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On the Electric Field of Moving Magnets

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ON THE ELECTRIC FIELD OF MOVING MAGNETS

By

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R. H. W.

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INTRODUCTION

The following study arose from the discussion of a problem in a course in theoretical physics at South Dakota State College during the Winter quarter 1956-1957. The problem is stated as follows:

> A symmetrically magnetized cylindrical magnet is rotating with constant angular velocity about its axis of symmetry. A straight wire at right angles to and in the plane of the axis of the magnet is arranged so that it can be rotated about the same axis. Is there an e.m.f. along the wire (a) when it is at rest, (b) when it is rotating with the magnet? Explain why. (5, p. 475)

In attempting to answer this question, one becomes involved in the question of whether or not the lines of magnetic induction rotate with the magnet since in elementary electromagnetic theory the e.m.f. induced depends upon the rate at which the lines cut through the wire.

Historically this rotation or non-rotation of the lines was at one time the primary point of interest in the so called "unipolar" induction experiments. The literature reveals an impressive list of experimenters who have investigated unipolar induction and have attempted to interpret their results in terms of an acceptable "cutting line" theory. This work culminated in a comprehensive paper by Tate (8) in which he summarizes the experimental work and follows Swann (7) in explaining the observed results and in proposing an acceptable "moving line" theory.

The unfamiliar nature of some of their conclusions and the lack of experimental verification for others justifies the following study of the electric field of moving magnets. The theory of Swann is developed and his "moving line" theory is stated. Experimental verification of this theory is investigated and questioned. Then an experiment designed to test this theory is described and its results are discussed.

HISTORY[1]

The original unipolar induction experiment was performed by Faraday (3) in 1831. He rotated a cylindrical bar magnet about its axis of symmetry and observed a current in a stationary wire whose ends were pressed to the surface of the spinning magnet. Similar experiments were performed by others including W. Weber who first termed such induction as "unipolar".

The induction of the electromotive force (e.m.f.) which caused the current was explained in two ways. Faraday, Pleucker and Lecher considered that the lines of magnetic induction did not rotate with the magnet; thus, the e.m.f. was induced in the magnet itself as it rotated through the lines (this will be referred to as the stationary line theory). Weber, Preston, Hertz, Lodge and Rayleigh thought of the lines as rotating with the magnet, so as to induce the e.m.f. in the stationary circuit (this will be referred to as the moving line theory). Both points of view, however, predict the same e.m.f. around the closed circuit. Therefore, the closed circuit system could not establish which theory was correct.

In the 1890's, experiments to examine the electrostatic field about a rotating magnet were proposed to solve the

[1] Much of the information given here is taken from the more detailed history of unipolar induction as given by Tate (8).

problem but were given up because of inadequate experimental facilities.

Finally, in 1912 independent experiments by Kennard and Barnett eliminated the rotating line theory. In their experiments a cylindrical bar magnet was rotated inside a concentric cylindrical condenser while the condenser plates were electrically connected. The connection was broken and then the magnet stopped. A rotating line point of view would predict a charge on the condenser. No such charge was found. Since it had been assumed that either the lines must rotate with the magnet or remain stationary, negative results for the Kennard (4) and Barnett (1) experiments tended to establish the stationary line theory.

However, Swann (7, p. 379) in 1920 proposed a new moving line theory which also explains the negative results. In this theory the lines of magnetic induction from each elementary magnetic pole participate in the translational part of the motion only. A development of this theory is found in the next section.

THEORY[1]

At one time many persons believed that there was an inherent uncertainty in electromagnetic theory when dealing with the problem of unipolar induction. This was because Faraday's law gave only the integrated value for the field around a closed circuit; thus, it had nothing to say about the field at each point around the circuit. However, Swann points out that since electromagnetic theory contains two circuital equations they will, when integrated, give an expression for the electric field at every point. Stated in Heaviside-Lorentz units[2] they are:

$$\nabla \times \bar{H} = \frac{1}{c}\left(\rho \bar{u} + \frac{\partial \bar{E}}{\partial t}\right) \tag{1}$$

and

$$\nabla \times \bar{E} = -\frac{1}{c}\frac{\partial \bar{H}}{\partial t} \tag{2}$$

which along with

$$\nabla \cdot \bar{E} = \rho \tag{3}$$

and

$$\nabla \cdot \bar{H} = 0 \tag{4}$$

form the basis of electromagnetic theory. Following the method of Maxwell these may be integrated to give

$$\bar{E} = -\frac{1}{c}\frac{\partial \bar{U}}{\partial t} - \nabla \psi \tag{5}$$

where

$$\psi = \frac{1}{4\pi}\iiint \frac{\rho}{r}\, d\tau \tag{6}$$

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[1] For the most part, the work of this section follows Swann (7).

[2] A discussion of Heaviside-Lorentz units may be found in (5, p. 503, 517). The symbols are defined in the appendix of this paper.

and $$\bar{U} = \frac{1}{4\pi c}\iiint \frac{\rho\bar{u} + \frac{\partial \bar{E}}{\partial t}}{r}\, d\tau \qquad (7)$$

Here, $\bar{E}$ is the force per unit charge on a charge fixed relative to the observer. When the charge is moving with a velocity $\bar{v}$ relative to the observer, the force per unit charge is given by:

$$\bar{F} = \bar{E} + \frac{\bar{v} \times \bar{B}}{c} \qquad (8)$$

the second term of the right hand member of equation 8 being referred to as the motional intensity.

Swann then points out that a charge moving in uniform translation with a magnetic doublet (which might be an Amperian whirl of an electron in orbit about a nucleus) should, for the observer at rest, experience a force given by equation 8. In determining this force the tendency is to omit the $-\nabla\psi$ term in equation 5 since there is no net electric charge distribution in a magnetic doublet when at rest relative to the observer. If this were done, the $-\frac{1}{c}\frac{\partial \bar{U}}{\partial t}$ term would combine with the motional intensity to give the resultant force on the moving charge. The motional intensity has no component in the direction of motion while the other term, in general, does. This means that for the observer at rest there will be a net force on the moving charge in the direction of motion. However, relativistic considerations require that there be no net force on the charge for either the observer at rest or the observer in motion, just as if the doublet and the charge were at rest. Evidently it is not correct to assume that $-\nabla\psi$ will be zero for the observer at rest.

Swann goes on to show that in order for the force on the charge to be zero for the observer at rest the term $-\nabla\psi$ must appear as the field of an electric doublet of electric moment $\bar{N}$ such that

$$\bar{N} = \frac{\bar{v} \times \bar{M}}{c} \tag{9}$$

where $\bar{M}$ is the magnetic moment of the magnetic dipole. The center of the electric doublet coincides with that of the magnetic doublet. This field combines with the $-\frac{1}{c}\frac{\partial \bar{U}}{\partial t}$ term in equation 5 to create about the magnetic dipole a field

$$\bar{E} = - \frac{\bar{v} \times \bar{B}}{c} \tag{10}$$

which just cancels the force due to the motional intensity. Swann shows this to be true by electromagnetic reasoning, but he also points out that this result may be obtained directly from the relativistic field transformations. Tate (8, p. 82) also obtains this result for a true magnetic doublet in translation and concludes it to be true for any magnetic system in uniform translatory motion.

