Effect of Poisson’s Ratio on Young's Modulus Characterization Using Ultrasonic Technique by Modeling

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EFFECT OF POISSON’S RATIO ON YOUNG’S MODULUS CHARACTERIZATION USING ULTRASONIC TECHNIQUE BY MODELING

BY

ONYETUBE MICHAEL ABUCHI

A thesis submitted in partial fulfilment of the requirements for the

Master of Science

Major in Mechanical Engineering

South Dakota State University

2018
EFFECT OF POISSON'S RATIO ON YOUNG'S MODULUS CHARACTERIZATION USING ULTRASONIC TECHNIQUE BY MODELING

This dissertation is approved as a credible and independent investigation by a candidate for the Master of Science degree and is acceptable for meeting the requirement for this degree. Acceptance of this design paper does not imply that the conclusions reached by the candidate are necessarily the conclusions of the major department.

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Lastly, I fought a lot with my dad about going to college 10 years ago, because he believed we were too poor to attain higher education. It’s a pity he passed 9 years ago. I dedicate this work to his memory, Mr. Peter Onyetube.
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LIST OF ABBREVIATIONS

FE     Finite Element
FEA    Finite Element Analysis
MPI    Magnetic particle inspection
NDE    Non-Destructive Evaluation
NDT    Non-Destructive Testing
RT     Radiographic testing
2D     Two Dimensional
3D     Three Dimensional
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ABSTRACT

EFFECT OF POISSON’S RATIO ON YOUNG’S MODULUS CHARACTERIZATION USING ULTRASONIC TECHNIQUE BY MODELING
ONYETUBE MICHAEL ABUCHI

2018

The past 27 years has witnessed a revolutionary growth in the progress of material development and application in almost all industry and business sectors, and this seems to be continuing even today. So many material-driven innovations have enabled the global spread in technology and improvements in capability, ranging from communications to aerospace and healthcare, to automotive and agriculture

Mechanical behavior of elastic materials is modeled by two main independent constants; Young’s modulus and Poisson’s ratio. An accurate measurement of both constants is necessary in most engineering applications, for example, the standard materials used for the calibration of some equipment, quality control of other mechanical materials.

In this work, 7075-T6 Aluminum and carbon fiber-epoxy composite were used to study the effect of change in Poisson’s ratio on the Young’s modulus of a material using non-destructive testing (NDT). Ultrasonic simulation was used because Lamb wave velocity is dependent on the elastic properties of the transmission medium. Also, ultrasonic simulation shows more accuracy and non-destructive advantages over the tensile and indentation test. The theoretical and experimental results were used to validate the results of simulation before using it for this study. ‘ANSYS mechanical’ software was used to simulate the process.
CHAPTER 1: INTRODUCTION

1.1 Problem Statement

The intent of this work is to study the effect of change in Poisson’s ratio on the Young’s modulus of materials. First, T-7075 Aluminum was used as specimen for this study, and the study was attempted on carbon fiber-epoxy composite.

1.2 Mechanical properties of materials

1.2.1 Poisson’s ratio

Poisson’s effect is physically obvious in rubber, which becomes noticeably thinner when stretched. Poisson’s ratio (ν) is a measure of the tendency of a material to expand in directions perpendicular to the direction of compression, and contract in directions perpendicular to the direction of tension. Some materials, however, show an expansion in the transverse direction under tension, and a contraction in the transverse direction under compression.

It is common to see a Poisson’s ratio range between 0.0 and 0.5. A perfectly incompressible material under elastic deformation has a Poisson’s ratio value of 0.5. For isotropic linear elastic materials, Poisson’s ratio value range between -1.0 to 0.5 because of the requirement for Young’s modulus, Shear modulus and Bulk modulus to have positive values [1]
Figure 1. 1: Material under axial tension

Strain ($\varepsilon$) is defined in elementary form as the change in length divided by the original length.

$$\varepsilon = \frac{\Delta L}{L} \quad (1.1)$$

Poisson’s ratio for a material stretched or compressed along the x-axis is given by

$$\nu = -\frac{d\varepsilon_{\text{transverse}}}{d\varepsilon_{\text{longitudinal}}} = -\frac{d\varepsilon_y}{d\varepsilon_x} = -\frac{d\varepsilon_z}{d\varepsilon_x} \quad (1.2)$$

Where

$\nu$ is the Poisson’s ratio

$\varepsilon_{\text{transverse}}$ is transverse strain

$\varepsilon_{\text{longitudinal}}$ is axial strain

The minus sign accounts for the sign change between longitudinal and lateral strains. The ‘constitutive’ or stress-strain law of material much be extended to include the Poisson’s
effect, since the strain in a given direction is influenced not only by the stress in that
direction, but by strain contributed by the stresses in the other two directions.

Figure 1.2: Longitudinal and lateral strain

A material subjected to a uniaxial stress $\sigma_x$ in x-direction will experience a resulting
strain in same direction of $\varepsilon_x = \sigma_x/E$. An additional stress $\sigma_y$ in the y direction will
introduce a new component of x-direction strain because of the ‘Poisson’s effect’ of $\varepsilon_x =
-\nu \varepsilon_y = -\nu \left( \frac{\sigma_y}{E} \right)$. If the material is under biaxial stress condition only, the strain effects can
be linearly superimposed to give:

In x direction

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \left( \frac{\sigma_y}{E} \right) = \frac{1}{E} (\sigma_x - \nu \sigma_y) \quad (1.3)$$

In y direction

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu \left( \frac{\sigma_x}{E} \right) = \frac{1}{E} (\sigma_y - \nu \sigma_x) \quad (1.4)$$
In a stress situation where there is no third stress in the z direction ‘plane stress’, there is a strain in the z direction from the stresses $\sigma_x$ and $\sigma_y$ and is given by

$$\varepsilon_z = \frac{\sigma_y}{E} - \nu \left( \frac{\sigma_x}{E} \right)$$  \hspace{1cm} (1.5)

Poisson’s ratio is a dimensionless parameter which helps in understanding the properties of a material. It is worth to note that brittle materials have lower Poisson’s ratio values compared to flexible or ductile materials. An approximated value of Poisson’s ratio for some of the major classes of engineering materials are given in the table below:

Table 1. 1: Poisson’s ratio values of engineering materials [1]

<table>
<thead>
<tr>
<th>Material Class</th>
<th>Poisson’s Ratio $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramics</td>
<td>0.2</td>
</tr>
<tr>
<td>Metals</td>
<td>0.3</td>
</tr>
<tr>
<td>Plastics</td>
<td>0.4</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The ability of a material to exhibit Poisson’s effect is directly related to its molecular mobility, with rubber being liquid-like and ceramic being very tightly bonded [1]. Compressibility is related to Poisson’s ratio in the formula below

$$K = \frac{-p}{\Delta V/V}$$  \hspace{1cm} (1.6)

Where

$K$ is the bulk modulus (modulus of Compressibility)
and $\Delta V/V$ is the relative volume decrease

The minus sign indicates that a compressive pressure (positive) causes a negative change in volume of the material.

Poisson’s ratio value cannot be greater than 0.5, since it will indicate that volume will increase under compressive pressure. Ceramics are held together by strong covalent bonds that makes it unable to rearrange its molecules to “fill the holes” that are created when they are put in tension. It is important to mention that the most common method of Poisson’s ratio measurement is the tensile test method.

1.2.2 Young’s modulus

Stiffness of a material is a measure of the load needed to induce a given deformation in that material. It is measured by applying loads below the fracture limit of the material and measuring the resulting deformation. One of the major experimental problems is measuring changes in length accurately, since deformation in most materials are very small.

