Investigation of the Dielectric Properties of Grain at Microwave Frequencies

Arthur A. Kruse

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INVESTIGATION OF THE DIELECTRIC PROPERTIES
OF GRAIN AT MICROWAVE FREQUENCIES

BY

ARTHUR A. KRUSE

A thesis submitted
in partial fulfillment of the requirements for the
degree Master of Science, Department of
Electrical Engineering, South Dakota
State University

1969

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INVESTIGATION OF THE DIELECTRIC PROPERTIES OF GRAIN AT MICROWAVE FREQUENCIES

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser

Date

Head, Electrical Engineering
Department

Date
ACKNOWLEDGMENT

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AAK
ABSTRACT

The components of the complex dielectric constant of feed grains are obtained for the 8.5 to 12 GHz frequency range. The short-circuited wave guide method is employed to provide null-shift data for computer calculation of the first-order approximation of the real part of the complex dielectric constant under the simplifying assumption of negligible loss. Correction to this value is then made, where appreciable, by calculating the loss tangent from attenuation data independently obtained for the grains. The imaginary part of the complex dielectric constant then is obtained by multiplying the loss tangent by the real part.

Curves of the real and imaginary components are plotted versus frequency for two moisture levels each of corn, soybeans, wheat, and barley.
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CHAPTER I

DIELECTRICS

Historical Survey and Fundamental Properties

Much of the foundation of modern knowledge and theory of electromagnetic phenomena was laid by Michael Faraday in the first half of the nineteenth century. His brilliant experimental researches into the nature of electricity and magnetism and the relation between them opened the door for the hundreds of exceptional minds of succeeding years to make further advances and to approach ever more closely to the truth behind natural phenomena. Faraday is perhaps most noted now for his discovery that a changing magnetic field causes charges to flow in a conductor present in the field. This is the famous "Law of Induction" later to be quantitatively formulated and incorporated into a unified theory of electromagnetism by James Clerk Maxwell. Likewise, many of the other basic discoveries of Faraday were drawn upon, mathematically justified, and masterfully made a part of the modern electromagnetic theory by Maxwell, a great admirer of Faraday.

It is not the purpose here to dwell on the broad development of electromagnetic theory and knowledge, but rather to reconstruct the origins and to present the state of knowledge of the central element of this work—dielectrics. It has not been merely in passing, however, that the work of Faraday has been discussed, because it was he that provided the term "dielectric" and conducted the first.
researches into their behavior. Basically, his experiments revealed that materials varied greatly in their ability to deny movement of electric charge through them. His experiments in static electric effects (there were no sources of continuous electric current at that time) involved charging of a vessel having inner and outer elements, then measuring the potential difference between the elements. Knowing in advance the amount of charge transferred to the vessel, the "capacity" of the arrangement was determined by using the defining relation,

\[ C = \frac{Q}{V} \]  

(1-1)

where \( C \) denotes capacity, \( Q \) denotes charge, and \( V \) denotes potential difference. The unit of capacity in the MKS (meter-kilogram-second) system of units is the Farad in honor of his recognition of this property.

Initially, the space between the two elements was only air, but eventually other materials were inserted to see what effect they would have on the capacity of the vessel. Large variations were observed, especially when a metallic substance was made to touch the two elements, which caused the potential difference to drop immediately to zero. This indicated to Faraday that the charges moved through the metal so as to attain an equilibrium distribution producing neutralization. This action was called conduction of charge and the metal was classified as a conductor. On the other hand, if any of the non-metallic materials were inserted between the elements,
the potential would remain at a sensibly constant value for a considerable time although the value would be different for each material. These materials thus prevented the movement of charge from one element to the other to a large extent and were termed dielectrics. Maxwell later defined dielectric materials as "all bodies whose insulating power is such that when they are placed between two conductors at different potentials the electromotive force acting on them does not immediately distribute their electricity so as to reduce the potential to a constant value." The fact that each dielectric material caused the vessel (Faraday called it an "accumulator" because charge could be accumulated on it) to have a unique capacity, always greater than that for only air in the interelement space, revealed to Faraday that this was a measure of the grade or "dielectricity" of the material. He therefore defined a parameter of a dielectric as the ratio of the capacity of the accumulator with the material to the capacity with only air, and he called this parameter the "specific inductive capacity" of the material. Today it is commonly called relative dielectric constant or just dielectric constant.

