1969

Piecwise-linear Models for MOSFET Large-signal Operation

Marlin H. Golnitz

Follow this and additional works at: https://openprairie.sdstate.edu/etd

Recommended Citation
PIECEWISE-LINEAR MODELS FOR MOSFET LARGE-SIGNAL OPERATION

BY

MARLIN H. GOLNITZ

A thesis submitted in partial fulfillment of the requirements for the degree Master of Science, Department of Electrical Engineering, South Dakota State University

1969
This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Advisor Date

Head, Electrical Engineering Date
Department
ACKNOWLEDGMENTS

The author wishes to express his appreciation and gratitude to Dr. F. C. Fitchen, whose guidance and advice made this investigation possible, and to the National Science Foundation for financial support of this investigation.

M.H.G.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. HARMONIC ANALYSIS</td>
<td>5</td>
</tr>
<tr>
<td>III. DEVELOPMENT OF A PIECE-WISE LINEAR MODEL</td>
<td>18</td>
</tr>
<tr>
<td>IV. EXPERIMENTAL DATA AND DISCUSSION OF RESULTS</td>
<td>25</td>
</tr>
<tr>
<td>A. The MOSFET Stage</td>
<td>25</td>
</tr>
<tr>
<td>B. Model Example</td>
<td>27</td>
</tr>
<tr>
<td>C. Experimental Results</td>
<td>29</td>
</tr>
<tr>
<td>D. Straight-Line Representation</td>
<td>35</td>
</tr>
<tr>
<td>V. MODEL MODIFICATIONS</td>
<td>50</td>
</tr>
<tr>
<td>A. Transfer Characteristic Shift</td>
<td>50</td>
</tr>
<tr>
<td>B. Floating-Gain Model</td>
<td>57</td>
</tr>
<tr>
<td>C. Actual Small-Signal Gain</td>
<td>63</td>
</tr>
<tr>
<td>VI. CONCLUSIONS</td>
<td>66</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>69</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>71</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>78</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1.</td>
<td>Drain characteristic curves for the RCA 3N142 MOSFET</td>
<td>4</td>
</tr>
<tr>
<td>2-1.</td>
<td>Output waveform vs. input waveform for large-signal operation</td>
<td>6</td>
</tr>
<tr>
<td>3-1.</td>
<td>Symmetrical transfer characteristic of the MOSFET</td>
<td>19</td>
</tr>
<tr>
<td>3-2.</td>
<td>Non-symmetrical transfer characteristic of the MOSFET</td>
<td>19</td>
</tr>
<tr>
<td>3-3.</td>
<td>Location of cutoff and saturation boundaries on the drain characteristic of the MOSFET</td>
<td>22</td>
</tr>
<tr>
<td>4-1.</td>
<td>Circuit diagram of MOSFET stage</td>
<td>26</td>
</tr>
<tr>
<td>4-2.</td>
<td>Representative example of the determination of model elements from the drain characteristic curves</td>
<td>28</td>
</tr>
<tr>
<td>4-3.</td>
<td>Experimental large-signal behavior of MOSFET with operating point near the saturation region</td>
<td>30</td>
</tr>
<tr>
<td>4-4.</td>
<td>Experimental large-signal behavior of MOSFET with operating point near the cutoff region</td>
<td>33</td>
</tr>
<tr>
<td>4-5.</td>
<td>Experimental large-signal behavior of MOSFET with operating point in the middle of the active region</td>
<td>34</td>
</tr>
<tr>
<td>4-6.</td>
<td>Comparison between an experimental case and its corresponding model representation</td>
<td>36</td>
</tr>
<tr>
<td>4-7.</td>
<td>Comparison between an experimental case and its corresponding model representation with misleading expressions of error noted</td>
<td>38</td>
</tr>
<tr>
<td>4-8.</td>
<td>Comparison between an experimental case and its corresponding model representation, in which the error is largely due to the method of representation</td>
<td>42</td>
</tr>
<tr>
<td>4-9.</td>
<td>Drain characteristic curves of MOSFET with operating points indicated</td>
<td>44</td>
</tr>
</tbody>
</table>
Figure 4-10. Comparison between an experimental case and its corresponding model representation, using a 10 KΩ load.

5-1. Transfer characteristic for large-signal operation.

5-2. Incorrect location of straight-line transfer characteristic which results when the model of Chapter IV is used.

5-3. Comparison of results obtained before and after the transfer characteristic shift modification.

5-4. Transfer characteristic for the floating-gain model.

5-5. Comparison of results obtained before and after the floating-gain modification.

5-6. Comparison of results obtained before and after the actual small-signal gain modification.
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-1. Error between the fundamental component obtained with the model and that obtained experimentally for the cases where a load resistance of 1 KΩ was used</td>
<td>45</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

The large-signal behavior of electronic devices has long been a topic of concern to the electronic engineer. This large-signal behavior is an important characteristic of digital, pulse and switching circuits as used in computers and data processing systems. During the past few years, considerable attention has been given to the derivation of computer models for the solid state devices used in digital systems. By using such models, one can predict the behavior of a device under given conditions without relying exclusively upon experimental observations.

A device which is currently coming into its own is the metal-oxide semiconductor field-effect transistor, or MOSFET. In the MOSFET, surface conductivity is controlled by a strong electric field; thus it is a voltage-dependent device rather than a current-amplifying structure as is the conventional junction transistor. Although the theory of operation dates back to the work of Lilienfeld in 1930, a method of fabricating the device was not developed until the 1950's and the MOSFET did not become commercially available until the early 1960's. Some of the characteristics which make the MOSFET superior to the vacuum tube and bipolar transistor are a very large input impedance, low self-generated noise, and superior thermal stability. The MOSFET also holds an advantage over its counterpart, the junction FET, because its input impedance is independent of the applied voltage,
even at very high temperatures. Thus it can function with very large bias resistors in analog circuits and direct coupling in digital circuits is simplified. 6

Realizing that the MOSFET is becoming increasingly important and that considerable attention is being given to the development of models for computer-aided circuit analysis and design, it is desirable to develop a large-signal model for the device. The object of this study is to develop a simple model for the MOSFET and to determine the degree of accuracy that can be expected with its use. A piecewise-linear model of the MOSFET which is suitable for use with the transient analysis program of the IBM Electronic Circuit Analysis Program (ECAP) was developed by Roberts and Harbourt. 14 Their complex model was subsequently simplified for use in pulse-inverter circuit analysis. The subject of this study is also a piecewise-linear model but one of simpler form.

Large-signal operation is distinguished from small-signal operation by the presence of distortion in the output waveform. Accordingly, under large-signal conditions, the larger the input signal, the greater the percentage of distortion in the output. This distortion is characterized by the presence of harmonics in the output voltage, and therefore, large-signal behavior may be evaluated in terms of these harmonics. 11 Accordingly, a desirable large-signal model would be one which would predict the values of the fundamental, second, and third harmonic components of the output voltage for any given value of input voltage.
Since this is to be a simple model, the number of model elements should be a minimum. Ideally, the model should have relatively few elements which must be determined by experimental means. A considerable amount of information about any MOSFET is available from the drain characteristic curves of the device. The drain characteristic curves for MOSFET type 3N142 are given in Fig. 1-1. It is the object of this study to develop a model that is based upon values which can be determined from these curves. Any user of the model will then only need to determine the drain characteristics of his device and he will then be able to predict the large-signal behavior for any combination of bias voltages and load resistances.

This study is developed around the RCA 3N142 MOSFET. However, since it is the development of a method of representation and not a study of the particular device, any MOSFET type could have been used.
Fig. 1-1. Drain characteristic curves for the RCA 3N142 MOSFET.
CHAPTER II

HARMONIC ANALYSIS

A piecewise-linear model that can be used to represent large-signal behavior is one in which the voltage-transfer characteristic consists of three straight-line segments. One segment represents the active region of operation and the other two represent the cutoff and saturation regions. The transfer characteristic may be used to determine the waveform of the output voltage for a given input. By using an assumed input waveform, a Fourier series representing the output voltage may be determined. The coefficients of the harmonic components can be obtained directly from the Fourier series representation.

