Ensemble of Score Likelihood Ratios under the common source problem

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Agenda for today

- The source problem in forensics comparison.
- Developing SLR: the issue with using pairwise comparisons.
- A remedy through sampling.
- Strengthening SLR system using an ensemble approach.
The common source problem refers to an inferential problem where an expert’s objective is to provide some probabilistic statement regarding the origin of two items given the observed evidence.

In forensics we usually contrast two propositions

- $H_p$: Item 1 and Item 2 come from the same source.
- $H_d$: Item 1 and Item 2 come from two different sources.

Our conclusions will be based on measurements (features) taken from each item.

- $u_x$: Measurement(s) from Item 1
- $u_y$: Measurement(s) from Item 2
Score Likelihood Ratios

- We can use score-based likelihood ratios as an alternative way to measure the evidence's weight.

$$SLR(\delta) = \frac{g(\delta | H_p)}{g(\delta | H_d)}$$

- $\delta$: A comparison computed between $u_x$ and $u_y$
- $g$: conditional probability density function.

- Interpretation:
  - Top: the probability of the score under $H_p$
  - Bottom: the probability of the score under $H_d$

- Hence:
  - $SLR(\delta) > 1$ supports $H_p$ (Score more likely under $H_p$ than $H_d$)
  - $SLR(\delta) < 1$ supports $H_d$ (Score more likely under $H_d$ than $H_p$)
Developing a Score Likelihood Ratio consists of two steps:

- Constructing a comparison metric: $\delta(.,.)$.
- Estimating the distribution of the score under both propositions $g(\delta \mid .)$ or a density ratio estimator $r(\delta)$

Popular methods used for creating the comparison metric and density estimation required an independence assumption that is not met in forensic comparison.
It is often assumed that a set of background population samples (A) is available to construct the SLR system

\[ A = \{ A_{ij} : i^{th} \text{ Source, } j^{th} \text{ Item} \} \]

Researchers create all possible pairwise combinations from A.

Categorize them into known matches (KM) when two items originate from the same source or known non-matches (KNM) when two items originate from different sources.
The issue with the pairwise comparison.

▶ We have an imbalanced sample. Known Non-Matches are going to outnumber Known Matches.
▶ For a set $A$ with 10 sources ($N_s$) and 3 items each ($N_i$).

\[
N_{KM} = N_s \binom{N_i}{2} \quad N_{KNM} = \binom{N_s N_i}{2} - N_{KM}
\]

There are $N_{KM} = 30$ pairs and $N_{KNM} = 405$ pairs
▶ We can accommodate this, for example, by downsampling the KNM.

▶ However, we aren’t acknowledging how the data was generated (Sources $\rightarrow$ Items). We are learning from dependent data, assuming they are independent.
The Dependence Problem

At source level:
- Multiple KM comparisons use the same source.
- Multiple KNM comparisons use the same source.

At item level:
- Items are used multiple times.

Figure 1: Dependence structure
To solve this issue, we introduce a sampling step to make sure that the training/estimation sets have independent observations.

SSSA: Strong Source Sampling Algorithm

Figure 2: Dependence structure
To make the most out of our data, we propose an ensembling learning approach for SLRs.

We build multiple Base SLRs (BSLR) over a set where assumptions are met.

Expert’s opinions (BSLR) are aggregated into a final SLR score.

Figure 3: Workflow structure
Testing Our Approach

Our Data and methods:
- CSAFE London Letter as background measurements.
- CVL (Computer Vision Lab) set for validation.
- Random forest-based comparison metric.
- Logit-based density ratio estimator.
- Naive Aggregators: Mean, median, majority voting.

Our evaluation metrics:
- Rate of misleading evidence.
- Discriminatory power.

SLR output interpretation

- Strong evidence towards defense
- Inconclusive range
- Strong evidence towards prosecutor

Consensus - over verbal sets - and distance in value of evidence.

$Cllr$, as a forensic cost function
We repeated 500 times the following experiments:

- Generate 50 BSLR following our ESLR workflow.
  - SSSA+ ensemble step.
- Generate a traditional SLR following the traditional workflow.
  - Splitting sources + Down sampling step.
- Generate a validation set of 1000 known matches and 1000 known non-matches from the CVL dataset.
- Computed performance metrics to evaluate traditional SLR and ESLRs (Exp. 1); and held the same validation set across repetition to evaluate consensus (Exp. 2).
### Performance Metric - Experiment 1

<table>
<thead>
<tr>
<th>Metric</th>
<th>Statistic</th>
<th>SLR</th>
<th>Mean ESLR</th>
<th>Median ESLR</th>
<th>V. ESLR</th>
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<tbody>
<tr>
<td>RME KM</td>
<td>Mean</td>
<td>14.1450</td>
<td>12.6382</td>
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#### Main takeaways:
- ESLR resulted in less misleading evidence for KM (2%) at the cost of a small increase of misleading evidence for KNM (.5%).
- More discriminatory evidence for KM and KNM.
- Overall reduction in the cost incurred.
<table>
<thead>
<tr>
<th>Metric</th>
<th>Statistic</th>
<th>SLR</th>
<th>Mean ESLR</th>
<th>Median ESLR</th>
<th>V.ESLR</th>
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</table>

**Main takeaways:**

- Ensemble SLR resulted in similar conclusions for the same holdout evidence.
- Higher consensus: ESLR conclusions tend to fall in the same category.
- Smaller distance: ESLR showed smaller distances in terms of probative value for the same hold-out pair.
Our results show that ESLRs (SSSA+Ensembling):

- Reduced the rate of misleading evidence for KM at the cost of a small increase in misleading evidence for KNM.
- Enhanced the discriminatory power, as traditional SLR tends to produce more inconclusive results.
- Can reduce the overall error, as measured by the cost function.
- Produced more consistent conclusion, more similar probative value.

Our approach is not limited to handwriting or forensics; it can be used whenever SLRs are built over pairwise comparisons.
Thank you

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