Fundamental as these early observations were, Faraday contributed something even more important to the understanding of dielectrics. It was not discovered through a single experiment or a series of experiments; nor was it developed as a relationship

\[ \text{Bracketed numbers refer to sources listed in the bibliography.} \]
between physical quantities. It was simply his recognition that electrical actions occurring between two points are transmitted through the interposing medium, that is, that there is a modification of the action as it travels through the medium and is finally felt at the other point. The majority of the eminent mathematicians and physicists of his time and following years subscribed to the action-at-a-distance viewpoint in attempting to describe observed phenomena. To them whatever existed between the two points was of no interest, the resultant effect of the one quantity on the other being of sole importance. Where they saw empty space Michael Faraday saw lines of force and interacting matter. Both approaches gave accurate predictions of the observed results, but the field approach permitted better visualization of the physical configuration and led to more practical analysis. This kind of treatment was adopted by Maxwell to derive the essential behavior of dielectrics in electric fields; indeed, he used it in the entire development of his electromagnetic theory. It is apparent now that the concept of a force field existing between electrified points is one of the most important inventions in electrical science.

Maxwell, then, used his considerable mathematical ability to describe what happens when a dielectric body is placed in an electric field. He conceived of the existence of dipoles in the material (either permanently there or induced by the external field) even though the electron theory or atomic theory had not been presented yet. He envisioned the rearrangement or alignment of these dipoles
under the force of the applied field so as to modify the total field into a resultant. This alignment amounted to a displacement of the dipoles and caused polarization of the material. The degree of polarization depended upon the intrinsic character of the material. The resistance to being polarized was defined as the ratio of the local electric intensity to the electric displacement. This dielectric parameter was called the "coefficient of electric elasticity" of the material. The inverse of this quantity is more commonly used now and is called the permittivity of the dielectric, apparently because it is a measure of how much it permits polarization. Symbolically, it is written as

$$\varepsilon = \frac{D}{E} .$$

(1-2)

E denotes the electric intensity, D denotes the displacement, and $\varepsilon$ denotes the permittivity.

Evidently, this property of the dielectric is directly related to the specific inductive capacity or dielectric constant earlier mentioned since both are measures of susceptibility to charge displacement under the coersion of an electric field. In elementary electrostatics this relationship is shown to be

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} ,$$

(1-3)

where $\varepsilon$ is the permittivity of the dielectric, $\varepsilon_0$ is the permittivity of "free space" (vacuum or, nominally, air at atmospheric pressure), and $\varepsilon_r$ is the "relative permittivity" or dielectric constant of the dielectric.
Equation (1-3) is the defining statement of the primary identifying property of an insulating material; it is a comparison of the effect of a static electric field on the material to that on empty space. (Presumably, there would be no effect on empty space since no charges are assumed to reside there, however this condition of no effect is used as a reference for gauging the level of the effects in other media.) In the rationalized MKS system

$$\varepsilon_0 = \left(\frac{1}{36\pi}\right) \times 10^{-9} \text{ Farads per meter or, approximately, } 8.854 \times 10^{-12} \text{ Farads per meter.}$$

In simplified terms, the dielectric constant gives the fraction of the empty space electric field intensity which exists in the dielectric substance. This statement is better represented by rearranging equation (1-2) as

$$E = \frac{D}{\epsilon} = \frac{D}{\varepsilon_0 \varepsilon_r}.$$

It should be pointed out that the simple relation expressed in equation (1-3) does not describe completely all dielectric bodies. The additional mention must be made that some dielectric materials are anisotropic, that is, their dielectric properties are different for different directions through them; some are non-homogeneous, that is, their dielectric properties vary from point to point because of nonuniform composition; and some contain free charges so that they behave as semiconductors or plasma. Naturally the simple scalar relation given does not apply in these cases because permittivity is not a real constant for these materials, but is rather a complex quantity. Thus, it must be stipulated here that the
dielectrics to be considered are assumed to be isotropic, source-free, and homogeneous. Complex permittivity, however, will be defined and utilized in the following pages and will form a very important part of the analysis in this thesis.

