Optimization of Cover Plated Steel Frames by Linear Programming

Chong Soo Cho

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OPTIMIZATION OF COVER PLATED STEEL FRAMES BY LINEAR PROGRAMMING

BY

CHONG SOO CHO

A thesis submitted in partial fulfillment of the requirements for the degree Master of Science, Major in Civil Engineering, South Dakota State University

1968

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OPTIMIZATION OF COVER PLATED STEEL FRAMES BY LINEAR PROGRAMMING

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser

Head, Civil Engineering Department

Date

Date
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This thesis is dedicated to the author's mother and father for their deep encouragement and long sacrifice.

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A. General Introduction

Structural engineers are required to design economical, reliable, and aesthetic structures which conform to given requirements and constraint conditions. To do this requires a knowledge of structural analysis methods and behavior of structural materials. In principle, structural engineers can work with any kind of structural material and any type of structure such as a two dimensional structure (frames, trusses, ... , etc.) or a three dimensional structure (shells, plates, ... , etc.) in order to fulfill the requirements. There exists an infinite number of possible ways to design a proposed structure as well as a multitude of difficulties in design.

The determination of an indeterminate structural system of specified topology that can withstand prescribed loading or environmental conditions is a problem without a unique solution. However, if the system satisfies some criterion, such as minimum weight or minimum cost, then a unique design is sought and the problem becomes correspondingly more difficult. The usual method of designing a statically indeterminate structure consists of a trial and error procedure in which a first design is assumed, the structure is then analyzed, and the design is revised on the basis of the results of the analysis. This cycle may be repeated until, hopefully, each
member is fully stressed under at least one loading condition. This procedure will not necessarily result in the best design, and it is even possible that the best design may not be fully stressed. The purpose of the present study is to develop a method by which an optimum design can be obtained with a reasonable mathematical certainty. The greatly expanded computational facilities and optimizing procedures\(^1\) being used in management and in other areas of engineering are providing structural engineers with possibilities of new approaches to optimum structural design.

With this background the theoretical description of a method for optimizing a structural design with particular reference to the conventional methods of analysis is presented. In this thesis, the optimization of a framed structure using wide flange sections and cover plates is considered. The working stress method of analysis and the American Institute of Steel Construction code have been adopted throughout this investigation.

B. Problem Background

Several recent investigations have demonstrated the feasibility of using several techniques for obtaining minimum weight or minimum cost in structural designs. One of the first papers on

this subject was written by Gary², who described his work at the University of Illinois in designing intersecting roof trusses wherein minimum weight was used as the criterion for determining truss spacing and patterns. The stress and weight relationships were programmed for a digital computer. The program was then started by using a small value for truss spacing. This initial value was then incrementally increased with each cycle of calculations. After a comparatively large number of cycles, the minimum linear roof system weight was reached.

Anaston³, in a paper at the second American Society of Civil Engineers conference on Electronic Computation in September, 1960, provided a conventional approach for designing a transmission line tower for its minimum weight. With the aid of an electronic computer the design was repeated for different configurations. The weight for the transmission tower was then calculated in each case and the minimum weight was obtained among them. This method showed that the computer can decrease the design time necessary to arrive at the


optimum design in terms of weight; it can relieve the engineer from repetitious work.

Several previous investigators employed the linear programming technique for obtaining the minimum weight of plastically designed steel frames. They considered the design criteria to be safety against collapse and a limitation on lateral deflections under service loads in the elastic range. The method of linear programming was used to generate the solution to the preliminary design. Plastic hinge theory was used to generate the equations of constraint on the moment carrying capacity of members. The equations of constraint were found to be compatible with the design criteria.

Schmit and Moses applied a similar technique of linear programming for finding optimum designs for a wide class of elastic structures. The optimization was accomplished by transforming the

---


analysis and design cycles into the solution of a series of linear programming problems.

Although these previous studies have dealt with a very small number of variables, they showed that the determination of minimum weight design can be formulated as a linear programming model and that the existence of an optimal solution among many other possibilities can be established.

C. Object and Scope of Investigation

The main objective of this investigation is to determine the optimum design variables which will minimize the weight of structure by using the linear programming technique. A systematic method is presented for finding optimum design solutions for the elastic structure. As the nature of the problem allows an infinite number of possible designs, a nonlinear programming model is constructed and used to determine the design variables. Because solutions of a set of nonlinear restrictions and nonlinear objective function do not necessarily optimize, the restrictions are approximated by the linear restrictions. This is called the cutting plane method. For the solution of the linear programming problem the simplex method is employed to obtain the optimum design variables. At each simplex solution, the linear approximations to the nonlinear restrictions are adjusted to minimize the differences. The entire process cannot be carried out without a digital computer.
As an illustration, the optimum dimensions for a one-bay, one-story rigid framed structure subjected to a uniformly distributed load were obtained. The cross sectional dimensions of beam, columns and cover plates and the ratios of cover plate length to member length were considered as the design variables (see Figure 1). The main constraint conditions on the design variables were those imposed by the AISC specification. The entire process was programmed in Fortran II language for calculation on IBM 1620 digital computer. The programs are shown in Appendix A.

This proposed method of solution is applicable to the determination of optimum designs for framed structures, trusses, plate girders, and other conventional structures. Also, this basic technique should be applicable to a broad class of design problems.

---

CHAPTER II

STRUCTURAL ANALYSIS

A. Design Variables

When a structure is to be designed under several loading conditions, there are certain items that need to be determined first. Some of these items are determined as the prescribed parameters according to the designer's intention. The others are then considered as the design variables during the design procedure. These items are

1. type of structure (frame, truss, shell, etc.)
2. topology and geometry of structure (configuration, that is, rectangular, trapezoid, triangular, the length of members, etc.)
3. proportioning of elements (wide flange section, channels, tubular section, etc.)
4. physical properties of material (mild steel, high tension steel, reinforced concrete, etc.)

If a large number of design variables are selected, the time required to obtain the optimum design will be relatively long even when the use of the electronic digital computer is available. Therefore, the design variables should be selected after prudent consideration.

In this thesis, the design variables are the cross sectional dimensions of wide flange beam, cover plate, and the ratios of cover plate length to member length. These variables are shown in Figure 1.
in which

\[ b_f = \text{width of flange} \]
\[ t_f = \text{thickness of flange} \]
\[ d_w = \text{depth of web} \]
\[ t_w = \text{thickness of web} \]
\[ b_p = \text{width of cover plate} \]
\[ t_p = \text{thickness of cover plate} \]
\[ r_i = \text{ratios of cover plate length to member length} \]

Figure 1. Design Variables
Whenever the term, design variables, is used, it will refer to those variables previously mentioned.

B. Analysis of Structure

When a statically indeterminate framed structure is composed wholly or partly of nonprismatic members, there are several methods that can be used to solve the structure. The well known moment distribution method is used in this thesis. The detailed computations prerequisite for moment distribution for an actual frame are quite different than those computation for a comparable frame composed entirely of prismatic members. Fixed-end moments, stiffnesses, and carry-over factors for nonprismatic members are evaluated by methods and formulas unlike those which are applicable to prismatic members. Also, the change in the stiffness at one end of nonprismatic member, when the far end is reduced from a fixed condition to a pinned condition, is not the same as in the case of a prismatic member. Methods and formulas\textsuperscript{9} necessary for computing these data are introduced in Appendix C. Program I in Appendix A illustrates the method of analysis for a typical example problem.

C. Allowable Stresses of Members

Any structural member under any kind of loading will be subjected to the following stresses:

a. pure bending stress
b. combined axial and bending stress
c. shear stress

All these stresses should lie within the allowable values of the specification and limitation of the code under consideration. These allowable stress limitations are treated as constants in this thesis. According to AISC specifications, the allowable stresses are defined as follows:

1. Pure Bending

   For compact and adequately braced members having an axis of symmetry in the plane of loading

   \[ F_b = 0.66F_y \]  \hspace{1cm} (1)

   where \( F_y \) is the yield strength of the material. If there is compression on extreme fibers of members having an axis of symmetry in the plane of their web, the larger value computed by formulas (2) and (3), but not more than 0.60\( F_y \) is used.

   \[ F_b = \left[ 1.0 - \frac{(l/r)^2}{2c_c c_b} \right] 0.60F_y \]  \hspace{1cm} (2)

   \[ F_b = \frac{12,000,000}{l d/A_f} \]  \hspace{1cm} (3)

   where

   \( l \) = unbraced length of the compression flange

   \( r \) = governing radius of gyration
\[ C_c = \text{column slenderness ratio dividing elastic and inelastic buckling equal to} \]

\[ C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \] (4)

c.f. if \( F_y = 36,000 \text{ psi}, C_c = 128.4 \)

\( E = \text{modulus of elasticity of steel} \)

\( C_b = \text{bending coefficient dependent upon moment gradient} \)

\( d = \text{depth of beam} \)

\( A_r = \text{area of compression flange} \)

2. Combined Axial and Bending

Members subjected to both axial compression and bending stress shall be proportioned by using the following interaction equations. The value of the interaction equation must be equal to or less than unity.