An input voltage waveform, a straight-line transfer characteristic, and the corresponding output voltage waveform are shown in Fig. 2-1. The input voltage is assumed sinusoidal and is of the form $V_{\text{in}} = V \sin \theta$. The transfer characteristic has a negative slope which results in a $180^\circ$ phase shift between the output and input voltages. The quantities $y_1$ and $y_2$ represent the negative and positive swings, respectively, of the output voltage with respect to the dc level. The time-varying value of the output voltage for each portion of a cycle is:

$$V_{\text{out}} = gV_{\text{in}}, \quad 0 < \theta < \theta_1$$
$$V_{\text{out}} = -y_1, \quad \theta_1 < \theta < \theta_2$$
Fig. 2-1. Output waveform vs. input waveform for large-signal operation.
\[ V_{\text{out}} = gV_{\text{in}}, \quad \theta_2 < \theta < \theta_3 \]  
\[ V_{\text{out}} = y_2, \quad \theta_3 < \theta < \theta_4 \]  
\[ V_{\text{out}} = gV_{\text{in}}, \quad \theta_4 < \theta < 2\pi \]  

But,

\[ V_{\text{in}} = V \sin \theta \]  

and

\[ -y_1 = gV \sin \theta_1 \] \hspace{1cm} (2-2) 
\[ y_2 = gV \sin \theta_3 \]  

By substituting the set of equations (2-2) into the set (2-1), we obtain:

\[ V_{\text{out}} = gV \sin \theta, \quad 0 < \theta < \theta_1 \] \hspace{1cm} (2-3a)  
\[ \theta_2 < \theta < \theta_3 \]  
\[ \theta_4 < \theta < 2\pi \]  

\[ V_{\text{out}} = gV \sin \theta_1, \quad \theta_1 < \theta < \theta_2 \] \hspace{1cm} (2-3b)  

\[ V_{\text{out}} = gV \sin \theta_3, \quad \theta_3 < \theta < \theta_4 \] \hspace{1cm} (2-3c)  

Except for the special case when the transfer characteristic is symmetrical about the operating point, \( y_1 \) does not equal \( y_2 \).

Therefore, in general, \( \theta_1 \) does not equal \( \theta_3 \). We assume that there
is no hysteresis present in the transfer characteristic; then,

$$\theta_2 = \pi - \theta_1$$ \hspace{1cm} (2-4a)

and

$$\theta_4 = 3\pi - \theta_3$$ \hspace{1cm} (2-4b)

Also,

$$\sin n\theta_2 = (-1)^{n+1} \sin n\theta_1$$ \hspace{1cm} (2-4c)

$$\cos n\theta_2 = (-1)^n \cos n\theta_1$$ \hspace{1cm} (2-4d)

$$\sin n\theta_4 = (-1)^{n+1} \sin n\theta_3$$ \hspace{1cm} (2-4e)

$$\cos n\theta_4 = (-1)^n \cos n\theta_3$$ \hspace{1cm} (2-4f)

Since the coefficients of the harmonic voltages present in the output are desired, the output voltage can be written in a Fourier series:

$$V_{out} = A_0 + A_1 \sin \theta + A_2 \cos 2\theta + A_3 \sin 3\theta$$ \hspace{1cm} (2-5)

$A_0$ represents the dc level that results from the distortion of the ac voltage and does not include the bias voltage. $A_1$, $A_2$ and $A_3$ represent the coefficients of the fundamental, second and third harmonic components, respectively. As can be seen, only the odd harmonic sine terms and the even harmonic cosine terms are used. The remaining terms are all equal to zero, the proof of which is
given in appendix A. By using eq. (2-5), it is assumed that the fourth and all subsequent higher harmonics are negligible.

The dc term, $A_0$, may be determined from,

$$A_0 = \frac{1}{2\pi} \int_{0}^{2\pi} f(\theta) \, d\theta$$  \hspace{1cm} (2-6)$$

Since $f(\theta)$ is the ac value of the output voltage, eqs. (2-3) may be substituted into eq. (2-6).

$$A_0 = \frac{1}{2\pi} \left[ \int_{0}^{\theta_1} gV \sin \theta \, d\theta + \int_{\theta_1}^{\theta_2} gV \sin \theta_1 \, d\theta + \int_{\theta_2}^{\theta_3} gV \sin \theta \, d\theta + \int_{\theta_3}^{2\pi} gV \sin \theta \, d\theta \right]$$  \hspace{1cm} (2-7)$$

$$A_0 = \frac{gV}{2\pi} \left[ \int_{0}^{\theta_1} \sin \theta \, d\theta + \sin \theta_1 \int_{\theta_1}^{\theta_2} d\theta + \int_{\theta_2}^{\theta_3} \sin \theta \, d\theta + \sin \theta_3 \int_{\theta_3}^{\theta_4} d\theta + \int_{\theta_4}^{2\pi} \sin \theta \, d\theta \right]$$

$$A_0 = \frac{gV}{2\pi} \left[ -\cos \theta_1 + (\theta_2 - \theta_1) \sin \theta_1 - \cos \theta_3 + \cos \theta_2 + (\theta_4 - \theta_3) \sin \theta_3 + \cos \theta_4 \right]$$  \hspace{1cm} (2-8)$$
By substituting eqs. (2-4) into eq. (2-8), \( A_0 \) may be obtained in terms of \( \theta_1 \) and \( \theta_3 \).

\[
A_0 = \frac{gV}{2\pi} \left[ -\cos \theta_1 - \cos \theta_3 - (\pi - 2\theta_1) \sin \theta_1 - \cos \theta_3 \\
- \cos \theta_1 + (3\pi - 2\theta_3) \sin \theta_3 \right]
\]

or

\[
A_0 = \frac{gV}{\pi} \left[ -\cos \theta_1 + \left( \frac{\pi}{2} - \theta_1 \right) \sin \theta_1 - \cos \theta_3 \\
+ (\frac{3\pi}{2} - \theta_3) \sin \theta_3 \right] \quad (2-9)
\]

The coefficient of the fundamental component may be determined from,

\[
A_1 = \frac{1}{\pi} \int_{0}^{2\pi} f(\theta) \sin \theta \, d\theta \quad (2-10)
\]

Substituting eqs. (2-3) for \( f(\theta) \) in Eq. (2-10) yields,
\[
A_1 = \frac{gV}{\pi} \left[ \int_0^{\theta_1} gV \sin \theta \sin \theta \, d\theta + \int_{\theta_1}^{\theta_2} gV \sin \theta \sin \theta \, d\theta + \int_{\theta_2}^{\theta_3} gV \sin \theta \sin \theta \, d\theta + \int_{\theta_3}^{\theta_4} gV \sin \theta \sin \theta \, d\theta \right] + \int_{\theta_4}^{2\pi} gV \sin \theta \sin \theta \, d\theta \right) \tag{2-11}
\]

\[
A_1 = \frac{gV}{\pi} \left[ \int_0^{\theta_1} \sin^2 \theta \, d\theta + \sin \theta_1 \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta + \int_{\theta_2}^{\theta_3} \sin^2 \theta \, d\theta + \sin \theta_3 \int_{\theta_3}^{\theta_4} \sin \theta \, d\theta + \int_{\theta_4}^{2\pi} \sin^2 \theta \, d\theta \right] \tag{2-12}
\]

\[
A_1 = \frac{gV}{\pi} \left[ \frac{\theta_1}{2} - \frac{\sin 2\theta_1}{4} \right] - \sin \theta_1 (\cos \theta_2 - \cos \theta_1) + \frac{\theta_2}{2} - \frac{\sin 2\theta_2}{4} - \frac{\theta_3}{2} + \frac{\sin 2\theta_3}{4} - \sin \theta_3 (\cos \theta_4 - \cos \theta_3) + \frac{\theta_4}{2} + \frac{\sin 2\theta_4}{4} \right] \]
By substituting eqs. (2-4) into Eq. (2-12), $A_1$ is obtained in terms of $\theta_1$ and $\theta_3$.

$$A_1 = \frac{gV}{\pi} \left[ \theta_1 + \theta_3 - \pi + \sin \theta_1 \cos \theta_1 + \sin \theta_3 \cos \theta_3 \right] \quad (2\text{-}13)$$

Similarly,

$$A_2 = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos 2\theta \, d\theta \quad (2\text{-}14)$$