It must, however, be realized that the creation of the electric dipole is not predicted by electromagnetic theory alone. It is the additional assumption of relativity which necessitates that result.

The creation of the field $\bar{E} = -\frac{\bar{v} \times B}{c}$ about a translating elementary magnetic dipole is the basis of Swann's moving line theory. Here, $\bar{v}$ is the velocity of the lines of induction relative to the observer. The lines are considered to be attached to the dipole and to partake of its translational motion only. For a charge moving with respect to the observer the field $E = -\frac{\bar{v} \times \bar{B}}{c}$ would then combine with the motional intensity to give the resultant force on the charge as in equation 8. In the special case where the dipole and the charge have the same velocity the forces cancel one another, but in general there would be a resultant force per unit charge given by

$$\bar{F} = \frac{\bar{v}_c \times \bar{B}}{c} - \frac{\bar{v}_1 \times \bar{B}}{c} \qquad (11)$$

Where there are many elementary dipoles the force per unit charge would become

$$\bar{F} = \frac{\bar{v}_c \times B}{c} - \left(\frac{\bar{v}_1 \times \bar{B}_1}{c} + \frac{\bar{v}_2 \times \bar{B}_2}{c} + \cdots + \frac{\bar{v}_n \times \bar{B}_n}{c} \right) \qquad (12)$$

Here, $\bar{v}_c$ is the velocity of the charge, $\bar{B}$ the total magnetic induction at the charge, $\bar{B}_1$ the portion of the magnetic induction at the charge due to the dipole with velocity $\bar{v}_1$ etc.

It might be inquired how this theory applies to the case where the dipole is rotating. If the elementary magnetic dipole is rotating about its axis of symmetry the term $-\frac{1}{c}\frac{\partial \bar{A}}{\partial t}$ in equation 5 is zero. If in addition the dipole is not translating, so also is the term $-\nabla\varphi$ zero. So for this case, no electric field is to be expected.

For the case when the dipole rotates about an axis parallel to its axis of symmetry but not coincident with it, the motion may be resolved into two parts - one a rotation of the dipole about its axis of symmetry and the other a motion of the center of the dipole about the axis of rotation. The first gives no electric field. The second gives the dipole a translational velocity at any instant of size $\bar{\omega} \times \bar{r}$ where $\bar{\omega}$ is the angular velocity of rotation and $\bar{r}$ is the distance of the center of the dipole from the axis of rotation. However, the direction of this velocity is continually changing and there is no way of knowing how this acceleration may affect the dipole.

Swann assumes that the dipole is unaffected by the acceleration[1] and makes this the basis of his treatment of unipolar induction. With this assumption the dipole has a field

$$\bar{E} = -\frac{\bar{v} \times \bar{B}}{c} = -\frac{(\omega \times r) \times \bar{B}}{c} \qquad (13)$$

One mode of rotation of the magnetic dipole is left to be considered - that of rotation about an axis perpendicular to the axis of symmetry. Swann treats this problem in terms of a true magnetic doublet, but argues that since only the magnetic moment appears in the result it applies equally well to an Amperian whirl. If the axis

[1]It is unaffected to a high degree of approximation with only the radiation field being neglected.

of rotation passes through the center of the doublet, then the doublet is not translating so that $-\nabla\varphi$ is zero and the only electric field is due to $-\frac{1}{c}\frac{\partial \overline{U}}{\partial t}$. By placing the center of the doublet at the origin and making the Y axis the axis of rotation, Swann shows that at the instant when the doublet is oriented so that its magnetic axis coincides with the Z axis the electric field is given by

$$E_x = -\frac{1}{c}\frac{\partial U_x}{\partial t} = 0$$

$$E_y = -\frac{1}{c}\frac{\partial U_y}{\partial t} = -\frac{w\mu}{c}\frac{\partial}{\partial z}\left(\frac{1}{r}\right) = \frac{w\mu}{c}\frac{n}{r^2} \qquad (14)$$

$$E_z = -\frac{1}{c}\frac{\partial U_z}{\partial t} = +\frac{w\mu}{c}\frac{\partial}{\partial y}\left(\frac{1}{r}\right) = -\frac{w\mu}{c}\frac{m}{r^2}$$

where μ is the magnetic moment, w is the angular velocity of rotation, r is a vector from the origin to the point where the field has the values shown and l, m and n are the direction cosines of r.

Swann then shows that this is also the field obtained from a moving line theory in which each magnetic pole carries its lines of magnetic induction with it in its translatory motion but not its rotatory motion. This then is the final form of his moving line theory. It should be added that there is nothing fundamental in this moving line theory. It is justified only because it gives results in agreement with accepted electromagnetic theory. In this form much of its value is lost in the case of rotating magnetic materials in bulk due to the difficulty in applying it.

In this section theory has been developed which can determine the electric field about a magnetic dipole for any type of motion, for, in general, any motion of the dipole can be resolved into the types of motion here discussed.[1] In the next section this theory is applied to particular cases of interest.

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[1]The results discussed apply to the non-rotating observer. Trocheris (9) has developed a transformation which will give the field for a rotating observer.

APPLICATI N OF THEORY TO EXPERIMENT

For the most part the theory developed in the previous section has been soundly based on accepted electromagnetic theory. The assumptions that were made seemed quite logical. Still, some sort of direct experimental verification of its predictions is desirable.

It is easily shown (7, p. 387) (8, p. 84) that this theory predicts results in agreement with the original unipolar induction experiments and the Kennard-Barnett experiments. However, these experiments are equally well explained by the stationary line theory.

Tate (8, p. 93) declares that a non-conducting magnetic material is necessary to distinguish between these two theories. He then refers to a work by M. Wilson and H. A. Wilson (10) in which they make a non-conducting magnetic material for use in an experiment which decides in favor of Swann's theory. However, this writer feels that the makeup of the Wilson's magnetic material is such as to render their experiment incapable of distinguishing between the two theories. Part A of this section will present an argument to show that a true dielectric magnetic material with a permeability appreciably greater than one is necessary in an experiment of that type if it is to decide between the two theories.

It appears that, in the absence of such material, these theories may not be distinguishable in an experiment where the magnetic material is rotated about its axis of symmetry. Unfortunately, the stationary line theory is not easily carried to the case of rotation about an axis other than the axis of symmetry. What, if any, meaning does it have in this case to say that the lines of the magnet as a whole do not rotate?

This writer has performed an experiment in which a bar magnet is rotated about an axis perpendicular to its axis of magnetization. Such an arrangement does not deal at all with the stationary line theory as applied to rotation about an axis of symmetry. In part B of this section Swann's theory is applied to this type of experiment. Also, a possible method of applying a stationary line theory to this case is considered.