When \( \frac{f_a}{F_a} \leq 0.15 \),

\[ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \] (5)

When \( \frac{f_a}{F_a} > 0.15 \),

\[ \frac{f_a}{F_a} + \frac{C_m f_b}{(1.0 - \frac{f_a}{F_y})} \leq 1.0 \] (6)
and, if \( k\ell/r < C_c \),

\[
F_a = \frac{1.0 - (k\ell/r)^2/2C_c^2}{F.S.} F_y
\]  
(7)

If \( k\ell/r > C_c \),

\[
F_a = \frac{149,000,000}{(k\ell/r)^2}
\]  
(8)

\[
C_m = 0.6 - 0.4 \frac{M_1}{M_2} \text{ but not less than 0.4}
\]  
(9)

\[
F_e = \frac{149,000,000}{(\ell/r)^2}
\]  
(10)

\[
F.S. = \frac{5}{3} + \frac{3(k\ell/r)^2}{8C_c^2} - \frac{(k\ell/r)^3}{8C_c^3}
\]  
(11)

where

- \( F_a \) = axial stress that would be permitted if axial force alone existed
- \( F_b \) = compressive bending stress that would be permitted if bending moment alone existed
- \( f_a \) = computed axial stress
- \( f_b \) = computed bending stress at the point under consideration
- \( M_1, M_2 \) = smaller and larger end moment on unbraced length of beam-column, respectively
- \( k \) = effective length factor
3. Shear Stress

\[ F_y = 0.40F_y \]  \hspace{1cm} (12)

D. Formulation of Constraints and Requirements

In general, a design is called acceptable if it fulfills a set of design requirements. These requirements for the structure can be classified as follows:

1. Geometric Requirements

These requirements are usually arbitrary in character and can occur to meet architectural, mechanical, or durability requirements. They also frequently arise as a result of specification limitations. These requirements are that the sizes of certain elements in the structure should be greater than a specified minimum, or less than a specified maximum. Such limitations are necessary because it is possible that an optimizing process will cause certain elements to vanish or become very large. In this thesis the main geometric requirements are assumed to be similar to AISC specification requirements such as

\[
d_w \leq \frac{14 \times 10^6}{\sqrt{F_y(16500 + F_y)}} t_w
\]  \hspace{1cm} (13)

and

\[
b_f \leq \frac{6000t_f}{\sqrt{F_y}}
\]  \hspace{1cm} (14)
However, the AISC specifications do not cover all of the elements. In this case, with the designer's experience, new requirements on the elements can be inserted in the form of minimum or maximum elements sizes. For example, the flange thickness of a wide flange beam should be equal to or greater than five sixteenths of an inch and the width of the cover plate should be equal to or greater than ten times the thickness of the cover plate. The general form of the geometric requirements is

\[ x_{\text{min}} \leq x \leq x_{\text{max}} \]  

in which \( x \) is a design variable and the subscripts max and min refer to the upper and lower bound, respectively.

2. Stress Requirements

Provision for the bending stress and the combined axial and bending stress induced in a frame by applied loading is usually the primary criterion for the design of the structure. Subsequent to the selection of the individual members, the problems of shear and deflection may need to be considered.

a. Bending Stress Requirements

\[ \frac{M_n c_n}{I_n} \leq F_b \]  

in which

\( n = \text{key point at which stresses are checked} \)
\[ M_n = \text{the design moment at point n, expressed as a function of a design variable, } r_i \]

\[ M_n = M_n(r_i) \quad (17) \]

\[ I_n = \text{moment of inertia of beam section at point n, expressed as a function of} \]

\[ I_n = I_n(b_f, t_f, d_w, t_w, b_p, t_p) \quad (18) \]

\[ c_n = \text{distance from neutral axis to stressed fiber at point n, expressed as a function of} \]

\[ c_n = c_n(d_w, t_f, t_p) \quad (19) \]

b. Combined Stress Requirements

If we express the interaction equations in terms of design variables, equations (5) and (6) will be reduced to equations (20) and (21) respectively.

\[ \frac{P}{A_n F_a} - \frac{M_n c_n}{I_n F_b} \leq 1.0 \quad (20) \]

\[ \frac{P}{A_n F_a} - \frac{c_n M C_{n \infty}}{I_n (1.0 - P/A_n F')} F_b \leq 1.0 \quad (21) \]

in which

\[ P = \text{axial force} \]

\[ A_n = \text{cross sectional area at point n, expressed as a function of} \]
\[ A_n = A_n (b_f, t_f, d_w, t_w, b_p, t_p) \]  \hspace{1cm} (22)

c. Shearing Stress Requirements

\[ \frac{Q_n}{A_w} \leq F_v \]  \hspace{1cm} (23)

in which

\( Q_n \) = the design shear at point n

\( A_w \) = area of web at point n, expressed as a function of

\[ A_w = A_w (d_w, t_w) \]  \hspace{1cm} (24)

3. Deflection Constraints

The deflection at any point of the beam should be within the allowable limits. These allowable limits may depend on the length of the span and can be expressed in the form

\[ w_{i \text{ min}} \leq w_i \leq w_{i \text{ max}} \]  \hspace{1cm} (25)

in which

\( w_i \) = maximum deflection at some point along the \( i^{th} \) span

\( w_{i \text{ max}} \) = upper bound on the deflection in \( i^{th} \) span

\( w_{i \text{ min}} \) = lower bound on the deflection in \( i^{th} \) span

However, due to the capacity limitation of IBM 1620 digital computer, the constraints on deflection have not been considered in the numerical example.
E. The Objective Function of Structure

The objective function is defined as the function which represents the cost required for a structure. This is also called the cost function or the merit function. But if the real cost evaluation of the structure should be carried out, the cost of fabrication and erection of the structure should be considered as well as the material cost. Moreover, the evaluation from the aesthetic and functional points of view may sometimes have to be added, though it is very difficult to estimate them. In this thesis, the objective function considered is to minimize the total weight of structure.

Since design variables are the cross sectional dimensions of the member, the objective function, \( F(x) \), can be given by the following equation.

\[
F(x) = \text{Weight} = \rho \sum_{i=1}^{n_m} A_i \ell_i = \text{Minimum} \tag{26}
\]

in which

\( \ell_i = \) length of the \( i \)th member

\( n_m = \) number of members in the structure

\( \rho = \) density of the material

\( \rho \) (steel) = 490/1728 lb/in\(^3\)

If the total cost of a structure must be considered as the objective function, the previous equation would be replaced by the following equation (27).
\[ F(x) = \text{Cost} = \rho \sum_{i=1}^{n_m} A n_i C_i + C_o = \text{Minimum} \] (27)

in which

\[ C_i = \text{the cost of material, the cost of fabrication,} \]
\[ \text{the cost of erection, . . . , etc.} \]
\[ C_o = \text{the almost constant common expenses, i.e., design cost,} \]
\[ \text{connection of joints, . . . , etc.} \]

It should be noticed that minimizing the objective function given by equation (26) is equivalent to minimizing the total cost if this cost is closely approximated by equation (27).
CHAPTER III

ITERATIVE DESIGN PROCEDURE

A. The Optimum Design

The optimum design can be defined as the design which gives the best possible structure under various constraints. In other words, an optimum design leads to the final dimensions of the member which will satisfy all predicted restrictions and, at the same time, minimize the given objective function.

The weight of the structure and the constraints on the stresses and on the sectional properties are expressed as continuous functions of the design variables. The relationships between the values of the constraint equations and the design variables are usually nonlinear as is the relationship between the total weight of structure and design variables. In order to apply linear programming procedure for this optimum design, the nonlinear constraint equations and the objective function equation have to be linearized. The method pursued herein is akin to the cutting plane method.10 This method involves the approximation of the nonlinear equations by the linear terms of the Taylor series. The nonlinear programming problem has then been effectively converted to a linear programming problem which

is solved by the simplex method. The nonlinear relations are
reapproximated and the next approximately optimized structure
is computed. This sequence of design procedures will usually
converge to the optimal solution after the sequences of computations
have been repeated many times. The designer may accept any level
of approximation desired.

B. Linear Approximations to Nonlinear Equations by Cutting Plane
   Method

Because of nonlinearities which usually arise in even the
simplest of design situations, linearization of these equations should
be done with high approximation. One method to solve this problem
has been developed, whereby, a curve may be represented to a high
accuracy by a series of intersecting straight lines. Such lines
are illustrated in Figure 2. Then the linear program may give a
solution at a vertex which is the intersection of two linearized
representations of the same constraint. These multiple linear
representations of same nonlinear constraints are conveniently
obtained by using linear approximations based on the Taylor series
terms. If there are enough of these intersections, the point can be

11. S. I. Gass, Linear Programming, Methods and Applications,
    p. 50-69.

12. K. F. Reinschmidt, C. Allin Cornell, and J. F. Brotchie,
    "Iterative Design and Structural Optimization"; Journal of the
    Structural Division, American Society of Civil Engineers, Vol. 92,
    No. ST.6, December, 1966, p. 305.
found with a high accuracy. This technique is known as the cutting plane method. The objective function can be treated in a similar manner.

The linear terms of the Taylor series for a nonlinear function in n variables are

$$F(x) = F(x^0) + \frac{\partial F}{\partial x_1}(x^0)(x_1 - x_1^0) + \frac{\partial F}{\partial x_2}(x^0)(x_2 - x_2^0) + \cdots + \frac{\partial F}{\partial x_n}(x^0)(x_n - x_n^0) \quad (28)$$
The expansion point \( x^0 \) for the Taylor series in any design cycle is the solution to the linear programming problem which has been obtained in the previous cycle. The expansion point in the first cycle can be the trial design set by the designer.

