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Study of a Variable Feedback Oscillator and its Use in Automatic Temperature Control

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STUDY OF A VARIABLE FEEDBACK OSCILLATOR AND ITS USE IN AUTOMATIC TEMPERATURE CONTROL

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STUDY OF A VARIABLE FEEDBACK OSCILLATOR AND ITS USE IN AUTOMATIC TEMPERATURE CONTROL

This thesis is approved as a creditable, independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser

Head of the Major Department

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CHAPTER I

INTRODUCTION

Many methods and devices have been utilized in the automatic control of temperature. However, the accuracy requirement of a temperature controlling device depends upon its use. The purpose of this work was to design a device to obtain accuracy of control to the extent of better than one tenth of a degree centigrade of deviation of the specified temperature. This can be accomplished by using a vacuum tube device. The theoretical analysis of the nonlinear characteristic of the electronic circuit will constitute the main part of the work.

The complete control system consisted of a feedback type oscillator, a power amplifier, a switching circuit and heating elements. The principle idea here was to put a thermistor into the feedback network so that the oscillator would have output whenever the ambient temperature of the thermistor is lower than the specified value. This output is being amplified and fed into a relay circuit to switch on the heating elements so that the temperature can be raised. Because of the high sensitivity of the bridge circuit and the critical response of oscillation, a high accuracy of control can be successfully achieved.

CHAPTER II

LITERATURE REVIEW

An oscillator is a circuit that converts d.c. power into a.c. or signal power. Whenever an analysis is made of an electronic circuit capable of producing a sustained oscillation, it is found that the circuit exhibits a negative resistance as a part of its static or dynamic characteristics.

This chapter is devoted to a review of the nonlinear behavior of oscillations, the feedback theory and the characteristic of thermistors.

1. Nonlinear Oscillations

A. A nonlinear system[1]

It is convenient to proceed by generalizing the system to include a single nonlinear negative resistance. Such a circuit is shown in Figure I. The negative resistance may be identified with a dynatron

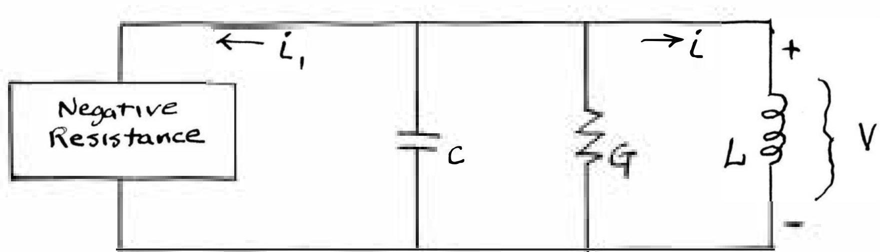

Figure I. Nonlinear oscillatory circuit

[1]This section and sections B and C are cited from: Edson, William A., "Vacuum-Tube Oscillator," Wiley, 1953, pp. 42-44.

or a pentode connected as a transitron. All known negative resistance devices have the property of nonlinearity if the amplitude of oscillation is sufficiently large. Otherwise, an infinitely large amount of power could be drawn by a suitable load, a violation of the principle of conservation of energy.

The characteristic of a typical negative resistance device is shown in Figure II. The curve, which may be obtained experimentally, is relatively complicated and is not representable by any simple equation. For the present purpose it is sufficient to represent this characteristic symbolically as

$$i_1 = F(v) \qquad (1)$$

where v represents the difference between the instantaneous potential and a bias voltage v_o. The other elements of Figure I are readily

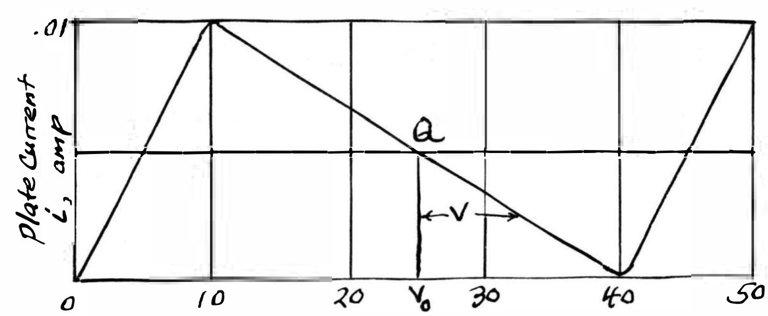

Figure II. Characteristic of a tetrode as a dynatron

identified with the passive tank circuit. All capacitances, including those of the tube, coil, and wiring, are lumped in C. All losses, including those of coil, condenser, and any useful load, are accounted for by the shunt conductance G. The inductance of the system is represented by L.

B. The differential equation

The differential equation which results from application of Kirchhoff's current law

$$F(v) + c(dv/dt) + Gv + i = 0 \qquad (2)$$

where i, the current through L, is related to the voltage across the system by the auxiliary equation

$$v = L(di/dt). \qquad (3)$$

C. Solution by isoclines

The differential equations above involve both current and voltage which vary with respect to time. In solving these equations, the undefined function F greatly complicates the procedure of eliminating either v or i between these equations. Accordingly, it is expedient to eliminate the time variable and study the relationship between v and i. One practical method is called the method of isoclines. From the plot, we may obtain full information about the build-up period and the steady state operation of the oscillation.

The elimination of t is accomplished by use of the derivative identity

$$dv/dt = (dv/dx)\cdot(dx/dt) \qquad (4)$$

where x is any variable. In the present case it is convenient to use

$$x = i. \qquad (5)$$

In addition, it is desirable to use a constant multiplier to change the voltage variable such that

$$v = kU. \qquad (6)$$

With these substitutions, equation 2 becomes

$$F\,(kU) + (k^2c/L)\cdot(dU/di)U + GkU + i = 0. \qquad (7)$$

By choosing $$k = \sqrt{L/c} \qquad (8)$$

and

$$f(U) = F(kU) + GkU = F(v) + Gv \qquad (9)$$

and transposing, it then becomes

$$dU/di = -\left[i + f(U)\right]/U \qquad (10)$$

or $$di/dU = -U/\left[i + f(U)\right]. \qquad (11)$$

In equation 11, the slope di/dU is determined as soon as the variables i and U are specified. Moreover, the form is such that this slope may be determined very rapidly on a graphical basis.

The basic idea is simple and may be stated as follows: If the current and voltage at any instant assumes values i and U, then from equation 11, we can readily calculate the slope di/dU and, hence, the incrementally different values which i and U will have some short time later. By sufficient repetition of this process and use of finite increments, it is possible to determine completely the variation of i and U from any assumed initial conditions.

In practice it is much more convenient to construct slope lines, called isoclines, from a large number of arbitrarily chosen starting points. Because these lines form a characteristic pattern, it is relatively easy to trace out the curve which will develop from any chosen starting point.

D. Isocline diagram[1]

An isocline diagram having coordinates U and i is shown in Figure III. This is accomplished as follows:

(1) Carefully plot the current-voltage characteristic of the negative resistance device.

(2) Locate the approximate center of the negative resistance region. This locates the Q point. Measure all voltage increments from this point.

(3) Assume a series of values for Δv. For each value assumed:

(a) Compute $\Delta U = \Delta e/k$.

(b) Determine $\Delta f(v) = \Delta i_1$ from the curve drawn in (1).

(c) Calculate $G \Delta v$.

(d) Compute $\Delta F(U) = \Delta f(v) + G \Delta v$.

(e) Plot $-F(U)$ against U shown in Figure III.

Use the same scale calibration for both the ordinate and the abscissa. The numerical values used in Figure III correspond to those of Figure II with the additional parameters $G = 2 \times 10^{-4}$ mho, $L = 2.5 \times 10^{-3}$ henry, $C = 4 \times 10^{-10}$ farad, and $K = \sqrt{L/C} = 2500$. Some additional graphical construction is now required, as follows:

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[1]This section is cited from: Martin, Thomas L. Jr., "Electronic Circuits," Prentice-Hall 1955, pp. 364-367.

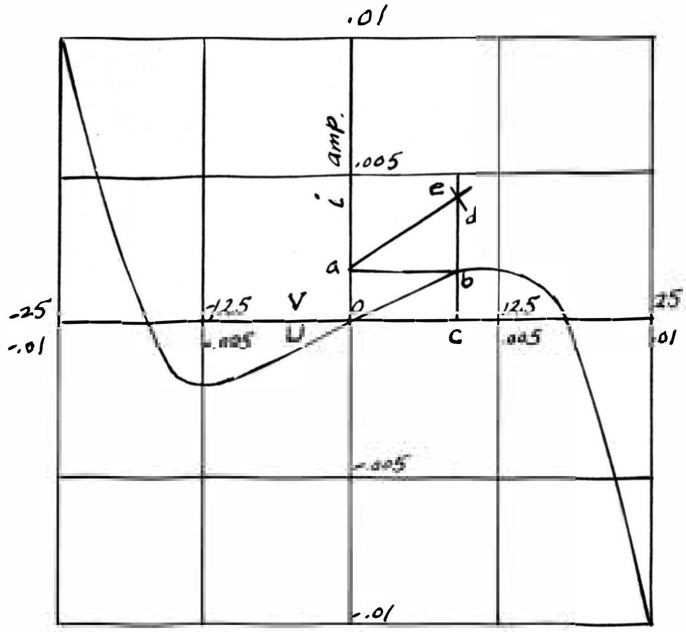

Figure III. Construction of isoclines[1]

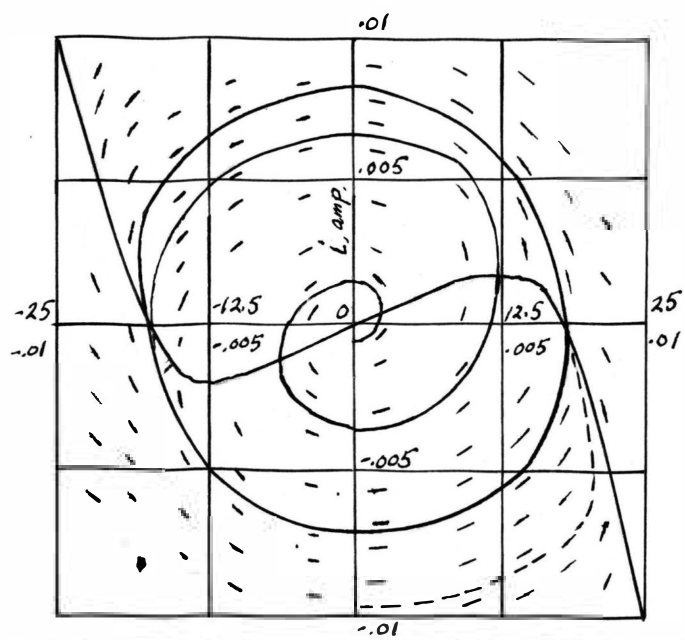

Figure IV. Isocline diagram for harmonic oscillation[2]

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[1]Edson, William A., "Vacuum-Tube Oscillator," Wiley, 1953, p. 45, Fig. 4.3.