Part A: Theory Applied to the Wilson and Wilson Experiment

In order to discuss the Wilson and Wilson experiment it is necessary to apply the theory of the preceding section to a cylindrical system of magnetic material in bulk rotating about its axis of symmetry. The system then consists of many elementary magnetic dipoles rotating about an axis parallel to their axis of magnetization.

If each of these were treated individually the total electric field about the magnet would be obtained by adding up the contribution of each elementary magnet as given by equation 13. However, the symmetry of the system allows a simpler method of attacking the problem. Because of the symmetry, the term $-\frac{1}{c}\frac{\partial \bar{U}}{\partial t}$ in equation 5 will be zero for the system as a whole. Whatever electric field that exists about the system will be given by the $-\nabla\psi$ term in equation 5.

Part of this term will be contributed by an apparent charge distribution due to the motion of the elementary magnetic dipoles which appear as electric dipoles as shown in equation 9. In this system, $\bar{v} = \bar{\omega} \times \bar{r}$, so that

$$\bar{N} = \frac{(\bar{\omega} \times \bar{r}) \times \bar{M}}{c} = \frac{\omega r M}{c} \qquad (15)$$

where $\bar{N}$ is radial. The net result of this for the magnet as a whole is that there is in the magnet an apparent or fictitious polarization (magnetic moment per unit volume) $\bar{P}_o$ which is also radial such that

$$\bar{P}_o = \frac{(\bar{\omega} \times \bar{r}) \times \bar{I}}{c} = \frac{\omega r I}{c} \qquad (16)$$

where $\bar{I}$ is the magnetization (magnetic moment per unit volume) of the magnetic material. The polarization is termed fictitious because it is not due to a true separation of charge in the material but rather to an apparent distribution as observed by the non-rotating observer.

The observable effects of this fictitious polarization are the same as a true polarization. It results in an apparent surface charge density, σ_o , on the magnetic material given by

$$\sigma_o = P_{o_n} \qquad (17)$$

where P_{o_n} is the component of $\bar{P}_o$ normal to the surface; an apparent volume charge density, ρ_o, such that

$$\rho_0 = -\nabla \cdot P_0 \qquad (18)$$

and an electric field inside the magnetic material given by $-\bar{P}_0$.

If the magnetic material is a conductor, the free charges of the conductor will redistribute themselves in the material in such a way as to annul the field of the fictitious polarization. The charge distribution necessary to create a field equal but opposite to $-\bar{P}_0$ is a surface charge density

$$\sigma = -P_{0_n} \qquad (19)$$

and a volume charge density

$$\rho = \nabla \cdot \bar{P}_0 \qquad (20)$$

Thus this redistribution of charge is such as to completely shield any other effect due to the fictitious polarization. The only effect which remains in the case of a conducting magnetic material is that due to the motional intensity.

If the magnetic material is a dielectric there are no free charges which can completely annul the effects of the fictitious polarization. Instead there would also be a true polarization $\bar{P}$ in the dielectric given by

$$\bar{P} = (\epsilon - 1)\bar{F} = (\epsilon - 1)\left(\bar{E} + \frac{\omega\, r\, B}{\epsilon}\right) \qquad (21)$$

where ϵ is the dielectric constant of the material. Here $\bar{E}$ is made up of a field $-\bar{P}_0$ due to the fictitious polarization and a field $-\bar{P}$ due to the true polarization, so

$$\bar{P} = (\epsilon - 1)\left(-\bar{P} - \bar{P}_o + \frac{w r B}{c}\right) \tag{22}$$

Then from equation 16

$$\bar{P} = (\epsilon - 1)\left(-\bar{P} - \frac{w r I}{c} + \frac{w r B}{c}\right) \tag{23}$$

Next, solving for $\bar{P}$ gives

$$\bar{P} = \frac{\epsilon - 1}{\epsilon}\left(\frac{w r B}{c} - \frac{w r I}{c}\right) = \frac{\epsilon - 1}{\epsilon}\,\frac{w r H}{c} \tag{24}$$

by using the usual constitutive relationship

$$\bar{B} = \bar{H} + \bar{I} \tag{25}$$

Now

$$\bar{E} = -\bar{P} - \bar{P}_o = -\frac{\epsilon - 1}{\epsilon}\,\frac{w r H}{c} - \frac{w r I}{c} \tag{26}$$

Then using $\bar{I} = (\mu - 1)\bar{H}$ gives

$$\bar{E} = -\frac{\epsilon\mu - 1}{\epsilon}\,\frac{w r H}{c} = \frac{\mu\epsilon - 1}{\mu\epsilon}\,\frac{w r B}{c} \tag{27}$$

where μ is the permeability of the material. This is the method used by Tate (8, p. 94). The results are in agreement with those obtained by Einstein and Laub (2) from relativity considerations.

Equation 27 gives the field inside a dielectric, magnetic material. The external field, as pointed out by Swann (7, p. 385), is the same as in the case of the conducting material.

In the Wilson and Wilson experiment a dielectric magnetic material composed of steel balls imbedded in paraffin was molded into a hollow cylinder which was mounted to rotate inside a solenoid about its axis of symmetry. The inner and outer faces of the cylinder were covered with conducting plates, thus making a cylindrical condenser. The plates were connected to a quadrant electrometer and a deflection of the electrometer was looked for as the current in the electrometer was reversed while the cylinder continued to rotate. The amount of deflection was determined by the electric field in the dielectric cylinder.

Swann's theory predicts a field given by equation 27. The stationary line theory predicts a field in the dielectric given by

$$\bar{E} = -\bar{P} \tag{28}$$

Here

$$\bar{P} = (\epsilon-1)\left(\bar{E} + \frac{\omega r B}{c}\right) = (\epsilon-1)\left(-\bar{P} + \frac{\omega r B}{c}\right) \tag{29}$$

Solving for $\bar{P}$ gives

$$\bar{P} = \frac{\epsilon-1}{\epsilon} \frac{\omega r B}{c} \tag{30}$$

Then substituting in equation 29 gives

$$\bar{E} = -\frac{\epsilon-1}{\epsilon} \frac{\omega r B}{c} = -\frac{\mu(\epsilon-1)}{\epsilon} \frac{\omega r H}{c} \tag{31}$$

The results of the experiment decided in favor of Swann's theory.

However, closer examination of the ability of the steel balls imbedded in paraffin to approximate a true dielectric, magnetic material for this experiment leads this writer to question the results of the experiment. Since all of the magnetic dipoles exist in the steel balls it would seem that the free charge of the steel would redistribute itself in such a manner as to cancel any external effect of the fictitious polarization.