Within the elastic limit of a material, the Hooke’s law [2] can be written algebraically as

$$P = k\delta$$

(1.7)

Where $P$ is the load applied to the material with units N, $\delta$ is the resulting deformation units m, $k$ is a constant of proportionality called the stiffness with units N/m. Stiffness is not a function of the material alone, it is also dependent on the specimen shape.
A way to make stiffness strictly a function of material property is by including cross-sectional area is to consider the stress on the material rather than load. These changes are represented mathematically below

\[ \frac{P}{AL} = \frac{k\delta}{AL} \]  

(1.8)

\[ \sigma = \frac{k+L}{A} \times \frac{\delta}{L} \]  

(1.9)

Or

\[ \sigma = E\varepsilon \]  

(1.10)

The constant of proportionality \(E\) known as Young’s modulus [3] or modulus of elasticity, is a major parameter used in the description of a material’s mechanical property. It has same units as stress (\(\sigma\)), Pa. Both versions of the Hooke’s law can be represented graphically as

Figure 1.3: Hooke’s law in terms of (a) load-displacement and (b) stress-strain

The stiffness \(k\) is related to the Young’s modulus \(E\) as
In the Hooke’s law experiment, note that the stress \((\sigma)\) developed in a tensile specimen is independent of the material properties, while the deformation depends on the material property \(E\). Hence, the stress would be the same on different materials but the strain \(\varepsilon\) will differ depending on if the specimen is made of polystyrene or steel; polystyrene will experience larger deformation since the modulus of elasticity is less than that of steel.

\[ k = \frac{AE}{L} \quad (1.11) \]

Also,

\[ \delta = \frac{PL}{AE} \quad (1.12) \]

1.3 Composites

Composites are structural materials that contain two or more constituents with significantly different physical and chemical properties combined at macroscopic level and are not soluble in one another. Composite fabrication usually involve wetting, mixing or saturating the reinforcement with the matrix, and then causing the matrix to bind together into a solid structure. Usually, we have one of the constituents to be the matrix phase, and one or more of the remaining constituents as the reinforcing phase(s). Examples of materials that could form the reinforcing phase are fibers, particles or flakes, etc. Examples of composite materials are alloy, epoxy reinforced with fiber, wood, bone, reinforced concrete [4]. Some of the uses include bridges, buildings and structures.
Factors that contribute to the mechanical performance of composites include length, orientation, shape and mechanical properties. Generally, mechanical properties of composite are dependent upon the following

- Reinforcing and matrix material
- The fiber orientation in the matrix
- Mechanical properties of the constituents
- Method of manufacturing

In an epoxy and fiber composite, the fiber carries most of the load, improves stiffness, strength, thermal stability electrical conductivity and other structural properties of the composite. The matrix (epoxy) isolates the fiber from each other and provides rigidity and shape to the composite.

Fabrication of composites can be done through one of the variety of techniques which include, advanced fiber placement (Automated fiber placement), tailored fiber placement, fiberglass spray lay-up process, filament winding, lanxide process, tufting and z-pinning.

Fiber reinforce composite materials are classified into two major categories; short fiber-reinforced material and continuous fiber-reinforced material. For this paper, the fiber used are continuous.

1.3.1 Applications of fiber-epoxy composite

Fiber reinforced composites basically improves rigidity and impedes crack propagation. The major benefit of epoxy-composites includes ability to be used to manufacture a lot of different products. Some of the features of carbon fiber that makes it
suitable for variety of applications include, it have the highest specific modulus and specific strength of all fiber reinforcing materials. At room temperature, carbon fiber is resistant to moisture and other variety of solvents, acids and bases, fiber composites manufacturing processes are common and are relatively inexpensive and cost effective, they retain high mechanical strength at elevated temperatures, exhibits diverse physical and mechanical characteristics, allowing composites of wide variety of properties to be engineered from it, hence, a wide range of areas where fiber composites can be used. It is relevant to note that high temperature oxidation is a challenge too.

There are a wide range of applications for fiber-epoxy composites because of its improved physical and chemical properties. Some of the common applications are as follows:

- Electricity industry: Composites are used as insulators for electric conductors, support for circuit barkers, support for printed circuits, armors, boxes covers, windmills, top of television power, etc.

- Aircraft: Composites are used to reduce the weight of aircrafts. In helicopters composites are used as blades for increased speed and life. Its durability in extreme conditions are also harnessed here

- Building: Bathrooms, Furniture, partitions

- Mechanical applications: Bearings, gears, car jack, prosthetics, pipes, tubes and casings for flow systems, housing etc.

- Sport and recreation: Bicycle frames, golf clubs, surf boards, fishing poles and skis, tennis and rachets, etc.
1.3.2 Mechanical properties of composites

Physical properties of composites are anisotropic (differ depending on the direction of the applied load). For example, the stiffness of a composite specimen will depend on the direction of applied force.

![Figure 1.4: (a) Transversely loaded composite (iso-stress) (b) Longitudinally loaded composite (Iso-strain)](image)

For a composite material with load applied parallel to the direction the fiber is aligned, there is a situation of iso-strain (the deformation of both matrix and fiber will be the same), this condition provides the higher strength of composites is found in longitudinally loaded composite materials. The concept behind iso-strain is that both phases experience the same strain but will feel different stress. The effective composite Young’s modulus is given in the equation below
\[ E_c = \sum_{i=1} V_i E_i \]  
\[ \text{(1.13)} \]

Where, \( V_i \) is the volume fraction of the composite phase

And \( E_i \) is the Young’s moduli of the composite phase

This can be rewritten as

\[ E_c = V_a E_a + V_b E_b \]  
\[ \text{(1.14)} \]

Where \( V_a \) and \( V_b \) respectively are the volume fractions for each phase

However, the lower strength of composites is found in transversely loaded composite materials (iso-stress). For the iso-stress condition, both phases feel the same stress but the strain on each of them differ. The mathematical representation of the effective composite Young’s modulus:

\[ \frac{1}{E_c} = \sum_{i=1} V_i \frac{1}{E_i} \]  
\[ \text{(1.15)} \]

For a composite material made of phases a and b as shown in figure 2.4 (b), \( E_c \) can be represented mathematically as

\[ E_c = \frac{E_a E_b}{V_a E_a + V_b E_b} \]  
\[ \text{(1.16)} \]

1.4 Non-destructive Testing

Nondestructive testing (NDT) is a range of analysis techniques used in science and industry to evaluate the properties of a material, component or system without damaging
the specimen [5]. NDT can also be defined as use of scientific ways to analyze a material without causing harm to it.

Some of the applications of nondestructive testing are in flaw detection, flaw evaluation, location detection, leak detection, microscopic characterization, dimensional measurements, mechanical and physical property estimation [6]. Most common areas of application of NDT, however, are as follows:

- Automobiles
- Airplanes
- Nuclear power plants
- Pipeline maintenance

### 1.4.1 Introduction to Methods of NDT

Test methods are often named the type of penetrating medium or the equipment used to perform that test. Here is a list of some of the current NDT methods:

- Film radiography
- Computed Radiography
- Computed Tomography
- Liquid penetrants
- Laser testing methods
- Tap testing
- Eddy current
1.4.1.1 Radiographic testing (RT)

Radiography involves exposing a test object to penetrating radiation so that it passes through the object being inspected and a recording medium is placed against the opposite side of that object [7]. Electricity generated radiations are often used for thinner less dense material, and for thicker or denser materials, gamma radiation is preferred. The most common sources of gamma radiation are the radioactive decay of Iridium-192(Ir-192) and Cobalt-60(Co-60). Ir-192 is used for steel between $2\frac{1}{2}$-3 inches, depending on the strength of the source, and Co-60 is used for denser material that require greater penetration ability.
Several types of radiation detectors can be used as the recording media, or the X-ray film. Having both, radiation passing through the test object exposes the media, avoiding the effect of having darker areas where more radiation has passed through the part and lighter areas where less radiation has penetrated.