Substituting eqs. (2-3) for $f(\theta)$ in eq. (2-14) yields,

$$A_2 = \frac{1}{\pi} \left[ \int_0^{\theta_1} gV \sin \theta \cos 2\theta \, d\theta + \int_{\theta_1}^{\theta_2} gV \sin \theta_1 \cos 2\theta \, d\theta ight. \\
\left. + \int_{\theta_2}^{\theta_3} gV \sin \theta \cos 2\theta \, d\theta + \int_{\theta_3}^{\theta_4} gV \sin \theta_3 \cos 2\theta \, d\theta \\
+ \int_{\theta_4}^{2\pi} gV \sin \theta \cos 2\theta \, d\theta \right] \quad (2\text{-}15)$$
\begin{align*}
A_2 &= \frac{gV}{\pi} \left[ \int_0^{\theta_1} \sin \theta \cos 2\theta \, d\theta + \sin \theta_1 \int_{\theta_1}^{\theta_2} \cos 2\theta \, d\theta \\
&\quad + \int_{\theta_2}^{\theta_3} \sin \theta \cos 2\theta \, d\theta + \sin \theta_3 \int_{\theta_3}^{\theta_4} \cos 2\theta \, d\theta \\
&\quad + \int_{\theta_4}^{2\pi} \sin \theta \cos 2\theta \, d\theta \right] \\
&= \frac{gV}{\pi} \left[ \frac{\cos \theta_1}{2} - \frac{\cos 3\theta_1}{6} - \frac{1}{2} + \frac{1}{6} + \frac{\sin \theta_1}{2} (\sin 2\theta_2 - \sin 2\theta_1) \\
&\quad + \frac{\cos \theta_3}{2} - \frac{\cos 3\theta_3}{6} - \frac{\cos \theta_2}{2} + \frac{\cos 3\theta_2}{6} + \frac{\sin \theta_3}{2} (\sin 2\theta_4 - \sin 2\theta_3) \\
&\quad - \sin 2\theta_3 + \frac{1}{2} - \frac{1}{6} - \frac{\cos \theta_4}{2} + \frac{\cos 3\theta_4}{6} \right] \\
&= \frac{gV}{\pi} \left[ \cos \theta_1 - \frac{\cos 3\theta_1}{3} - \sin \theta_1 \sin 2\theta_1 + \cos \theta_3 \\
&\quad - \frac{\cos 3\theta_3}{3} - \sin \theta_3 \sin 2\theta_3 \right] \quad (2-17)
\end{align*}

By substituting eqs. (2-4) into eq. (2-16), \( A_2 \) may be obtained in terms of \( \theta_1 \) and \( \theta_3 \).
This may be simplified to,

\[ A_2 = \frac{2gV}{3\pi} \left[ \cos^3 \theta_1 + \cos^3 \theta_3 \right] \quad (2-18) \]

In the same manner,

\[ A_3 = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin \theta \, d\theta \quad (2-19) \]

Substituting eqs. (2-3) for \( f(\theta) \) in eq. (2-19) yields,

\[
A_3 = \frac{1}{\pi} \left[ \int_0^{\theta_1} gV \sin \theta \sin 3\theta \, d\theta + \int_{\theta_1}^{\theta_2} gV \sin \theta \sin 3\theta \, d\theta \\
+ \int_{\theta_2}^{\theta_3} gV \sin \theta \sin 3\theta \, d\theta + \int_{\theta_3}^{\theta_4} gV \sin \theta \sin 3\theta \, d\theta \\
+ \int_{\theta_4}^{2\pi} gV \sin \theta \sin 3\theta \, d\theta \right] \quad (2-20)
\]
\[ A_3 = \frac{gV}{\pi} \left[ \frac{\sin 2\theta_1}{4} - \frac{\sin 4\theta_1}{8} + \frac{\sin \theta_1}{3}(-\cos 3\theta_2 + \cos 3\theta_1) \right. \\
\left. + \frac{\sin 2\theta_3}{4} - \frac{\sin 4\theta_3}{8} - \frac{\sin 2\theta_2}{4} + \frac{\sin 4\theta_2}{8} + \frac{\sin \theta_3}{3}(-\cos 3\theta_4) \right. \\
\left. + \cos 3\theta_3) - \frac{\sin 2\theta_4}{4} + \frac{\sin 4\theta_4}{8} \right] \\
(2-21) \]

By substituting eqs. (2-4) into eq. (2-21), \( A_3 \) may be found in terms of \( \theta_1 \) and \( \theta_3 \) only.

\[ A_3 = \frac{gV}{\pi} \left[ \sin 2\theta_1 \right. \\
\left. \frac{\sin 4\theta_1}{2} - \frac{\sin 4\theta_1}{4} \right] + \frac{2 \sin \theta_1 \cos 3\theta_1}{3} + \frac{\sin 2\theta_3}{2} \\
- \frac{\sin 4\theta_3}{4} + \frac{2 \sin \theta_3 \cos 3\theta_2}{3} \right] \\
(2-22) \]

This may be simplified to,

\[ A_3 = \frac{2gV}{3\pi} \left[ \sin \theta_1 \cos^3 \theta_1 + \sin \theta_3 \cos^3 \theta_3 \right] \\
(2-23) \]

All of the harmonic coefficients are given in terms of \( \theta_1 \) and \( \theta_3 \). Therefore it is necessary to determine \( \theta_1 \) and \( \theta_3 \) in terms of known values. Since \( x_1 = V \sin \theta_1 \) and \( x_2 = V \sin \theta_3 \) (Fig. 2-1), \( \theta_1 \) and \( \theta_3 \) may be given by:
In summary, the coefficients of the harmonics of the output voltage are as follows:

\[ A_0 = \frac{gV}{\pi} \left[ -\cos \theta_1 + \left( \frac{\pi}{2} - \theta_1 \right) \sin \theta_1 - \cos \theta_3 + \left( \frac{3\pi}{2} - \theta_3 \right) \sin \theta_3 \right] \]  \hspace{1cm} (2-9)

\[ A_1 = \frac{gV}{\pi} \left[ \theta_1 + \theta_3 - \pi + \sin \theta_1 \cos \theta_1 + \sin \theta_3 \cos \theta_3 \right] \]  \hspace{1cm} (2-13)

\[ A_2 = \frac{2gV}{3\pi} \left[ \cos^3 \theta_1 + \cos^3 \theta_3 \right] \]  \hspace{1cm} (2-18)

\[ A_3 = \frac{2gV}{3\pi} \left[ \sin \theta_1 \cos^3 \theta_1 + \sin \theta_3 \cos^3 \theta_3 \right] \]  \hspace{1cm} (2-23)

Where:

\[ g = \text{small signal gain} \]

\[ V = \text{peak value of input voltage} \]

\[ \theta_1 = \arcsin \frac{x_1}{V} \]

\[ \theta_3 = \arcsin \frac{x_2}{V} \]
The elements $x_1$ and $x_2$ are the values of the input voltage where saturation and cutoff, respectively begin.
CHAPTER III

DEVELOPMENT OF A PIECE-WISE LINEAR MODEL

The analysis given in Chapter II provides a means for determining the harmonic content of the output waveform of an amplifier in terms of the following quantities: the peak value of the input voltage \( V \), the small signal gain \( g \), the value of the input which will drive the MOSFET into saturation operation \( x_1 \), and the value of the input which will drive the MOSFET into cutoff operation \( x_2 \). The quantities \( x_1 \) and \( x_2 \) are shown in Fig. 3-1. The figure shows the orientation of the three-segment straight line characteristic that will be used to represent the actual voltage transfer characteristic of the MOSFET stage. The ordinate of the figure represents the instantaneous level of the output voltage with \( V_{DS} \) being the drain bias point of the MOSFET. The abscissa represents the instantaneous time-varying value of the gate input voltage about its quiescent point.

Several different methods were employed in an attempt to experimentally determine the values of \( x_1 \) and \( x_2 \). In one method, the voltage-transfer characteristic was observed on an oscilloscope and the values were determined directly from the display. Another method involved the observation of the output waveform on an oscilloscope. The ac input voltage was monitored as it was slowly increased from zero and the values at which the two different
Fig. 3-1. Symmetrical transfer characteristic of the MOSFET.

Fig. 3-2. Non-symmetrical transfer characteristic of the MOSFET.
halves of the output waveform began to distort were recorded as $x_1$ and $x_2$. Both of these methods were somewhat erroneous because of the broad transition areas that exist between the active region and each of the other regions of operation. Therefore it was difficult to obtain exact values with either of these methods. The same quantities determined on different occasions, under the same conditions, would seldom be in agreement.

Since $x_1$ and $x_2$ could not be satisfactorily determined directly, a method involving the measurement of the output voltage levels was utilized. The voltage transfer characteristic was again observed on the oscilloscope. However, in this case the saturated values of the output voltage were recorded. It was possible to obtain quite accurate measurements of these levels as can be seen in Fig. 3-1 ($V_1$ and $V_2$). The quantities $x_1$ and $x_2$ could then be determined from the following relationships:

$$x_1 = \frac{V_{DS} - V_1}{g}$$

$$x_2 = \frac{V_{DS} - V_2}{g}$$

(3-1)

From these equations it is evident that $x_1$ is a positive quantity while $x_2$ is negative, since $V_1 < V_{DS} < V_2$.