This is possible if the necessary distribution is such as to leave the total charge on the ball zero. The necessary charge distribution is given by equations 19 and 20. The total volume charge, Q_v, will be

$$Q_v = \int_\tau \rho_o \, d\tau = \int_\tau \nabla \cdot \bar{P}_o \, d\tau \tag{32}$$

where τ indicates integration throughout the volume of the conductor, in this case the steel ball. The total surface charge, Q_s, will be

$$Q_s = \int_S \sigma_o \, dS = \int_S -P_{o_n} \, dS = -\int_S \bar{P}_o \cdot d\bar{S} \tag{33}$$

where S indicates integration over the entire surface of the conductor. Applying Gauss's theorem gives

$$Q_v = \int_\tau \nabla \cdot \bar{P}_o \, d\tau = \int_S \bar{P}_o \cdot dS \tag{34}$$

so that the total charge, Q_T, on the conductor is

$$Q_T = Q_v + Q_s = \int_S \bar{P}_o \cdot d\bar{S} - \int_S \bar{P}_o \cdot d\bar{S} = 0 \tag{35}$$

Therefore, the necessary charge distribution is possible for any size, shape, or position of conductor.

It can then be concluded that all other effects of the fictitious polarization will be canceled in each steel ball by the charge distribution which this polarization causes.

This means that for the Wilson and Wilson experiment $\bar{P}_0$ should be zero in equation 22 and the subsequent equations. The result would then be a field in the cylinder given by equation 31, the same as for the stationary line theory. Thus, their experiment is not capable of distinguishing between that theory and Swann's theory. Only the use of a truly non-conducting magnetic material could do this.

It would not be just to conclude on the basis of this argument alone that the Wilsons' experiment was in error. However, it does raise sufficient doubt in the mind of this writer to justify a further investigation of this matter to the point of repeating the experiment. This may be done at a later date.

Part B: Theory Applied to a Magnet Rotating About an Axis Perpendicular to Its Magnetic Axis

Before considering the effect of rotating a magnet in bulk, it is instructive to consider the rotation of a hypothetical line magnet. Suppose a line magnet with magnetic moment per unit length J to be rotating about the Y axis with angular velocity $\bar{\omega}$. Consider it at the instant when the axis of the magnet coincides with the Z axis as shown in figure 1. The magnet extends from $-\ell$ to $+\ell$ so its overall length is 2ℓ.

The motion of each elementary magnet of which the line magnet is composed may be resolved into a rotation about an axis passing through its center and parallel to the Y axis as well as motion of the center about the Y axis. The latter gives rise to the field of an electric dipole as shown in the section on theory. It need not be considered here for reasons given later. The former gives rise to a $-\frac{1}{c}\frac{\partial \overline{U}}{\partial z}$ field as given by equation 14, and it is this effect which will be considered here.

Only the Y component of this field at a point P in the XZ plane is of interest here. An elemental length dz of the magnet will contribute an amount dE_y according to equation 14. Here

$$dE_y = \frac{\omega J\, dz}{c}\,\frac{\cos\Theta}{r^2} = \frac{\omega J}{c}\,\frac{(Z_0-Z)\,dZ}{[X_0^2+(Z_0-Z)^2]^{3/2}} \qquad (36)$$

Adding up the contributions for the whole magnet gives

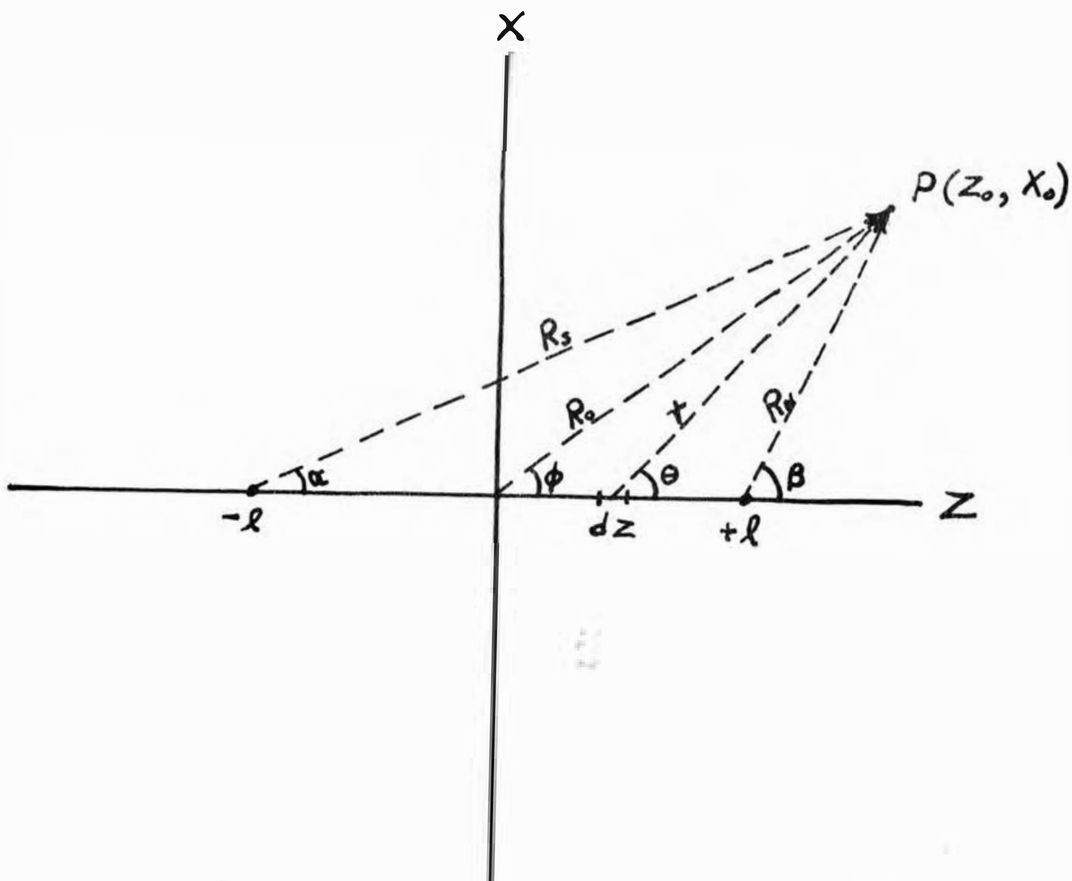
$$E_y = \int_{-\ell}^{\ell} \frac{\omega J}{c}\,\frac{Z_0-Z}{[X_0^2+(Z_0-Z)^2]^{3/2}}\,dZ$$

$$= \frac{\omega J}{c}\left[\frac{1}{[X_0^2+(Z_0-\ell)^2]^{1/2}} - \frac{1}{[X_0^2+(Z_0+\ell)^2]^{1/2}}\right]$$

$$= \frac{\omega J}{c}\left[\frac{1}{R_N} - \frac{1}{R_S}\right] \qquad (37)$$

Equation 37 gives the force in the Y direction on a unit positive charge at P due to this effect for the entire magnet.