1.4.1.2 **Acoustic emission**

Acoustic emission testing is carried out by applying a localized external force such as an abrupt mechanical load or rapid temperature or pressure change to the part being tested. The stress waves generated in turn generates short-lived, high frequency elastic waves in the form of material displacements, or plastic deformation, on the part surface that are detected by sensors attached to the part surface.
1.4.1.3 Visual inspection

Almost all specimens can be visually inspected [8]. Visual inspection is the first stage of examination carried out on a specimen to determine the soundness for its intended application. It is also the most common nondestructive testing (NDT) technique [9]. Some of the examples of visual inspection applications are; to determine whether the correct fabrication dimensions were achieved, whether the parts are complete, whether all the parts are appropriately incorporated into the device [10]. Visual inspection may be by Direct viewing, using line-of-sight vision, or may be enhanced with use of optical instruments such as fiberscopes, borescopes, magnifying glasses, charge-coupled devices (CCDs) and computer assisted viewing systems (remote viewing). Corrosion, misalignment of parts, physical damage and cracks are some of the discontinuities that may be detected by visual inspection.

1.4.1.4 Ultrasonic testing

Ultrasonic testing uses sound energy with frequency above audible range (20kHz) to conduct examinations and make measurements. The basic principle is that reflected sound energy is displayed versus time, and the inspector can visualize a cross-section of the specimen showing the depth of features that reflect sound [11].
Sound is introduced into the specimen using an ultrasonic transducer that converts the electrical impulses into sound waves, the sound wave either goes through the specimen to another transducer or is reflected to the sending transducer, then it converts the sound back to electric impulses that can be displayed as a visual representation on a digital or LCD screen.

In a case where the transducer is used to detect a defect, usually the wave is received back to the sending transducer, the equation that helps determine the location of defect in this process is
\[ d = \frac{tc}{2} \]  \hspace{1cm} (1.17)

where \( d \) is the distance between transducer and defect, \( c \) is the wave velocity, \( t \) is the time for signal to return to transducer.

Advantages of Ultrasonic testing

- It can be applied in flaw detection in both surface and submerged defects
- Better penetration power compared to other NDT techniques
- Ultrasonic testing requires minimal specimen preparation before use
- Results are quick
- Diverse applications in both flaw detection and thickness measurement
- Available in different sizes which makes it portable for uses
- Relatively high accuracy

Disadvantages of Ultrasonic testing

- Requires accessible surface to conduct test
- Since it is a surface dependent test, material shape, roughness, size, and homogeneity can affect test feasibility
- Linear defects parallel to the wave beam might be difficult to detect
- Requires coupling

Applications of Ultrasonic testing

- Material characterization
- Flaw detection and evaluation
- Dimensional measurement
Medical field; therapeutic ultrasound, sonography, etc.

To calculate the ultrasonic speed in an isotropic material, the following set of equations used for this calculation:

Lamé constants,

\[ \mu = \frac{E}{2(1+\nu)} \]  \hspace{1cm} (1.18)

\[ \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \]  \hspace{1cm} (1.19)

The longitudinal velocity,

\[ C_L = \sqrt{\frac{2\mu + \lambda}{\rho}} \]  \hspace{1cm} (1.20)

Time of travel,

\[ t = \text{thickness} / C_L \]  \hspace{1cm} (1.21)

1.4.1.5 Infrared Thermography

Infrared thermography is used to map surface temperatures. It is a thermal NDE technique which combines the external heating phenomena with the infrared imaging to evaluate subsurface structure [6]. This is achieved by analyzing the response obtained from the thermography test. This method has the capability to convert any small temperature changes between sides of the inspected material into images.
Thermographic cameras detect radiation in the long-infrared range of the electromagnetic spectrum (9000-14000 nanometers). The device converts the radiated heat in the form of infrared energy into electronic signals that are amplified and transmitted to a monitor. The images are sometimes accompanied by their calculations. Various temperature levels are represented by different color shades.

1.4.1.6 Microwave transmission

Microwave transmission is not one of the frequently used NDT techniques. It can be defined as the inspection and characterization of materials and structures using high-frequency electromagnetic energy [6]. An estimated frequency range of a microwave regime is 300MHz-300GHz. Microwave imaging can be either quantitative or qualitative. Quantitative techniques give the electrical and geometrical parameters of the image object,
while qualitative techniques reveal parameters like reflectivity function to represent the hidden object.

Figure 1. 9: A general view of a microwave imaging system

One of the most common applications of microwave systems is inspecting food products transferred by a conveyer belt in real time. Components of a microwave system include [12]

- Transmitter; for transmitting continuous microwave
- Receiver; for receiving microwave that went through the specimen
- Scanner; for directing a microwave beam
- Waveform extractor; for extracting received signal in waveform
- Central processing unit; for analysis of waveform signals

Research work has been carried out that proves microwave method a better NDT technique than radiography, ultrasonic, and eddy current in terms of cost, good penetration ability in nonmetallic materials, good resolution and contactless feature [13]

1.4.1.7 Magnetic particle inspection (MPI)

This is an NDT process used on ferromagnetic materials to detect defects that are at the surface or shallow subsurface. The underlying principle is that the magnetized specimen will attract the ferromagnetic fine particles towards the defect [6].

Figure 1. 10: Magnetic particle inspection
The steps involved in conducting an MPI test include;

- Magnetize the area to be inspected through direct or indirect magnetization; direct magnetization occurs when electric current is passed through the material itself, and indirect magnetization is when a magnetic field is applied from an outside source.
- Apply ferrous particles (iron fillings) to the part. These fillings are attracted to the areas where there is flux leakage, which indicates flaw in those areas.

Some of the desirable features of MPI include; it is quick and simple, it gives instantaneous indication (result), relatively inexpensive and can be conducted on materials irrespective of size. Some of the disadvantages of MPI include; it requires electricity, it cannot be used on insulator coated materials, only usable on ferromagnetic materials, etc.

1.5 **Fundamentals of wave propagation**

The science that describes the phenomenon of production of mechanical vibrations, effects and their propagation through solids, liquids and gases is termed acoustics [14]. Sound cannot be transmitted in a vacuum since it requires particles of matter to vibrate. In acoustics, materials are assumed to be made up of particles that are maintained at their state of equilibrium by forces [12]. Many different patterns of vibrational motion exist at the particulate level, most of which are not related to acoustics. The combination of the displacement of particles when a force is applied and the restoration of equilibrium by these particles cause oscillatory motions in a specimen.
Ultrasonic is a branch of acoustics that deals with vibrations of high frequencies. It is based on time-varying deformations in materials. Depending on pattern of particle oscillation, sound can propagate as bulk waves; longitudinal waves and shear waves, or guided waves; surface waves. Longitudinal waves involve a wave situation where the oscillation is in same direction as the wave propagation. Transverse wave (shear wave) occur when particles oscillate in a direction perpendicular to the direction of wave production. Transverse and longitudinal waves are the most common modes of propagation in ultrasonic testing.
1.5.1 Transducers

A transducer consists a piezoelectric element, which can convert electrical signal into mechanical vibration, and back to electrical signal. Piezoelectric transducers are the most common method used today for creating or detecting ultrasonic waves [15]. The transducer is the most basic component of an ultrasonic testing setup. It is a piece of material with one part positively charged and the other negatively charged, with electrodes attached to these opposite faces. When an electric signal is received, the charges align themselves with the electric field, resulting in induced dipole in the crystal structure of the material. Electrostriction occurs when a change in dimension occurs because of passage of electric signal through a transducer.
Ultrasonic measurement is most commonly used in determination of material thickness. This feature has applications in pipeline industries, construction, storage tanks. The underlying equation for this measurement is given as

$$Thickness = \frac{V_{\text{specimen}}}{2 \cdot f}$$ (1.22)

Where $V_{\text{specimen}}$ is the sound velocity in the specimen, $f$ is the frequency with wavelength equaling twice the thickness of the specimen. $d$ is the thickness of the specimen. At a set transducer frequency, the thickness of the specimen is measured. This is done by making a thin wafer element vibrate at a frequency twice the thickness of the specimen. Higher frequency transducers are often difficult to produce because the specimen is always thin and fragile [15][5] [17]
Other ways a transducer can be used is on the angle beam assembly. The setup is made up of a transducer and a wedge. Angle beam assembly has a wide range of applications which include weld inspection, crack detection in metals, pipes, tanks, etc.