[2]Edson, William A., "Vacuum-Tube Oscillator," Wiley, 1953, p. 47, Fig. 4.5.

(1) Draw a vertical line, as at C in Figure III. This intersects the $-F(U)$ characteristic at a point marked as b.

(2) From b project horizontally to the vertical axis and locate the point a.

(3) Take any point e on the original vertical axis and draw in the line $\overline{ae}$.

(4) Construct the perpendicular to $\overline{ae}$ at point e as shown by the line $\overline{de}$. Thus, $\overline{de}$ is tangent to an arc drawn from a as center and through e. Then $\overline{de}$ is an isocline.

From the preceding construction, which is shown in Figure III, it is clear that

$$\overline{ab} = U \qquad \overline{cb} = -F(U)$$

$$\overline{ce} = i \qquad \overline{be} = \overline{ce} - \overline{cb} = i + F(U). \qquad (12)$$

The slope of the isocline $\overline{de}$ is

$$di/dU \text{ (slope of isocline)} = -1/\tan\theta$$

but $$\tan\theta = \overline{be}/\overline{ab}, \qquad (13)$$

so $$di/dU = -\overline{ab}/\overline{be}. \qquad (14)$$

An isocline diagram is constructed by simply drawing a number of vertical lines for arbitrary values of U. Then locate points b and a for each such line. From each point use a compass to draw a series of short arcs intersecting the corresponding vertical line. These arcs are all isoclines, and the overall result is an isocline diagram as shown in Figure IV.

E. The cyclogram[1]

It remains to determine the direction of rotation which corresponds to an increase of time variable. This is found by reference to equation 3, which shows that an increase, that is, positive increment, in time requires an increase, that is, positive increment, in i whenever v and, hence, U are positive. This requires upward motion in the right half plane. Hence, counterclockwise rotation in Figure IV corresponds to increasing time.

The entire performance of the system, including the build-up from arbitrary starting conditions and the steady state, is described by isoclines such as those of Figure IV, which shows the behavior that follows from two different starting conditions. These curves are called cyclograms. Note in particular that the steady state corresponds to a closed curve which is nearly symmetrical and approximately circular. It approaches a circle as the function -f(U) approaches the horizontal axis.

2. Positive Voltage Feedback

A. Negative resistance produced by feedback

For a feedback circuit as shown in Figure V [2], it can be shown that the output impedance of such a circuit is

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[1]This section is cited from: Edson, William A., "Vacuum-Tube Oscillators," Wiley, 1953, p. 46.

[2]Martin, Thomas L. Jr., "Electronic Circuits," Prentice-Hall, 1955, p. 370.

$$Z_{out} = Z_c = Z_o/(1-\beta A_o) \qquad (15)$$ [1]

where A_o = open loop voltage gain of the amplifier, β = voltage transfer function of the feedback circuit, Z_o = output impedance of the open loop amplifier. It is very clear from this equation that the output impedance of the feedback amplifier can be made to have a negative resistance component by using positive feedback in an amount sufficient to make βA_o larger than 1. This will be discussed in more detail in the next chapter.

B. Bridge circuit

A common bridge circuit is shown in Figure VI, in which the bridge elements are made of resistors. Let R_1, R_2 and R_3 have equal value, R, and let R_a be the variable resistor. V_1 is the voltage applied to the bridge circuit. V_2 is the voltage difference between c and d. When the bridge is in a balance condition, V_2 is zero. Now if $R_a \neq R$, then $i_1 \neq i_2$. Therefore, $V_2 = R(i_1 - i_2) \neq 0$. The polarity of V_2 will depend on the direction of change of R_a from the balanced value. For the circuit connection shown in Figure VI, $i_1 > i_2$ when $R_a > R$, then point d will be positive and C will be negative. When $R_a < R$, $i_1 < i_2$, then point d will be negative and C will be positive.

When this bridge is used as a part of the feedback circuit, the positive or negative feedback can be easily controlled by simply changing R_a to the appropriate position. It is, therefore, clear that the value of β is also proportional to the value of the deviation of R_a from R.

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[1]This section is cited from: Tele-Tech, "Electronic Industries," April, 1954, pp. 72-78.

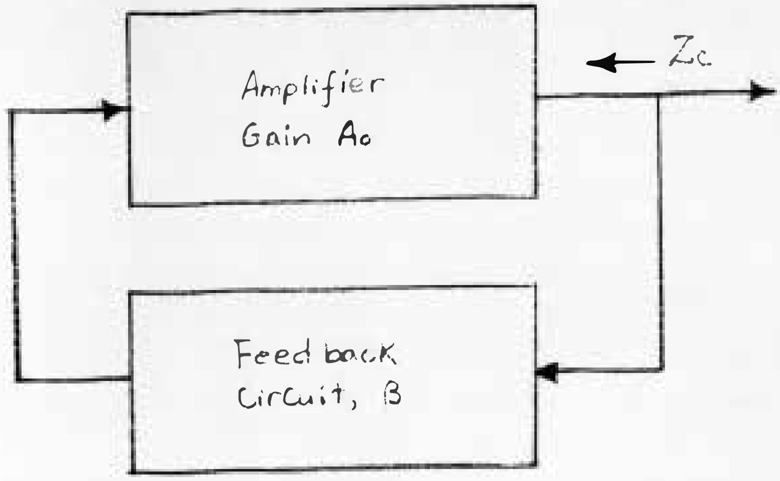

Figure V. Feedback Circuit

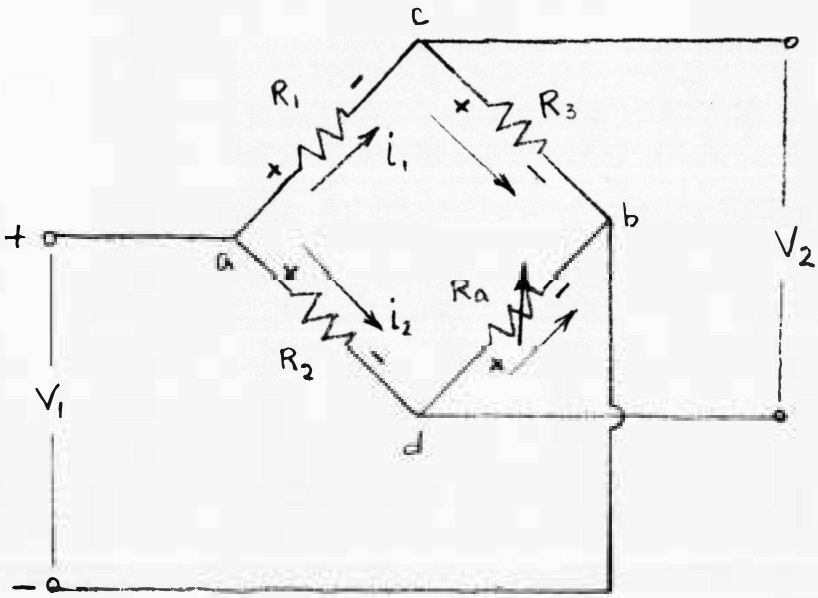

Figure VI. Bridge Circuit

3. Thermistors[1]

Thermistors are thermally sensitive resistors made of a class of ceramic-like semiconducting materials such as metallic oxides. They do not rectify. Because they have a negative temperature coefficient, as they become heated their resistance decreases. There are three primary ways of changing thermistor temperature: externally, directly, and indirectly. In the externally heated method, resistance changes as the ambient temperature varies. In the directly heated method, an electric current passing through the thermistor raises its temperature. The indirectly heated method depends upon a separate heating coil in thermal, though not electrical, contact to maintain constant temperature irrespective of ambient.

One or more of three main characteristics are utilized in most applications. The resistance-temperature characteristic as shown in Figure VII provides large changes in resistance for small changes in temperature, and over the range $-100^{\circ}C$ to $4000^{\circ}C$ the ratio of cold to hot resistance may conceivably decrease several million to one.

The voltage-current characteristic as shown in Figure VIII follows Ohm's law provided the power dissipated in the thermistor does not raise its temperature measurably above ambient. Therefore, increasing the current up to a point will produce a proportional voltage drop, just as in a conventional resistor. A further current increase will cause the voltage drop to remain constant, and a still further current increase will

[1] This section is cited from: Tele-Tech, "Electronic Industries," April, 1954, pp. 72-78.

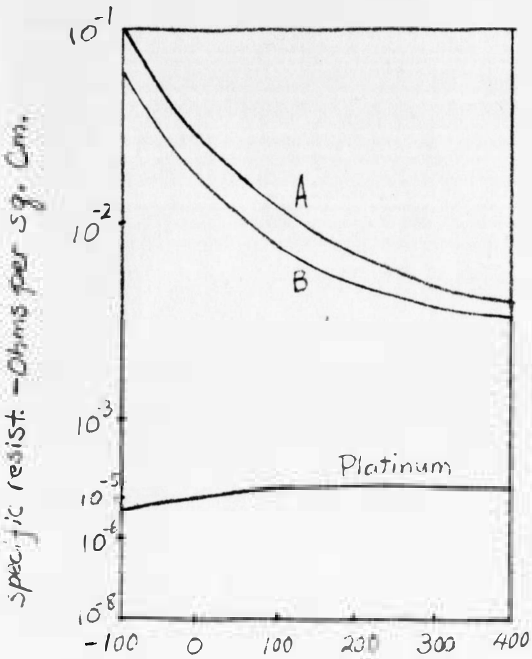

Figure VII. Thermistor resistance-temperature characteristic compared with Platinum.