Dielectrics in Alternating Fields and Complex Permittivity

The foregoing discussion presented the principal aspects of dielectrics in stationary (constant) electric fields. This topic is important in obtaining an understanding of the fundamental mechanisms involved, but of much greater importance in applications is the behavior of dielectrics in alternating fields.

Among the many contributions Maxwell made to the understanding of electromagnetic phenomena one of the most significant was the origination of "displacement current". Having imagined the existence of dipoles in dielectrics and their displacement by electric stress, he saw that the logical consequence of their movement was a net passage of charge across an arbitrary reference plane. This constituted a transient displacement current in the material as long as the dipoles were in the act of rotating or aligning. If the impressed electric field were continuously varying as in an alternating current field, the dipoles would be constantly moving, first in one direction then in the opposite direction, and an alternating displacement current would exist in the dielectric body. The magnitude of this current would of course depend on the number of dipoles in the material and the facility with which they could be aligned with the
external field, that is, it would depend upon the permittivity of the medium. Accordingly, Maxwell derived this current to be proportional to \( \frac{dD}{dt} \), the rate of change of the electric displacement, which can be written, \( \varepsilon \frac{dE}{dt} \), by equation (1-2). The introduction of this current factor was crucial in the development of the famous "Maxwell's Equations" which predicted the possibility of electromagnetic wave propagation through a dielectric medium. And, as will be seen in later chapters, propagation through dielectric material is an essential principle to the objective of this report.

The mechanism of polarization requires expenditure of energy by the polarizing agent. As noted by Maxwell, the process is elastic; the charges return to their normal positions as determined by existing molecular or atomic forces once the polarizing stress is removed. This means that there must be an overcoming of these forces in the process of inducing dipoles and causing their alignment. These forces acting on the dipoles so as to restrict their movement are analogous to friction in a mechanical system, consequently they are termed "damping forces". The work done by the external field in overcoming the damping forces constitutes energy lost by the field and dissipated in the dielectric.

Another mechanism of energy absorption in dielectrics is conduction current. Although dielectrics have been treated thus far as if they were perfect insulators, in practice most of them do have some conductivity. The current due to this conductivity (often
called "leakage current") produces heat in the dielectric and represents energy lost by the field.

The usual equivalent circuit representation of a capacitor is shown in Figure 1-1. C represents a capacitor with a lossless dielectric and $R$ represents both the conduction current loss and the loss due to damping forces. The equivalent impedance of the circuit is seen to be

$$Z = \frac{R}{1 + j\omega RC} \quad (1-4)$$

Neither $C$ nor $R$ is a constant with respect to frequency. This might have been anticipated since the mechanism of charge displacement and dipole alignment is affected by the rapidity of alternations of the electric field. The way the losses and polarization vary with frequency depend upon the molecular forces within the material. It can be shown that by ascribing a complex permittivity to the material the behavior of the dielectric at different frequencies can be satisfactorily described. [2,3] The complex permittivity and complex dielectric constant are defined in equation (1-5) below.

$$\hat{\varepsilon} = \varepsilon' - j \varepsilon'' \quad (1-5)$$

$$\hat{\varepsilon}_r = \frac{\varepsilon'}{\varepsilon_0} - j \frac{\varepsilon''}{\varepsilon_0} = \varepsilon'_r - j \varepsilon''_r$$