If a unit positive charge at P is traveling with the



$$\sin\theta = \frac{X_0}{r} \qquad \cos\theta = \frac{Z_0 - Z}{r}$$

$$\sin\phi = \frac{X_0}{R_0} \qquad \cos\phi = \frac{Z_0}{R_0}$$

$$\sin\beta = \frac{X_0}{R_N} \qquad \cos\beta = \frac{Z_0 - \ell}{R_N}$$

$$\sin\alpha = \frac{X_0}{R_s} \qquad \cos\alpha = \frac{Z_0 + \ell}{R_s}$$

$$r = \sqrt{X_0^2 + (Z_0 - Z)^2} \qquad R_0 = \sqrt{X_0^2 + Z_0^2}$$

$$R_N = \sqrt{X_0^2 + (Z_0 - \ell)^2} \qquad R_s = \sqrt{X_0^2 + (Z_0 + \ell)^2}$$

Figure 1. Line Magnet on Coordinate Axes

magnet so as to maintain a constant position with respect to the magnet the charge will experience an additional force due to the motional intensity $\frac{\bar{v} \times \bar{B}}{c}$. The magnetic induction, $\bar{B}$, is that of a magnetic dipole of length 2ℓ and magnetic moment $2\ell J$. Thus, J corresponds to the pole strength of the magnet and

$$\bar{B} = i\, J\left(\frac{\sin\beta}{R_N^2} - \frac{\sin\alpha}{R_s^2}\right) + k\, J\left(\frac{\cos\beta}{R_N^2} - \frac{\cos\alpha}{R_s^2}\right) \tag{38}$$

The charge would move in a circular path of radius R_0 so

$$\bar{v} = i\, \omega R_0 \cos\phi - k\, \omega R_0 \sin\phi \tag{39}$$

The result is a motional intensity in the Y direction given by

$$\frac{\bar{v} \times \bar{B}}{c} = \frac{J\omega}{c}\left[\frac{1}{R_s^3}\left(R_0^2 + z_0\ell\right) - \frac{1}{R_N^3}\left(R_0^2 - z_0\ell\right)\right] \tag{40}$$

using the identities given with figure 1. This gives the force in the Y direction due to the motional intensity acting on a unit positive charge moving at P.

Now adding equations 37 and 40 gives the force in the Y direction due to these two effects.

$$E_Y + \frac{\bar{v} \times \bar{B}}{c} = -\frac{\omega J\ell}{c}\left[\frac{z_0 - \ell}{R_N^3} + \frac{z_0 + \ell}{R_s^3}\right] \tag{41}$$

By approximating a magnet in bulk to be made up of many line magnets grouped together parallel to one another we can expand equation 41 to apply approximately to a real magnet in a similar state of rotation. If the magnet is non-conducting, an additional term due to the translational motion of the elementary dipoles would have to be added.

However, as pointed out in part A, if the magnet is a conductor, this effect is canceled by a charge distribution in the magnet.

Additional charge distributions in a conducting magnet will create an electrostatic field about the magnet. However, a field of this type when integrated around a closed circuit contributes nothing to the e.m.f. Thus, as will be seen in the next section, it need not be considered further here.

Only forces of the type considered in equation 41 will give rise to e.m.f.'s around a closed circuit. These will be considered further in the next section in connection with the apparatus there described.

Equation 41 is the result of applying Swann's theory. How might the stationary line theory be applied to this type of rotation? Both Swann's theory and the stationary line theory consider that the lines of induction do not rotate. Their chief difference is that Swann applies this idea to the elementary magnetic particles while the other theory applies it to the magnet as a whole. Since Swann pictures the magnetic poles of the elementary dipoles as carrying their fields in their translational motion, this same idea applied to the poles of the magnet as a whole might be the true representative of the stationary line theory as applied to rotation of the type being considered. It is instructive to apply this to the line magnet.

Consider again the line magnet shown in figure 1. The field of this magnet as a whole is that of a pole of strength $+J$ placed at $+\ell$ and a pole of strength $-J$ placed at $-\ell$. The electric field at P will be due to a motion of the lines from each pole and is given by

$$\bar{E} = -\frac{\bar{v}_N \times \bar{B}_N}{c} - \frac{\bar{v}_S \times \bar{B}_S}{c} \tag{42}$$

where $\bar{v}_N$ is the velocity of the lines from the north pole; thus, it is the velocity of the pole itself. $\bar{B}_N$ is the magnetic induction at P due to the north pole. The "s" subscripts refer to the corresponding quantities for the south pole. For the line magnet in figure 1 the field becomes

$$\bar{E} = E_y = \frac{wJ\ell}{c}\left[\frac{z_0-\ell}{R_N^3} + \frac{z_0+\ell}{R_S^3}\right] \tag{43}$$

There are many objections to this method of applying the stationary line theory. For one thing, it gives too much reality to the poles of magnet. Another objection is that it does not consider the motion of the charge at P.

The results of that application are included here for two reasons. One is that equation 43 differs only in sign from equation 41. The second is found in comparing its predictions with the results of the next section.

EXPERIMENTAL APPARATUS, PROCEDURE AND RESULTS

This section contains a description of an experiment performed during the Summer of 1958. Its purpose was to test Swann's theory which is developed in an earlier part of this paper. In so far as the writer has been able to determine from the physics abstracts, the results of an experiment of this type have not been published prior to this time.

The results are more qualitative than quantitative, but they do appear to represent a definite electromagnetic effect. It would seem desirable to refine the apparatus and investigate this effect further at some future time. Thereupon, the effect could be reported and discussed more conclusively.

The present work was as follows.

Part A: Apparatus

A cylindrical bar magnet was fitted crosswise into a brass collar which, in turn, was fitted onto the shaft of an electric motor as shown in figure 2. The magnet was 11 cm. long and 1.5 cm. in diameter and made of alnico alloy. With the arrangement described, the motor could rotate the magnet about an axis perpendicular to its length.

Two thin copper plates were mounted on masonite slabs, and these were arranged parallel to one another on either side of the magnet so that the magnet could rotate between the copper plates as shown in figure 3a. With this arrangement,