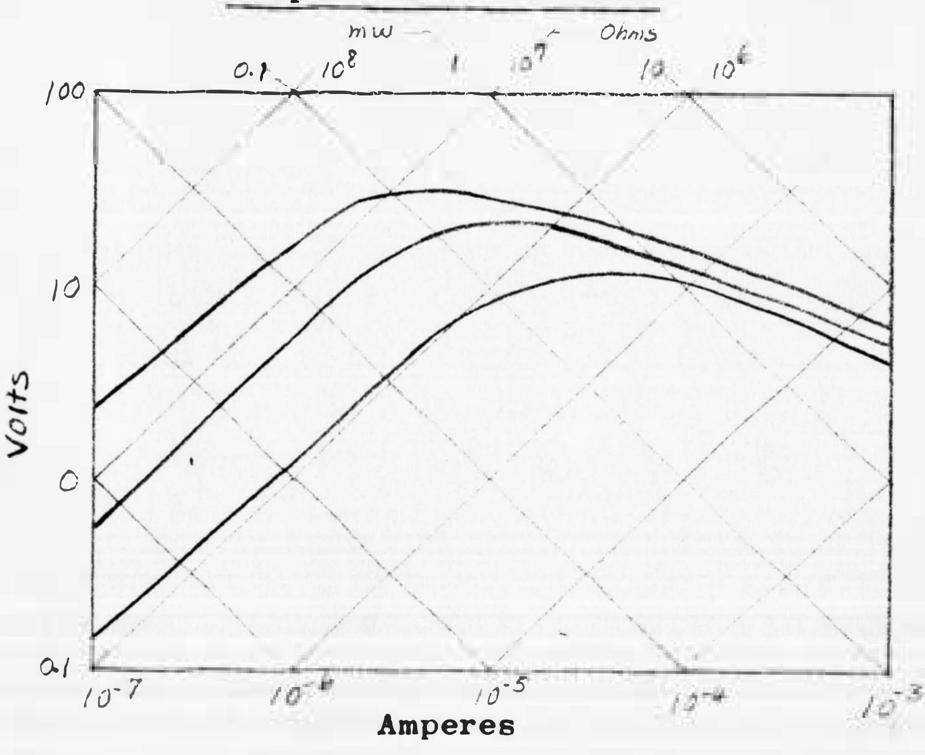

Figure VIII. Logarithmic plot of voltage-current characteristics

cause the voltage to decrease because of the negative resistance characteristic.

The rate of thermistor temperature rise is dependent upon power dissipation and thermal mass. For a particular applied voltage, the current will rise to a maximum and remain essentially constant. The time required for the current to go from minimum to maximum determines the thermistor's dynamic current-time characteristic.

There are several important symbols which represent thermistor functions. R_o is the cold resistance, measured at a specified temperature with a power small enough to keep the thermistor from heating. At 25°C, different thermistors may generally be obtained R_o of 2 ohms to as high as 5 megohms. Typical tolerance values are between 5% and 20%.

The temperature coefficient of resistance, α , is the ratio of the rate of change of resistance with temperature to the resistance of the thermistor. Represented values are from 3.0 to 5.5%/°C. B is a material constant, and is usually about 3500 to 4000°K.

The dissipation constant, C, is a proportionality factor of power dissipation to consequent temperature change. C may be as low as 60 w/°C to as high as 30mw/°C. Special mountings may increase this value several times.

The highest rating for maximum continuous ambient temperature, T, is usually about 300°C, but it could be much higher. Maximum current may range from 25ma to 7 amps in different types.

The thermal time constant, τ , is the time required for a thermistor to change 63% of the difference between its initial value and that of its surroundings when no electrical power is being dissipated in it. Depending on conditions, it may be anywhere from several milliseconds to five minutes, or even longer.

Although circuit designers usually use the manufacturer's performance curves, it may be helpful to approximate a thermistor's resistance, R, at an absolute temperature, T.

$$R = R_o \, e^{\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)} \qquad (16)$$

where e is 2.7183.

Another useful relationship is

$$\alpha = \frac{\beta}{T^2}. \qquad (17)$$

There are several different types of physical construction of thermistors: such as, washer type, disc type, rod type, bead type, flake type and printed circuit thermistor.

CHAPTER III

THEORETICAL ANALYSIS

The negative resistance characteristics of an oscillator can be achieved by the positive feedback method. To obtain the nonlinear characteristic curve, it is necessary to determine the circuit constants. It is then possible for the oscillation to be analyzed by a graphical method.

This chapter is an attempt to predict the variation of output voltage associated with the change of positive feedback. The V_p vs. β curve will give a full picture of how well oscillation is controlled by the amount of feedback and how precisely the temperature could be controlled by using a thermistor as a system element.

1. Negative resistance character of a feedback oscillator

The equivalent circuit of a feedback type oscillator is shown in Figure IX. Where Z_ℓ is the load impedance, g_m is the tube transconductance, γ_p is the tube plate resistance and Z_b is the impedance looking into the left of the dashed line.

If e_g is related to e_p by the equation $e_g = \beta e_p$, then

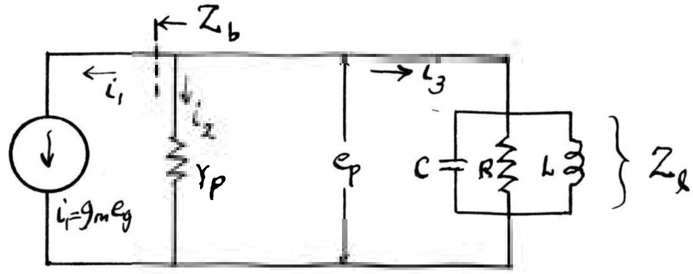

Figure IX. Equivalent circuit of feedback oscillator

$$Z_b = e_p/i_1 = e_p/g_m e_g = e_p/g_m \beta e_p = 1/g_m\beta \qquad (18)$$

Z_b is positive when β is a positive number. In this case, there is no oscillation of the circuit. Now if the polarity of the feedback network is reversed to make β a negative number, then Z_b becomes a negative resistance. Before proceeding to the theoretical analysis, it is necessary to determine Z_ℓ and g_m of the circuit.

2. Method of determining Z

The load impedance Z_ℓ in Figure IX included the impedance of the resonant circuit and the reflected impedance from the bridge circuit through the transformer action shown in Figure XIX. The equivalent impedance of Z_ℓ could be represented by a RLC parallel circuit. The tank circuit was disconnected from the plate circuit in Figure XIX and a frequency generator was connected to these two terminals. The resistor R_a in Figure VI was set at 68 kilohms and also the bridge circuit was disconnected from the tube. This was shown in Figure X, where G was the frequency generator, V_1 and V_2 were a.c. voltmeters. If the applied voltage V_1 for various frequencies and the corresponding current I delivered to the network were recorded, it was then possible to determine Z_ℓ by the equation $Z_\ell = V_1/I$. Instead of using an a.c. ammeter to measure the current I, a 1 ohm resistor was placed in series with the generator as shown and the a.c. voltage across it was measured. Then the voltage reading was equal to the magnitude of the current flowing through the resistor and into the network. The data of V_1, V_2, I, $|Z_\ell|$ were listed in Table 1.

For a RLC parallel circuit, the magnitude of the impedance was determined by the following equation:

$$|Z_\ell| = wLR/\sqrt{(wL)^2 + R^2 \quad (1-w^2LC)^2}. \qquad (19)$$

From Table 1, it was evident that the resonant frequency of the circuit was 650 cps. At resonant condition, $R = |Z_\ell| = 10500$ ohms and

$$1/LC = (2\pi f)^2, \text{ or } C = 1/(2\pi f)^2L.$$

When f = 650 cps,

$$C = 1/(2\pi \times 650)^2L = 0.06 \times 10^{-6}/L \text{ farads.}$$

Next, L was calculated by using f = 300 cps and $|Z_\ell| = 7500$ ohms as listed in Table 1. Substituting into the equation 19, the resultant was L = 4.54 henrys. Therefore,

$$C = 0.06 \times 10^{-6}/4.54 = 0.0132 \times 10^{-6} \text{ farads.}$$

Using the values of R, L, C obtained above, and equation 19, $|Z_\ell|$ was calculated at f = 1300 cps to be equal to 8000 ohms. This checked exactly with the value obtained in Table 1, which meant that the values of R, L, C calculated were excellent approximations to the actual circuit of Figure X.

3. Determine g_m of tube 12AU7

The transconductance of a vacuum tube is represented by the curve of e_g vs. I_p plot. The circuit used for determining g_m of tube 12AU7 is shown in Figure XI. E was a variable d.c. voltage source which was equivalent to the voltage applied to the bridge from the output of the tank circuit (refer to Figure XIX), and the series 68 kilohms resistance was equivalent to the bridge resistance viewed from the grid terminal.

By changing the voltage of E, which was considered as e_g, in

Table 1. Data for Calculating Z

f(cps)	V_1(volts)	I(amp)	$(Z_\ell) = \frac{V_1}{1}$ (ohms)
200	4	0.00073	5,480
300	4	0.00530	7,550
400	4	0.00480	8,340
500	4	0.00400	10,000
600	4	0.00384	10,400
650	4	0.00380	10,500
700	4	0.00384	10,400
800	4	0.00390	10,250
900	4	0.00410	9,750
1,000	4	0.00425	9,420
1,100	4	0.00450	8,900
1,200	4	0.00475	8,420
1,300	4	0.00500	8,000

Table 2. Data for Determining gm of 12AU7

e_g(v)	I_p(ma)	e_g(v)	I_p(ma)
-16.5	0	1.3	29
-10.7	2	8.4	30
-7.8	5	28.0	31
-5.5	10	38.5	32
-3.7	15	45.0	33
-2.2	20	55.0	34
-1.0	25	65.0	35
0.0	28		

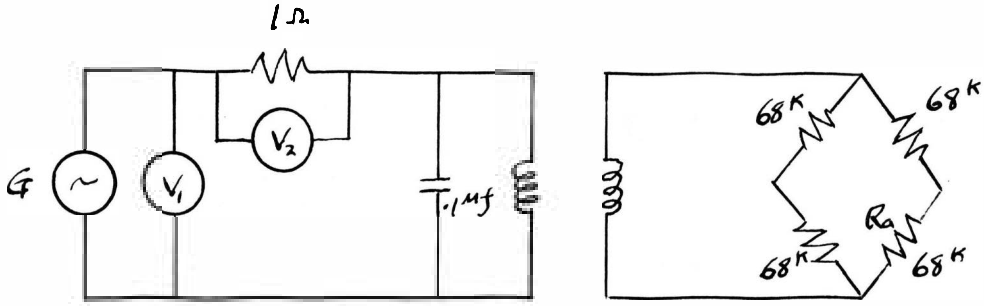

Figure X. Circuit for measuring Z_l

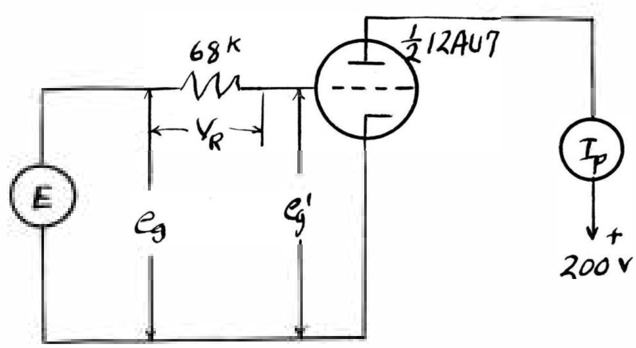

Figure XI. Circuit for measuring g_m

steps, recording I_p in correspondence, Table 2 was obtained. Then e_g vs. I_p curve was plotted in Figure XII, the g_m curve was determined.