C. Linear Programming with the Application of the Iterative Design Method

The general linear programming problem is to find a vector \((x_1, x_2, \ldots, x_j, \ldots, x_n)\) which will minimize the linear form of the objective function

\[
c_1 x_1 + c_2 x_2 + \ldots + c_j x_j + \ldots + c_n x_n
\]  

subject to the linear constraints

\[
x_j \geq 0 \quad \text{for} \quad j = 1, 2, \ldots, n
\]  

and

\[
\begin{array}{cccccc}
a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_{ij} & a_{ij} & \cdots & a_{ijn} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \\
\end{array}
\begin{array}{c}
x_1 \\
x_2 \\
\vdots \\
x_j \\
\vdots \\
x_n \\
\end{array}
\begin{array}{c}
b_1 \\
b_2 \\
\vdots \\
b_i \\
\vdots \\
b_m \\
\end{array}
\]  

\[
i = 1, 2, \ldots, n
\]
where the $a_{ij}$, $b_i$ and $c_j$ are given constants. The vector $x$ can be found by the simplex technique (see Appendix B for a numerical example).

The problem of structural optimization is then as follows:

minimize

$$F(x_j) \quad j = 1, 2, \ldots, n$$

subject to

$$G_i(x_j) \leq b_i \quad i = 1, 2, \ldots, m$$

and

$$x_j \geq L_j$$

$$x_j \leq U_j$$

in which $F(x_j)$ is the objective function which would be nonlinear; $G_i(x_j) \leq b_i$ represents the constraint equation, including nonlinear form; and $L_j$ and $U_j$ are lower and upper bounds, respectively, on the design variables.

When dealing with the linear programming problem, the inequalities of the $\geq$ type such as equation (34) are sometimes encountered. For the particular simplex linear programming algorithm employed in this thesis, it is necessary to change the form from the $\geq$ type to the $\leq$ type. This change is easily accomplished by changing all the signs in the inequality. This then produces negative elements for the right hand side of this inequality, which are in turn unacceptable for the same algorithm. In this case,
the situation is resolved by making a linear transformation of all design variables in the objective function and in the constraint set according to the following relation:\textsuperscript{13}

\[ x'_j = U_j - x_j \]  

(36)

The linear programming problem is then solved for the new variables \( x'_j \). The variables of interest, \( x_j \), are subsequently obtained with a reverse transformation. The \( U_j \) can usually be chosen so that the elements of \( b_1 \) vector are all positive. This transformation provides the designer with the convenient capability of being able to place an upper limit on any dimension.

After changing all these inequalities into the form of the \( \leq \) type, it is also required to change the inequation into the form of equality by adding a slack variable\textsuperscript{14} to each inequation. However, the slack variables have no meaning and should be eliminated before the final solution basis\textsuperscript{15} is determined. Therefore, it is recommended to assign a large value of coefficient to each of these slack variables in the objective function so that they will be driven


\textsuperscript{15}Ibid., p. 54.
out of the solution basis. A numerical example of the simplex linear programming algorithm appears in Appendix B with detailed explanation.

As mentioned before, equations (32) and (33) are usually described by a number of nonlinear equations expressed in terms of design variables. For the application of the linear programming method, a linearization is done at the \((x_j^0)\) by replacing nonlinear equations (32) and (33) by their linear first order Taylor series term. Thus, the optimization problem of minimizing the objective function is approximated by the following linear programming problem:

\[
\begin{align*}
\text{minimize} & \quad F(x_j^0) + \sum_{j=1}^{n} \frac{\partial F(x_j^0)}{\partial x_j} (x_j - x_j^0) \\
\text{subject to} & \quad \sum_{i=1}^{n} \frac{\partial G_i}{\partial x_j} (x_j - x_j^0) \leq b_i \\
& \quad x_j \geq L_j \\
& \quad x_j \leq U_j
\end{align*}
\]

From equations (37), (38), (39) and (40), the sensitivity coefficients \(a_{ij}, b_i\) and \(c_j\) must be calculated in order to apply the linear programming problem.
The solution from the first cycle is a point \( [X'] \) giving design variables that minimize equation (37) and satisfy equations (38), (39) and (40), but it does not necessarily satisfy the original equation (33). In structural applications, the new design variables given by \( [X'] \) are retained and the true design variables associated with these new design variables are computed by solving equation (33). A new point \( [X''] \) is thus found from the second cycle that differs from \( [X'] \) in the value of the design variables.

If any of the design variables are violating the constraints of equations (34) and (35), then their values are increased or decreased throughout the several cycles until the violations are eliminated. Thus, a final point \( [X^k] \) from the \( k^{th} \) cycle may be obtained, which satisfies equations (33), (34) and (35). Therefore, it can be considered an acceptable design point. The obtained point \( [X^k] \) is checked to see if this point can be considered to have reached the optimized point or not. When the differences of the design variables are smaller than the values which are considered as the conditions for convergence, the computations are completed and the results are printed out. If these results are not satisfactory, the above sequence of computations will be repeated after substituting new design variables. At each cycle, the new design variables are those obtained from the previous cycle.

If the relationships between the design variables and the constraint equations or the objective function were linear, the linear programming procedure would derive the exact optimized point
in the first iteration set. But since most of the structural optimizing problems are nonlinear, the solutions of the linear programming technique as applied are always approximate ones. There is the reason why iterative design method is used. As the new initial design variables become close to the optimized point, changes will be smaller and the approximations to the real constraint equations will be better. Repeating this sequence, the optimized point will be generally obtained. The flow chart shown in Figure 3 illustrates this sequence.

D. Aids for the Convergence

1. Bounds

Bounds are defined as the restrictions on the design variables. In general the bounds have two main purposes:

a. Practicality:

The cross sectional dimensions of the members should be within the practical values and limits given by the current codes and specifications.

b. Convergence of the iteration:

Since the linear approximations to the nonlinear restrictions are used (Figure 4), it is quite possible that the design point obtained may not satisfy some of the given nonlinear constraint equations. These kinds of problems usually happen during the optimizing process. In this case, tighter restrictions on the
Figure 3. Flow Chart of Iterative Design Sequence
Figure 1. Approximately Optimized Point
upper or lower bounds of the design variables should be properly given by the designer, thereby restricting the movements of the design variables. Otherwise the convergence may not be reached even after a large number of cycles and the approximated optimum point may be far from the feasible region.

2. Iteration Determination

When an iteration method is employed, certain conditions on the judgment need to be set. These conditions indicate when the results obtained are considered satisfactory. These conditions also have a great effect on the computation time and the preciseness of the solution. Consequently, these conditions have to be determined after some consideration.

In this thesis, the solution was considered to be satisfactory and the answers were printed out, when the change of design variables obtained in two successive cycles was less than 0.001. At this point the iteration was terminated.
A. The Problem

It is required to design a one-bay, one-story framed structure with cover plates as shown in Figure 5. The entire structure would be built of structural steel plates welded by E-60 electrodes. A nonlinear programming model would be constructed and used to determine all cross sectional dimensions of the beam, column and cover plate, as well as the cover plate length and position, thus minimizing the weight of structure. The design should conform with the AISC specifications. It would be assumed that the structure is subjected to a uniformly distributed load and has full lateral support.

B. Problem Solution

The objective function to be minimized for the stated problem would be in the form:

\[
    f(x) = \text{Weight} = \left[ (2b_1^p t_f + d_w t_w) (l + 2h) \right. \\
    + \left( 2b_2^p t_p \right) (2r_1 l + r_2 l + 2r_3 h) \right] \cdot \sqrt{\cdot} \quad (41)
\]

The constraint equations are formulated according to the method explained in Chapter II. The constraints on the stresses are (see Figures 5 and 6):
Figure 5. Frame with Cover Plates

Figure 6. Moment Diagram
1. Bending stresses

\[ \frac{M_B c_o}{I_o} \leq F_b \]  (42)

in which

\[ c_o = \frac{d_w}{2} + t_f + t_p \]

\[ I_o = \frac{t_w d_w^3}{12} + 2b_t t_f \left( \frac{d_w + t_f}{2} \right)^2 + 2b_p t_p \left( \frac{d_w + t_p}{2} + t_f \right)^2 \]

\[ \frac{M_F c_w}{I_w} = \frac{\left[ \frac{v l^2}{2} \left( r_1 - r_1^2 \right) - K_B \right] c_w}{I_w} \leq F_b \]  (43)

in which

\[ c_w = \frac{d_w}{2} + t_f \]

\[ I_w = \frac{t_w d_w^3}{2} + 2b_t t_f \left( \frac{d_w + t_f}{2} \right)^2 \]

\[ \frac{M_G c_w}{I_w} = \frac{\left[ \frac{v l^2}{8} \left( 1 - R_2^2 \right) - K_B \right] c_w}{I_w} \leq F_b \]  (44)

\[ \frac{M_H c_o}{I_o} = \frac{\left( \frac{v l^2}{8} - K_B \right) c_o}{I_o} \leq F_b \]  (45)

2. Combined axial and bending stress (refer to equations (20) and (21))

\[ \frac{P}{f_a (2b_t t_f + d_w t_w)} + \frac{N_B c_w}{F_b I_w} \leq 1.0 \]  (46)
3. Shear stress

\[
\frac{w_0}{2c w t_w} \leq F_V \tag{48}
\]

Also the constraints on the geometric requirements are expressed as follows:

\[
10 t_f \leq b_f \leq \frac{6,000 t_f}{\sqrt{t_f}} \tag{49}
\]

\[
t_f \geq \frac{5}{16} \tag{50}
\]

\[
d_{w \text{ low}} \leq d_w \leq \frac{(14 \times 10^6) t_w}{\sqrt{F_y (16,500 - F_y)}} \tag{51}
\]

\[
t_{w} \geq \frac{5}{16} \tag{52}
\]

\[
b_p \geq 10 t_p \tag{53}
\]

\[
t_p \geq \frac{5}{16} \tag{54}
\]

The computations of the required sensitivity coefficients of \( a_{ij} \) and \( b_i \) are shown within Program I which accompanies the
linearization of the nonlinear expressions (42) through (48). These particular statements also include the linear transformation described in equation (36). The change of the sign in $a_{ij}$ matrix created by the transformation is subsequently made by statement 309 in Program I.