It is helpful in gaining an understanding of the complex permittivity and dielectric constant to relate its components to the equivalent circuit parameters of Figure 1-1. This can be done
by writing an expression for the impedance of a capacitor in terms of the dielectric constant of its dielectric, which is,

$$ Z_c = \frac{1}{j\omega \hat{\epsilon}_r C_0} $$  \hspace{1cm} (1-6)

and where $\hat{\epsilon}_r$ is the dielectric constant of the dielectric and $C_0$ is the capacitance of the capacitor with air as dielectric. Substituting equation (1-5) into equation (1-6) and rearranging, the capacitor impedance becomes,

$$ Z_c = \frac{\epsilon''_r}{\omega C_0(\epsilon''_r^2 + \epsilon'_{r2}^2)} - j \frac{\epsilon'_r}{\omega C_0(\epsilon''_r^2 + \epsilon'_{r2}^2)} $$ \hspace{1cm} (1-7)

If the real and imaginary parts of equations (1-4) and (1-7) are equated, (which is valid since both are expressions for the impedance of the same capacitor) the following pair of equations are extracted:

$$ \frac{\epsilon''_r}{\omega C_0(\epsilon''_r^2 + \epsilon'_{r2}^2)} = \frac{R}{1 + \omega^2 R^2 C^2} $$ \hspace{1cm} (1-8)

$$ \frac{\epsilon'_r}{\omega C_0(\epsilon''_r^2 + \epsilon'_{r2}^2)} = \frac{\omega R^2 C}{1 + \omega^2 R^2 C^2} $$ \hspace{1cm} (1-9)

By inspection of these simultaneous equations,

$$ \epsilon''_r = \frac{\epsilon'_r}{\omega R C} $$ \hspace{1cm} (1-10)
Figure 1-1. Equivalent circuit of a capacitor with lossy dielectric

Figure 1-2. Relationship of currents in a capacitor with lossy dielectric
Substituting equation (1-10) into equation (1-9) gives,

\[ \varepsilon'_r = \frac{\omega^2 R^2 C_0 (1 + \frac{1}{\omega R^2 C^2}) \varepsilon''_r}{1 + \omega^2 R^2 C^2} \]  

(1-11)

or

\[ \varepsilon'_r = \frac{\varepsilon}{\varepsilon_0} \]  

(1-12)

Equation (1-12) only verifies the fact that the real part of the complex dielectric constant is the dielectric constant as originally defined. That is, \( \varepsilon'_r \) is similar to \( \varepsilon_r \). The imaginary part of the complex dielectric constant is obtained by substituting (1-12) into (1-10), giving,

\[ \varepsilon''_r = \frac{1}{\omega R C_0} \]  

(1-13)

This states that the "loss factor" (\( \varepsilon''_r \)) decreases if \( R \) increases, as would happen if the conductivity or the damping became less. Additionally, the \( \frac{1}{\omega C_0} \) part of the expression tends to make the loss factor go down with increasing frequency, however, the manner in which \( R \) varies with frequency is a complicated function of atomic and molecular structure. In general it can be said that conductivity decreases with frequency, but the variation of the so-called "dielectric conductivity" (damping) yields to no such generalization. Therefore, the overall behavior of the loss factor versus frequency cannot be simply stated.
The frequency dependence of the real part of the complex dielectric constant is not evident from equation (1-12). It must be noted in connection with that expression that $C$ is not constant but rather varies as the permittivity of the dielectric material, which decreases as the frequency increases. This is as might be expected since the charges would have less time in which to be displaced during half-cycles of the external field and thus the level of polarization would be reduced somewhat. This of course assumes that the frequency is sufficiently high so as to be somewhere near the natural relaxation frequency of the material. For most dielectrics this would be at least in the microwave frequency range or higher. At lower frequencies not much variation in permittivity would be expected. At extremely high frequencies on the other hand it would be expected that little polarization would occur, and consequently the permittivity would approach that of free space.

