frac{dU(\Pi)}{d\Pi} \cdot \frac{d\Pi}{dx}\right]$$

$$\frac{dU(\Pi)}{d\Pi} = U'(\Pi)$$

$$\frac{d\Pi}{dx} = P \cdot \frac{dh(x_1)}{dx_1} \cdot \frac{dx_1}{dx} - r$$

$$\frac{d\Pi}{dx} = P \cdot h'(x_1) \cdot \frac{dx_1}{dx} - r$$

$$\frac{dx_1}{dx} = v$$

$$\frac{d\Pi}{dx} = P \cdot v \cdot h'(x_1) - r$$

$$EQ. 8: \frac{dE[U(\Pi)]}{dx} = E[U'(\Pi) \cdot (P \cdot v \cdot h'(x_1) - r)]$$

$$EQ. 9: \frac{d^2 E[U(\Pi)]}{dx^2} = E\left[\frac{d^2 U(\Pi)}{d\Pi^2} \cdot \frac{d\Pi}{dx} + \frac{d^2 \Pi}{dx^2} \cdot \frac{dU(\Pi)}{d\Pi}\right]$$

In the paper EQ. 9 is derived in the same manner as EQ. 8. EQ. 9 in the paper can be derived following the mathematical expression given above.

EQ. 10: EQ(10) is just EQ (8) rearranged.

EQ. 11: Horowitz, p. 364-367 uses the following definition  $E(xy) = E(y) \cdot E(x) + COV(x,y)$ . Thus the left hand side of EQ. 10 is equalivant to  $E\{U'(\Pi)\} \cdot E[P \cdot v \cdot h'(x_1)] + COV[U'(\Pi), v \cdot h'(x_1)]$  replacing the LHS with this equalivant expression and solving for  $E[P \cdot v \cdot h'(x_1)]$  gives us EQ. 11.

EQ. 12: Equation (12) gives the standard procedure for deriving an elasticity coefficient.

$$EQ. 13: \frac{d[v \cdot h'(x_1)]}{dv} = h'(x_1) + v \cdot \frac{dh'(x_1)}{dv}$$

$$\frac{dh'(x_1)}{dv} = \frac{dh'(x_1)}{dx_1} \cdot \frac{dx_1}{dv}$$

$$\frac{dh'(x_1)}{dx_1} = h''(x_1)$$

$$\frac{dx_1}{dv} = X$$

$$\frac{dh'(x_1)}{dv} = h''(x_1) \cdot X$$

$$\frac{d[v \cdot h'(x_1)]}{dv} = h'(x_1) + v \cdot [h''(x_1) \cdot X] : \text{ where } x_1 = v \cdot X$$

$$= h'(x_1) + h''(x_1) \cdot X_1$$

$$= h'(x_1) \cdot [1 + \epsilon] > 0$$

$$EQ. 14: \frac{dU'(\Pi)}{dv} = \frac{dU'(\Pi)}{d\Pi} \cdot \frac{d\Pi}{dv}$$

$$\frac{dU'(\Pi)}{d\Pi} = U'(\Pi)$$

$$\frac{d\Pi}{dv} = \frac{d\Pi}{dx_1} \cdot \frac{dx_1}{dv} : \text{ where } \frac{d\Pi}{dx_1} = p \cdot h'(x_1) \wedge \frac{dx_1}{dv} = x$$

$$EQ\ 14: \frac{dU'(\Pi)}{dv} = U'' \cdot P \cdot X \cdot h'(x_1)$$

Given that  $P$ ,  $X$ ,  $h'(x_1)$  are all positive, the sign of EQ (14) is the same as the sign of  $U''(\Pi)$ .

EQ. 15: Eq. (15) is expressing the implications coming from EQs. 12 - 14 on EQ. 11.

Equations 16 - 18 should be clear.

After proposition I, it is stated that if  $h'''(x_1) < 0$ , then  $d\mathcal{E}/dx_1 < 0$ .

$$\begin{aligned} \frac{d\mathcal{E}}{dx_1} &= d \left[ \frac{x_1 \cdot h''(x_1)}{h'(x_1)} \right] \\ &= \frac{h'(x_1) [h''(x_1) + h'''(x_1) \cdot x_1] - [h''(x_1) \cdot x_1 \cdot h''(x_1)]}{h'(x_1) \cdot h'(x_1)} \\ &\quad - \quad (+) \quad - \quad (-) \quad + \\ &= \frac{h''(x_1)}{h'(x_1)} + \frac{h'''(x_1) \cdot x_1}{h'(x_1)} - \frac{[h''(x_1)]^2 \cdot x_1}{[h'(x_1)]^2} < 0 \end{aligned}$$

Thus  $h'''(x_1) < 0$  assures that  $\frac{d\mathcal{E}}{dx_1} < 0$ .