When the objective function given by equation (41) is linearized and all extraneous constant terms are dropped, the objective function appears as

$$F(x) = 2t_f(\ell + 2h)b_f + 2b_f(\ell + 2h)t_f + t_w(\ell + 2h)d_w$$

$$+ d_w(\ell + 2h)t_w + 2t_p(2r_1\ell + r_2\ell + 2r_3h)b_p$$

$$+ 2b_p(2r_1\ell + r_2\ell + 2r_3h)t_p + 4b_p r_1 + 2b_p r_2$$

$$+ 4hb_p r_3$$

Equation (55) is used only in solution of the linear programming problem. The actual weight of the structure is computed from equation (41).

The linear transformation of variables must also be applied to the objective function (Equation (55)) as well as to the constraints before the simplex linear programming routine is implemented. The negative coefficients for $c_j$ are then computed in statement 301 in Program I. The slack variables which were added to each constraint appear as a unit matrix in statement 545 in Program II.
**TABLE I. OBTAINED DESIGN VARIABLES**

<table>
<thead>
<tr>
<th>CYCLE</th>
<th>BF (IN)</th>
<th>TF (IN)</th>
<th>DW (IN)</th>
<th>TW (IN)</th>
<th>BP (IN)</th>
<th>TP (IN)</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
</tr>
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<tr>
<td>1 CYCLE</td>
<td>5.00</td>
<td>0.50</td>
<td>32.00</td>
<td>0.50</td>
<td>6.00</td>
<td>0.60</td>
<td>0.100</td>
<td>0.300</td>
<td>0.300</td>
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<td>0.31</td>
<td>30.21</td>
<td>0.31</td>
<td>3.13</td>
<td>0.31</td>
<td>-0.002</td>
<td>0.270</td>
<td>0.088</td>
</tr>
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<td>0.31</td>
<td>27.29</td>
<td>0.31</td>
<td>10.618</td>
<td>0.31</td>
<td>0.022</td>
<td>0.500</td>
<td>0.110</td>
</tr>
<tr>
<td>4 CYCLE</td>
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<td>0.31</td>
<td>28.23</td>
<td>0.31</td>
<td>10.580</td>
<td>0.31</td>
<td>0.016</td>
<td>0.500</td>
<td>0.151</td>
</tr>
<tr>
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<td>0.31</td>
<td>27.94</td>
<td>0.31</td>
<td>10.708</td>
<td>0.31</td>
<td>0.018</td>
<td>0.500</td>
<td>0.121</td>
</tr>
<tr>
<td>6 CYCLE</td>
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<td>0.31</td>
<td>28.21</td>
<td>0.31</td>
<td>10.608</td>
<td>0.31</td>
<td>0.016</td>
<td>0.500</td>
<td>0.123</td>
</tr>
<tr>
<td>7 CYCLE</td>
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<td>0.31</td>
<td>28.62</td>
<td>0.31</td>
<td>10.458</td>
<td>0.31</td>
<td>0.012</td>
<td>0.500</td>
<td>0.074</td>
</tr>
<tr>
<td>8 CYCLE</td>
<td>9.88</td>
<td>0.31</td>
<td>29.25</td>
<td>0.31</td>
<td>10.238</td>
<td>0.31</td>
<td>0.007</td>
<td>0.500</td>
<td>0.012</td>
</tr>
<tr>
<td>9 CYCLE</td>
<td>9.88</td>
<td>0.31</td>
<td>29.25</td>
<td>0.31</td>
<td>10.238</td>
<td>0.31</td>
<td>0.007</td>
<td>0.500</td>
<td>0.012</td>
</tr>
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<td>CYCLE</td>
<td>FBB (KSI)</td>
<td>FBF (KSI)</td>
<td>FBG (KSI)</td>
<td>FBH (KSI)</td>
<td>INTER-*</td>
<td>SHEAR (KSI)</td>
<td>WEIGHT (LBS)</td>
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<td></td>
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<td>21.595</td>
<td>15.382</td>
<td>0.707</td>
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<td>26.410</td>
<td>27.535</td>
<td>26.673</td>
<td>1.000</td>
<td>9.533</td>
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<td></td>
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<tr>
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<td>21.029</td>
<td>22.869</td>
<td>22.710</td>
<td>1.042</td>
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</tr>
<tr>
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<td>21.654</td>
<td>0.975</td>
<td>10.201</td>
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<td></td>
<td></td>
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<tr>
<td>5 CYCLE</td>
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<td>22.108</td>
<td>21.985</td>
<td>1.000</td>
<td>10.310</td>
<td>4027.18</td>
<td></td>
<td></td>
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<tr>
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<td>21.892</td>
<td>21.862</td>
<td>0.995</td>
<td>10.062</td>
<td>4017.48</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>21.726</td>
<td>21.901</td>
<td>21.872</td>
<td>1.000</td>
<td>10.003</td>
<td>3994.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* INTERACTION MEANS THE VALUES OF THE INTERACTION EQUATION OF MEMBER, ACCORDINGLY, IT MUST NOT BE OVER UNITY.

** THE VALUE OF WEIGHT IS NEGLECTED BECAUSE OF THE NEGATIVE VALUE OF R1.
All computations mentioned above have been carried out on an IBM 1620 computer available at the university computing center. The input consisted of the following data:

- $l = 360.00$ in.
- $w = 500.00$ lbs/in.
- $E = 29 \times 10^6$ psi
- $n = 9$
- $m = 14$

- upper limit of $b_r = 18.00$ in.
- upper limit of $d_w = 65.00$ in.
- upper limit of $b_p = 14.00$ in.
- upper limit of $r_1 = 0.20$
- upper limit of $r_3 = 0.70$

and the following values were assigned as the trial design values at the first cycles:

- $b_r = 5.00$ in.
- $d_w = 10.00$ in.
- $b_p = 6.00$ in.
- $r_1 = 0.10$
- $r_3 = 0.20$

- $t_r = 0.50$ in.
- $t_w = 0.50$ in.
- $t_p = 0.60$ in.
- $r_2 = 0.30$

The final optimum solution was obtained after 9 cycles of iteration. Tables I and II gives the successive results obtained from each cycle.

*Difference in a variable between successive cycles*
CHAPTER V

SUMMARY AND CONCLUSIONS

A. General Summary

The optimum design of a framed structure using wide flange sections has been developed in this thesis by using simplex linear programming and an iterative method. The entire process was started by assuming some initial trial values first. The nonlinear equations were then approximated by linear equations. After establishing the objective function and the constraints, the problem became analogous to a linear programming problem. By using the simplex method, a solution was obtained. This solution minimized the objective function subject to the linearized equations. The new design was used as a reference point for another linearization. Additional cycles of linearization and simplex programming yielded a solution that converged to the optimum design. The entire process mentioned above was automatically carried on a digital computer (see Programs I and II in Appendix A).

The following conclusions may be drawn:

1. The mere formulation of the problem provides several benefits for the engineer. It impresses on him the role of specifications as a restraining influence on imprudent design, and at the same time forcefully establishes the concept of the existence of an optimal condition from among many possibilities.
2. The method of iterative design in obtaining an optimum solution to the indeterminate, elastic structures can be considered as a powerful tool. The convergence may be slow but stable. In the stated problem, the bounds were the most effective factors for convergence. Whenever the convergence was not reached after a large number of cycles, tighter restrictions were applied to the upper or lower limits of the design variables.

3. The cutting plane method appeared to be advantageous for choosing the design variables that tend to reduce linearization errors in the constants, even at the expense of increased nonlinearity in the objective function.

4. Numerical results showed that the optimum solution for a statically indeterminate structure did not necessarily correspond to the minimum weight or maximum load carrying capacity in each member.

5. Since the IBM 1620 digital computer had limited core memory, the size of the structure to be solved was limited. As the simplex algorithm program occupied a relatively large number of locations in the core memory of any computer, it is highly recommended to use a computer with a large storage capacity. A large computer will enable the designer to include all possible constraints and to obtain a more accurate solution.
6. The application of the foregoing technique for optimization seems also feasible for other structures such as trusses, bridges, dams and airplanes.

B. Future Areas of Study and Research

1. Further efforts are needed to develop applications for buckling and nonlinear load-deflection problems.

2. A mathematical proof showing that successive cycles of redesign will converge to an absolute optimum solution is required. Such a proof must be based on the characteristics of the equations describing the design variables.