A set of different operating points was chosen for the evaluation of the MOSFET. In several cases the MOSFET would be biased very close to either cutoff or saturation. In these cases,
the operating point actually would be within one of the curved transition areas of the transfer characteristic, as is shown in Fig. 3-2. Because of this situation, the small-signal gain at the operating point would not be the same as that in the linear portion of the transfer characteristic. Therefore, if the gain at the operating point were used for the model, erroneous results would be obtained for many of the cases. In order to eliminate this problem, the value of the small signal gain at the midpoint of the middle segment of the transfer characteristic was determined to be the best value to use in all cases. The drain bias voltage at the midpoint was determined from

\[ V'_{DS} = \frac{V_1 + V_2}{2} \]  \hspace{1cm} (3-2)

With the MOSFET biased at the operating point and with no input signal applied, the gate bias voltage was adjusted to obtain a drain-source voltage equal to \( V'_{DS} \), the value calculated in eq. (3-2). The small-signal gain at this point was then determined experimentally and this was the value used in the harmonic analysis equations.

The drain characteristic curves of an RCA 3N142 MOSFET were shown in Fig. 1-1. These same characteristics are shown in Fig. 3-3 with several operating points and the corresponding load lines identified. On each load line the values of \( V_1 \) and \( V_2 \) are indicated by small squares. The locus of the \( V_2 \) points almost coincides with
Fig. 3-3. Location of cutoff and saturation boundaries on the drain characteristic of the MOSFET.
the horizontal axis of the graph. Actually the locus has a very small slope (about 0.01 ma/volt) which is a result of the leakage current in the cutoff mode, but this is negligible when one considers the experimental error that may be present. Therefore it will be assumed that the horizontal axis provides an accurate boundary between the active and the cutoff regions. The pinchoff voltage \( V_p \) of the MOSFET is the value of the gate voltage at which the channel conductance is reduced to zero; its locus very nearly coincides with the horizontal axis. In view of this, the boundary between the active and cutoff regions can be said to be represented by \( V_p \).

As the gate voltage becomes more positive, the characteristic curves of the MOSFET begin to approach a limiting slope. If a tangent to the curves is drawn through the origin of the graph as a representation of the limiting slope, this tangent line is also the locus of the \( V_1 \) values. This line can then be used to represent the boundary between the active and the saturation regions for analysis.

The drain characteristics of the MOSFET may now be divided into the three regions for behavior studies. With known values of \( V_{DD} \) and \( R_L \), an operating point may be picked on the characteristic curves and the desired load line may be drawn through the point. The values of the drain voltage at which the load line intersects the saturation and the cutoff boundaries are then recorded as \( V_1 \).
and $V_2$, respectively. With $V_{DS}$, $V_1$, and $V_2$ known, $V_{DS}$ may be calculated and $g$ can be determined experimentally. These values can then be used with the computer program for the model to determine the harmonic behavior of the output voltage under the given conditions.
CHAPTER IV

EXPERIMENTAL DATA AND DISCUSSION OF RESULTS

In order to make a complete comparison between experimental results and those obtained for the model, tests were made over a wide range of conditions. Operating points were chosen such that all types of operation would be well represented. Some points correspond to operation in the center of the active region with equal swings in both the saturation and the cutoff directions. Others result in operation using either primarily active and saturation conditions or primarily active and cutoff conditions. Since this was not intended to be a study of heat effects or one of power output, the operation of the amplifier was limited to low power conditions. By doing this, several variables could be eliminated.

A. The MOSFET Stage

The circuit diagram of the amplifier analyzed is shown in Fig. 4-1. The circuit is a simple, resistively loaded MOSFET stage. A Hewlett-Packard Model 300A harmonic wave analyzer was used to measure the harmonic content of the output voltage. With this instrument, a direct measurement of the rms value of each individual harmonic component up to a frequency of 16 KHz may be made. This frequency limitation greatly restricts the value of the fundamental frequency if many harmonics are to be observed. Therefore,
Fig. 4-1. Circuit diagram of MOSFET stage.
an input signal of 1 KHz was used for the experimentation. The 1 µF capacitor in the output circuit was used because the wave analyzer does not have a capacitive input. The 10 µF capacitor was used to eliminate the drain bias supply and the ammeter from the ac load. Because of the very thin insulation layer between the gate and the channel of the MOSFET, a 10 MΩ resistor was connected between the gate and source terminals to prevent any damage which might occur to the MOSFET because of static charges. This resistor provides a discharge path for any static potential which may exist on the MOSFET leads, and therefore prevents the insulation from being punctured. Three different load values (1 K, 2.5 K, and 10 KΩ) were used so as to provide extensive comparisons.

B. Model Example

For each set of experimental output voltage components obtained, a corresponding set was calculated with the computer. The computer program used can be found in Appendix B. A representative example of the determination of $V_1$ and $V_2$ is shown in Fig. 4-2. The operating point for this case is,

$$V_{GS} = -2.0 \text{ volts}$$

and

$$V_{DS} = 4.0 \text{ volts}.$$
Fig. 4-2. Representative example of the determination of model elements from the drain characteristic curves.
A load line with a slope of 1 KΩ was drawn through the operating point. The points of intersection of the load line with the saturation and cutoff boundaries are $V_1$ and $V_2$, respectively. In this particular case,

$$V_1 = 0.8 \text{ volts}$$

and

$$V_2 = 5.7 \text{ volts}.$$ 

Accordingly,

$$V_{DS'} = \frac{V_1 + V_2}{2} = 3.25 \text{ volts}.$$ 

This is the voltage at which the small signal gain is measured for the model. With $V_{DS'}$, $V_1$, $V_2$, and $g$ determined, all of the elements needed for the computer program are known and the harmonic coefficients may therefore be obtained.

C. Experimental Results

A typical example of the experimental behavior of the MOSFET is shown in Fig. 4-3. The fundamental component of the output voltage increases somewhat linearly at first, but then saturates and begins to approach a limiting value of approximately 3 volts. The values of the other harmonic components are negligible for the very small magnitudes of input voltage, but become quite significant as the input voltage increases. The second harmonic
Fig. 4-3. Experimental large-signal behavior of MOSFET with operating point near the saturation region.

\[ R_L = 1 \, \text{k} \Omega \]
\[ V_{GS} = -1.0 \, \text{V} \]
\[ V_{DS} = 2.0 \, \text{V} \]

3rd Harmonic
2nd Harmonic
increases quite rapidly at first to about 0.75 volt, then decreases and slowly approaches a smaller limiting value. The third harmonic remains negligible through a greater range of input voltage and then increases with a moderate slope. It eventually approaches a limiting value of about 1 volt, which is approximately one-third of the limiting value of the fundamental component.

The fundamental component represents the entire output voltage for the small-signal condition where there is no appreciable distortion. For this reason the slope is fairly linear in this region. As the input voltage is increased to large-signal proportions, distortion appears and the output voltage can no longer be represented exclusively by the fundamental component; it then depends upon all the harmonic voltages.

The second harmonic has a peak value which occurs at a relatively small value of input voltage. The peak occurs in the area which represents the maximum curvature of the transfer characteristic. Since this is the area where the gain of the MOSFET is changing most radically, one would expect that the second harmonic would be quite significant in this area. As the input voltage increases to larger values, the output becomes independent of the input; the second harmonic becomes negligible and the third harmonic becomes quite significant. This is the expected behavior, since the output waveform is approximately a square wave for large values of input voltage, and, for a pure square wave, the Fourier series
predicts that the third harmonic will be equal to one-third of the fundamental component and the second harmonic will be equal to zero.\textsuperscript{10}

The results shown in Fig. 4-3 were for a case where the operating point of the MOSFET was near the saturation region. A case where the operating point was near the cutoff region is shown in Fig. 4-4. In the cases observed, the results had the same general trend when the operating point was near the cutoff region as for when it was near the saturation region.