Measurement of Dielectric Properties at Low Frequencies

At low frequencies, up to the limit of where reliable capacitors can be designed and constructed, the expressions of equations (1-12) and (1-13) are useful for determining the dielectric constant and loss factor of materials. Capacitance can be measured quite accurately on a bridge circuit or "Q-meter" and $R$ can be found by measuring the power factor of the capacitor. Figure 1-2 shows the relationship between currents in a capacitor with lossy dielectric. The loss current $I_R$ is proportional to $\varepsilon_r^n$ and the
displacement current $I_c$ is proportional to $\varepsilon'_r$. Thus, it is seen that

$$\tan \delta = \frac{I_R}{I_c} = \frac{\varepsilon''}{\varepsilon'_r} \tag{1-14}$$

Tan $\delta$ is called the loss tangent of the dielectric. For a good dielectric the loss factor is much less than the dielectric constant so that

$$\tan \delta = \cos \theta \tag{1-15}$$

very closely. Loss tangent is more commonly used to describe a dielectric than is loss factor.

Determination of the components of the complex dielectric constant at frequencies much above 100 megaHertz by the means discussed in the preceding paragraph becomes quite susceptible to error because of fringing fields, stray capacitance, and mutual inductance. At even higher frequencies, say above 1000 megaHertz, the wavelength becomes so short that the concept of a capacitor is no longer useful. The problem then is one of field theory rather than circuit theory. It is the purpose of this thesis to show a simple method for determining the approximate values of the components of the complex dielectric constant at these frequencies. Many methods have previously been described $[2,4,5,6,7]$, but the procedures to
be presented in the following chapters are adaptations centered around utilization of a digital computer and focused on the dielectric properties of feed grains.

It might be asked, "Why is it necessary to measure the dielectric constant of materials?" From the discussion of capacity and dielectric constant earlier, it is apparent that the design of capacitors requires knowledge of the dielectric constant of the dielectric to be used. At higher frequencies the need is perhaps greater because of the close relation of field quantities to dielectric properties. For instance, reflection and refraction of an electromagnetic wave at a dielectric boundary depend upon the dielectric constant of the two regions; also, the velocity of propagation of the wave through a dielectric is governed by its dielectric constant. Practical problems relating to these effects are illustrated by the design of radomes for high-frequency antennas [8], and by the design of pressure windows in wave guides. Reflection and phase shift are important factors influencing operation of systems in these examples. Other examples could be given to illustrate the need to know the dielectric constant of a material. The imaginary part of the complex dielectric constant is not usually of concern in this type of application because dielectrics having very small loss factors are normally employed and the thickness is kept as small as mechanical considerations will allow. Absorption of power from the wave therefore is negligible.
It isn't always the aim to minimize the effects of dielectrics on electromagnetic waves. Sometimes it is desired to achieve optimum interaction of the wave with the dielectric. Perhaps the best example of this is in microwave heating. In this process the loss factor of the material to be treated is of primary concern, as contrasted to the applications just mentioned above. The purpose is to dissipate as much high frequency energy as possible within the dielectric and thereby increase its temperature. In general, the rate of absorption of energy by the material increases with frequency, but it has been found that most dielectrics have certain ranges of frequencies for which absorption is very large compared to the rest of the spectrum. These regions roughly coincide with those for which the dielectric constant drops to a lower level. The typical behavior of the components of the complex dielectric constant are illustrated in Figure 1-3. The frequency ranges where these phenomena occur are near the characteristic oscillation frequencies corresponding to the polarization mechanisms occurring in the particular dielectric. Electronic and ionic polarization resonance frequencies typically exist in the vicinity of infrared radiation and ultraviolet light and the related absorption is not as large as that caused by the third type of polarization—orientational polarization. This mode of polarization (which occurs only in substances having permanent dipoles) produces an absorption band of large amplitude often in the microwave region for many substances.
Figure 1-3. Typical characteristic of $\varepsilon'_r$ and $\varepsilon''_r$ near the dipole relaxation frequency for an idealized dielectric.
To achieve the most efficient expenditure of high frequency power in heating the dielectric material for whatever purpose, it is necessary to know where this absorbing band is. This information can be obtained from a plot of $\varepsilon_\prime$ and $\varepsilon_\prime\prime$ across a wide range of frequencies, and this has been done for many substances. Obtaining this information for application of microwave heating to cereal grains is the main objective of the research for which this report is prepared.