3. Recently several types of high strength steels have been developed and made available to the structural engineer at a relatively small premium. An optimum design can be obtained by using these different steels at the different places and locations in the structure under consideration. In this case, materials might be considered as the design variables and the cost of structure should be estimated instead of the weight.
NOTATION

The following symbols have been adopted for use in this thesis:

\( A_f \) = area of compression flange
\( a_{ij} \) = coefficient matrix for left side of constraint set
\( A_n \) = cross section area at point \( n \)
\( A_w \) = area of web
\( b_f \) = width of flange
\( b_i \) = column matrix of elements on right side of constraint set
\( b_p \) = width of cover plate
\( C_b \) = bending coefficient
\( C_c \) = column slenderness ratio dividing elastic and inelastic buckling
\( C_i \) = the cost of material, the cost of fabrication, the cost of erection, ... , etc.
\( c_j \) = objective function coefficient
\( C_m \) = coefficient applied to bending term in interaction formula
\( c_n \) = distance from neutral axis to stressed fiber at point \( n \)
\( C_o \) = the constant common expenses, i.e., design cost, connection of joints, ... , etc.
\( d \) = depth of beam
\( d_v \) = depth of web
\( d_{v, low} \) = lower limit on web depth
E = modulus of elasticity of steel
F_a = axial stress that would be permitted if axial force alone existed
f_a = computed axial stress
F_b = compressive bending stress that would be permitted if bending moment alone existed
f_b = computed bending stress at the point under consideration
F'_e = Euler stress divided by factor of safety
F.S. = factor of safety
F_v = allowable shear stress
F_y = yield strength of material
h = height of frame
I_n = moment of inertia of beam section at point n
k = effective length factor
\ell = length of member
L_j = lower bound on variable \( x_j \)
M_n = design moment at point n
\( M_1, M_2 \) = smaller and larger end moment on unbraced length of beam-column respectively
n = key point at which stresses are checked
P = axial force
per = difference in a variable between successive cycles
C_n = design shear at point n
r = governing radius of gyration
\( r_i = \text{ratios of cover plate length to member length} \)
\( t_f = \text{thickness of flange} \)
\( t_p = \text{thickness of cover plate} \)
\( t_w = \text{thickness of web} \)
\( U_j = \text{upper limit on variable } x_j \)
\( w = \text{external uniform load} \)
\( W_i = \text{maximum deflection at some point along the } i^{th} \text{ span} \)
\( x_j = \text{design variables} \)
\( \rho = \text{density of the material} \)
BIBLIOGRAPHY


APPENDIX A

COMPUTER PROGRAMS


PROGRAM I

C A FORTRAN II PROGRAM FOR ANALYZING ONE-BAY ONE-STORY
C NONPRISOMATIC FRAMED STRUCTURE AND GENERATING
C SENSITIVITY COEFFICIENTS A(I,J), B(I) AND C(J) FOR
C LINEAR PROGRAMMING IN TERMS OF TRANSFORMED VARIABLES
C UNITS USED IN THIS PROGRAMMING ARE POUND AND INCH
C N IS THE NUMBER OF DESIGN VARIABLES
C M IS THE NUMBER OF CONSTRAINT EQUATIONS
C DIMENSION A(20,20), B(20), C(30), EM(30)
23 FORMAT (5E16.4)
1001 FORMAT (5E16.8)
2001 FORMAT (2F4)
5003 READ 1001, XL, H, W, FY, PER, EX
READ 1001, UBF, UTF, UDW, UTW, UBP, UTP, UR1, UR2, UR3, DWL
READ 1001, YBF, YTF, YDW, YTW, YBP, YTP, YR1, YR2, YR3
READ 2001, N, M
BF = YBF
TF = YTF
DW = YDW
TW = YT W
BP = YBP
TP = YTP
R1 = YR1
R2 = YR2
R3 = YR3
ABC = FY / 1.65
SHE = 0.4 * FY
P = W * XL / 2.
CC = SQRT (19.7 * EX / FY)
C INDETERMINATE ANALYSIS FOR MOMENTS AND STRESSES
31 WI = (TW * DW * 3 / 12.) + 2. * BF * TF * ((DW + TF) / 2.) ** 2
CI = WI + 2. * BP * TP * (DW / 2. + TF + TP / 2.) ** 2
X1 = R1 * XL
X2 = XL / 2. - R2 * XL / 2.
X3 = (XL + (R2 * XL)) / 2.
X4 = XL - (R1 * XL)
Y1 = H - (R3 * H)
FSB = 2. * XI / CI + 2. * (X2 - X1) / WI + R2 * XL / CI
SO = (X4 ** 2 - X3 ** 2) / WI + (XL ** 2 - X4 ** 2) / CI
SNB = (X1 ** 2 / CI) + (X2 ** 2 - X1 ** 2) / WI + (X3 ** 2 - X2 ** 2) / CI + SO
C / 2.
TO = (X4 ** 3 - X3 ** 3) / WI + (XL ** 3 - X4 ** 3) / CI
TRB = ((X1 ** 3 / CI) + (X2 ** 3 - X1 ** 3) / WI + (X3 ** 3 - X2 ** 3) / CI + TO
C / 3.
FEM = W * (XL * SNB - TRB) / (2. * FSB)
CBC = (XL * SNB / TRB) - 1.
SBC = 1. / (FSB - (1. + CBC) * SNB / XL))

-CONTINUE-
\[
\begin{align*}
FSC &= R3H/C1 + (Y1/W1) \\
SNCB &= ((R3H)**2/C1 + (H**2 - (R3H)**2)/W1) / 2. \\
SNCA &= (Y1**2/W1 + (H**2 - Y1**2)/C1) / 2. \\
TRCB &= ((R3H)**3/C1 + (H**3 - (R3H)**3)/W1) / 3. \\
TRCA &= (Y1**3/W1 + (H**3 - Y1**3)/C1) / 3. \\
CBA &= (H*SNCB/TRCB) - 1. \\
CAB &= (H*SNCA/TRCA) - 1. \\
SBA &= 1.0/(FSC - ((1 + CBA)*SNCB/H)) \\
SBAA &= SBA*(1.0 - CBA*CAB) \\
SST &= SBC + SBAA \\
DFB &= SBC/SST \\
DFC &= SBAA/SST \\
UBM &= FEM*DFC \\
FM &= 0.0 \\
27 \text{ DO } 48 \text{ I} = 1, 30 \\
EM(1) &= UBM*(DFB**I)*(CAB**I) \\
FM &= FM + EM(1) \\
\text{IF}(EM(1) - 0.01)20, 20, 48 \\
48 \text{ CONTINUE} \\
\text{GO TO} 27 \\
20 \text{ XMB} &= ABSF(UBM + FM) \\
XME &= ABSF(XMB*(1.0 - R3)) \\
XMF &= ABSF(((W*XL**2)*(R1 - R1**2)/2.) - XMB) \\
XMG &= ABSF(((W*XL**2)*(1.0 - R2**2)/8.) - XMB) \\
XMH &= ABSF(W*XL**2/8. - XMB) \\
FBB &= XMB*(DW/2.0 + TF + TP)/C1 \\
FBF &= XMF*(DW/2.0 + TF)/WI \\
FBG &= XMG*(DW/2.0 + TF)/WI \\
FBH &= XMH*(DW/2.0 + TF + TP)/C1 \\
FVV &= P/(DW*TW) \\
Q &= 2.0*SF*TF+DW*TW \\
AX &= P/Q \\
BX &= XME*(DW/2.0 + TF)/WI \\
RX &= SQRTF(WI/Q) \\
SL &= 2.0*H/RX \\
FE &= 149.0E6/SL**2 \\
FS &= 1.067*(0.375*SL/CC) - (0.125*SL**3/(CC**3)) \\
\text{IF}(SL - CC)172, 172, 173 \\
172 \text{ FA} &= (1.0 - (SL**2/(2.0*CC**2)))*FY/FS \\
\text{GO TO} 142 \\
173 \text{ FA} &= 149.0E6/(SL**2) \\
C \text{ ADJUST VARIABLES TO REDUCE STRESSES TO ALLOWABLE RANGE} \\
142 \text{ IF}(AX/FA - 0.15)145, 145, 146 \\
145 \text{ FOR} &= AX/FA + BX/ABC \\
\text{GO TO} 272 \\
146 \text{ FOR} &= AX/FA + (0.6*BX/(ABC*(1.0 - AX/FE))) \\
272 \text{ IF}(FBG - ABC)33, 33, 62 \\
33 \text{ IF}(FBG - ABC)34, 34, 62 \\
\text{ -CONTINUE-}
34  IF(FBB-ABC) 36, 36, 62
36  IF(FBF-ABC) 74, 74, 62
74  IF(FVV-SHE) 35, 35, 62
62  DW=DW+0.25
    GO TO 31
35  IF(FOR-1.) 37, 37, 28
28  IF(R3-1.) 63, 62, 62
63  R3=R3+0.01
    GO TO 31
37  NM=NM+M