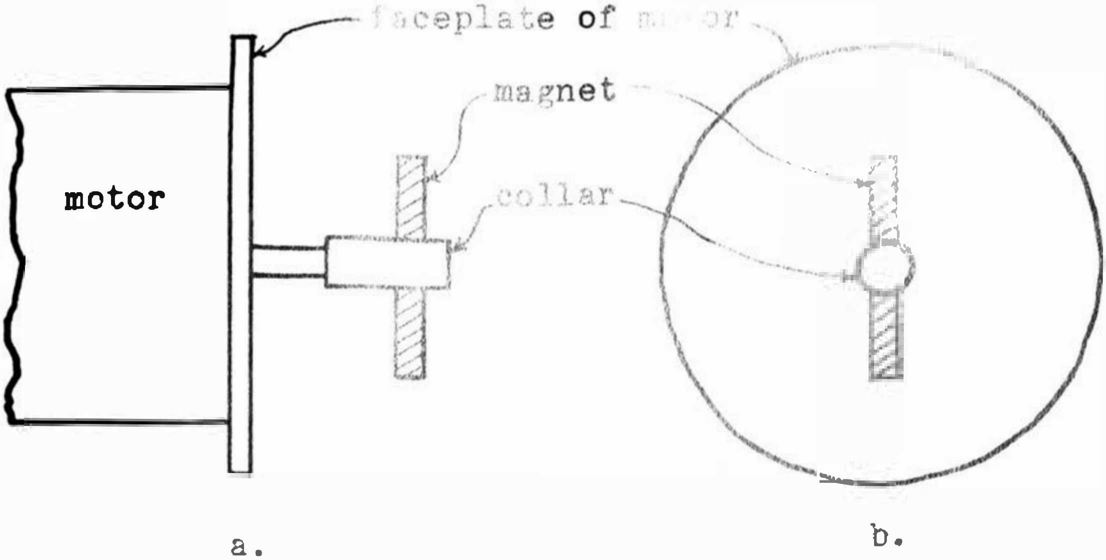


Figure 2. Magnet as Mounted on Motor

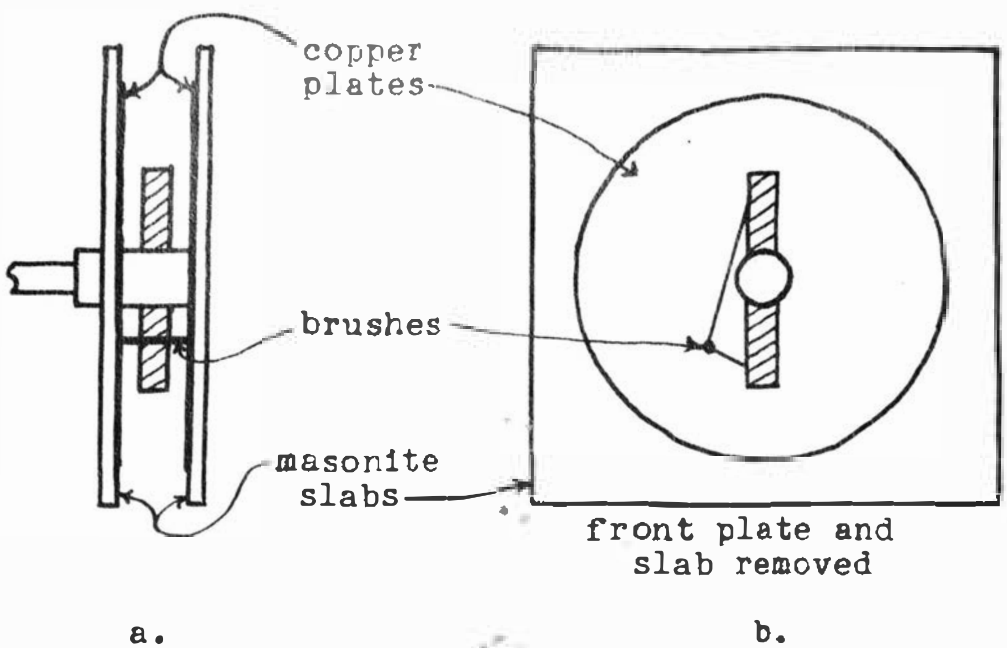


Figure 3. Plates and Brushes as Mounted About Magnet

brushes rigidly attached to the magnet, as shown in figure 3, could maintain a constant electrical connection between the two plates as the magnet rotated. The brushes were mounted so that they could be fixed at various positions with respect to the magnet.

A Weston model 699 galvanometer was connected across the two plates to form a closed circuit.

Part B: Procedure

With the apparatus as described, the magnet was set in rotation by the motor. At first, erratic deflections of the galvanometer were observed. These, however, were attributed to thermal e.m.f's at the brush contacts. When a thin film of oil was placed on the copper plates these erratic effects disappeared.

With the brush contact effects eliminated, it was observed that when the brushes had certain positions with respect to the magnet, the galvanometer would deflect in one direction during rotation of the magnet. For certain other positions of the brushes, the galvanometer deflected in the other direction and for other positions, no deflection was obtained. These positions will be described in more detail in part C of this section.

In order to ascertain that the currents causing these deflections were due to the magnet, a number of control procedures were used:

1. An unmagnetized steel cylinder was used in place of the magnet--no deflections of the galvanometer were observed.

2. A wooden piece was used in place of the magnet -- no deflections were observed.

3. With the magnet rotating, the entire apparatus was given various orientations in the earth's magnetic field -- no change in the deflection was observed for various orientations.

4. The magnet was fixed at various positions with respect to the motor shaft -- the observed deflections were the same for the different positions.

It appeared that the deflections were definitely due to the magnet.

Other checks on extraneous effects were as follows:

1. In order to determine the effects of eddy currents in the copper plate, the solid copper plates were temporarily replaced with circular strips of copper, mounted concentrically. The strips were about one-half inch wide. The observed deflections followed the same pattern.

2. The spacing between the plates was changed to various values between about one and one-half inches and three-fourths inch. The deflections increased when the spacing decreased.

3. The galvanometer leads were originally attached to the copper plates at points opposite each other on the outer edge of the plates. These points of contact were varied to other possible positions on the back of the plates. The deflection of the galvanometer was the same for all combinations of contact points for the galvanometer leads.

4. When the brushes were positioned along the magnet, the deflection seemed to be the same whether or not the magnet itself was a part of the conducting circuit.

There were many inductive effects involved in this arrangement. A change in flux through the closed circuit was expected to induce an e.m.f.; however, this was alternating and did not read on the direct current galvanometer. Such alternating e.m.f. did show on the oscilloscope.

Other possible inductive effects in the copper plates were such as to cancel themselves because of the symmetry of the plates about the magnet.

That left only the inductive effects in the brushes to contribute to a direct e.m.f. around the circuit. These are the effects considered in equation 41.

The positioning of the brushes and the reading of the deflections were not done with great precision. The observed results differed radically from those expected; therefore, it was felt that, until there was a plausible explanation of the effect to guide the investigation, greater precision was not necessary. Many refinements in procedure could be made to test such an explanation.

Part C: Results

The significance of the experimental results can best be emphasized by comparing them with the results of equations 41 and 43. In order to do this, figure 4 shows the magnet at the instant it is rotating through the Z axis just as is the line magnet in figure 1. The lines about the magnet are the looi of points where the brushes were positioned to give zero deflection on the galvanometer. These lines will be referred to as nodal lines. It should be emphasized that these lines represent positions of the brushes with respect to the magnet. When the brushes were positioned on one side of the nodal line, the deflection was in one direction and when positioned on the other side of the line the deflection was in the opposite direction.

To establish a sign convention, consider a deflection to be positive when it is caused by a conventional current coming out of the paper in figure 4. The plane of the page is divided into definite areas by the nodal lines. The direction of the deflection is the same throughout each area. These areas are marked with a plus or minus sign to indicate the direction of the deflection when the brushes are within that area.

The magnitudes of the deflections vary greatly within each area. Some magnitudes are indicated in figure 4 at the position they were measured when the plates were one inch apart. The numbers are in galvanometer divisions. Each division of deflection indicates approximately 2×10^{-4} volts e.m.f. around the circuit.