![Beam angle assembly](image1.png)

Figure 1. 14: Beam angle assembly [16]

Snell’s law of refraction is the law that governs the behavior of a wave as it crosses from one material to another. Refraction is the bending of a wave as it passes through a boundary between two materials of different velocities.

![Snell’s law for angled beam analysis](image2.png)

Figure 1. 15: Snell’s law for angled beam analysis
This bend when a cross is made into a different medium is governed by the equation below:

\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{V_{L1}}{V_{L2}} \tag{1.23}
\]

where,

\(\theta_2\) = angle of refracted beam

\(\theta_1\) = incident angle from normal of beam in the wedge

\(V_{L1}\) = velocity of the incident beam in the wedge

\(V_{L2}\) = velocity of refracted beam

When the refracted angle is 90 degree, it is called the critical angle. At this point \(V_{L1} = V_{L2}\sin \theta_1\). Two conditions at which critical angle can be reached are; when all the longitudinal energy has been totally reflected, and when the shear angle of refraction is 90 degrees.
1.6 Research Objectives

The basic objective of this study is to generate a plot of the Young’s modulus versus Poisson’s ratio of T6-7075 Aluminum and carbon fiber-epoxy composite. This was achieved through simulating an ultrasonic wave travel through these materials on ANSYS. The following are summarized steps towards achieving this objective (i) development of understanding of guide waves; (ii) writing an ANSYS batch file for simulating the process (iii) performing basic Lamb wave experiments; and (iv) running finite element simulations.

- Guided waves were understood by presenting extensive literature review of the propagation characteristics of the different types of waves commonly used for non-destructive testing techniques.
- ANSYS mechanical batch file was prepared.
- First, finite element simulations in ‘ANSYS mechanical’ were run to determine the effect of element size on the wave travel, the effect of length of specimen, and the thickness of the material. Findings from these simulations were applied in further finite element simulations to determine the wave time of travel for T6-7075 aluminum and carbon fiber-epoxy composite materials.
- Ultrasonic experimental testing was carried out in the lab for validation purposes. The specimen (T-7075 Aluminum) was tested for different thicknesses by passing an ultrasonic wave through its thickness and recording the time of travel.
- Further simulations were run to achieve the goals of this work, which are to determine the effect of change in Poisson’s ratio and Young’s modulus on wave
travel time, and eventually to study the effect of Poisson’s ratio on the Young’s modulus of engineering materials.

1.7 Research Contribution

The process of economic and safe manufacturing of new products is complicated. There is need to understand the constituent material not only after production but also in processing. The increased competition amongst material producing industries to manufacture better products has brought stringent requirements for process and quality control. This demands the characterization of materials. The topic material characterization essentially includes the evaluation of elastic behavior, associated mechanical properties etc.

Literature reports from various vendors of the elastic properties (Young’s modulus and Poisson’s ratio) of solids (especially composites) are widespread due to the slight differences in the material properties. Another reason for this variation is the precision of the mechanical testing process itself, which depends on the quality and accuracy of the experimental setup. This study will be able to show at a glance the precision and error for any values of Young’s modulus and Poisson’s ratio of engineering materials. It also provides an easy and cost-effective way of characterizing the mechanical properties of new engineering materials.

1.8 Organization of report

This work investigates the effects of Poisson’s ratio on Young’s modulus of isotropic and composite materials via ultrasonic wave propagation using finite element
analysis. ‘ANSYS mechanical’ software is used as the simulation tool for this research. A rundown of the proceeding part of the report are as follows:

Chapter one contains a brief introduction to the research, discusses Poisson’s ratio, Young’s modulus, Non-destructive testing, composite materials, an overview of ultrasonic wave propagation. Explained the significance of this study in real life engineering applications, objectives of this work, and the organization of this thesis.

The second chapter shows the fundamentals FEA analysis on ANSYS, basics of piezoelectricity, equations wave travel on a plate and finite element analysis of wave propagation.

Chapter three shows the properties of 2D T6-7075 Aluminum model, the results and validation using experimental and theoretical results from same specimen. Also, the plot of the dependency of young’s modulus on poison’s ratio.

Chapter four contains the results and validation of the FEA analysis on ANSYS for composite material.

The last chapter is the conclusion and recommendation on this research area, and possible future studies.
2.1 Introduction

Finite element analysis (FEA) is an engineering solution process that involves the use of numerical methods to solve complex engineering problems. It uses modern computers to handle enormous computational problems. Some of the engineering areas FEA can be applied are structural analysis, fluid flow, mass transport, heat transfer, electromagnetic potential. The analytical solution to these problems usually involve setting a boundary condition and solving partial differential equations. For example, the Navier-Stokes equation (differential equation) is used to simulate fluid flow.

This work used FEA to investigate the propagation of ultrasonic wave through a 2D aluminum plate and 2D fiber-epoxy composite. The ANSYS version used was ANSYS 18.2, a commercially available finite element analysis software owned by ANSY Inc. Some of the basic steps involved in running a simulation in ANSYS are discussed below

2.1.1 Preprocessor

The preprocessor consists of mainly three stages, build a model, assign attributes, and mesh model.

Building a model is the first stage of all engineering simulations. Here the engineer creates the geometry of the real-life situation he is trying to simulate. This could be a solid component for a structural analysis or the air volume for a fluid or electromagnetic study [17]. There are a few ways a model could be built for use in ANSYS, these include, writing a batch file code that creates the geometry, creating the geometry model within the software
(GUI), or importing an already created model from computer aided design (CAD) software, for example, AutoCAD, Solid works, etc. In this work, the model was created using batch file code. Some of the steps that could come before building a model in ANSYS simulation include:

- Setting up the unit system that suits your work, for example, SI units, cgs, or user defined as the case may be
- Activate the appropriate coordinate system
- Determining the approach that will suit your simulation
- Establish work plane
- Setting up analysis type, for example structural, thermal, etc.
- Result configuration set up

Assigning attributes involves specifying some attributes of the model created, such attributes include

- Element type
- Property of interest, for example, plane strain for structural simulation
- Material properties, for example, density, Young modulus, etc.
- Real constants

Meshing is the process of dividing up a model into smaller sized elements so that a load is applied and distributed more evenly on the model. ANSYS Meshing is a general-purpose, intelligent, automated high-performance product. It produces the most appropriate mesh for accurate, efficient multiphysics solutions [17].
Generally, preprocessor is used to define the model geometry first, then mesh the geometry using appropriate element types, element real constants, and material properties [17].

2.1.2 Solution and load application

The next step after the preprocessor stage will be to apply loads and solve. Structural analysis in ANSYS is basically an investigation of the reaction of a structure to a given loading condition. It is relevant to ensure that the loads are applied correctly. Some of the activities on this stage of simulation are select analysis type; for example, transient, set boundary conditions, apply load on the node or area of interest; loads apply to different disciplines such as structural, thermal, magnetic, electric or fluid, set initial conditions, set up load step and time step size. A load step is a sized arrangement of loads for which solution is obtained. It could be linear, stepped, steady or transient analysis, finally, solve; here the computer automatically solves the simultaneous set of equations that the finite element method generates.

2.1.3 Postprocessor

The postprocessor here is discussed in two stages, the general postprocessor and the time-history postprocessor.

- General postprocessor; It uses POST1 to review analysis results over the entire model, or selected portions of the model for specifically defined combination of loads at a single time (or frequency) [17]. There are a variety of tasks POST1 can carry out which include; simple graphic displays, tabular listings, complex data manipulations etc. Graphic display is an effective way to display the results of a
simulation before going to look at the data. It can show the contour displays, deformed shape displays, path plots reaction force display, vector display, and particle flow traces.

- Time-history postprocessor; Like the general postprocessor, the time-history postprocessor is used for analysis of result. The difference is time-history postprocessor uses POST26 to review results not the entire model but at specific points as a function of time, frequency, etc. It has a range of capabilities which include; simple graphic display, tabular listings, differential calculus and response spectrum generation [17]. The most common application of POST26 is to graph result items versus time in transient analysis, or to graph force versus deflection in a nonlinear structural analysis [17]. In this work, POST26 is used to get graph result items versus time in transient analysis of the specimen.