The theoretical expressions to be used for determination of the dielectric parameters by microwave methods will be derived in the next chapter. Description of the measurements and calculations on actual samples of the grains will be covered in chapter three.
CHAPTER II
THE MEASUREMENT OF DIELECTRIC PROPERTIES AT MICROWAVE FREQUENCIES

There are numerous methods for measuring the dielectric properties of materials at microwave frequencies. The quantities that are measured in these methods vary; some require determination of phase shift, others require determination of amplitude change, and some depend upon frequency change required to maintain a stable reference phase. A comprehensive presentation of methods and criteria for their use is given by R. M. Redheffer \[4\].

Although the measuring techniques in each method differ, they all depend upon the same principles: that the dielectric constant of a material dictates the degree of reflection of an electromagnetic wave from its boundary, that the phase velocity of an electromagnetic wave traveling through a dielectric is governed by the dielectric constant of the medium, and that the decrease in amplitude of the wave in passing through the medium is determined by the loss factor of the medium. The field quantity or quantities which can be measured and related to these basic principles depend upon, among other things, the available equipment, the amount of the dielectric obtainable, and the shape or configuration of the material.
Standing Waves Near a Dielectric Boundary

Before going into the details of the method employed in this investigation, the derivation of expressions relating electromagnetic wave parameters to the dielectric properties discussed earlier will be given. To do this, a plane wave is assumed to be normally incident on a boundary between two dielectrics characterized by complex dielectric constants $\varepsilon_{r1}$ and $\varepsilon_{r2}$. Dielectric region 2 is considered to be infinite in dimensions to the right of the boundary so that only a single traveling wave exists in it. See Figure 2-1. The incident plane wave is assumed to be uniform and linear, with its E-field in the $y$ direction, which can be mathematically identified as

$$E^i_1 = E_1 e^{j\omega t - \gamma_1 x}, \quad (2-1)$$

where $\gamma_1$ is the propagation constant in region 1, and $x$ is the distance along the direction of propagation in the cartesian coordinate system. Similarly, the reflected wave due to the discontinuity of permittivities at the boundary has the form,

$$E^r_1 = E'_1 e^{j\omega t + \gamma_1 x}. \quad (2-2)$$

The resultant standing wave in region 1 is

$^{1}$A major part of this development is due to A. R. von Hippel in Dielectrics and Waves. See bibliography reference 2.
The ratio \( \frac{E_2'}{E_1} \) is defined as the voltage reflection coefficient at the boundary and is designated \( \rho_\varepsilon \). It can be shown by simplification of Fresnel’s equation from optics for this perpendicular incidence case that the reflection coefficient is given by

\[
\rho_\varepsilon = \frac{Z_2 - Z_1}{Z_2 + Z_1},
\]

where \( Z_1 \) and \( Z_2 \) are the intrinsic impedances of regions 1 and 2, respectively, which are given by

\[
Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_{r1} \varepsilon_0}}
\]

and

\[
Z_2 = \sqrt{\frac{\mu_0}{\varepsilon_{r2} \varepsilon_0}}.
\]

As indicated in Figure 2-1, the permeabilities of both regions can be considered to be the same as that for free space, because all dielectrics other than ferrites, certain garnets, and magneto-plumbites display negligible magnetic activity. The permeability of free space, \( \mu_0 \), is \( 4\pi \times 10^{-7} \) Henry/meter in the MKS system.
In terms of the reflection coefficient (2-3) becomes

\[ E_{1T} = E_1 e^{i\omega t} (e^{-\gamma_1 x} + \rho_e \gamma_1 x) \]

\[ = E_1 e^{i\omega t} - \gamma_1 x (1 + \rho_e e^{2\gamma_1 x}) \]

\[ = E_1 (1 + \rho_e e^{2\gamma_1 x}). \quad (2-6) \]