4. Isocline diagram and cyclograms construction

Equation 18 showed that the negative resistance of the tube was $Z_b = -1/g_m\beta$. Since $Y_b = 1/Z_b$, then

$$Y_b = -g_m\beta = -\frac{I_p}{e_g/\beta} = \frac{I_p}{-e_p} .$$

Therefore, from the above evaluation, the negative resistance curve could be determined by merely changing the ordinate e_g of the g_m curve to $-e_g/\beta$ and plotting I_p vs. $(-e_p)$. As soon as this was completed, isograms could be drawn as described in Chapter II, and then the amplitude of oscillation could be determined from the cyclograms.

The following paragraph will describe the construction of one of the isograms and cyclograms in detail. First $\beta = 0.25$ was selected, then Table 3 was made from Table 2 and Figure XIII was plotted. A point at $I_p = 13$ ma and $e_p = 17.5$V in Figure XIII was taken as the middle point of the curve. Then the values of v and f(v) were determined.

From the vacuum tube manual[1], the plate resistance $r_p = 7300$ ohms of 12AU7 at $E_p = 200$V, $I_p = 5$ ma, $E_c = -8$V was obtained; therefore, the total conductance in Figure IX was

$$G = 1/r_p + 1/R = 1/7300 + 1/10500$$
$$= 0.232 \times 10^{-3} \text{ mhos}$$

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[1]General Electric, "Electronic Tube," Vol. 2.

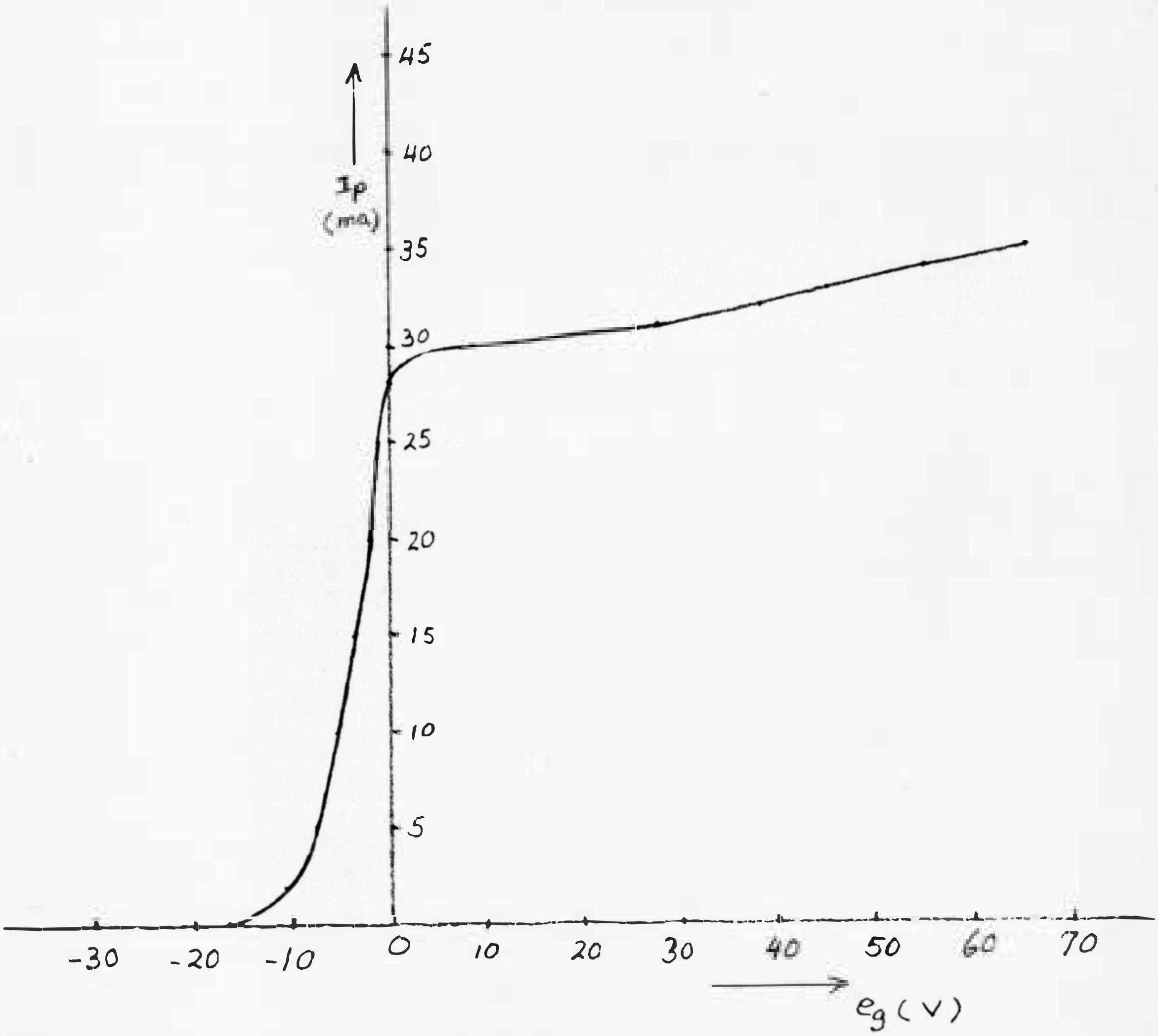

Figure XII. gm curve of 12AU7

Table 3. Plate Characteristic of 12AU7 for β = 0.25, K = 18.5 x 10^3 ohms, G = 0.232 x 10^{-3} mhos

I_p	e_g	$-e_p = -e_g/$
0	-16.5	66.0
2	-10.7	42.8
5	-7.8	31.2
10	-5.5	22.0
15	-3.7	14.8
20	-2.2	8.8
25	-1.0	4.0
28	0.0	0.0
30	8.4	-33.6
31	28.0	-112.0

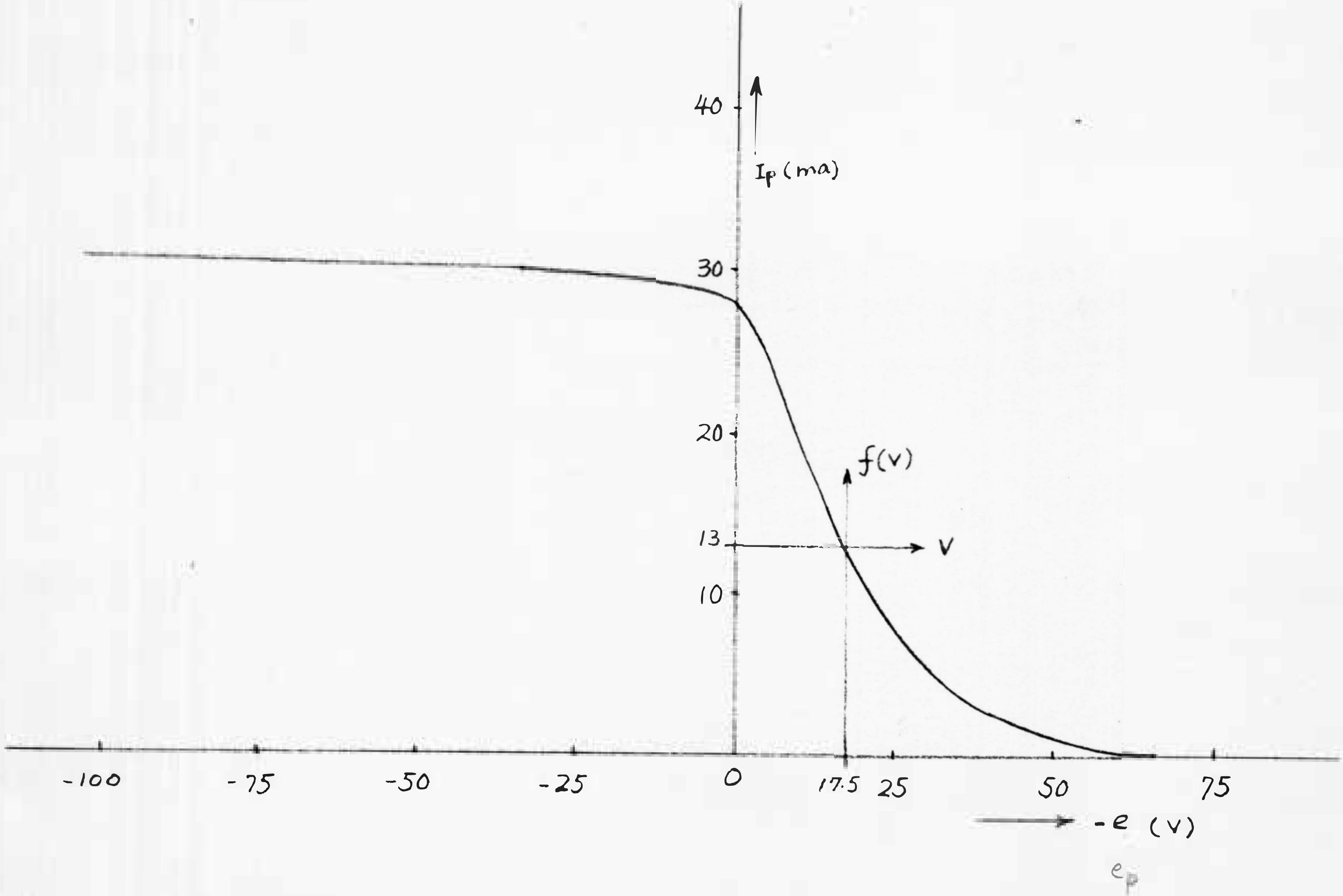

Figure XIII. Negative Resistance Characteristic Curve for $\beta = 0.25$

and from equation (8)

$$K = \sqrt{\frac{L}{C}} = \sqrt{\frac{4.54}{0.0132 \times 10^{-6}}} = 18.5 \times 10^{3}.$$

By using G and K, U, GV and f(U) were calculated from equations 6 and 9. Table 4 was then completed. The U vs. -f(U) plot is shown in Figure XIV.