C  COMPUTATION OF OBJECTIVE FUNCTION COEFFICIENTS
C  INITIALIZE AT LARGE VALUES TO PREVENT SLACK VARIABLES
C  FROM ENTERING INTO THE SOLUTION BASIS
D0 301 J=1,NM
301  C(J)=1.E+12
    C(1)=-4.*TF*(0.5*X+H)
    C(2)=-4.*BF*(0.5*X+H)
    C(3)=-Tw*(0.5*X+H)*2.
    C(4)=-DW*(0.5*X+H)*2.
    C(5)=-4.*TP*(R1*X+0.5*R2*X+R3*H)
    C(6)=-4.*BP*(R1*X+0.5*R2*X+R3*H)
    C(7)=-4.*XL*BP*TP
    C(8)=-XL*BP*TP*2.
    C(9)=-4.*H*BP*TP
    D1=Tw*DW**3/12.
    D2=DW/2.+TF
    D3=(DW+TF)/2.
    D4=2.*BF*TF*D3*D3
    D5=D1+D4
    D6=(DW+TP)/2.+TF
    D7=W*X*X/8.
    D8=D2+TP
    D9=2.*BP*TP*D6*D6
    D10=D5+D9
    CH=P/(FA*Q*Q)
    CO=0.6*XME*FE/ABC
    CG=FE*Q-P

C  COMPUTATION OF A(I,J) COEFFICIENT MATRIX
D0 302 I=1,M
B(I)=0.
D0 302 J=1,N
302  A(I,J)=0.
    A(1,1)=-XMB*D8*D4/(BF*D10*D10)
    A(1,2)=XMB*(1./D10-D8*(D4/TF+D4/D3+2.*D9/D6))
    C(D10*D10))
    A(1,3)=XMB*(0.5/D10-D8*(Tw*DW*Dw/4.*D4/D3+D9/D6))
    C(D10*D10))
    A(1,4)=-XMB*D8*D1/(Tw*D10*D10)
    -CONTINUE-
A(1,5) = -XMB*D8*D9/(BP*D10*D10)
A(1,6) = XMB*(1/D10-D8*(D9/TP+D9/D6)/(D10*D10))
A(2,1) = -XMF*D2*D4/(BF*D5*D5)
A(2,2) = XMF*(1/D5-D2*(D4/TF+D4/D3)/(D5*D5))
A(2,3) = XMF*(0.5/D5-D2*(TW*Dw*Dw/4*D4/D3)/(D5*D5))
A(2,4) = -XMF*D2*D1/(TW*D5*D5)
A(2,7) =ABSF(4*7*(1-2*R1)*D2/D5)
A(3,1) = -XMG*D2*D4/(3F*D5*D5)
A(3,2) = XMG*(1/D5-D2*(D4/TF+D4/D3)/(D5*D5))
A(3,3) = XMG*(0.5/D5-D2*(TW*Dw*Dw/4*D4/D3)/(D5*D5))
A(3,4) = -XMG*D2*D1/(TW*D5*D5)
A(3,8) = -ABSF(2*7*R2*D2/D5)
A(4,1) = -XMH*D8*D4/(BF*D10*D10)
A(4,2) = XMH*(1/D10-D8*(D4/TF+D4/D3+2*D9/D6)/
           C(D10*D10))
A(4,3) = XMH*(0.5/D10-D8*(TW*Dw*Dw/4*D4/D3+D9/D6)/
           C(D10*D10))
A(4,6) = -XMH*D8*D1/(TW*D10*D10)
A(4,5) = -XMH*D8*D9/(BP*D10*D10)
A(4,6) = XNM*(1/D10-D8*(D9/TP+D9/D6)/(D10*D10))
IF (AX/FA=0.15) 443, 443, 444
443 A(5,1) = -2*CH*TF-(XME*D2*D4)/(ABC*BF*D5*D5)
A(5,2) = -2*BF*CH+(XME/ABC)*(1/D5-D2*(D4/TF+D4/D3)/
           C(D5*D5))
A(5,3) = -TW*CH+(XME/ABC)*(0.5/D5-D2*(TW*Dw*Dw/4*D4/
           CD3)/(D5*D5))
A(5,4) = -Dw*CH+(XME/ABC)*D1*D2/(TW*D5*D5))
A(5,9) = -XMB*D2/(ABC*D5)
GO TO 474
444 A(5,1) =-2*CH*TF+CO*((2*TF*D2/(D5*CG))-(2*TF*FE*
           CD5+(D4*CG/3BF)))/(D2*Q)/(CG*CG*D5*D5))
A(5,2) = -2*CH*BF+CO*((Q+2*D2*BF)/(CG*D5)-D2*Q*(2*
           C*BF*FE*D5+CG*1*D4/TF+D4/D3))/(CG*CG*D5*D5))
A(5,3) = -CH*TW+CO*((0.5*Q+D2)/(CG*D5)-D2*Q*(FE*D5*TW+
           CCG*(TW*Dw*Dw/4*D4/D3))/(CG*CG*D5*D5))
A(5,4) = -CH*Dw+CO*((D2/(CG*D5)-D2*Q*(FE*D5*Dw+CG*D1/
           CTw)/(CG*CG*D5*D5))
A(5,9) = -0.6*FE*XMB*D2*Q/(ABC*CG*D5)
474 A(6,2) = -1*
A(7,4) = -1*
A(8,6) = -1*
A(9,1) = 1*
A(9,2) = -6000/SQRT(FY)
A(10,3) = 1*
A(10,4) = -14*E6/SQRT(FY*(16500+S*FY))
A(11,1) = -1*
A(11,2) = 10*
A(12,3) = -1*
-CONTINUE-
\[ A(13,5) = -1 \]
\[ A(13,6) = 10 \]
\[ A(14,3) = -P \cdot TW / (DW \cdot TW)^2 \]
\[ A(14,4) = -P \cdot DW / (DW \cdot TW)^2 \]

**C**  
COMPUTATION OF CONSTANT TERMS IN TAYLOR SERIES  
\[ G_1 = XMB \cdot D_8 / D_{10} - ABC \]
\[ G_2 = XMF \cdot D_2 / D_5 - ABC \]
\[ G_3 = XMG \cdot D_2 / D_5 - ABC \]
\[ G_4 = XMH \cdot D_8 / D_{10} - ABC \]
\[ G_5 = \text{FOR} - 1 \]

**C**  
COMPUTATION OF COEFFICIENTS OF \( B(I) \) VECTOR  
\[ B(1) = -G_1 + A(1,1) \cdot (BF - UBF) + A(1,2) \cdot (TF - UTF) + A(1,3) \cdot (DW - UDW) + A(1,4) \cdot (TW - UTW) + A(1,5) \cdot (BP - UBP) + A(1,6) \cdot (TP - UTP) \]
\[ B(2) = -G_2 + A(2,1) \cdot (BF - UBF) + A(2,2) \cdot (TF - UTF) + A(2,3) \cdot (DW - UDW) + A(2,4) \cdot (TW - UTW) + A(2,5) \cdot (R_1 - UR_1) \]
\[ B(3) = -G_3 + A(3,1) \cdot (BF - UBF) + A(3,2) \cdot (TF - UTF) + A(3,3) \cdot (DW - UDW) + A(3,4) \cdot (TW - UTW) + A(3,5) \cdot (R_2 - UR_2) \]
\[ B(4) = -G_4 + A(4,1) \cdot (BF - UBF) + A(4,2) \cdot (TF - UTF) + A(4,3) \cdot (DW - UDW) + A(4,4) \cdot (TW - UTW) + A(4,5) \cdot (BP - UBP) + A(4,6) \cdot (TP - UTP) \]
\[ B(5) = -G_5 + A(5,1) \cdot (BF - UBF) + A(5,2) \cdot (TF - UTF) + A(5,3) \cdot (DW - UDW) + A(5,4) \cdot (TW - UTW) + A(5,9) \cdot (R_3 - UR_3) \]
\[ B(6) = UTF - 0.3125 \]
\[ B(7) = UTF - 0.3125 \]
\[ B(8) = UTF - 0.3125 \]
\[ B(9) = A(9,2) \cdot (-UTF) - UBF \]
\[ B(10) = A(10,4) \cdot (-UTW) - UDW \]
IF (UBF = 10.3125) 303, 304, 305

303 UTF = UBF / 10.
304 IF (UBF = UTF) 305, 306, 307
305 UTP = UBP / 10.

307 B(11) = UBF - 10.3125
308 B(12) = UDW - DWL
309 B(13) = UBP - 10.3125
310 B(14) = -(FV - SHE) + A(14,3) \cdot (DW - UDW) + A(14,4) \cdot (TW - UTW)

DO 309 I = 1, M
DO 309 J = 1, N

309 A(I, J) = -A(I, J)
I = 1

3242 IF (B(I)) 3237, 3238, 3238
3237 PRINT 3002
3002 FORMAT(47HNEGATIVE ELEMENT IN B VECTOR: CHECK INPUT CDA)
GO TO 5003
3238 IF (I - M) 3992, 3999, 3999
3992 I = I + 1
GO TO 3242
3999 CONTINUE
P(1) = BF

---CONTINUE---
P(2) = TF
P(3) = DW
P(4) = TW
P(5) = BP
P(6) = TP
P(7) = R1
P(8) = R2
P(9) = R3
PRINT 1001, FBB, FBF, FBG, FBH, FOR, FVV
PUNCH 1001, BF, TF, DW, TW, BP, TP, R1, R2, R3
PUNCH 1001, UBF, UTF, UDW, UTW, UBP, UTP, UR1, UR2, UR3, DWL
PUNCH 23, ((A(I, J), J=1, N), I=1, M)
PUNCH 23, (B(I), I=1, M)
PUNCH 23, (C(J), J=1, NM)
PUNCH 23, (P(I), I=1, N)
STOP
END
PROGRAM II