2.2 Theoretical analysis of piezoelectricity on ANSYS

It is relevant to discuss the equations that govern electrodynamics for waves in ANSYS. When a voltage potential is noticed on a piezoelectric material (transducer), it creates a displacement or vibration, and vice versa. The equation of elasticity, charge equations from electrostatics and piezoelectric constants are combined to form the set of equations that analyze piezoelectricity in ANSYS as follows

\[
\{T\} = [e^E]\{S\} - [\varepsilon]\{E\} \quad (2.1)
\]

\[
\{D\} = [e]^T\{S\} + [\varepsilon^\text{T}]{E} \quad (2.2)
\]
Rearranging in matrix form,

\[
\begin{bmatrix}
\{T\} \\
\{D\}
\end{bmatrix} =
\begin{bmatrix}
[c^E] & [e] \\
[e]^T & -[\varepsilon^S]
\end{bmatrix}
\begin{bmatrix}
\{S\} \\
\{E\}
\end{bmatrix}
\] (2.3)

Where:

\[c^E\] = elasticity matrix (evaluated at constant electric field (referred to as \([D]\) elsewhere in this manual))

\[e\] = piezoelectric stress matrix

\[\varepsilon^S\] = dielectric matrix (evaluated at constant mechanical strain)

\([T]\) = stress vector (referred to as \(\sigma\) elsewhere in this manual)

\([D]\) = electric flux density vector

\([S]\) = strain vector (referred to as \(\varepsilon\) elsewhere in this manual)

\([E]\) = electric field intensity vector

Applying finite element discretization and variation principle, the finite element matrix equation becomes

\[
\begin{bmatrix}
[M] & [0] \\
[0] & [0]
\end{bmatrix}
\begin{bmatrix}
\{\ddot{u}\} \\
\{\ddot{v}\}
\end{bmatrix} +
\begin{bmatrix}
[C] & [0] \\
[0] & [\psi R]
\end{bmatrix}
\begin{bmatrix}
\{\ddot{u}\} \\
\{\ddot{v}\}
\end{bmatrix} +
\begin{bmatrix}
[K] & [K^z] \\
[K^z]^T & -[K^d]
\end{bmatrix}
\begin{bmatrix}
\{u\} \\
\{v\}
\end{bmatrix} =
\begin{bmatrix}
\{F\} \\
\{L\}
\end{bmatrix}
\] (2.4)

Where:

\([C]\) = element structural damping matrix

\([F]\) = vector of nodal and surface forces

\([K]\) = element stiffness matrix
\[ [M] = \text{element mass matrix} \]

\[ [K^d] = \text{element of dielectric permittivity coefficient matrix ([}K^{ys}\text{] in or [}K^{vh}\text{] in} \]

\[ \{L\} = \text{vector of nodal, surface, and body charges} \]

\[ [K^Z] = \int_{vol}(B)^T[e][B]d(vol) = \text{piezoelectric coupling matrix} \]

\[ [B] = \text{strain displacement matrix} \]

\[ [C^{yh}] = \text{element dielectric damping matrix} \]

To iterate this system a transient model is setup using ANSYS 18.2

2.3 Theoretical wave equations on a plate

It is relevant to emphasize the fundamental difference between bulk waves and guided waves. While bulk waves travel through the bulk of the material and interacted with the boundary in form of reflection and refraction, guided waves are waves with boundaries. However, bulk waves and guided waves are governed by same partial differential equations. Some examples of guided waves are Rayleigh waves, Lamb waves and Stanley waves [ANSYS database].

The governing partial differential equation which contains only particle displacement is as given in equation below
This equation can be solved using the method of potential if the displacement vector is decomposed according to Helmholtz decomposition and substituted into (2.5), the resulting equation will be;

\[
\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_3^2} = \frac{1}{c t^2} \frac{\partial^2 \phi}{\partial t^2}
\]

for longitudinal waves; (2.6)

and,

\[
\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2} = \frac{1}{c t^2} \frac{\partial^2 \psi}{\partial t^2}
\]

for shear waves (2.7)

Plane strain analysis is greatly simplified in this here. Achenbach shows that beginning with the general state of strain as a starting point gives the same set of solutions presented here.

Because of our assumption of plane strain or plain stress, the displacements can be written in terms of potentials as

\[
u_1 = u = \frac{\partial \phi}{\partial x_1} + \frac{\partial \psi}{\partial x_3}
\]  

\[
u_2 = v = 0
\]

\[
u_3 = w = \frac{\partial \phi}{\partial x_3} - \frac{\partial \psi}{\partial x_1};
\]
\[
\sigma_{31} = \mu \left( \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \right) = \mu \left( \frac{\partial^2 \psi}{\partial x_1 x_3} - \frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2} \right), 
\] 
(2.9a)

\[
\sigma_{32} = \lambda \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right) + 2\mu \frac{\partial u_3}{\partial x_3}
\]

\[
= \lambda \left( \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_3^2} \right) + 2\mu \left( \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1 x_3} \right) 
\] 
(2.9b)

where \( \lambda \) and \( \mu \) are Lamé constants

We begin the analysis by using solutions to equations 2.6 and 2.7 as

\[
\phi = \Phi(x_3) \exp[i(kx_1 - \omega t)] 
\] 
(2.10)

\[
\psi = \Psi(x_3) \exp[i(kx_1 - \omega t)] 
\] 
(2.11)

Substitutions of these assumed solutions into 2.6 and 2.7 yields equations governing the unknown functions \( \Phi \) and \( \Psi \). The solutions to these equations are

\[
\phi(x_3) = A_1 \sin(px_3) + A_2 \cos(px_3) 
\] 
(2.12)

\[
\psi(x_3) = B_1 \sin(qx_3) + A_2 \cos(qx_3) 
\] 
(2.13)

where,
The wave number $k$ is numerically equal to $\frac{\omega}{c_p}$, and $C_p$ is the phase velocity of the Lamb wave mode and $\omega$ is the angular frequency. The phase velocity is related to the wavelength ($\lambda$) by the simple relation $C_p = (\omega/2\pi)/\lambda$.

With these results displacements can be obtained directly from (2.8) and (2.9)

$$u_2 = [ik\Phi + \frac{\partial \Psi}{\partial x_3}]$$  (2.15)

$$u_3 = [\frac{\partial \Phi}{\partial x_3} - ik\Psi]$$  (2.16)

$$\sigma_{33} = [\lambda(-k^2 \phi + \frac{\partial^2 \phi}{\partial x_3^2} + 2\mu \left(\frac{\partial^2 \phi}{\partial x_3^2} - ik \frac{\partial \Psi}{\partial x_3}\right)]]$$  (2.17)

$$\sigma_{31} = \mu \left(2ik \frac{\partial \phi}{\partial x_3} + k^2 \Psi + \frac{\partial^2 \phi}{\partial x_3^2}\right).$$  (2.18)

The solutions can be split into symmetric and anti-symmetric modes as follows:

**Symmetric Modes:**

$$\Phi = A_2 \cos(px_3)$$
\[
\Psi = B_1 \sin(q x_3)
\]

\[
u = u_1 = i k A_2 \cos(p x_3) + q B_1 \cos(q x_3)
\]

\[
w = u_3 = -p A_2 \sin(p x_3) + i k B_1 \sin(q x_3)
\]

(2.19)

\[
\sigma_{31} = \mu[-2i k p A_2 \sin(p x_3) + (k^2 - q^2) B_1 \sin(q x_3)]
\]

\[
\sigma_{33} = -\lambda(k^2 + p^2) A_2 \cos(p x_3) - 2\mu[p^2 A_2 \cos(p x_3)
\]

\[
+ i q B_1 \cos(q x_3)]
\]