The standing wave of the magnetic fields in region 1 corresponding to equation (2-6) is

\[ H_{1T} = H_1 (1 + \rho_H e^{2\gamma_1 x}), \quad (2-7) \]

but

\[ \rho_H = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -\rho_E, \quad (2-8) \]

so equation (2-7) can be revised to

\[ H_{1T} = H_1 (1 - \rho_E e^{2\gamma_1 x}). \quad (2-9) \]

Equations (2-6) and (2-9) govern the form of the electric and magnetic fields in region 1. It is seen that the total field at a given point is composed of the incident field multiplied by the operator in parenthesis, which determines the amplitude and phase of the field at any point along the direction of propagation. At the boundary \( x = 0 \), these fields reduce to

\[ E_1(0) = E_1^i (1 + \rho_E) \] \quad (2-10)
Figure 2-1. Electric waves at boundary between dielectrics

Figure 2-2. Electric waves at two boundaries
and

\[ H_1(0) = H_1^i (1 - \rho_E). \]  \hspace{1cm} (2-11)

The ratio of \( E_1(0) \) to \( H_1(0) \) is defined as the wave impedance \( Z(0) \) at the boundary and is seen to be

\[ Z(0) = \frac{E_1(0)}{H_1(0)} = \frac{E_1^i (1 + \rho_E)}{H_1^i (1 - \rho_E)} \]

\[ = Z_1 \frac{(1 + \rho_E)}{(1 - \rho_E)}. \]  \hspace{1cm} (2-12)

It is desired to convert (2-12) into terms involving the normally measured quantities of the standing wave in region 1. These are voltage standing wave ratio (VSWR), location of the first minimum from the boundary \( (x_0) \), and wavelength \( (\lambda_1) \). To do this, the reflection coefficient \( \rho_E \) must be converted to polar form,

\[ \rho_E = e^{-2a} e^{-j2\Psi}, \]  \hspace{1cm} (2-13)

where \( 2\Psi \) is the phase angle and \( e^{-2a} \) is the amplitude of the coefficient, as determined by equation (2-4). The expression for VSWR in terms of the voltage reflection coefficient is

\[ VSWR = \frac{1 + |\rho_E|}{1 - |\rho_E|}, \]  \hspace{1cm} (2-14)

which becomes, by using (2-13),

\[ VSWR = \frac{1 + e^{-2a}}{1 - e^{-2a}} = \coth a. \]  \hspace{1cm} (2-15)
Using (2-13) the expression for the wave impedance at the boundary is changed to

\[ Z(0) = Z_1 \frac{(1 + e^{-2a}e^{-j2\psi})}{(1 - e^{-2a}e^{-j2\psi})} \]

\[ = Z_1 \frac{(1 + e^{-2(a + j\psi)})}{(1 - e^{-2(a + j\psi)})} = Z_1 \coth(a + j\psi). \quad (2-16) \]

Using the expansion of \( \coth(a + j\psi) \) as given in equation (2-17), \( Z(0) \) can also be written as shown in (2-18).

\[ \coth(a + j\psi) = \frac{\tanh a - j\cot \psi}{1 - j\tanh a \cot \psi} \quad (2-17) \]

\[ Z(0) = Z_1 \frac{\tanh a - j\cot \psi}{1 - j\tanh a \cot \psi}. \quad (2-18) \]

\( \tanh a \) and \( \psi \) in the foregoing expression may be replaced by measurable quantities from the standing wave pattern. The equivalent for the former has already been presented in equation (2-15), and that for the latter can be shown to be \( (2-19) \)

\[ \psi = \frac{\pi}{2} - \frac{2\pi x_0}{\lambda_1}, \quad (2-19) \]

where \( x_0 \) indicates the distance in region 1 of the first minimum of the standing wave pattern from the boundary. Introducing these two relations causes (2-18) to be
\[ Z(0) = Z_1 \frac{1}{\text{VSWR}} - j \cot \left( \frac{\pi}{2} - \frac{2\pi x_0}{\lambda_1} \right) \]
\[ \frac{1 - j \frac{1}{\text{VSWR}} \cot \left( \frac{\pi}{2} - \frac{2\pi x_0}{\lambda_1} \right)}{1 - j \frac{1}{\text{VSWR}} \tan \frac{2\pi x_0}{\lambda_1}} \]

The wave impedance at the boundary can thus be determined by measurements on the standing wave pattern in region 1. However, no knowledge of the dielectric constants of either medium is obtainable from (2-20) alone. What is needed is a second expression for \( Z(0) \) in terms of the parameters of region 2.