For comparison, the nodal lines predicated by equations 41 and 43 are shown in figure 5. These nodal lines correspond to lines of zero force in the plane of the page. If the hypothetical line magnet to which the equations apply were used in the experiment in place of the real magnet, then the predicted e.m.f. around the circuit could be obtained by integrating the force in equation 41 along the brushes from plate

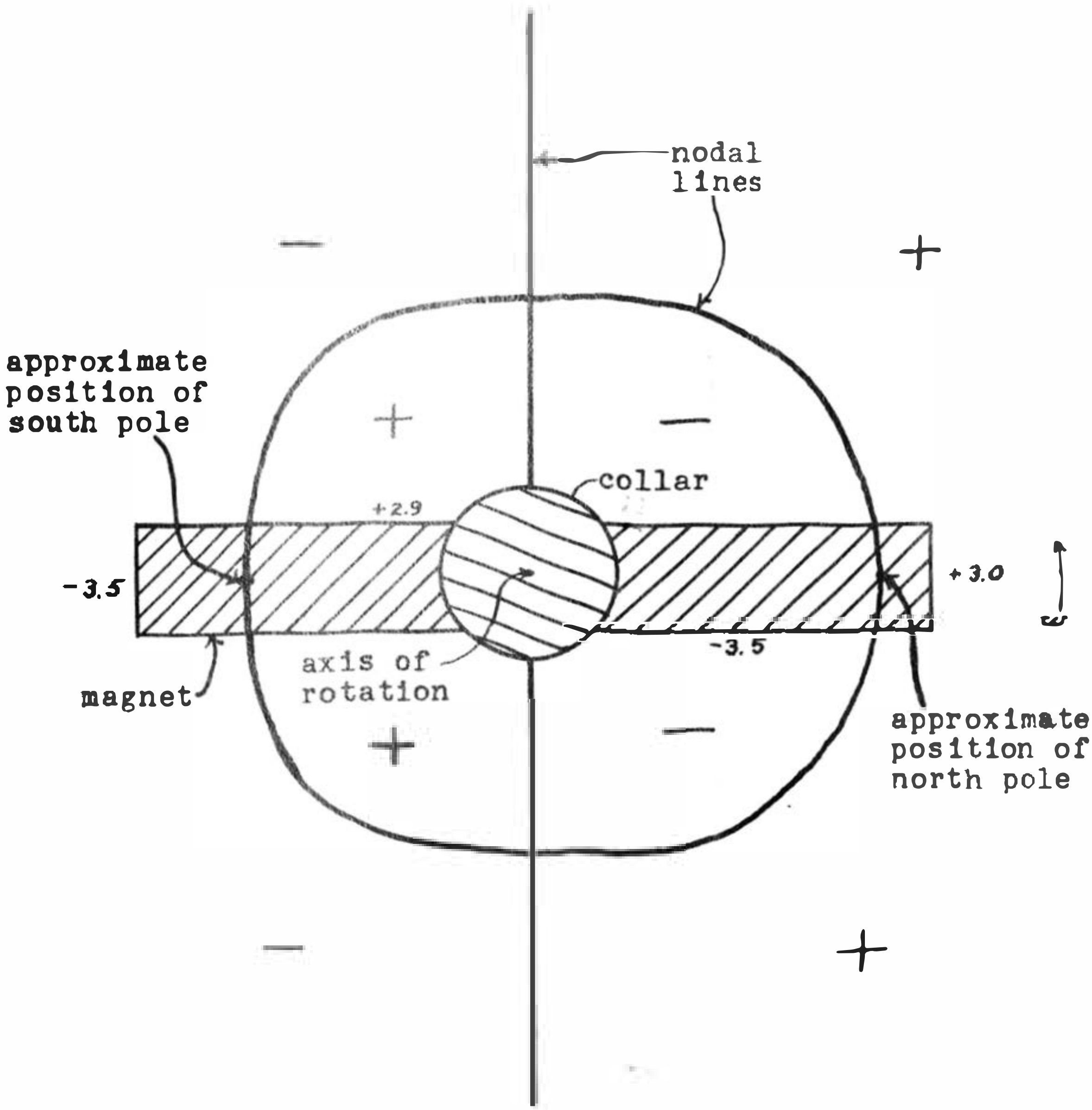


Figure 4. Locations of Experimental Nodal Lines

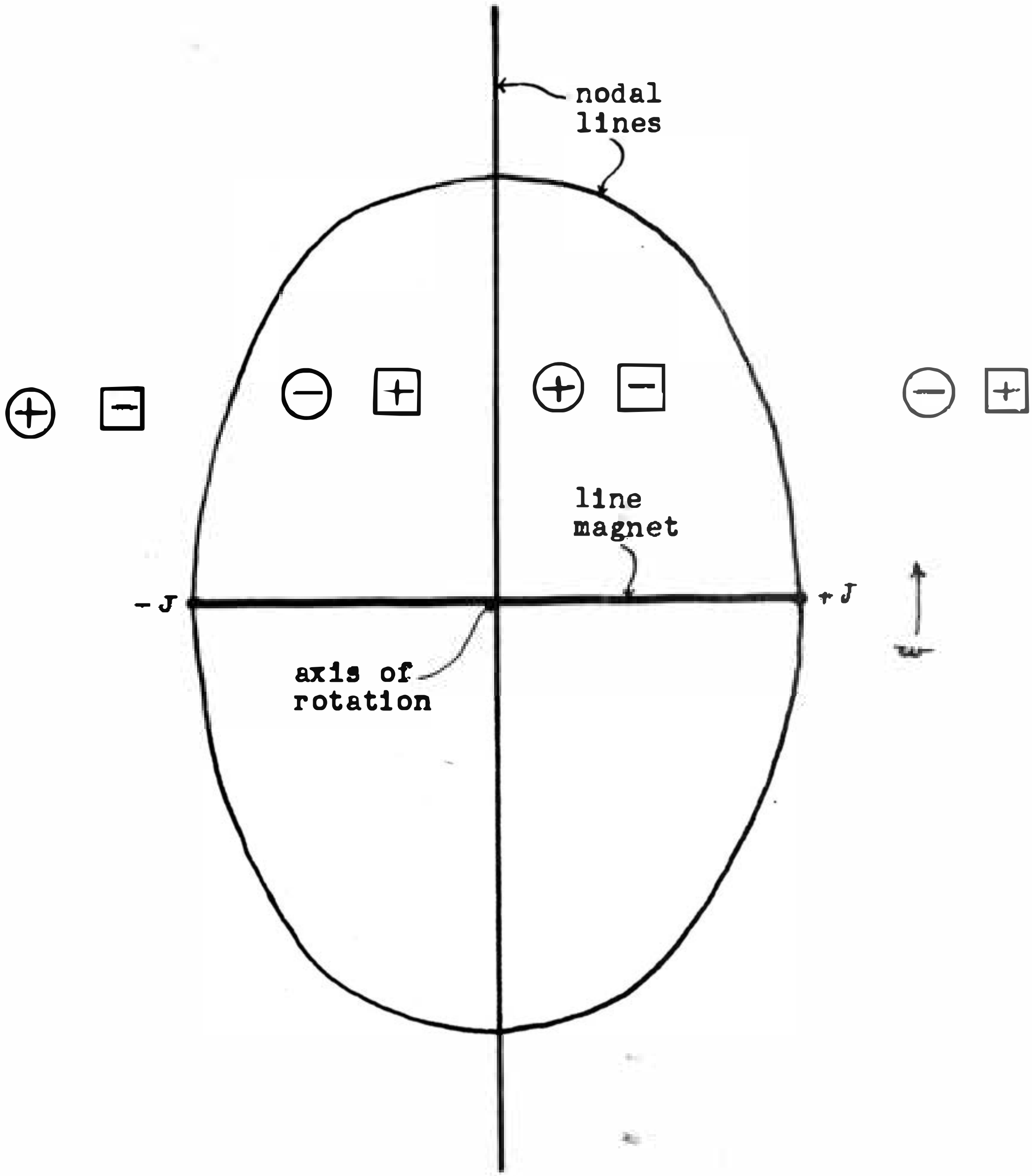


Figure 5. Locations of Theoretical Nodal Lines

to plate. The nodal lines in figure 5 do not show the exact position of the lines along which the brushes could be placed to give zero deflection because equation 41 does not apply off the plane of the page. However, the important comparison is not the size or shape of these areas, but it is the fact that these equations predict that such areas should exist.

The magnitude of the expected deflection due to the line magnet can be calculated. In order to have a comparable value the line magnet was given the approximate dimensions and strength of the real magnet. They are as follows:

$J = 100$ unit poles

$l = 4.3$ centimeters

A point out on the axis of the magnet 2 cm. from the pole was chosen for the brush position. Then using 1 inch for the separation of the plates and a rotational speed of 1780 revolutions per minute, a rough value of 4×10^{-4} volts was calculated for the e.m.f. This compares with an experimental value of 7×10^{-4} volts for a similar point. The values agree in order of magnitude which is all that could be expected considering the many approximations involved.

Next, it is worthwhile to note the direction of the deflection to be expected when the line magnet is used. The Directions predicated by equation 41 are enclosed in circles in figure 5; those predicted by equation 43 are enclosed in squares. The directions predicated by equation 41 are in each area opposite those found in the experiment. This is the radical difference spoken of previously. No way has been found to resolve this difference.

Conversely, the directions of the deflections predicted by equation 43 agree with those of the experiment. However, that equation is based on very unsound theory. The fact that it does predict results in agreement with this experiment is its only justification here.

This dilemma is the reason for the lack of greater refinement in the results of this experiment.

CONCLUSIONS

Before concluding, it seems desirable to answer the question that prompted this study. Part (a) of the question: Is there an e.m.f. along the wire when it is at rest? Yes, there is. It is due to the electrostatic field created by a charge distribution inside the magnet. This charge distribution is such as to cancel the force of the motional intensity produced when the magnet rotates in its own field. The total charge on the magnet remains zero so, if the magnet were infinitely long, the external field and the e.m.f. in the wire would be zero. The external field that exists for a real magnet is an end effect so that, near the middle of the magnet, the field and, consequently, the e.m.f. will be small.

Part (b) of the question: Is there an e.m.f. along the wire when it is rotating with the magnet? Yes, there is. The field considered for part (a) will still be effective in causing an e.m.f. In addition, the motional intensity in the wire itself contributes to the e.m.f. The motional intensity does not become small near the middle of the magnet.

Therefore, the e.m.f.'s for part (a) and for part (b) are different and depend upon where the wire is placed.

The answer given here is from the point of view of the non-rotating observer. For the point of view of the rotating observer the transformations of Trocheris (9, p. 1148) may be applied.

Beyond answering that question, the purpose of this work was threefold:

1. To become familiar with Swann's theory and to transcribe his main contentions.

2. To question the experimental verification of Swann's theory in the Wilson and Wilson experiment.