*Anti-Symmetric Modes:*

\[
\Phi = A_1 \sin(p x_3)
\]

\[
\Psi = B_2 \cos(q x_3)
\]

\[
u = u_1 = i k A_1 \sin(p x_3) - q B_2 \sin(q x_3)
\]
\[ w = u_3 = -pA_1 \cos(px_3) - kB_2 \cos(qx_3) \quad (2.20) \]

\[ \sigma_{31} = \mu[2ikpA_1 \cos(px_3) + (k^2 - q^2)B_2 \cos(qx_3)] \]

\[ \sigma_{33} = -\lambda(k^2 + p^2)A_1 \sin(px_3) - 2\mu[p^2A_1 \sin(p_3) \]
\[ + ikqB_2 \sin(qx_3)] \]

The constants \( A_1, A_2, B_1, B_2 \) as well as the dispersion equations, are still unknown. They can be determined by applying the traction-free boundary condition, which reduces to

\[ \sigma_{31} = \sigma_{33} \equiv 0 \text{ at } x3 = \pm \frac{d}{2} = \pm h \text{ (for convenience)} \quad (2.21) \]

Applying the boundary conditions will give a homogenous system of two equations for the approximate two constants \( A_2, B_2 \) (anti-symmetric case). For homogenous equations we require that the determinant of the coefficient matrix vanish to ensure solutions other than the trivial one. From (2.21) we thus have

\[ \frac{(k^2 - q^2) \sin(qh)}{2ikp \sin(ph)} = \frac{-2uikq(\cos(qh))}{(\lambda k^2 + \lambda p^2 + 2\mu p^2) \cos(ph)} \quad (2.22) \]

After some manipulations, this may be written as
\[
\frac{\tan(qh)}{\tan(ph)} = \frac{4k^2 q\mu}{(\lambda k^2 + \lambda p^2 + 2\mu p^2)(k^2 - q^2)} \tag{2.23}
\]

We can further simplify using the wave velocities and definitions of \( p \) and \( q \). we obtain:

\[
\lambda = c_L^2 \rho - 2\mu \tag{2.24}
\]

Then

\[
\lambda k^2 + \lambda p^2 + 2\mu p^2 = \lambda(k^2 + p^2) + 2\mu p^2 \tag{2.25}
\]

\[
= (c_L^2 \rho - 2\mu)(k^2 + p^2) + 2\mu p^2
\]

\[
\lambda k^2 + \lambda p^2 + 2\mu p^2 = \rho c_L^2(k^2 + p^2) - 2\mu k^2 \tag{2.26}
\]

Using equation 2.14 and \( C_T^2 = \frac{\mu}{\rho} \) yields

\[
\lambda k^2 + \lambda p^2 + 2\mu p^2 = \rho \omega^2 - 2\rho c_T^2 k^2 \tag{3.27}
\]

Which therefore implies:
\[ \rho c \tau^2 \left[ \left( \frac{\omega}{c \tau} \right) - 2k^2 \right] = \rho c \tau^2 (q^2 - k^2) = \mu (q^2 - k^2) \quad (2.28) \]

Now substituting (2.28) into an initial form of the dispersion equation (2.23) gives

\[ \frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2 qp}{(q^2 - k^2)^2} \quad (2.29) \]

\[ \frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2 qp (q^2 - k^2)^2}{4k^2 qp} \quad (2.30) \]
CHAPTER 3: TRANSIENT FINITE ELEMENT ANALYSIS FOR 2-D ALUMINUM

3.1 Introduction

Here we discuss how the modeling and simulation of the 2D T6-7075 Aluminum (specimen) was done in details. Transient analysis was used to determine the time of travel through the thickness of the specimen. The basics steps of building a model, applying load and solution, and time-history postprocessor analysis were followed.

It is relevant to reiterate the objectives in this study here, which are to conduct an element size test to determine the best mesh size which does not affect the travel time of ultrasonic wave, conduct specimen length test to notice the length that prevents boundary reflection from interfering with result, validate the result using experimental and formula methods, study the effect of increase in Young’s modulus on travel time, study the effect of increase in Poisson’s ratio on time of travel, lastly, the effect of change in Poisson’s ratio on Young’s modulus characterization.

A detailed description of how the ultrasonic simulation of the 2D aluminum on ANSYS is as follows

3.1.1 Model

As we mentioned earlier, building the model is an important part of simulation. For this problem, it is necessary to build the model in such a way that the load is applied on a node. As it was stated earlier in this thesis, this problem was modeled using a batch file method. The properties of the material; T6-7075 Aluminum used are as follows
Table 3.1: T6-7075 aluminum material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>T6-7075 Aluminum</td>
<td>Geometry</td>
</tr>
<tr>
<td></td>
<td>• Width (wₘ) = 38.1 mm</td>
</tr>
<tr>
<td></td>
<td>• Thickness (tₘ) = 12.7 mm</td>
</tr>
<tr>
<td></td>
<td>Physical properties</td>
</tr>
<tr>
<td></td>
<td>• Density = 2810 Kg/m³</td>
</tr>
<tr>
<td></td>
<td>Mechanical properties</td>
</tr>
<tr>
<td></td>
<td>• Young’s modulus = 7.17e10Pa</td>
</tr>
<tr>
<td></td>
<td>• Poisson’s ratio = 0.33</td>
</tr>
</tbody>
</table>

Some of the properties of the simulation were defined before the geometry, which includes the units, SI units. The maximum number of result (NRES) to be displayed was set at 5000. The filename was defined for each specific simulation.

The element type information was added to the batch file to create the 2D aluminum specimen PLANE182, with key options KEYOPT (1) = 1, this ensures full integration with B-bar method. KEYOPT (3) = 2 makes sure that strain in the z-direction equals zero for the two-dimensional simulation.
PLANE182 is used for 2-D modeling of solid structures. The element can be used either as a plane element (plane stress or plane strain) or as an axisymmetric element. The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, stress stiffening, large deflection, and large strain capabilities [17].

The element size for meshing the model was obtained from an element size test. Various element sizes were tested to obtain their effect of the time of travel of ultrasonic wave through the specimen. Details will be shown later in the thesis. An element size of 0.4233 mm was found most suitable for this project.
Figure 3. 2: 2D Aluminum model of T6-7075 Aluminum

4.1.2 Applying loads and solution

Force is applied at the symmetric bottom of the model and the signal is received at the symmetric top to obtain the time of travel. Number of cycles chosen was 5 and at a frequency of 500kHz. For the actuation of the signal, the following formula was used

\[
U(t) = 0.5 \left(1 - \cos \left(\frac{2\pi f_0 t}{n_0}\right)\right) \cdot \cos\left(2\pi f_0 t\right) \quad (3.1)
\]

where \(n_0\) number of cycles

\(f_0\) is frequency

\(t\) is time which range between zero and 0.001 was used at a time step of 5E-06 seconds
The analysis type is transient. The 2D model was also restricted all degrees of freedom on the edges, this is to ensure that the specimen is in static position throughout the period of load application.

3.1.3 Postprocessor analysis

Time-history postprocessor was the major tool used here, the displacement signal over time was studied for each scenario.

![Figure 3.3: Time-history plot at the loading node](image)
3.2 Result, validation and discussion

3.2.1 Elements size test result

Figure 3.4: Input and output signal thru thickness at 2.116 mm element size
Figure 3.5: Input and output signal thru thickness at 1.27 mm element size
Figure 3. 6: Input and output signal thru thickness at 0.635 mm element size
Figure 3. 7: Input and output signal thru thickness at 0.423 mm element size
Figure 3. 8: Input and output signal thru thickness at 0.3175 mm element size
Figure 3.9: Input and output signal thru thickness at 0.254 mm element size
Figure 3.10: Dependence of time of travel on element size

The method of time of travel analysis used here involves getting signal at two points on the specimen, read off the time of the maximum amplitude on the signal, and find the time difference between that value for two different locations of interest. The signal for this work are read at the point of input of signal and at the direct opposite thickness.

Notice the time of travel increased with decrease in element size from the graph. However, the time of travel became constant from 0.423 mm and below. Hence, 0.423 mm was chosen for this research to be the suitable element size.
### 3.2.2 Length test result

The length test was necessary to determine the required length that enables us to read accurate information. Very short materials will read reflected signal at the thickness first before reading the signal that shows the accurate time of travel. The result of testing out different materials lengths of 8.467mm, 12.7mm, 25.4mm, 38.1mm, 50.8mm and 63.5mm are shown below.