After constructing isoclines according to the rules stated in Chapter II, a closed cyclogram was obtained by starting at an arbitrary point and proceeding counterclockwise along the tangent lines. The maximum variation along U axis was 4.8 ma to the left and 5 ma to the right, the total variation was 9.8 ma. This corresponded to a peak to peak voltage output of

$$v = KU = 18.5 \times 10^{3} \times 9.8 \times 10^{-3} = 181 \text{ volts.}$$

Following the same procedure, Figure XV and Figure XVI were constructed. Then the peak to peak output voltages were determined accordingly.

As discussed in Chapter II, the oscillation occurred when $(1 - \beta A_o) \leqq 1$. Therefore, the oscillation of this case started at $\beta_o = 1/A_o$ and would sustain for all values of β which were greater than β_o.

The open circuit gain of a triode is A_o which approaches μ when the load impedance is large. The value of μ for the 12AU7 tube given in the tube manual[1] at operating point of $e_b = 200v$, $I_p = 5$ ma., $E_c =$

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[1]General Electric, "Electronic Tube," vol. 2.

Table 4. Table of -f(U) and U, $\beta = 0.25$, $K = 18.5 \times 10^3$ ohms, $G = 0.232 \times 10^{-3}$ mhos

f(v)(ma)	v(v)	Gv(ma)	-f(U)(ma)	U(ma)
-13	60.0	14.0	-1.0	3.24
-13	48.5	11.3	1.7	2.62
-11	25.3	5.87	5.13	1.37
-8	13.7	3.18	4.82	0.74
-3	4.5	1.0	2.0	0.24
0	0	0	0	0
7	-8.7	-2.2	-4.8	-0.47
12	-13.5	-3.3	-8.7	-0.73
15	-17.5	-4.1	-10.9	-0.95
17	-51.1	-12.8	-4.2	-2.76
18	-129.5	-30.0	-12.0	-7.0

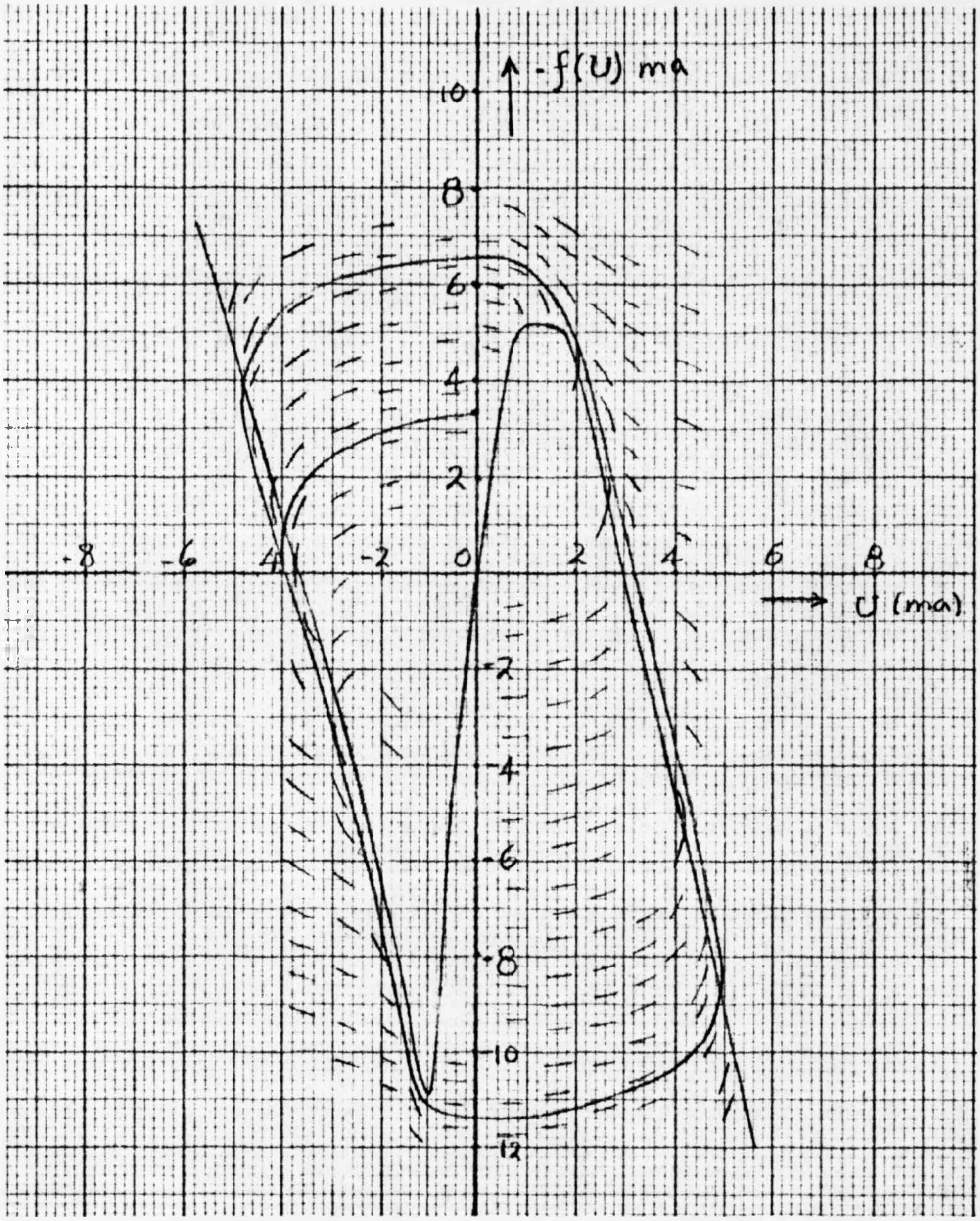

Figure XIV. Isocline diagram of $\beta = 0.25$, peak to peak voltage output is 18.5 x 9.8 = 181 volts

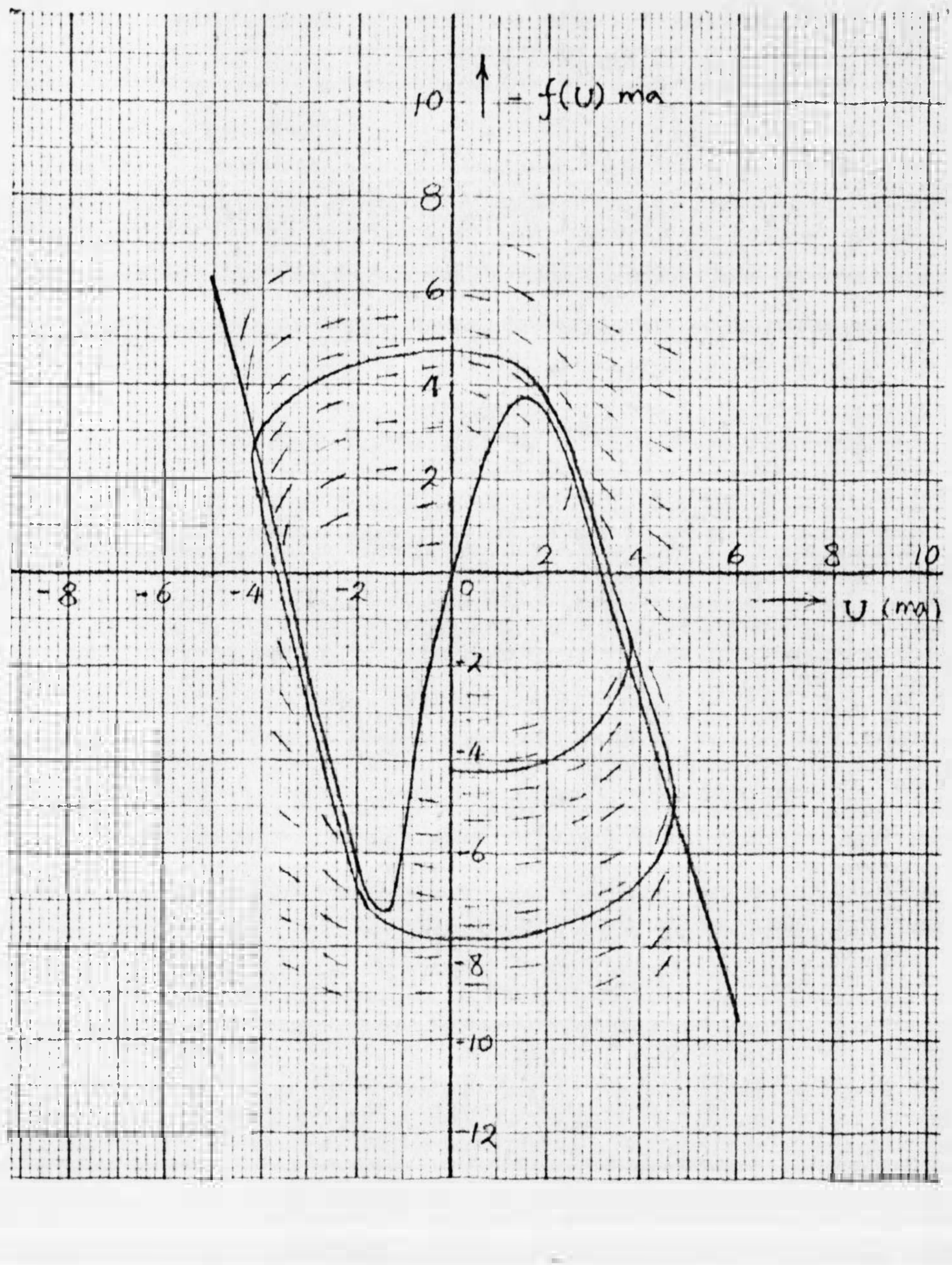

Figure XV. Isocline diagram of β = 0.15, peak to peak voltage output is 18.5 x 8.8 = 162 volts