An exception to the behavior noted in the previous examples is shown in Fig. 4-5. In this case, the second harmonic is negligible throughout the entire range of input voltage. The third harmonic behaves much the same as before except that the initial slope is somewhat greater. There is no noticeable difference in the pattern of the behavior of the fundamental component, however. This case occurs when the MOSFET is biased at an operating point in the middle of the active region, equidistant from the saturation and cutoff boundaries. This results in a symmetrical voltage transfer characteristic and, therefore, a symmetrical output waveform. Since the waveform is symmetrical, only even functions are present and therefore the second harmonic, which is a sine function, is essentially zero.\textsuperscript{10}
Fig. 4-4. Experimental large-signal behavior of MOSFET with operating point near the cutoff region.

\[ R_L = 1 \text{ K}\Omega \]
\[ V_{GS} = -2.5 \text{ V} \]
\[ V_{DS} = 8.0 \text{ V} \]
Fig. 4-5. Experimental large-signal behavior of MOSFET with operating point in the middle of the active region.
D. Straight-Line Representation

A comparison between an experimental case and its corresponding model representation is shown in Fig. 4-6. This particular example shows very good agreement between the two sets of results, especially when one allows for experimental error. The results of all of the cases were not so desirable, as will be explained later. However, in all of the cases studied the results obtained with the model always had the same general trends as those obtained experimentally, but the magnitudes differed.

The error between the experimental results and those of the model can be divided into two major categories. One source of error is the recognized fact that the actual transfer characteristic is not composed of straight lines. This study is directed toward the goal of determining how large this error is. The other type of error results from the approximations and graphically-determined values that are used with the model. The most significant of these is the value of small signal gain that is used. There is no convenient way to separate the two types of error, so they must be considered as a single entity. However, since it is assumed that the second type of error is due largely to the choice of small-signal gain, the analysis may be simplified somewhat. If, for a given case, there is no error between the experimental results and those obtained with the model for very small values of the input voltage, one can assume that a proper choice of small-signal gain
Fig. 4-6. Comparison between an experimental case and its corresponding model representation.
was made. One can then assume that the total error is due largely to the straight line assumption and an evaluation can be made on this basis.

The only other factor to be considered is that in a few of the cases there was significant error between the two sets of results at very large values of input voltage. Because of the nature of the model, this indicates that the graphically-determined values for either $V_1$ or $V_2$, or both, were incorrectly determined. Since this error occurred in only a small percentage of the cases, these will also be eliminated from the evaluation as a whole.

A true evaluation of the error incurred in the straight line representation is somewhat difficult to express. If the amount of error at a particular value of input voltage is given as a percentage of the experimental value of the component at that point, the results could be misleading, as is shown in Fig. 4-7. It is almost impossible to attach numerical values to the error without being somewhat misleading. In order to make a true comparison, one must personally observe the data rather than figures pertaining to the amount of error.

Consider the case that was shown in Fig. 4-6. The dc bias point was $V_{GS} = -2.0$ volts and $V_{DS} = 4.0$ volts. The load resistance was 1000 ohms. Since for very small values of input voltage and also for very large values the two sets of data are in very good agreement, this case is one in which the error is
Fig. 4-7. Comparison between an experimental case and its corresponding model representation with misleading expressions of error noted.
predominantly due to the straight line approximation. Also, since the second harmonic is rather small at its peak value, the bias point of the MOSFET was evidently somewhere near the midpoint of the active region. The largest error in the fundamental component is approximately 15 percent and occurs when the input voltage is 0.7 volts. The largest error for the second harmonic is 60 percent and also occurs with $V_{\text{in}}$ equal to 0.7 volt. For the third harmonic the largest error does not occur until $V_{\text{in}}$ equals 1.5 volts, and this error is approximately 20 percent. In spite of these figures, the model provides a very good representation of the actual output voltage for this particular case, as can be seen in the figure. As expected, the maximum error occurs when operation is in the transition area between the active and the other two regions. The experimental data is characterized by a gradual transition over a considerable range of input voltage. This is in keeping with the general transfer characteristic of the MOSFET. However, since there is essentially no transition area as such in the straight-line transfer characteristic, the changes in the output voltage components are rather abrupt. Since the changes in operation from the active region to the saturation region and from the active region to the cutoff region do not occur at the same value of input voltage, a transition region is apparent in the results of the model, but it is over a much smaller segment of input voltage than that of the experimental results.
The error in the fundamental and the third harmonic components is caused by the fact that the components of the model reached a particular value at a different value of input voltage than the experimental components did, rather than that the wrong level was reached. This is a characteristic of the fact that the transition region of the model is not the same as the actual transition region. Actually, if the transition area of the model was begun at the same value of input voltage but was twice as long in duration, these components would almost directly coincide with the experimental components throughout the entire range of input voltage.

The error in the second harmonic is one of a different nature than that of the fundamental and third harmonic. The second harmonic component of the model always reached a larger peak value than did the experimental component. Despite the fact that this error is of a different nature, it is also due to the length of the transition area. According to Alley and Atwood,¹ the second harmonic component occurs as a result of a changing $g_m$. The magnitude of the component is proportional to the rate of change. Since the rate of change in $g_m$ is much greater for the model than it is in the actual MOSFET operation, the second harmonic should correspondingly reach a larger peak value in the results obtained from the model.

So far only the error resulting from the use of a straight line approximation has been considered. Since this study involves
the development of a method of representing the MOSFET operation, the error resulting from the use of this method must also be considered. In the previous discussion, the cases where an inaccurate determination of $V_1$, $V_2$, or the small signal gain $g$ was made were eliminated from the evaluation. However, when the method as a whole is being considered, these cases must be included. The accuracy of the overall representation is a function of two main variables. One is the location of the dc bias point for any particular case and the other is the value of the load resistance. In the previous discussion only the error present in the transition area was considered so that the accuracy of the representation could be evaluated. Now the error for the very small and the very large values of input voltage will be considered. All of the cases where an error exists due to the method of representation are characterized by the general trend shown in Fig. 4-8. For very small values of input voltage the fundamental component of the model has a greater slope than that of the experimental component. However, as the value of the input voltage increases and reaches either $x_1$ or $x_2$, the slope of the model changes and becomes approximately equal to that of the experimental case. As a result, the curve for the model is approximately parallel to that of the experimental case for the next range of input values. As the input magnitude reaches the remaining boundary point ($x_1$ or $x_2$), the slopes of both curves change and begin to approach limiting values. In almost all of the
Fig. 4-8. Comparison between an experimental case and its corresponding model representation, in which the error is largely due to the method of representation.
cases, these limiting values were the same or approximately the same. This behavior can be discussed most easily by thinking in terms of the input voltage. The same results are obtained for the model as are for the experimental case, but they occur at a smaller level of input voltage. In other words,

\[ \text{Experimental } V_{out} = gV_{in} \]

and

\[ \text{Model } V_{out} = g(K + V_{in}) \]

The value of \( K \) varies, depending upon the location of the operating point. The actual cause of this situation is that the value of \( g \) used for the model is not the same as the small-signal gain at the operating point. However, the value of \( g \) is the same as the small-signal gain at the center portion of the transfer curve. The reason that the curves in Fig. 4-8 are parallel and not overlapping in this area is the fact that no compensation was made for the error which was incurred for very small values of input. The result is the apparent offset between the two representations.

Ten randomly chosen operating points are shown on the characteristic curves in Fig. 4-9. As can be seen, these points represent all modes of operation. The amount of error for these cases using a 1 KΩ load resistor is given in Table 4-1 and is summarized in the following paragraphs. This error is that of the fundamental component only and is separated into three different areas: the error
Fig. 4-9. Drain characteristic curves of MOSFET with operating points indicated.
Table 4-1. Error between the fundamental component obtained with the model and that obtained experimentally for the cases where a load resistance of 1 KΩ was used.

<table>
<thead>
<tr>
<th>Q Point</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear Portion</td>
</tr>
<tr>
<td>Vgs</td>
<td>Vds</td>
</tr>
<tr>
<td>-1.0 V</td>
<td>2.0 V</td>
</tr>
<tr>
<td>-1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>-1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>-1.5</td>
<td>6.0</td>
</tr>
<tr>
<td>-2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>-2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>-2.0</td>
<td>10.0</td>
</tr>
<tr>
<td>-2.5</td>
<td>4.0</td>
</tr>
<tr>
<td>-2.5</td>
<td>6.0</td>
</tr>
<tr>
<td>-2.5</td>
<td>8.0</td>
</tr>
</tbody>
</table>
at very large values of input voltage (the saturation level), the error in the transition region, and the error over the linear region of operation. The first two types are expressed as a percentage of the experimental output voltage, but the third is given as a segment of input voltage, or a $\Delta V_{in}$. This $\Delta V_{in}$ corresponds to the constant $K$ earlier referred to in this discussion of the results.