![Graph showing input and output signal through thickness at 8.467 mm specimen length](image)

Figure 3.11: Input and output signal thru thickness at 8.467 mm specimen length
Figure 3.12: Input and output signal thru thickness at 12.7 mm specimen length
Figure 3.13: Input and output signal thru thickness at 25.4 mm specimen length
Figure 3. 14: Input and output signal thru thickness at 38.1 mm specimen length
Figure 3.15: Input and output signal thru thickness at 50.8 mm specimen length
Figure 3.16: Input and output signal thru thickness at 63.5 mm specimen length
3.2.3 Validation of result

Validation of this simulation was done primarily using experimental ultrasonic testing of T6-7075 Aluminum using portable ultrasonic tester in the university Metlab. Two thicknesses where considered for validation purposes, 0.4 inch and 0.5 inch.

![Ultrasonic tester](image)

Figure 3. 17: Ultrasonic tester

A second validation method was the formula method. Since the point of receiving the signal is in same direction as the direction of the force, a longitudinal wave velocity situation is applied. Here the mechanical properties (Young’s modulus and Poisson’s ratio) of the specimen were used to calculate the lame constants $\mu$ and $\gamma$. The constants were used to determine the longitudinal ultrasonic wave speed.

The simulation results for thicknesses 0.4 inch and 0.5 inch are as follows:
Figure 3. 18: Input and output signal thru 0.4inch thickness validation
Figure 3. 19: Input and output signal thru 0.5inch thickness validation
The signal above obtained from simulation were used to obtain the tabulated data below, the experimental data from department material laboratory was compared to it, it gave an accuracy of less than 5% for both thicknesses.

Table 3.2: Comparing experimental result to simulation result

<table>
<thead>
<tr>
<th>Thickness(in)</th>
<th>Thickness(m)</th>
<th>Simulation Input time (s)</th>
<th>Simulation output time (s)</th>
<th>Simulation Time of travel(s)</th>
<th>Experimental time of travel(s)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.01016</td>
<td>5.15E-06</td>
<td>7.15E-06</td>
<td>2.00E-06</td>
<td>1.90E-06</td>
<td>4.9%</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0127</td>
<td>5.15E-06</td>
<td>7.55E-06</td>
<td>2.40E-06</td>
<td>2.30E-06</td>
<td>4.1%</td>
</tr>
</tbody>
</table>

The respective result using formula method for 0.4 inch and 0.5 inch are 1.78 $\mu$s and 2.20 $\mu$s.
3.2.4 Effect of change in Young’s modulus on wave travel time results

The signals obtained from repeating simulation at Young’s modulus values of 51.7Gpa, 61.7Gpa, 71.7Gpa, 81.7Gpa and 91.7Gpa are as follows:

Figure 3. 20: Input and output signal thru 0.5inch at 51.7Gpa Young’s modulus
Figure 3. 21: Input and output signal thru 0.5inch at 61.7Gpa Young’s modulus
Figure 3.22: Input and output signal thru 0.5 inch at 71.7 Gpa Young’s modulus
Figure 3. 23: Input and output signal thru 0.5 inch at 81.7 Gpa Young's modulus
Figure 3. 24: Input and output signal thru 0.5inch at 91.7Gpa Young’s modulus
From the plot of various Young’s modulus values and time of wave arrival, it was discovered that an increase in Young’s modulus causes the wave speed to increase. Hence, reducing the wave travel time.
3.2.5 Effect of change in Poisson’s ratio on wave travel time

The signals obtained from repeating simulation at Poisson’s ratio values of 0.1, 0.2, 0.3, 0.4 and 0.5 are as follows

Figure 3.26: Input and output signal thru 0.5 inch at 0.1 Poisson’s ratio
Figure 3.27: Input and output signal thru 0.5 inch at 0.2 Poisson’s ratio
Figure 3. 28: Input and output signal thru 0.5 inch at 0.3 Poisson’s ratio
Figure 3. 29: Input and output signal thru 0.5 inch at 0.4 Poisson’s ratio
Figure 3.30: Input and output signal thru 0.5 inch at 0.5 Poisson’s ratio
Table 3. 4: Data on the effect of varying Poisson’s ratio on travel time

<table>
<thead>
<tr>
<th>Poisson's ratio</th>
<th>Initial time (s)</th>
<th>arrival time (s)</th>
<th>TOA (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.00000515</td>
<td>0.0000079</td>
<td>0.00000275</td>
</tr>
<tr>
<td>0.2</td>
<td>0.00000515</td>
<td>0.0000078</td>
<td>0.00000265</td>
</tr>
<tr>
<td>0.3</td>
<td>0.00000515</td>
<td>0.00000765</td>
<td>0.0000025</td>
</tr>
<tr>
<td>0.4</td>
<td>0.00000515</td>
<td>0.0000075</td>
<td>0.00000235</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00000515</td>
<td>0.00000745</td>
<td>0.0000023</td>
</tr>
</tbody>
</table>

Figure 3. 31: Effect of varying Poisson’s ratio on travel time
From the plot of various Poisson’s ratio values and time of wave arrival, it was discovered that an increase in Poisson’s ratio causes the wave speed to increase. Hence, reducing the wave travel time.

### 3.2.6 Dependence of Young’s modulus on Poisson’s ratio

Using formula method as a guide to estimate the initial material properties (Young’s modulus and Poisson’s ratio) for simulations and carrying out a multiple trial to obtain a constant travel time, the effect of Poisson’s ratio on Young’s modulus data was obtained from ANSYS. For a Poisson’s ratio of 0.1, and correct value of Young’s modulus, the result signals is as follows
Figure 3. 32: Trial input and output signal thru 0.5 inch at 0.1 Poisson’s ratio
Poisson’s ratio

Figure 3. 33: A plot of the effect of Poisson’s ratio on Young’s modulus

From the results shown previously and the one shown in this section. It makes sense to conclude that an increase in Poisson’s ratio will result in a decrease in Young’s modulus of a material.
3.3 3D T6-7075 Aluminum model

This model was created as a check on the isotropic 2D aluminum specimen. The result of time of travel is expected to be the same since the same thickness of sample was used.

Table 3. 5: 3D Aluminum material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>T6-7075 Aluminum</td>
<td>Geometry</td>
</tr>
<tr>
<td></td>
<td>Width ((w) = 38.1\text{mm})</td>
</tr>
<tr>
<td></td>
<td>Thickness ((t) = 12.7\text{ mm})</td>
</tr>
<tr>
<td></td>
<td>Length ((l) = 38.1\text{ mm})</td>
</tr>
<tr>
<td></td>
<td>Physical properties</td>
</tr>
<tr>
<td></td>
<td>Density = 2810 Kg/m³</td>
</tr>
<tr>
<td></td>
<td>Mechanical properties</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus = 7.17e10Pa</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio = 0.33</td>
</tr>
</tbody>
</table>

The element type used to build this model was SOLID185
Figure 3. 34: 3D isotropic T6-7075 Aluminum
CHAPTER 4: TRANSIENT FINITE ELEMENT ANALYSIS FOR FIBER-EPOXY COMPOSITE

4.1 Introduction

In the past decade, the need of composite materials has increased exponentially, with specific applications in engineering areas like pipeline, aerospace, automotive, and electrical industries. Composites are widely accepted because of their wide variety of features. The mechanical and physical properties of any material can be improved significantly by mixing it with other material to create a composite. The physical properties of composites are generally anisotropic; it depends on the direction in which they are measured. The fundamental design concept of composites is that the bulk phase accepts the load over a large surface area, and transfers it to the reinforcement material, which can carry a greater load [18].