EQ. 20. The first order condition is rewritten to incorporate  $V^*$  and the assumption of a risk neutral firm.

$$20. \frac{dE(\Pi)}{dx} = E[p \cdot V^* \cdot h'(x_1) - r] = 0$$

Eq. 21 replaces  $V^*$  with  $(\alpha \cdot v + \beta)$  and rename the FOC:  $E[Z]$ .

$$21. E[Z] = E[p \cdot (\alpha \cdot V + \beta) \cdot h'(x_1) - r] = 0$$

EQ. 22 is the result of comparative static analysis. Taking the total differential of  $E[z]$  and setting all differentials to zero except  $dx$ ,  $d\alpha$ , and

remembering that  $dv^*/d\alpha = (v-1)$ , we have,

$$dE[z] = E[P \cdot [V^*]^2 \cdot h''(x_1)] dx + E[P \cdot v \cdot h'(x_1) - P \cdot h'(x_1) + P \cdot h''(x_1) \cdot (\alpha \cdot v + \beta) \cdot X \cdot (v-1)] d\alpha$$

Now the above equation reduces to:

$$dE[z] = E[P \cdot (V^*)^2 \cdot h''(x_1)] dx + E[P \cdot (v-1) \cdot h'(x_1) \cdot (1 + \theta)] d\alpha$$

Setting  $dE[z]$  to zero allows  $\partial x/\partial \alpha$  to be derived.

$$EQ. 22. \quad \frac{\partial x}{\partial \alpha} = - \left[ \frac{P \cdot E[(v-1) \cdot h'(x_1) \cdot (1 + \theta)]}{P \cdot E[(V^*)^2 \cdot h''(x_1)]} \right] < 0$$

**NOTE:** When doing the comparative statics one must remember that  $x_1 = (\alpha \cdot v + \beta) \cdot X$  so that

$$\frac{dx_1}{d\alpha} = (v-1) \cdot X$$

The sign of equation 22 is dependent on the numerator, since the denominator is negative and the entire expression has a negative sign. The key to signing the numerator is the following relationship:

$$E(x \cdot y) = E(x) \cdot E(y) + Cov(x, y) \text{ thus}$$

$$EQ. 23. \quad E[(v-1) \cdot h'(x_1) \cdot (1 + \theta)] = E[(v-1)] \cdot E[h'(x_1) \cdot (1 + \theta)] +$$

$$+ Cov[(v-1), h'(x_1) (1 + \theta)], \text{ but } E[(v-1)] = 0. \text{ So we have } EQ. 23.$$

$$23. \quad E[(v-1) \cdot h'(x_1) \cdot (1 + \theta)] = Cov[(v-1), h'(x_1) \cdot (1 + \theta)]$$

$$EQ. 24. \quad \frac{d[h'(x_1) \cdot (1 + \theta)]}{dv} = \frac{d[h'(x_1) + x_1 \cdot h''(x_1)]}{dv} =$$

$$\frac{d[h'(x_1) (1 + \theta)]}{dv} = h''(x_1) \cdot \alpha \cdot X + \alpha \cdot X \cdot h''(x_1) + x_1 \cdot h'''(x_1) \cdot \alpha \cdot X$$

given that  $h''$ ,  $h'''$  are negative, then  $\frac{d[h'(x_1) \cdot (1 + \theta)]}{dv} < 0$

$$\text{EQ. 25. } \frac{d(v - 1)}{dv} = 1 > 0$$

Thus equations 24 and 25 have opposite signs, so the covariance is negative, which means  $\partial x / \partial \alpha < 0 \therefore$

PRODUCTION UNCERTAINTY AND FACTOR PRICE DISPARITY  
IN THE SLAUGHTER CATTLE MARKET: THEORY AND EVIDENCE

Data Appendix

Detailed data were collected on 69 pens of steer calves in 1991, 84 pens of steer calves in 1992 and 65 pens of steer calves in 1993 as part of a retained ownership demonstration project. The summary statistics for variables of interest to this paper are included in Appendix Table A1.

These steers were marketed on a grade and yield basis in the spring of the year when three out of the five steers were estimated to be at 0.4 inches of fat over the 12th rib. The Choice market price and discounts for Select carcasses, Yield grade 4 carcasses (\$10-12/cwt), carcasses over 950 pounds (\$10/cwt), or carcasses under 550 pounds (\$12/cwt) were negotiated with a commercial cattle buyer in a competitive market. The average live and dressed weight market prices for similar types of steers were obtained from market quotes and revenue per head was calculated as if the steers had been sold under all three marketing methods (Appendix Table A1).

The data are most representative of the upper midwest/western corn belt region of the U.S. The data also are limited to the March through June marketing time frame. The empirical results generated are thought to be representative of this marketing area and time frame. However, additional research is needed to determine if similar results would occur in other marketing areas and time frames.



Appendix Table A1. Summary Statistics on 218 pens of Slaughter Steers used for the Empirical Analysis of this Paper.

Variable	Units	Mean	Std Dev
Live Slaughter Weight	Pounds	1146.15	90.8936
Hot Carcass Weight	Pounds	730.84	62.8868
Dressing Percentage	Percent	63.74	1.3883
Yield Grade	1 - 5	2.35	0.4457
Percentage Choice Grade	Percent	0.45	0.2867
Calculated Revenue			
Live Weight	\$/head	895.25	70.8036
Dressed Weight	\$/head	901.99	79.3132
Grade & Yield	\$/head	904.47	81.5949