3. To test Swann's theory experimentally.

The first has been accomplished unequivocally.

The second has been accomplished theoretically to this writers satisfaction. The final judgement necessarily lies in further experimental investigation.

The third has not been accomplished conclusively. The results of the experiment seem to indicate that Swann's theory is in error as it has been applied here. Yet, his ideas are so soundly based on accepted theory that it is hard to doubt his conclusions. Perhaps somewhere there is an error in an assumption made in applying the theory.

While they do not sound plausible, the following changes in the method of applying the theory lead to a possible explanation of the experimental results.

1. Neglect the motional intensity. For some reason this force on the charge in the moving brushes may not be effective in adding to the e.m.f. of the circuit.

2. Instead of considering the rotation of each elementary dipole as was done for equation 37, suppose that the lines of induction from the elementary dipoles link together so that only the

> motion of the apparent poles of the magnet as a whole need be considered. This was essentially what was done in deriving equation 43.

However implausible these may seem, they lead to results in approximate agreement with the experiment. A conclusive answer to this problem must be left for a future more advanced study.[1]

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[1] A variation of the experimental work reported here is discussed in the Appendix'. This work has been done since this paper was written and may give reason to doubt some of the conclusions stated here.

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APPENDIX

The terms found in equations 1 through 4 are defined as follows:

$\bar{H}$ ....magnetic intensity at a point
$\bar{E}$ ....electric intensity at that point
ρ ....charge density at that point
$\bar{u}$ ....velocity of the charge at that point

The bars above the symbols indicate vector quantities.

In equation 5, ψ is the true electrostatic potential and is defined by equation 6 where ρ is the charge density at a point at the instant when ψ is determined; r is the distance from the point where ψ is determined to the point where ρ is determined.

Also in equation 5, $\bar{U}$ is the Maxwellian vector potential and is defined by equation 7. There ρ is the charge density, $\bar{u}$ the velocity of the charge and $\frac{\partial \bar{E}}{\partial t}$ the time rate of change in electric intensity at a point at the instant $\bar{U}$ is determined; r is the distance from the point where $\bar{U}$ is determined to the point where ρ , $\bar{u}$ and $\frac{\partial E}{\partial t}$ are determined.

In both equation 6 and equation 7 the integration is throughout all space.

In equation 8, $\bar{B}$ is the magnetic induction at the point where the charge is moving.

APPENDIX'

A recent variation of the experiment was as follows. The copper plates were attached to the collar so that they rotated with the magnet. Then, instead of brushes, a conducting bar extending between the plates was used. The leads to the galvanometer were arranged to brush on the copper plates so that a closed circuit was formed.

When the magnet and plates were rotated, the galvanometer deflections were about one-tenth of a division on the Weston model 699 galvanometer. The deflections did not appear to have nodal lines; instead, the deflections were always in the same direction when the connecting bar was given various locations with respect to the magnet. The magnitude of the deflections seemed to depend on the distance of the bar from the axis of rotation. The amount of deflection also depended on the distance from the axis of rotation to where the galvanometer leads brushed on the copper plates.

Since no deflections were observed which are comparable to those observed when the plates did not rotate, the latter effect may have to be explained as being due to eddy current fields. The significance of this in terms of Swann's theory is not clear.