In this work, a fiber-epoxy composite made of a continuous fiber aligned in parallel lines in an epoxy matrix was modeled. Focus was on 2D model since it was difficult to get a 3D model with extremely small size of a fiber in an epoxy to create a reasonable matrix-reinforcement ratio without exceeding element restriction in ANSYS.
4.2 Model

The modeling concept here is to create a 2D rectangular epoxy material and fill in circular fiber material in it up to a 50 percent reinforcement-matrix composition. A batch file was used to build this model. The fiber and the matrix elements were glued together before meshing into one (composite) material. The fiber arrangement in the matrix is shown in figure 4.2, four fiber strands are arranged in an equidistant square-like shape with one sitting in the middle.

Figure 4. 1: 2D composite model with reactions
Figure 4. 2: Fiber arrangement in the matrix

Table 4. 1: Material properties of composite

<table>
<thead>
<tr>
<th>Material</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon fiber</td>
<td>Geometry</td>
</tr>
<tr>
<td></td>
<td>Radius = 3.25μm</td>
</tr>
<tr>
<td></td>
<td>Young’s moduli</td>
</tr>
<tr>
<td></td>
<td>$E_X = 22.4e3 , N/mm^2$</td>
</tr>
<tr>
<td></td>
<td>$E_Y = 22.4e3 , N/mm^2$</td>
</tr>
<tr>
<td></td>
<td>$E_Z = 1.25e3 , N/mm^2$</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Epoxy</td>
<td>Geometry</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Width = 0.562176 mm</td>
</tr>
<tr>
<td></td>
<td>Thickness = 0.183552 mm</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus</td>
</tr>
<tr>
<td></td>
<td>$E = 2.06e3 , N/mm^2$</td>
</tr>
<tr>
<td></td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td></td>
<td>$PRXY = 0.35$</td>
</tr>
<tr>
<td></td>
<td>Density</td>
</tr>
<tr>
<td></td>
<td>$\rho = 1.25 - 6 , Kg/mm^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>PLANE182</th>
</tr>
</thead>
</table>

Shear moduli

$G_{XY} = 8.3e3 \, N/mm^2$

$G_{YZ} = 22.1e3 \, N/mm^2$

$G_{XZ} = 22.1e3 \, N/mm^2$

Density

$\rho = 1.780e - 6 \, Kg/mm^3$
SOLID185 Structural Solid is suitable for modeling general 3-D solid structures. It allows for prism, tetrahedral, and pyramid degenerations when used in irregular regions. Various element technologies such as B-bar, uniformly reduced integration, and enhanced strains are supported [17]

Figure 4.3: SOLID185 Homogeneous Structural Solid Geometry [17]

The units for this simulation was set as user defined. A total of 1498 strands of fiber were created. The equation for getting the composition (50%) of the composite was
\[ Composition = \frac{2\pi l + r^2}{\sqrt{3} + L^2} \]  

(4.1)

Figure 4. 4: 2D composite mesh

Meshing was done affected gluing both materials together with an element size of 3.23\(\mu\)m.

4.3 Applying loads and solution

In this work, the analysis type used is transient analysis. Boundary condition was a restriction of all degrees of freedom on the edges of the model, this ensure that the composite is kept static when load is applied. Force is applied at the bottom on the node located at the point of symmetry of the material. Signal is picked up at the top symmetric
node from the model to determine the time of travel of ultrasonic wave through the material. The following formula was used for the actuation of the signal

\[ U(t) = 0.5 \left\{ 1 - \cos \left( \frac{2\pi f_0 t}{n_0} \right) \right\} \cos[2 \pi f_0 t] \]  \hspace{1cm} (4.2)

where \( n_0 \) number of cycles

\( f_0 \) is frequency

\( t \) is time which range between zero and 0.001 was used at a time step of 5E-06 seconds

4.4 Postprocessor analysis

The only analysis required here is the time-history analysis which studied the displacement versus time plot of the signal at the node of interest. For this composite model, our node of interest is the symmetric node at the top of the composite material. This is to ensure that the wave speed is longitudinal since that point is in same direction as the applied load.
Figure 4. 5: Time-history signal at the load point

4.5 Result and discussion

The result of the time of travel of ultrasonic signal through the composite model was gotten by picking up time-history signal at the load application node and the node through thickness, subtracting the time of the maximum signal at the input node from the time of the maximum signal at the output node.
Figure 4. 6: Input and output signal thru thickness of the fiber-epoxy composite

From the signal above, the travel time through the thickness of the composite is 3.25µs. Also, the travel time was calculated using formula method using the equation stated
in the literature. Basically, a transverse composite Young’s modulus and Poisson’s ratio were calculated, the values were then used in determining the lame constants $\mu$ and $\lambda$, the lame constants are in turn used to determine the longitudinal speed. Using this formula method to find the time of travel gave a value of 0.183 $\mu$s.

It was discovered that the formula method gave a time that is almost 18 times less than the simulation result. Hence, it proves that longitudinal wave speed calculation for our transverse composite model is not a suitable way to estimate the travel speed for the study of the effect of change in Poisson’s ratio on Young’s modulus.
CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In this paper, finite element analysis is used to characterize materials using ultrasonic speed through the materials. Validation of the finite element results was done using experimental and theoretical means. Interesting discoveries were made on both isotropic and anisotropic composite materials.

The model was validated by comparing the time of travel results to ones gotten from ultrasonic tester from the department material laboratory. It shows an acceptable level of discrepancy less than 5% for both 0.4inch and 0.5inch thicknesses. The theoretical results also helped in estimating the values of material properties used for the trial and error simulations to determine the effect of Poisson’s ratio on Young’s modulus.

We were able to establish from this work that for a 2D T6-7075 Aluminum, time of travel of an ultrasonic wave decreases with increasing Young’s modulus. Also, the time of travel of the wave decreases with increasing Poisson’s ratio. And lastly, the Young’s modulus of Aluminum increased with a decrease in Poisson’s ratio. These trends are seem clearly in the plots shown in the results section of this paper.

For composite material, it was discovered that use of theoretical formula was not an effective way of getting the time of wave travel through a transverse arrangement of continuous fiber in epoxy matrix. The result showed that the time of travel from FEA is 17.8 times the value gotten using formula method. This made it difficult to get a good estimate of the mechanical properties to go ahead and characterize the composite model.
I recommend that a direction-based study be carried out for composite materials to obtain an equation that will help in studying the effect of Poisson’s ratio on Young’s modulus in future works.

5.1 Future work

There are a few limitations and assumptions in this work that can be improved in future work. These are pointed out the following paragraphs.

Software limitations did not allow for the simulation of a 3D continuous fiber composite model. If newer versions of ANSYS permits more nodes, it will be interesting to see how the 3D composite model compares to the 2D model.

Discontinuous fiber has more applications in the current industry than continuous fiber. I will like to see a future work that is done on a discontinuous fiber composite in 3D.

The effect of attenuation was not considered in this work; however, this is not the case in real life. It will be interesting to see a future work that involves attenuation, as it will produce a result closer to what is obtainable. The batch file used here can be used to characterize and material (polymer) by merely changing the properties and geometry.
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http://www.ndt-ed.org/GeneralResources/IntroToNDT/GenIntroNDT.htm


http://www.ndt.net/article/wendt00/papers/idn251/idn251.htm


17. ANSYS database center

http://www.ansys.com/Products/

APPENDIX A: OTHER ELEMENT AND SIGNAL RESULT FROM SIMULATION

Figure A. 1: Simulation results for element size test 0.254mm
Figure A. 2: Simulation results for element size test 0.635mm
Figure A. 3: Simulation results for element size test 0.3175mm
Figure A. 4: Simulation results for element size test 0.4233mm
Figure A. 5: Simulation results for element size test 1.27mm
Figure A. 6: Simulation results for element size test 2.116mm
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Figure A. 16: Simulation results for varying Poisson’s ratio 0.1
Figure A. 17: Simulation results for varying Poisson’s ratio 0.2
Figure A. 18: Simulation results for varying Poisson’s ratio 0.3
Figure A. 19: Simulation results for varying Poisson’s ratio 0.4
Figure A. 20: Simulation results for varying Poisson’s ratio 0.5
Figure A. 21: Simulation results for thickness test of 0.4 inches