FORTRAN II SIMPLEX PROGRAM OF LINEAR PROGRAMMING
FOR OPTIMIZING THE FRAMED STRUCTURE

DIMENSION A(20,20), B(20), C(30), P(30), PP(30), IBA(30)
DIMENSION TAB(30,30)

505 FORMAT (5E16.4)
506 FORMAT (5E16.8)
507 FORMAT (6HWEIGHT F10.3)
604 FORMAT (2E16.4)
722 FORMAT (2I4)

READ 506, XL, H, W, FY, PER
READ 506, 3F, TF, DW, TW, BP, TP, R1, R2, R3
READ 506, UBF, UTF, UDW, UTW, UBP, UTP, UR1, UR2, UR3, DWL
READ 722, N, M
READ 505, ((A(I, J), J=1, N), I=1, M)
READ 505, (B(I), I=1, M)
READ 505, (C(J), J=1, NM)
READ 505, (P(I), I=1, N)

SETTING-UP OF THE MATRIX FOR SIMPLEX TABLEAU

NM = N + M
MP1 = M + 1
NCOL = NM + 1

DO 201 I = 1, MP1
  IBA(I) = 0
  PP(I) = 0.
  DO 202 J = 1, NCOL
    TAB(I, J) = 0.
  DO 203 J = 1, N
    TAB(I, J) = A(I, J)
  201

INSERT SLACK VARIABLES IN EACH CONSTRAINT EQUATION

NP1 = N + 1
545 TAB(I, NP1) = 1.
TAB(I, NCOL) = B(I)

202 IBA(I) = N + I
DO 204 J = 1, NM
204 TAB(MP1, J) = -C(J)
205 PLE = TAB(MP1, 1)
K = 1

CHOOSE THE MOST POSITIVE VALUE TO INSERT IN THE
SOLUTION BASIS

DO 206 J = 2, NM
  IF(TAB(MP1, J) - PLE) > 0.06, 0.06, 0.07
207 PLE = TAB(MP1, J)
K = J
206 CONTINUE
IF(PLE) > 0.08, 0.08, 0.09

--CONTINUE--
208 DO 210 I=1,M
   IIBA=IBA(I)
210 PP(IIBA)=TAB(I,NCOL)
   GO TO 154
209 I=1
215 IF(TAB(I,K))212,212,211
212 IF(I-M)214,213,213
214 I=I+1
   GO TO 215
213 PRINT 299
299 FORMAT(29H OBJECTIVE FUNCTION UNBOUNDED)
   GO TO 154
C
SELECT THE LEAST POSITIVE RATIO OF B(I) TO A(I,K)
211 RATIO=TAB(I,NCOL)/TAB(I,K)
   IRO=I
216 I=I+1
   IF(I-M)227,227,220
227 IF(TAB(I,K))218,218,217
217 RATE=TAB(I,NCOL)/TAB(I,K)
   IF(RATE-RATIO)219,218,218
219 RATIO=RATE
   IRO=I
218 IF(I-M)216,220,220
220 IBA(IRO)=K
   PIVT=TAB(IRO,K)
   DO 221 J=1,NCOL
221 TAB(IRO,J)=TAB(IRO,J)/PIVT
C
COMPUTATION OF NEW COEFFICIENTS FOR REST OF TABLEAU
   DO 222 I=1,MP1
      DELE=TAB(I,K)
      IF(DELE)223,222,223
223 IF(I-IRO)224,222,224
224 DO 225 J=1,NCOL
225 TAB(I,J)=TAB(I,J)-TAB(IRO,J)*DELE
   CONTINUE
336 FORMAT(3H PLE E20.6)
   PRINT 336,PLE
338 FORMAT(4HP IVT E20.6)
   PRINT 338,PIVT
   GO TO 205
C
CONVERT SOLUTION TO ORIGINAL VARIABLES
154 PP(1)=UBF-PP(1)
   PP(2)=UTF-PP(2)
   PP(3)=UDW-PP(3)
   PP(4)=UTW-PP(4)
   PP(5)=UBP-PP(5)
   PP(6)=UTP-PP(6)
   PP(7)=UR1-PP(7)
   PP(8)=UR2-PP(8)
PP(9)=UR3-PP(9)

C CHECK FOR SATISFACTORY CONVERGENCE
BDIF=0.
DO 115 I=1,N
IF (P(I))118,115,118
118 DIF=ABSF((PP(I)-P(I))/P(I))
IF(DIF-BDIF)115,115,116
116 BDIF=DIF
115 CONTINUE
WT=((2.*BF*TF+DW*TW)*(XL+2.*H)+2.*BP*TP*((2.*R1+R2)*
CXL+2.*R3*H))/490.*1728.
IF(BDIF-PER)108,108,91
108 PRINT 508
508 FORMAT(36HFINAL RESULTS BE OBTAINED AS FOLLOWS)
PRINT 506,BF,TF,DW,TW,BP,TP,R1,R2,R3
PRINT 507,WT
STOP
C CONTINUE FOR ANOTHER CYCLE IF CONVERGENCE TEST WAS
C NOT SATISFIED
91 DO 11 I=1,N
11 P(I)=PP(I)
XBF=PP(1)
XTF=PP(2)
XDW=PP(3)
XTW=PP(4)
XBP=PP(5)
XTP=PP(6)
XR1=PP(7)
XR2=PP(8)
XR3=PP(9)
PRINT 604,BF,TF,DW,TW,BP,TP,R1,R2,R3
PRINT 507,WT
PRINT 604,(P(I),I=1,N)
PRINT 604,XBF,XTF,XDW,XTW,XBP,XTP,XR1,XR2,XR3
STOP
END
APPENDIX B

NUMERICAL EXAMPLE OF

SIMPLEX METHOD
NUMERICAL EXAMPLE OF SIMPLEX METHOD

This section is devoted to an explanation of the simplex algorithm on a basic problem. The simplex procedure calls for the successive application of the following steps:

1. An initial computation tableau is constructed.

2. The testing of the $z_j - c_j$ elements to determine whether a minimum solution is found, i.e., whether $z_j - c_j \leq 0$ for all $j$.

3. The solution of the vector to be introduced into the basis if some $z_j - c_j > 0$, i.e., selection of the vector with maximum $z_j - c_j$.

4. The selection of the vector to be eliminated from the basis to ensure feasibility of the new solution, i.e., the vector with $\min \left( \frac{x_i}{x_{ik}} \right)$ for those $x_{ik} > 0$, where $k$ corresponds to the vector selected in step 3. If all $x_{ik} \leq 0$, then the solution is unbounded.

5. The transformation of the tableau by the complete elimination procedure to obtain the new solution and associated elements.

Each such iteration produces a new basis feasible solution, and we shall eventually obtain a minimum solution or determine an unbounded solution. An application of the simplex procedure to the initial tableau yields the transformed values of Table III.
Consider the following linear programming problem: minimize

\[ F(x) = -150x_1 - 200x_2 - 20x_3 \]

subject to

\[ x_1 + 5x_2 + 10x_3 \leq 1000 \]
\[ 10x_1 + 8x_2 + 5x_3 \leq 1500 \]
\[ 4x_1 + 5x_2 \leq 500 \]

and

\[ x_1 > 0, \quad x_2 > 0, \quad x_3 > 0 \]

To convert the inequalities to equalities a non-negative slack variable is added to each inequation and the model appears as follows:

minimize

\[ F(x) = -150x_1 - 200x_2 - 50x_3 + (0)x_4 + (0)x_5 + (0)x_6 \]

subject to

\[ x_1 + 5x_2 + 10x_3 + x_4 = 1000 \]
\[ 10x_1 + 8x_2 + 5x_3 + x_5 = 1500 \]
\[ 4x_1 + 5x_2 + x_6 = 500 \]

and

\[ x_1 > 0, \quad x_2 > 0, \quad x_3 > 0 \]

Our initial basis (see First Step, Table IV) consists of \( P_4, P_5, P_6 \).
### TABLE III

**INITIAL STEP OF COMPUTATIONAL SIMPLEX PROCEDURE**

<table>
<thead>
<tr>
<th>i</th>
<th>Basis</th>
<th>$P_0$</th>
<th>$P_{n+1}$</th>
<th>$P_{n+2}$</th>
<th>$P_{n+m}$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_k$</th>
<th>$P_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$P_{n+1}$</td>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$x_{i1}$</td>
<td>$x_{i2}$</td>
<td>$x_{ik}$</td>
<td>$x_{in}$</td>
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<tr>
<td>2</td>
<td>$P_{n+2}$</td>
<td>$x_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$x_{i1}$</td>
<td>$x_{i2}$</td>
<td>$x_{ik}$</td>
<td>$x_{2n}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>$P_{n+t}$</td>
<td>$x_t$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$x_{i1}$</td>
<td>$x_{i2}$</td>
<td>$x_{ik}$</td>
<td>$x_{jn}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>$P_{n+m}$</td>
<td>$x_m$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$x_{m1}$</td>
<td>$x_{m2}$</td>
<td>$x_{mk}$</td>
<td>$x_{mn}$</td>
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<tr>
<td>m+1</td>
<td>$z_s$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$z_j-c_i$</td>
<td>$z_j-c_2$</td>
<td>$z_j-c_k$</td>
<td>$z_j-c_n$</td>
</tr>
</tbody>
</table>