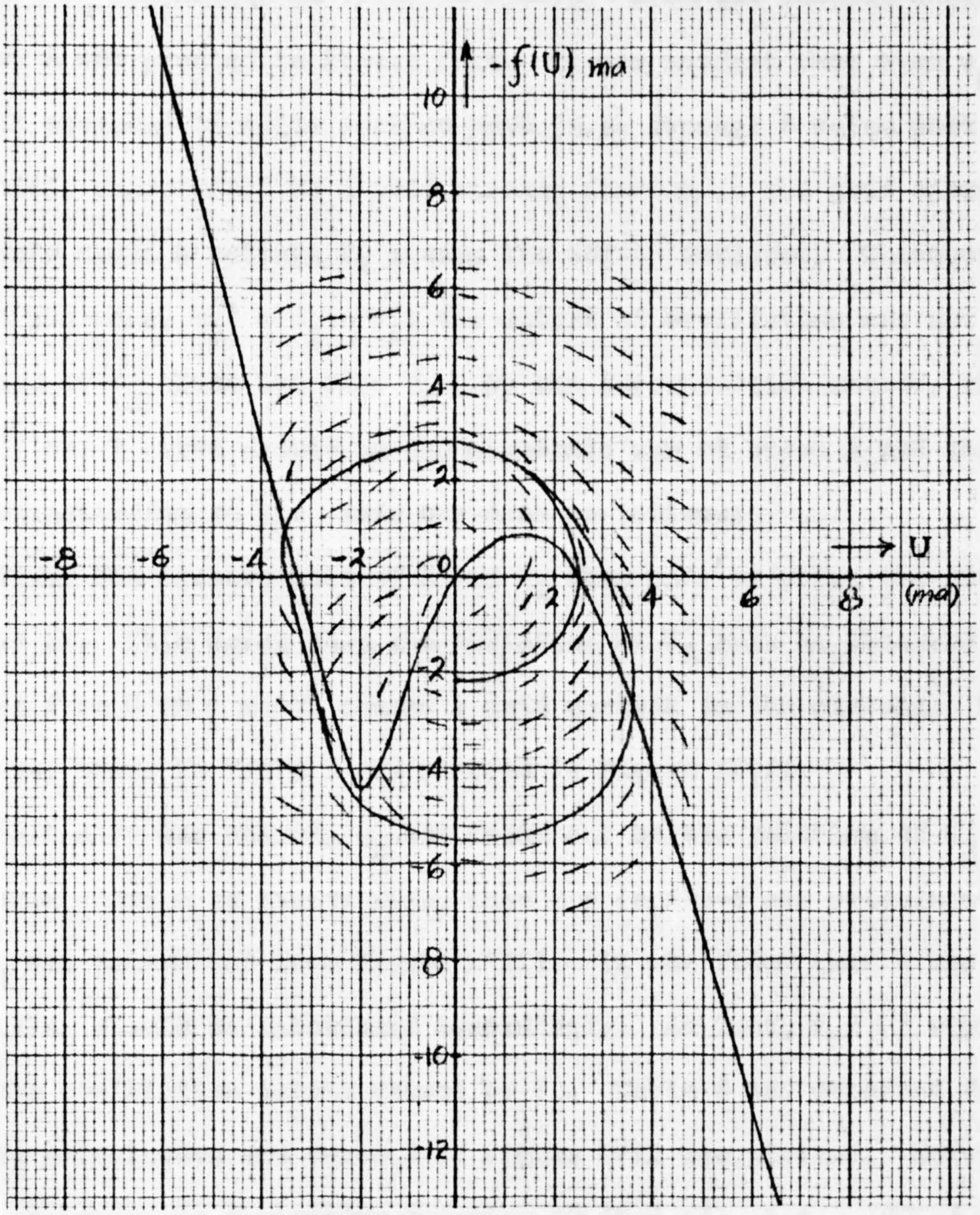

Figure XVI Isocline diagram of β = 0.1, peak to peak voltage output is 18.5 x 7.2 = 133 volts.

-8V is 15.3. Therefore, with a large load impedance A_o approaches to 15.3 and β_o approaches to 1/15.3 = 0.0653. With the results obtained, the curve A was plotted in Figure XVII. It was evident that when e_g in Figure XI exceeded 16 volts peak to peak (i.e. average 8 volts peak in half cycle), there would be a grid current flowing in the grid circuit which would cause a voltage V_R across the 68KΩ resistor. Whenever this was the case, $e_g' = e_g - V_R$, the output of oscillation was not only determined by e_g alone but also V_R. By the superposition method, the final β vs. V curve should be the algebraic sum of curve A and the curve corresponding to V_R. This was done as follows:

First the measurements of E and V_R were made by using the circuit of Figure XI. This meant that the variation of V_R must be observed as e_g varies from 0 to + which caused a grid current in the circuit. The results are shown in Table 5 and plotted in Figure XVIII.

Next a β was chosen, e_g was found out by the relation of $e_g = e_p/\beta$ from curve A in Figure XVII, then $E = (e_g/2)-8$ was calculated. Also, V_R was determined from Figure XVIII. It was found that a new $\beta' = (e_g - V_R)/e_p$ should be calculated accordingly. At last, V_R/β' represented the oscillation output caused by V_R. Table 6 was obtained from the above calculation and curve B in Figure XVII was plotted. Note that curve B was drawn as a negative function of (V_R/β'), i.e., $-f(V_R/\beta')$.

As shown $e_g' = (e_g - V_R)$, the resultant output of oscillation was determined by adding curves A and B, which was shown by curve C. This was the final picture of the theoretical analysis. From this analysis, the circuit output could be predicted. The cutoff value of β and the

Table 5. Measurement of V_R when E_g is positive

$E_g(V)$	$V_R(V)$	$E_g(V)$	$V_R(V)$
0	0.0	40	37.2
5	4.5	45	41.8
10	9.0	50	46.3
15	13.7	60	54.0
20	18.3	70	63.0
25	23.0	80	72.0
30	27.7	90	80.0
35	32.3	100	90.0

Table 6. Data of β' and V_R/β'

	$e_p(v)$	$e_g(e_p\ )v$	$E(v)$	$V_R(v)$	β'	$V_R/\beta'(v)$
0.10	131	13.1	1.0	0.0	0.100	0.0
0.12	148	17.8	0.9	0.9	0.114	7.8
0.15	162	24.3	4.1	3.7	0.127	29.1
0.20	174	34.8	9.4	8.5	0.151	56.3
0.25	181	45.3	14.6	13.4	0.177	77.5
0.30	184	55.2	19.6	18.0	0.203	88.8
0.35	186	65.0	24.5	22.6	0.228	99.2