Of the ten cases studied, the error at the saturation level was less than 2 percent for six of the cases; and was approximately 3 percent for two cases, 5 percent for another case, and 8 percent for the remaining case. These results are very favorable, since only one of the ten cases resulted in an error greater than 5 percent. This indicates a high degree of accuracy in the choice of values for $x_1$ and $x_2$ in the majority of the cases.

The error in the transition region is somewhat greater than the saturation level error, but this is to be expected, since this is the area in which the error due to the straight-line approximation is predominant. Of the ten cases; two had an error of approximately 5 percent, five were in the 8 to 10 percent range, two were approximately 15 percent, and the remaining case had an error of approximately 20 percent. The largest errors in this area do not necessarily correspond to the largest errors in the other two areas. In fact, the case studied earlier in the discussion was one in which there was no error for either the small
or large values of input voltage. All of the error was in the transition region and the largest was approximately 15 percent. This value is greater than the average error of all ten cases.

As was mentioned above, the error for the linear portion of the output voltage is expressed in terms of $\Delta V_{in}$. For the ten cases; $\Delta V_{in}$ equaled zero for three examples, 0.05 volts for one, 0.1 volts for two, and 0.15 volts for the remaining four cases. The amount of error in this region is directly proportional to the distance that the operating point is from the center of the transfer characteristic. For all four of the cases where the error was 0.15 volts, the MOSFET was biased very near either the saturation or the cutoff region. In the other two areas, there appears to be no definite correlation between the location of the operating point and the amount of error which resulted.

The value of the load resistance had a varying effect on the total error present. The error resulting from the straight-line approximation appeared to be significantly less for the higher values of load resistance. However, as the load resistance was increased, the error due to the method of representation increased. Most of the cases where a 10 $\text{k}\Omega$ load was used had a significant error at the large values of input voltage, indicating that the choices for $x_1$ and $x_2$ were incorrect. However, the results at small levels of input voltage were similar to those obtained with a 1 $\text{k}\Omega$ load. A case where $x_1$ and $x_2$ were apparently correct is
shown in Fig. 4-10. This case shows excellent agreement between the model and the experimental results, but this is an exception to the general cases where a 10 KΩ load was used.
Fig. 4-10. Comparison between an experimental case and its corresponding model representation, using a 10 KΩ load.
CHAPTER V

MODEL MODIFICATIONS

A very important factor that affects the accuracy of the model is the location of the operating point on the transfer characteristic. If the operating point is located on the linear portion of the characteristic, near the center of the output voltage variation, the highest degree of accuracy is obtained. This location is represented in Fig. 5-1 by point A. In this area, the straight-line representation and the actual characteristic coincide; therefore, the slope of the straight-line segment has the same value as the experimental small-signal gain. In contrast, if the operating point is located at point B a lesser degree of accuracy is obtained. This is largely because the slope of the straight-line segment for the active region is not equal to the small-signal gain at that point. Because of this discrepancy, several variations in the method of representation were considered in order to obtain the optimum representation of the actual operation.

A. Transfer Characteristic Shift

An examination of the method discussed in Chapter IV reveals that the middle segment of the straight-line characteristic represents the observed transfer characteristic only when the operating point is located on the linear portion of the characteristic.
Fig. 5-1. Transfer characteristic for large-signal operation.
Therefore, when the operating point is selected at another site, a modified procedure may be employed in order to shift the transfer characteristic of the model so that it more closely represents the characteristic that was observed experimentally. If the operating point is in a curved area, the representation resulting from the previous method is offset from the observed characteristic, as is shown in Fig. 5-2. The dashed line (A) represents the desired location for the straight-line representation, while the solid line (B) represents the location that results when the method of Chapter IV is used.

From the figure,

\[ (V_C - V_{GS}) - x_2' = \frac{(V_2 - V_1)/2}{g} \]  

(5-1)

Also,

\[ a = \frac{V_2 - V_{DS}}{g} \]  

(5-2)

and

\[ b = (V_C - V_{GS}) - x_2' - a \]  

(5-3)

Substituting eqs. (5-1) and (5-2) into eq. (5-3) yields,

\[ b = \frac{V_2 - V_1}{2g} - \frac{V_2 - V_{DS}}{g} \]

or

\[ b = \frac{2V_{DS} - V_1 - V_2}{2g} \]  

(5-4)
Fig. 5-2. Incorrect location of straight-line transfer characteristic which results when the model of Chapter IV is used.
From the figure,

\[ V_C - V_{GS} = \Delta V + b \]  \hfill (5-5)

\[ x_1' = \frac{V_{DS} - V_1}{g} + \Delta V \]  \hfill (5-6)

\[ x_2' = \frac{V_{DS} - V_2}{g} + \Delta V \]  \hfill (5-7)

Therefore, by combining eqs. (5-4) and (5-5), we obtain,

\[ \Delta V = V_C - V_{GS} - \frac{2V_{DS} - V_1 - V_2}{2g} \]  \hfill (5-8)

Substituting eq. (5-8) into eqs. (5-6) and (5-7) results in,

\[ x_1' = V_C - V_{GS} + \frac{V_2 - V_1}{2g} \]  \hfill (5-9a)

\[ x_2' = V_C - V_{GS} + \frac{V_1 - V_2}{2g} \]  \hfill (5-9b)

The quantities \( x_1' \) and \( x_2' \) may be substituted for \( x_1 \) and \( x_2 \) in the model of Chapter IV and then the middle segment of the straight-line characteristic will be shifted from location B to location A, the desired position (Fig. 5-2). Since \( \Delta V \) is equal to zero when the chosen operating point is in the linear portion of the experimental curve, the correction factor will have no effect upon the satisfactory results that have already been obtained for this area. The modification will have an effect only when the operating point is located on the curved portions of the experimental transfer
characteristic. The only other consideration is that additional data, the value of $V_c$, is necessary in order to determine $\Delta V$. $V_c$ represents the value of the gate bias at the midpoint of the drain voltage variation. This quantity may be approximated graphically from the characteristic curves of the MOSFET. However, since this is also the point at which the small-signal gain used for the representation is measured, $V_c$ may be measured at the same time with no additional inconvenience.

The results of this modification for the case shown in Fig. 4-8 are given in Fig. 5-3. The dashed lines represent the results before making the characteristic shift and the dotted lines represent the results obtained after the modification. The results are slightly better after the correction is made but the effect is small. The fundamental and the third harmonic components of the modified model are better representations of the experimental components than those of the original model. But a greater error exists between the second harmonic component of the modified model and the experimental component than for that of the original model. The slope of the fundamental component for very small values of input voltage is the same for the modified model as for the original model, and the modified model component is also parallel to the experimental component in the remainder of the small-signal area, as was that of the original model. The results of all other experimental cases in which the operating point was in a curved
Fig. 5-3. Comparison of results obtained before and after the transfer characteristic shift modification.
area of the transfer characteristic showed the same slight improvement as did those for the case shown in Fig. 5-3.

B. Floating-Gain Model

Since the above correction had a small effect on the results, one can reason that the discrepancies between the calculated and the experimental results at small values of input voltage are primarily the result of the choice of small-signal gain. The question then arises as to whether or not a more suitable value of \( g \) could be determined for use with the representation. One possible consideration is shown in Fig. 5-4. The linear segment of the transfer characteristic of the model, line A, is constructed such that it passes through both the operating point on the actual characteristic and also the midpoint of the output voltage variation. Line A has a smaller slope than line B and therefore represents a new small-signal gain \( (g') \), the value of which is less than the value of \( g \) used in the previous models.

This new value of small-signal gain \( g' \) can be geometrically determined from Fig. 5-4. From the figure,

\[
g' = \frac{b}{a}
\]

Therefore,

\[
g' = \frac{(V_2 - V_1)/2 - (V_2 - V_{DS})}{V_C - V_{GS}}
\]
Fig. 5-4. Transfer characteristic for the floating-gain model.
or

\[ g' = \frac{2V_{DS} - V_1 - V_2}{2(V_C - V_{GS})} \]  

(5-10)

If \( g' \) is substituted for \( g \) in the mathematical analysis, then \( x_1 \) and \( x_2 \) automatically become \( x_1'' \) and \( x_2'' \) respectively, which are better representations of the true cutoff and saturation boundaries for many of the cases. The results obtained by using this representation are compared to those of the model of Chapter IV in Fig. 5-5. The input elements of the modified model for this case are determined and compared to those of the original model in the following paragraph.