### TABLE IV

**COMPUTATIONAL SIMPLEX PROCEDURE**

<table>
<thead>
<tr>
<th>$c_j$</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>-150</th>
<th>-200</th>
<th>-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol</td>
<td>$P_0$</td>
<td>$P_4$</td>
<td>$P_5$</td>
<td>$P_6$</td>
<td>$P_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>0</td>
<td>$P_4$</td>
<td>1000</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$P_5$</td>
<td>1500</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>$P_6$</td>
<td>500</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$z_j$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$z_j-c_j$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150</td>
<td>200</td>
</tr>
</tbody>
</table>

(First Step)
### TABLE IV (Continued)

<table>
<thead>
<tr>
<th>$c_j$ →</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>-150</th>
<th>-200</th>
<th>-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$p_4$</td>
<td>500</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>$p_5$</td>
<td>700</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1.6</td>
</tr>
<tr>
<td>-200</td>
<td>$p_2$</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>$z_j$</td>
<td>-20000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-40</td>
<td>-160</td>
</tr>
<tr>
<td>$z_j-c_j$</td>
<td>-20000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-40</td>
<td>-10</td>
</tr>
</tbody>
</table>

(Second Step)

<table>
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<tr>
<th>$c_j$ →</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>-150</th>
<th>-200</th>
<th>-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-50</td>
<td>$p_3$</td>
<td>50</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>$p_5$</td>
<td>450</td>
<td>-0.5</td>
<td>1</td>
<td>0</td>
<td>-1.1</td>
</tr>
<tr>
<td>-200</td>
<td>$p_2$</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>$z_j$</td>
<td>-22500</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>-35</td>
<td>-145</td>
</tr>
<tr>
<td>$z_j-c_j$</td>
<td>-22500</td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>-35</td>
<td>+5</td>
</tr>
</tbody>
</table>

(Third Step)
TABLE IV (Continued)

<table>
<thead>
<tr>
<th>$c_j$</th>
<th>$0$</th>
<th>$0$</th>
<th>$0$</th>
<th>$-150$</th>
<th>$-200$</th>
<th>$-50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol</td>
<td>$P_0$</td>
<td>$P_4$</td>
<td>$P_5$</td>
<td>$P_6$</td>
<td>$P_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$-50$</td>
<td>$P_3$</td>
<td>$76.45$</td>
<td>$0.07$</td>
<td>$0.06$</td>
<td>$-0.17$</td>
<td>$0$</td>
</tr>
<tr>
<td>$-150$</td>
<td>$P_1$</td>
<td>$88.20$</td>
<td>$-0.10$</td>
<td>$0.20$</td>
<td>$-0.22$</td>
<td>$1$</td>
</tr>
<tr>
<td>$-200$</td>
<td>$P_2$</td>
<td>$29.50$</td>
<td>$0.08$</td>
<td>$-0.16$</td>
<td>$0.37$</td>
<td>$0$</td>
</tr>
<tr>
<td>$z_j$</td>
<td>$-22950$</td>
<td>$-4.45$</td>
<td>$-0.95$</td>
<td>$-34.1$</td>
<td>$-150$</td>
<td>$-200$</td>
</tr>
<tr>
<td>$z_j - c_j$</td>
<td>$-22950$</td>
<td>$-4.45$</td>
<td>$-0.95$</td>
<td>$-34.1$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

(Fourth Step)

and the corresponding solution $X = (x_4, x_5, x_6) = (1000, 1500, 500)$. Since $c_4 = c_5 = c_6 = 0$, the corresponding value of the objective function, $z_0$, equals zero. $c_2$ is selected to go into the basis, since \( \sum \alpha x_j (z_j - c_j) = z_2 - c_2 = 200 > 0 \). The ratio $\theta_x$ is the minimum of $x_j / x_{12}$ for $x_{12} > 0$, that is,

$$
\min \left( \frac{1000}{5}, \frac{1500}{8}, \frac{500}{5} \right) = \frac{500}{5}
$$

and hence column $P_2$ is eliminated according to the Gauss-Jordan elimination procedure. We transform the table (see Second Step, Table IV) and obtain a new solution $X' = (x_4', x_5', x_2') = (500, 700, 100)$, and the value of objective function is $-20000$. In the second
step, since

$$\max (z_j^I - c_j) = z_3^I - c_3 = 50 > 0$$

and

$$Q_0 = \min \left( \frac{500}{10}, \frac{700}{5} \right) = \frac{500}{10}$$

c_3 is introduced into the basis and column P_3 is eliminated. We transform the second step values of Table IV and obtain the third solution

$$x'' = (x_3, x_5, x_2) = (50, 450, 100)$$

and the value of objective function is -22500. Similarly the procedure is continued until all z_j - c_j becomes less than or equal to zero. Therefore, final value of the objective function is -22950 from the fourth step. The solution becomes

$$x''' = (x_3, x_1, x_2) = (76.45, 88.20, 29.50)$$
APPENDIX C

STRUCTURAL ANALYSIS OF
NONPRISMATIC MEMBERS
STRUCTURAL ANALYSIS OF NONPRISMATIC MEMBERS

A structure which is made up of nonprismatic members can be analyzed by the moment distribution method after stiffness, carry-over factors and fixed-end moments have been determined. These factors, the so called "beam constants" can be found from analytical expressions, as will be explained in the following.

A. The Fixed-End Moments

Figure 7-(a) shows a beam of varying cross section with both ends fixed and subjected to the bending action of a uniformly distributed load \( w \). General expressions for the fixed-end moments can be found by stating that the changes in slope and vertical deflection between the ends of the beam must both be zero. According to the moment-area principle this will be the case if

\[
\int_0^L \frac{Mdx}{EI} = 0 \tag{a}
\]

\[
\int_0^L \frac{Mdx}{EI} = 0 \tag{b}
\]

where both \( M \) and \( I \) are functions of \( x \). The bending moment is most conveniently expressed in two parts, namely the portion due to the uniform load acting on a simply supported beam (Figure 7-b), and the part caused by the two end moments (Figure 7-c). Due to the discontinuity of the moment curve for the simply supported beam, the integrations across the beam must be made in two steps.
Figure 7. Fixed-End Moments

(a) Fixed Beam

(b) Simply Supported Beam

(c) Restraining Moments
Equations (a) and (b) can be written

\[
\frac{w}{l} \int_0^l (lx - x^2) \, dx - \int_0^l [M_A - \frac{x}{l} (M_A - M_B)] \frac{dx}{I} = 0 \tag{c}
\]

\[
\frac{w}{2I} \int_0^l (lx - x^2) \, dx - \int_0^l [M_A - \frac{x}{l} (M_A - M_B)] \frac{xdx}{I} = 0 \tag{d}
\]

Eliminating \(M_A - M_B\) from equation (c) and (d) and solving for \(M_A\) will give

\[
M_A = \frac{w}{2} \frac{\int_0^l \frac{xdx}{I}}{\int_0^l \frac{x^2dx}{I}} \left\{ \frac{\int_0^l \frac{x^3dx}{I}}{\int_0^l \frac{x^2dx}{I}} \right\}^2 \tag{e}
\]

B. Carry-Over Factor

In Figure 8 is shown a beam of varying cross section, subjected to the action of a moment \(M\) at the simply supported end \(A\), and resisted by a moment of magnitude \(CM\) at the fixed end \(B\). As the deflection at \(A\) is zero it follows from equation (b) that

\[
\frac{1}{EI} \int_0^l (1.0 - \frac{x}{l}) M - \frac{x}{l} CM \, dx = C \tag{f}
\]

Integrating and solving for \(C\) gives for the carry-over factor from \(A\) to \(B\)

\[
C = \frac{l \int_0^l \frac{xdx}{I} - \int_0^l \frac{x^2dx}{I}}{\int_0^l \frac{x^2dx}{I}} \tag{g}
\]
Figure 8. Carry-over Factor

Figure 9. Modified Rotational Stiffness
C. Rotational Stiffness

The rotational stiffness factor $K$ at one end of a member can be defined as the moment required to produce unit rotation at one end which is assumed simply supported while the other end is fixed. It follows from equation (a) that

$$\int_0^l \frac{K}{EI} \left[ 1.0 - \frac{x}{l} (1.0 + C) \right] dx = \theta \quad \text{(h)}$$

If $\theta = 1.0$, $K$ may be substituted for $M$ and solved for in equation (i), thus

$$K = \frac{E \int_0^l dx - 1.0 + C}{\int_0^l x dx - \frac{1.0 + C}{l} \int_0^l x dx} \quad \text{(i)}$$

If the value of the carry-over factor $C$, as expressed by equation (h), is substituted in equation (i), the expression becomes

$$K = \frac{E \int_0^l x^2 dx}{\int_0^l dx - \frac{\int_0^l x^2 dx}{l} - \left[ \frac{\int_0^l x dx}{l} \right]^2} \quad \text{(j)}$$

The modified rotational stiffness for one end of a structural member if the other end is simply supported can be found, as shown in Figure 9, by first assuming this end fixed, then releasing it by balancing the moment by an equal and opposite amount and carrying over a portion of the latter. The modified rotational stiffness will be equal to

$$K' = K_1 \left( 1.0 - C_1 C_2 \right) \quad \text{(k)}$$