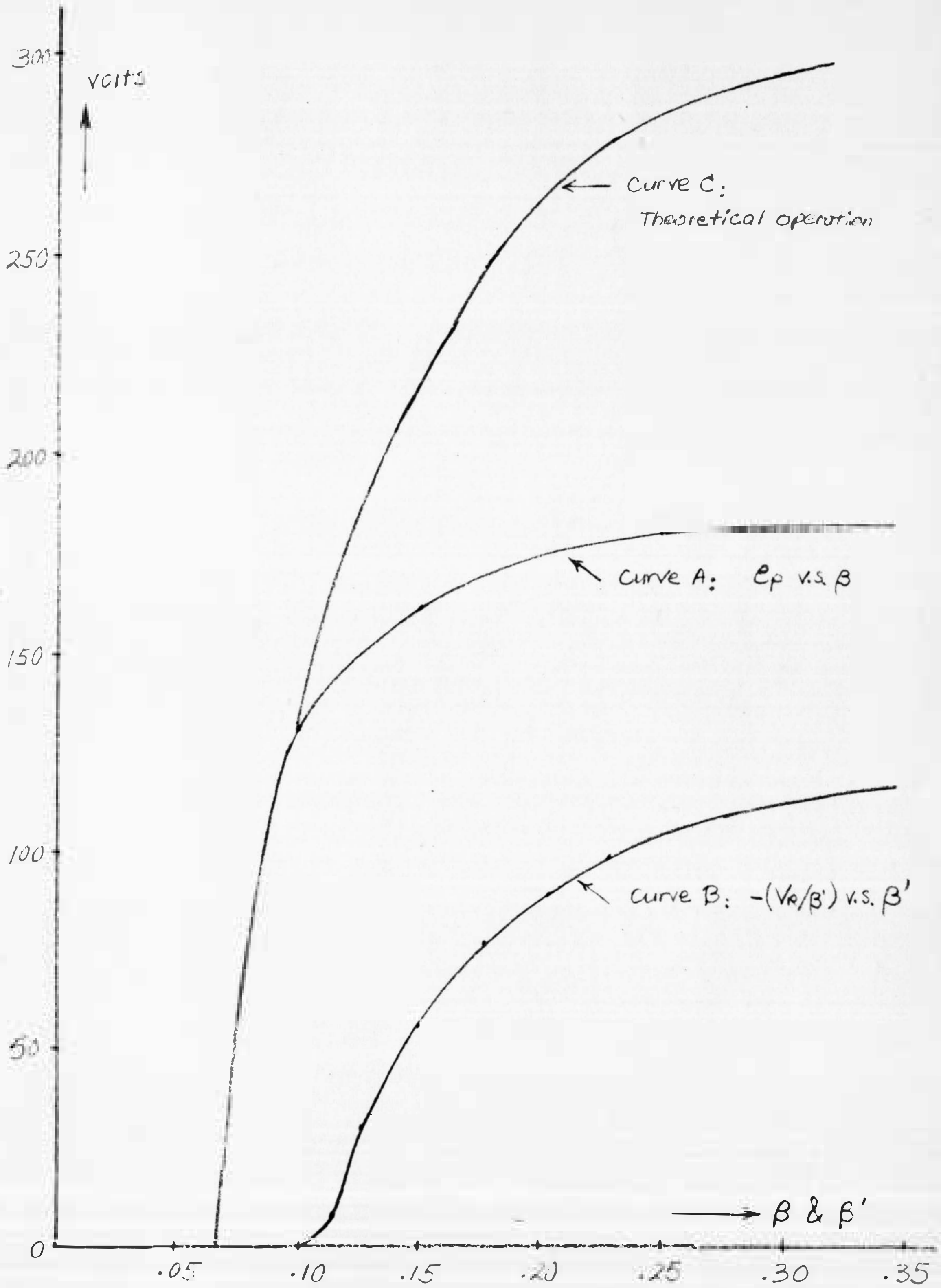

Figure XVII. Theoretical Analysis of Circuit Operation

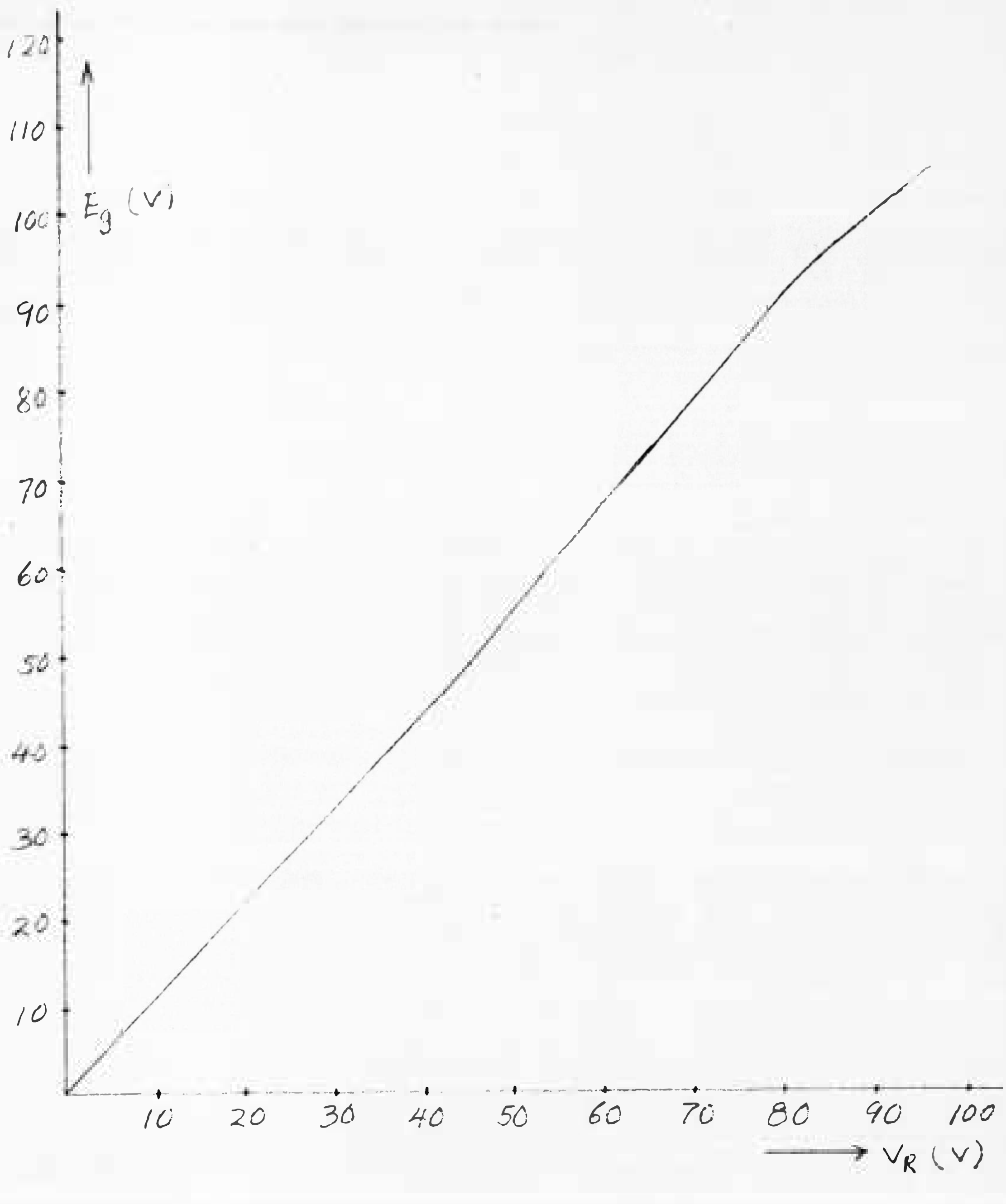

Figure XVIII E_g (positive grid voltage) v.s. V_R

corresponding resistance of the thermistor or the cutoff temperature could be controlled and predicted also.

CHAPTER IV

EXPERIMENTAL VERIFICATION

A feedback oscillator was set up and the characteristic of oscillation was determined experimentally. This checked and validated the theoretical results obtained in the last chapter. An attempt was made to show the practicability of this performance.

1. Circuit Construction

The complete circuit connection of the laboratory experiment is shown in Figure XIX. R_a was the variable resistor replacing the thermistor in the actual circuit for the convenience of analyzing the oscillation. The primary winding of the transformer was used as an inductance of the resonant circuit and the output of the secondary winding was stepped up by a ratio of 1:2.66. The connection of the transformer polarities was determined by contributing positive feedback to the tube whenever R_a was greater than 68 kilohms. This assured an output of the oscillator for decreasing ambient temperature because of the negative temperature coefficient of the thermistor.

V_p and V_g were the a.c. voltages in the plate and grid circuit respectively and were measured by using oscilloscopes for their high impedance (2.2 megohms) properties. At the same time, the change of wave forms during oscillation could also be observed.

2. Circuit Operation

With the circuit connected as above, data of V_p, V_g (peak to peak

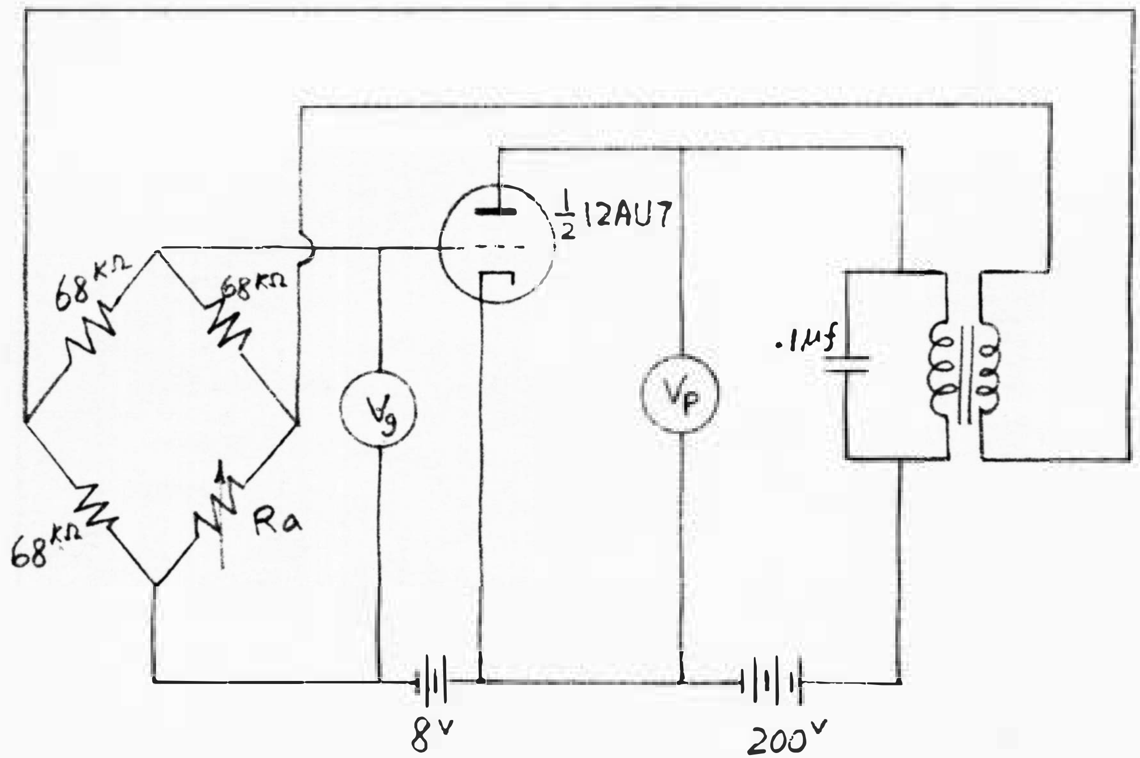

Figure XIX, Feedback Oscillator Circuit diagram values) and R_a were taken by increasing R_a from low resistance to high resistance in steps. Then, Table 7 was obtained.

From Table 7, V_p vs. R_a curve and V_p vs. β curve were plotted very easily as shown in Figure XX. These curves showed that the amplitude of oscillation increased rapidly near the cutoff point and was then saturated by the nonlinearity of the vacuum tube. This performed a very sharp switching function of the system.

Table 7. Data Taken from Figure XIX

R(ohms)	V_p(p-p volts)	V_g(p-p volts)	(V_g/V_p)
90,000	0	0.00	0.000
90,500	12	1.25	0.104
91,600	92	10.00	0.109
92,000	174	24.00	0.138
100,000	220	34.00	0.155
110,000	240	45.50	0.184
120,000	260	54.00	0.207
130,000	273	63.50	0.232
140,000	282	72.50	0.257
150,000	290	81.00	0.279
160,000	294	88.00	0.299
170,000	300	95.50	0.318
180,000	303	100.00	0.330