The bias voltages for this case are:

\[ V_{GS} = -1.0 \]

\[ V_{DS} = 2.0 \]

The values determined from the drain characteristic of the MOSFET by the method discussed in the first part of Chapter IV are:

\[ V_1 = 1.1 \]
\[ V_2 = 7.8 \]
\[ V_C = -1.63 \]
\[ g = 4.45 \]
Fig. 5-5. Comparison of results obtained before and after the floating-gain modification.
These values are substituted into eq. (5-10) to obtain $g'$:

$$g' = \frac{2V_{DS} - V_1 - V_2}{2(V_C - V_{GS})}$$  \hspace{1cm} (5-10)

$$g' = \frac{2(2) - 1.1 - 7.8}{2(-1.63 + 1)}$$

$$g' = 3.89$$

The equations for $x_1$ and $x_2$ are

$$x_1 = \frac{V_{DS} - V_1}{g'}$$  \hspace{1cm} (5-11a)

and

$$x_2 = \frac{V_{DS} - V_2}{g'}$$  \hspace{1cm} (5-11b)

Therefore,

$$x_1'' = \frac{V_{DS} - V_1}{g'}$$  \hspace{1cm} (5-12a)

$$x_2'' = \frac{V_{DS} - V_2}{g'}$$  \hspace{1cm} (5-12b)

The values of $x_1''$ and $x_2''$ are:

$$x_1'' = \frac{2.0 - 1.1}{3.89} = 0.23 \text{ V}$$

$$x_2'' = \frac{2.0 - 7.8}{3.89} = -1.49 \text{ V}$$
Whereas, the values for $x_1$ and $x_2$ where:

$$x_1 = 0.2 \text{ V}$$

$$x_2 = -1.3 \text{ V}$$

These are all peak values, not rms values. By using these numbers in the computer program, the results in Fig. 5-5 were obtained.

For very small values of input voltage, the slope of the fundamental component is closer to the experimental slope than is the slope of the original representation. Because the magnitudes of $x_1''$ and $x_2''$ are larger than $x_1$ and $x_2$, the small-signal gain is effective for a greater range of input voltage. Therefore, the first transition in the fundamental curve occurs at a greater value of input than it does for the original representation. However, after the first transition is reached, the slope of the fundamental component is again less than it was for the original model. This causes the fundamental curve of the floating-gain model to approach the experimental curve as the input voltage increases, while the component of the original model is parallel to the experimental curve in this area. The third harmonic component obtained with the floating-gain model is very similar to that obtained with the characteristic shift model, that is, a closer representation of the experimental component than that of the original model. The second harmonic is a better representation than that of the original
model in the small-signal area, but as the input increases, a small
error develops which was not present in results obtained with the
original model.

For the cases where the operating point of the MOSFET was
located in the linear portion of the experimental characteristic,
the results obtained by using the floating-gain model were the
same as those obtained with the original model. In these cases,
line A and line B in Fig. 5-4 are the same. Therefore $g$ and $g'$
have the same value. The results of the remaining cases were
similar to those shown in Fig. 5-5.

C. Actual Small-Signal Gain

When this study was begun, it was readily assumed that
using the small signal-gain measured at the operating point would
provide very inaccurate results for many of the cases. Only when
the operating point is near the center of the linear portion of
the transfer characteristic would one expect accurate results,
as was shown in Fig. 3-2. For the sake of comparison, a set of
data was obtained using the actual small-signal gain. As was
expected, the results were accurate only when the operating point
was in the center. As the point was moved away from the center,
in either direction, the results became less and less accurate.
Typical results are shown in Fig. 5-6. The dashed lines represent
the results obtained by using a small-signal gain of 4.45 which
was determined by the method discussed in the first part of Chapter
Fig. 5-6. Comparison of results obtained before and after the actual small-signal gain modification.
IV. The dotted lines represent the results obtained by using a gain of 2.35 which was the value measured at the dc operating point of the MOSFET. As can be seen, large errors are evident when the actual small-signal gain is used.
CHAPTER VI

CONCLUSIONS

A piecewise-linear model may be used to predict the large-signal behavior of the MOSFET. The three straight-line-segment model which was developed in this study will provide good representations of the harmonic components of the output voltage of a resistively loaded MOSFET stage.

The accuracy of the representation is directly related to the location of the quiescent point on the transfer characteristic of the MOSFET. If the quiescent point is located in the center of the middle segment of the characteristic, the greatest degree of accuracy is obtained. If the point is located on either of the curved portions of the characteristic, a lesser degree of accuracy results.

Another factor that affects the accuracy of the model is the value of the load resistance that is used. The best results were obtained by using a 1 KΩ load, as compared to 2.5 KΩ and 10 KΩ loads. The increased error that resulted when the larger resistances were used appears to be caused by the graphical method of representation and not by the straight-line approximations.

Three different modifications of the original model were considered in order to obtain greater accuracy. The transfer characteristic shift modification actually corrects an oversight
in the original model and therefore produces more accurate results. A distinguishing characteristic of this modification is that in the region of input voltage where the greatest error occurs the fundamental component of the model is graphically parallel to that obtained experimentally. Since this portion of the experimental component consists of a straight line which passes through the origin of the graph, its location may be determined from the results obtained with the model. By making this alteration, one can increase the accuracy of the results considerably.

The other modification which increased the accuracy of the model is the floating-gain method. This modification provides the best representation of the experimental voltages obtained in this study. A particular advantage of this method is that the small-signal gain is not determined experimentally. All of the elements needed for the model may be obtained from the drain characteristic of the MOSFET. However, with this method, no apparent alterations can be made to determine the actual location of the fundamental component as can be done for the previous modification.

The third modification of the original model, the actual gain method, can be eliminated from any further consideration because of the large errors that resulted from its use. Either of the other two, the transfer characteristic shift or the floating gain modification, may be used to obtain satisfactory results.
The individual may determine which method he prefers by referring to the results obtained in this study.

More accurate piecewise-linear models for the large-signal behavior of the MOSFET could be developed by using more than three straight-line segments. However, a larger number of segments would greatly increase the complexity of the representation.
REFERENCES


APPENDIX A

FOURIER SERIES FOR LARGE-SIGNAL BEHAVIOR

The Fourier series for \( f(x) \) in the interval from 0 to \( 2\pi \) is given by,

\[
f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)
\]  

(A-1)

where,

\[
a_0 = \frac{1}{2\pi} \int_{0}^{2\pi} f(x) \, dx
\]  

(A-2a)

\[
a_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \cos nx \, dx
\]  

(A-2b)

\[
b_n = \frac{1}{\pi} \int_{0}^{2\pi} f(x) \sin nx \, dx
\]  

(A-2c)

Let,

\[
f(x) = k \sin x, \quad 0 < x < x_1
\]

\[
x_2 < x < x_3
\]

\[
x_4 < x < 2\pi
\]

\[
f(x) = kw, \quad x_1 < x < x_2
\]

\[
f(x) = kz, \quad x_3 < x < x_4
\]

(A-3)
Substituting eqs. (A-3) into eq. (A-2b) yields,

\[ a_n = \frac{1}{n} \left[ \int_0^{x_1} k \sin x \cos nx \, dx + \int_{x_1}^{x_2} k w \cos nx \, dx \right. \]

\[ + \int_{x_2}^{x_3} k \sin x \cos nx \, dx + \int_{x_3}^{x_4} k z \cos nx \, dx \]

\[ + \left. \int_{x_4}^{2n} \sin x \cos nx \, dx \right] \]

\[ a_n = \frac{k}{n} \left[ \int_0^{x_1} \sin x \cos nx \, dx + \int_{x_1}^{x_2} \cos nx \, dx \right. \]

\[ + \int_{x_2}^{x_3} \sin x \cos nx \, dx + \int_{x_3}^{x_4} \cos nx \, dx \]

\[ + \left. \int_{x_4}^{2n} \sin x \cos nx \, dx \right] \]

(A-4)
\[ a_n = \frac{k}{\pi} \left[ \frac{-\cos (1-n) x_1 - \cos (1-n) x_3 + \cos (1-n) x_4 + \cos (1-n) x_2}{2(1-n)} \right. \]
\[ \left. - \frac{\cos (1+n) x_1 + \cos (1+n) x_3 - \cos (1+n) x_4 - \cos (1+n) x_2}{2(1+n)} \right] \]
\[ + \frac{w \sin nx_2 - w \sin nx_1 + z \sin nx_4 - z \sin nx_3}{n} \]  

(A-5)

where,

\[ n = 2, 3, 4, 5, \ldots \]

For \( n \) odd, \((1-n)\) and \((1+n)\) are both even. From eqs. \((2-4)\), for \( m \) odd,

\[ \sin mx_2 = \sin mx_1 \]  

\[ \sin mx_4 = \sin mx_3 \]