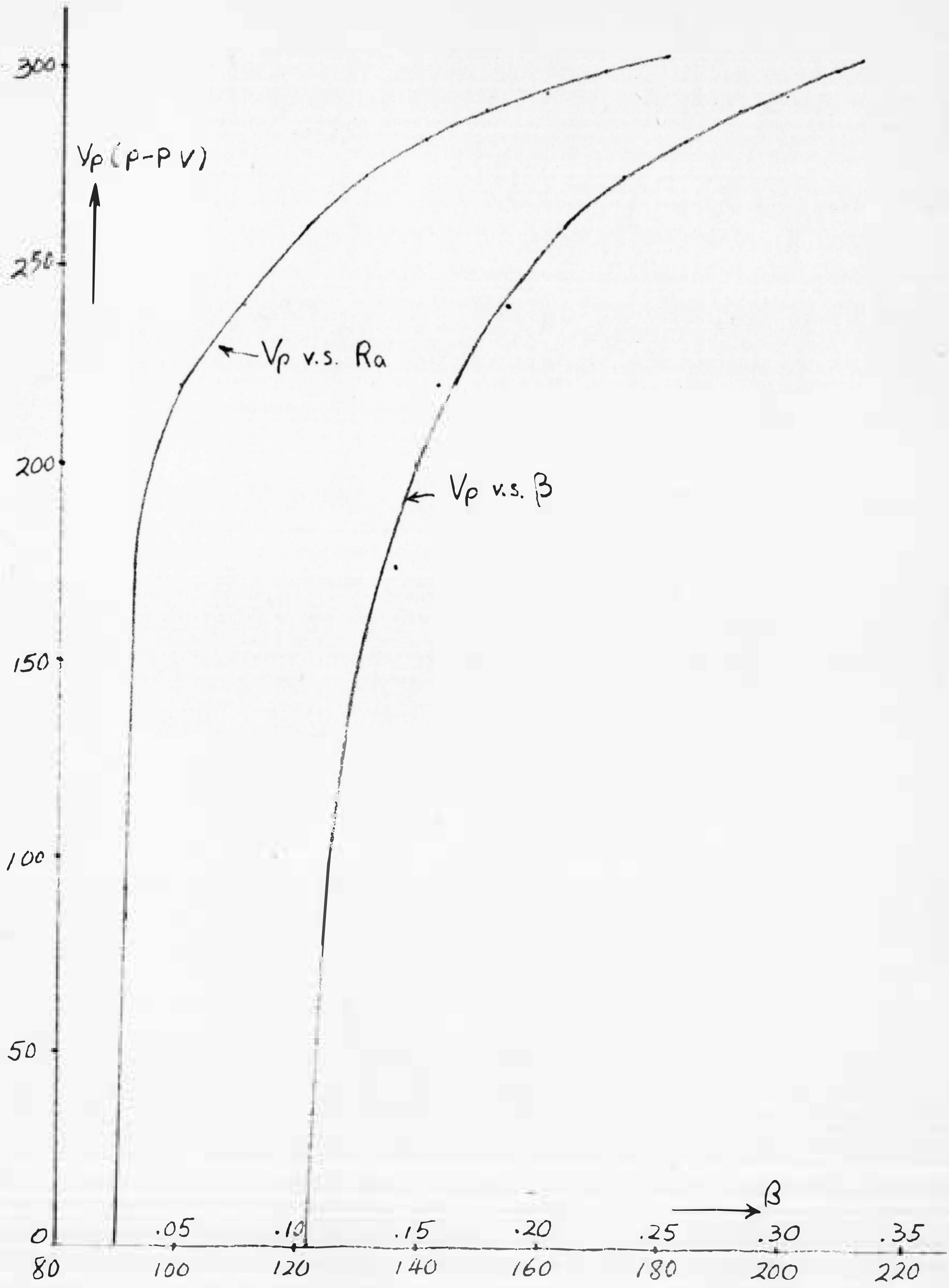

Figure XX. Circuit Characteristics of Fig. XIX

CHAPTER V

CONCLUSION

A complete theoretical analysis and experimental verification have been presented in the previous chapters. The graphical method which had been used to solve the nonlinear problem seemed very nice and convenient and was considered as the most powerful tool to analyze the oscillation. The experimental verification was straightforward and behaved very closely with the theoretical analysis. Now, a final discussion of the two results will conclude this topic.

1. Comparison of Results

In comparing the V_p vs. β curves shown in Figure XVII and Figure XX, it was clear that they were very similar in shape. However, there were some differences between the two results. These differences are discussed in the following:

A. There were differences between starting points of the value of β and the slopes of the linear portions of oscillation. In Figure XVII, β_o was determined by using $\mu = 15.3$, which was the amplification factor when the tube 12AU7 was new. But the one used in the experiment was a used one so that μ decreased. Therefore, β_o would become 0.091 at a smaller value of $\mu = 11.0$. Now, if $\beta_o = 0.091$ was used as the corrected starting point in Figure XVII, then the slope of this part became identical to that of Figure XX. Since the results of this correction seemed so perfect, the decreased value of μ assumed above was reasonable and the above correction was necessary and practical.

B. After the previous correction was made on curve C of Figure XVII, it was still necessary to shift the curve C to a position where β_o = 0.104 in order to agree perfectly with the curve in Figure XX. Fortunately, a certain shifting effect was found to actually exist, due to the connection of an oscilloscope in the grid circuit of Figure XIX for measuring V_g. The internal resistance of the oscilloscope was 2.2 magohms; therefore, using the Thevenin's theorem, the effective grid voltage in Figure XI was decreased by 3.5%, this would effect the value of β also. Consequently, every point on curve C in Figure XVII had to shift to the right along the β axis to a new position where the new value of β would be 1/0.965 times greater. Therefore, β_o was shifted from 0.091 to 0.094.

But the above correction was not enough for which β_o still had a difference of 0.104 - 0.094 = 0.01, which was about a 10% error. However, this error could be introduced by either the incorrect calibration of the two oscilloscopes so that a constant error in β did exist, or the incorrect values of the tank circuit elements determined in Chapter III.

2. Wave Forms of Output Voltage

The isocline diagrams shown in Figures XIV, XV, and XVI gave a clear picture that the smaller the β, the more circular in form. This meant that for smaller β, the oscillator featured a linear operation and the output was very close to a sinusoidal wave form. Then, the wave forms became gradually distorted as β increased.

CHAPTER VI

RESULTS

A thorough analysis and discussion of the oscillator operation had been presented. Then a check on the over-all performance of system was examined as follows:

The system was set up as shown in Figure XXI. The left lower branch of the bridge circuit consisted of a 25 kilohms variable resistor for the purpose of adjusting the controlled temperature through a considerable range.

A rod type thermistor with glass coating was used and placed into a tank of water. The thermistor was selected to have a resistance of around 70 kilohms at the desired temperature. The required accurate temperature was obtained by varying the 25 kilohms resistor as mentioned above.

The output of the oscillator was taken out from the third winding of the transformer as shown and fed into an audio amplifier to gain higher power. This output was then rectified and used to operate a relay. The relay was very sensitive and switched on the heater system whenever there was oscillation.

The heater was placed in the tank and water was kept circulating for good heat distribution. It was found that the temperature was accurately controlled with very critical switching action of the relay

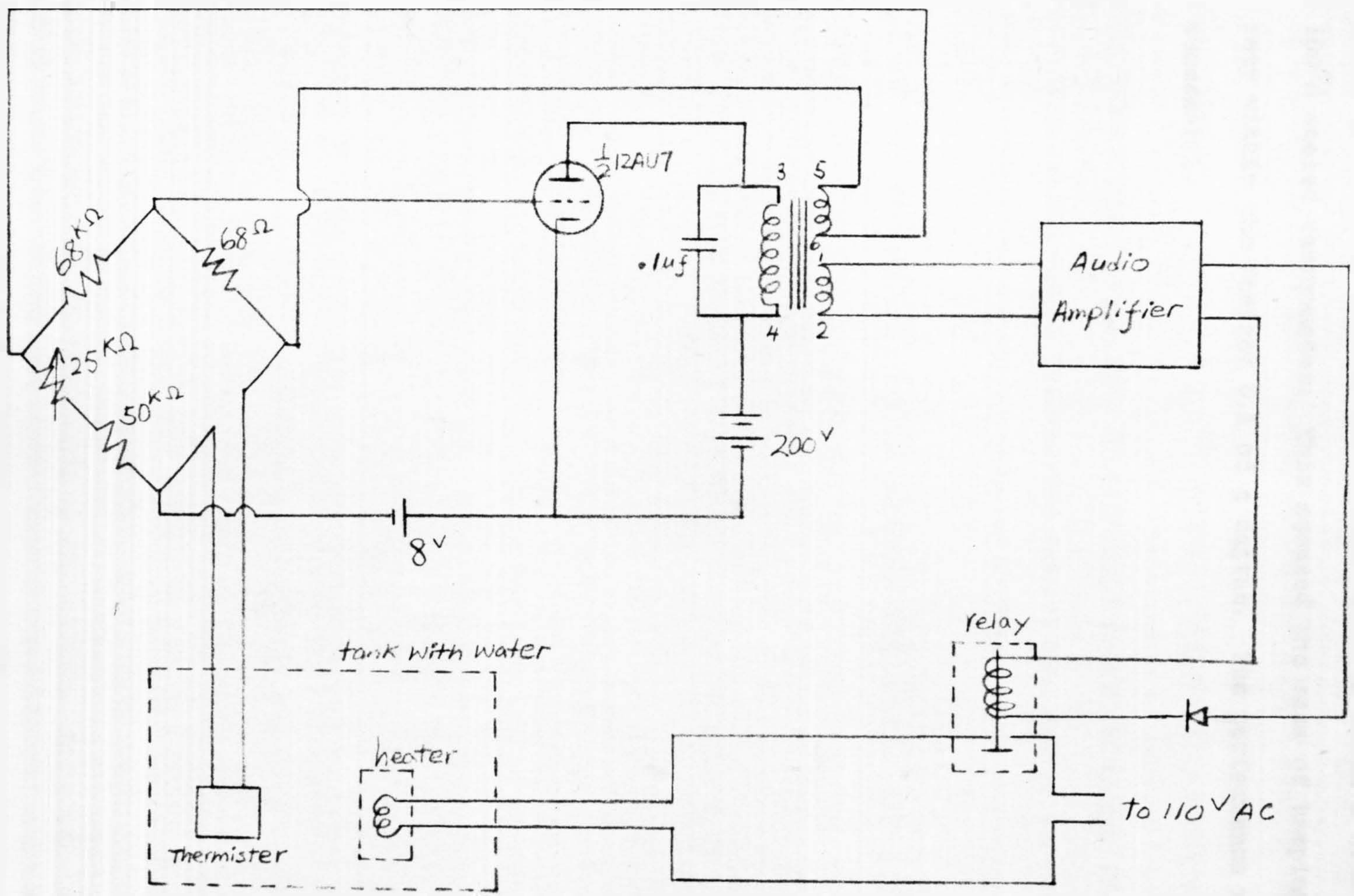

Figure XXI. System diagram

so that no fluctuation could be identified by using a general purpose 100°C scaled thermometer. This assured the ease of keeping the accuracy within the desired 0.1 of a degree. The performance was considered successful.

LITERATURE CITED

1. Edson, William A., "Vacuum Tube Oscillator," Wiley, 1953, pp. 42-47.

2. Martin, Thomas L. Jr., "Electronic Circuits," Prentice-Hall, 1955, pp. 364-367, p. 370.

3. Tele-Tech, "Electronic Industries," April 1954, pp. 72-78.

4. General Electric, "Electronic Tube," Vol. 2.