(A-6a)

(A-6b)

For \( m \) even,

\[ \cos mx_2 = \cos mx_1 \]  

\[ \cos mx_4 = \cos mx_3 \]

(A-7a)

(A-7b)

By substituting eqs. \((A-6)\) and \((A-7)\) into eq. \((A-5)\), \( a_n \) may be obtained in terms of \( x_1 \) and \( x_3 \) only.
\[ a_n = \frac{k}{\pi} \left[ \frac{-\cos (1-n)x_1 - \cos (1-n)x_3 + \cos (1-n)x_3 + \cos (1-n)x_1}{2(1-n)} \right. \]

\[ + \frac{\cos (1+n)x_1 + \cos (1+n)x_3 - \cos (1+n)x_3 - \cos (1+n)x_1}{2(1+n)} \]

\[ + \frac{w(\sin nx_1 - \sin nx_1) + z(\sin nx_3 - \sin nx_3)}{n} \right] \]

\[ (A-8) \]

or

\[ a_n = 0, \quad \text{for} \quad n = 3, 5, 7, 9, \ldots \ldots \ldots \ldots \]

Since the general form does not hold for \( n = 1 \), \( a_1 \) will be considered as a special case. Substituting 1 for \( n \) in eq. (A-4) yields,

\[ a_1 = \frac{k}{\pi} \left[ \int_{0}^{x_1} \sin x \cos x \, dx + \int_{x_1}^{x_2} \cos x \, dx + \int_{x_2}^{x_3} \sin x \cos x \, dx \right. \]

\[ + \int_{x_3}^{x_4} \cos x \, dx + \int_{x_4}^{2\pi} \sin x \cos x \, dx \] \]

\[ (A-9) \]

\[ a_1 = \frac{k}{\pi} \left[ \frac{\sin^2 x_1}{2} + w \sin x_2 - w \sin x_1 + \frac{\sin^2 x_3}{2} - \frac{\sin^2 x_2}{2} \right. \]

\[ + \int_{x_3}^{x_4} \sin x \, dx - z \sin x_3 - \frac{\sin^2 x_4}{2} \right] \]

\[ (A-10) \]
Substituting eqs. (2-4) into eq. (A-10) yields,

\[ a_1 = \frac{k}{\pi} \left[ \frac{\sin^2 x_1}{2} + w \sin x_1 - w \sin x_1 + \frac{\sin^2 x_3}{2} - \frac{\sin^2 x_3}{2} \right. \]

\[ + z \sin x_3 - z \sin x_3 - \frac{\sin^2 x_3}{2} \left. \right] \]

(A-11)

or,

\[ a_1 = 0 \]

Therefore,

\[ a_n = 0 , \quad \text{for } n = 1, 3, 5, \ldots \]

Now we will consider \( b_n \) for the same function. Substituting eqs. (A-3) into eq. (A-2c) yields,

\[ b_n = \frac{1}{\pi} \left[ \int_0^{x_1} k \sin x \sin nx \, dx + \int_0^{x_2} kw \sin nx \, dx \right. \]

\[ + \int_0^{x_3} k \sin x \sin nx \, dx + \int_0^{x_4} kz \sin nx \, dx \]

\[ + \int_0^{2\pi} k \sin x \sin nx \, dx \left. \right] \]

(A-12)
\[ b_n = \frac{k}{n} \left[ \frac{\sin (1-n)x_1 + \sin (1-n)x_3 - \sin (1-n)x_2 - \sin (1-n)x_4}{2(1-n)} \right. \]

\[ - \frac{\sin (1+n)x_1 + \sin (1+n)x_3 - \sin (1+n)x_2 - \sin (1+n)x_4}{2(1+n)} \]

\[ + \frac{w \cos nx_1 - w \cos nx_2 + z \cos nx_3 - z \cos nx_4}{n} \right] \quad (A-13) \]

where,

\[ n = 2, 3, 4, 5, \ldots \]

For \( n \) even, \((1-n)\) and \((1+n)\) are odd. Therefore, by substituting eqs. \((A-6)\) and \((A-7)\) into eq. \((A-13)\), \( b_n \) may be obtained in terms of \( x_1 \) and \( x_3 \) only.

\[ b_n = \frac{k}{n} \left[ \frac{\sin (1-n)x_1 + \sin (1-n)x_2 - \sin (1-n)x_1 - \sin (1-n)x_3}{2(1-n)} \right. \]

\[ - \frac{\sin (1+n)x_1 + \sin (1+n)x_3 - \sin (1+n)x_1 - \sin (1+n)x_3}{2(1+n)} \]

\[ + \frac{w(\cos nx_1 - \cos nx_1) + z(\cos nx_3 - \cos nx_3)}{n} \right] \quad (A-14) \]

Therefore,

\[ b_n = 0, \quad \text{for } n = 2, 4, 6, \ldots \]
Eq. (A-1) for the Fourier series of $f(x)$ may now be written as,

$$f(x) = a_0 + a_1 \cos x + b_2 \sin 2x + a_3 \cos 3x + b_4 \sin 4x + \ldots$$

or,

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left[ a_{2n} \cos 2nx + b_{(2n-1)} \sin (2n-1)x \right] \quad (A-15)$$

Eq. (A-15) describes the harmonic content of the output voltage under large-signal conditions. The odd harmonic components consist of cosine functions only, and the even harmonic components are sine functions only.
APPENDIX B

COMPUTER SOLUTION FOR HARMONIC COEFFICIENTS IN FORTRAN IV

1 FORMAT (1H ,7F10.3)
2 FORMAT (1H1,6X,2HVG,8X,2HVD,8X,2HV1,8X,2HV2,8X,2HX1,8X,2HX2,9X,1HG)
3 FORMAT (15)
4 FORMAT (1H0,6X,1HV,9X,2HB1,8X,2HB3,8X,2HAAO,8X,2HA1,8X,2HA2,8X,2HA3)
5 FORMAT (7F10.3)
6 WRITE(12,2)
   READ(11,5)VG,VD,V1,V2,VC,G,D
   READ(11,3)J
   X1=(VD-V1)/G
   X2=(VD-V2)/G
   Z=SQRT(2.)
   WRITE(12,1)VG,VD,V1,V2,X1,X2,G
   WRITE(12,4)
   DO 40 I=1,J,1
   VRMS=I
   VRMS=D*VRMS
   V=Z*VRMS
   IF(X1-V)10,8,8
8   B1=22./14.
    GO TO 15
10  B1=ATAN(X1/(SQRT(V**2.-X1*X1)))
15  IF(X2&V)20,20,25
20  B3=66./14.
    GO TO 30
25  B3=ATAN(X2/(SQRT(V**2.-X2*X2)))
    B3=22./7.-B3
30  S1=SIN(B1)
    C1=CO(S(B1)
    S3=SIN(B3)
    C3=CO(S(B3)
    A0=(G*V**7./14.)*(-2.*C1&(22./7.-2.*B1)*S1-2.*C3&(66./7.-2.*B3)*S3)
    A1=(G*V**7./22.)*(B1&B3-22./7.&S1*C1&S3*C3)/Z
    A2=(G*V**14./66.)*(C1*C1*C1&C3*C3*C3)/Z
    A3=(G*V**14./66.)*(S1*C1*C1&C1&S3*C3*C3*C3)/Z
40  WRITE(12,1)VRMS,B1,B3,A0,A1,A2,A3
    GO TO 6
END
Characteristic Shift Modification

Substitute the following statements for X1 and X2:

\[ X_1 = V_C - V_G + \frac{(V_2 - V_1)}{2 \cdot G} \]
\[ X_2 = V_C - V_G + \frac{(V_1 - V_2)}{2 \cdot G} \]

Floating-Gain Modification

Insert the following statement immediately following READ(11,3)J:

\[ G = \frac{(2 \cdot V_D - V_1 - V_2)}{(2 \cdot (V_C - V_G))} \]

Sample Data Cards

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.000</td>
<td>2.000</td>
<td>1.100</td>
<td>7.800</td>
<td>-1.630</td>
<td>4.450</td>
<td>0.250</td>
</tr>
</tbody>
</table>

20

Definition of Terms:

\[ V_D = V_{DS} \]
\[ V_G = V_{GS} \]
\[ V_C = V_C \]
\[ V_1 = V_1 \]
\[ V_2 = V_2 \]
\[ X_1 = x_1 \]
\[ X_2 = x_2 \]
\[ G = g \]