**Effect of a Second Boundary—the Conductor Simplification**

The dielectric of region 2 in Figure 2-1 was assumed to be infinite to the right of the boundary in order to simplify the derivation of equation (2-20). Actually, most samples of dielectrics to be used in practice will have finite dimensions. Possible reflections from a second boundary must therefore be introduced as shown in Figure 2-2. The standing wave in region 2 can be derived in the same manner as before and results in the following electric and magnetic fields

\[ E_{2T} = E_2 e^{j \omega t} (e^{-\gamma_2(x-d)} + \rho_{E2} e^{\gamma_2(x-d)}) \quad (2-21) \]

\[ H_{2T} = H_2 e^{j \omega t} (e^{-\gamma_2(x-d)} - \rho_{E2} e^{\gamma_2(x-d)}) \quad (2-22) \]
At the boundary between regions 1 and 2 (x = 0) these become

\[ E_2(0) = E_2 e^{j\omega t} (e^{\gamma_{2d}} + \rho_{E2} e^{-\gamma_{2d}}) \]  \hfill (2-23)

\[ H_2(0) = H_2 e^{j\omega t} (e^{\gamma_{2d}} - \rho_{E2} e^{-\gamma_{2d}}). \]  \hfill (2-24)

The ratio of (2-23) to (2-24) is again the wave impedance at the boundary between regions 1 and 2, \( Z(0) \), and is given by

\[ Z(0) = \frac{E_2(0)}{H_2(0)} = z_2 \frac{(e^{\gamma_{2d}} + \rho_{E2} e^{-\gamma_{2d}})}{(e^{\gamma_{2d}} - \rho_{E2} e^{-\gamma_{2d}})}. \]  \hfill (2-25)

In general the reflection coefficient \( \rho_{E2} \) at the second boundary (that is, between regions 2 and 3) is found by evaluating

\[ \rho_{E2} = \frac{Z_3 - z_2}{Z_3 + z_2}. \]  \hfill (2-26)

Two equations are now available for the wave impedance at the first boundary, (2-20) and (2-25). However, before equating these, further assumptions can be made in order to convert (2-25) to less cumbersome form. This is accomplished by making region 3 a conductor, for which the intrinsic impedance \( Z_3 = 0 \) because of its high conductivity. Thus, the reflection coefficient at the second boundary, \( \rho_{E2} \), reduces to

\[ \rho_{E2} = \frac{z_2}{z_2} = -1, \]  \hfill (2-27)
so that (2-25) simplifies to

\[ Z(0) = Z_2 \left( \frac{e^{\gamma_2 d} - e^{-\gamma_2 d}}{e^{\gamma_2 d} + e^{-\gamma_2 d}} \right) = Z_2 \tanh \gamma_2 d. \quad (2-28) \]

Equations (2-20) and (2-28) may now be equated since they both represent the wave impedance at the first boundary. Doing this produces

\[ Z_2 \tanh \gamma_2 d = Z_1 \frac{\frac{1}{\text{VSWR}} - j \tan \frac{2\pi x_0}{\lambda_1}}{1 - j \frac{1}{\text{VSWR}} \tan \frac{2\pi x_0}{\lambda_1}} \quad (2-29) \]

or

\[ \frac{Z_2}{Z_1} \tanh \gamma_2 d = \frac{\frac{1}{\text{VSWR}} - j \tan \frac{2\pi x_0}{\lambda_1}}{1 - j \frac{1}{\text{VSWR}} \tan \frac{2\pi x_0}{\lambda_1}}. \quad (2-30) \]

The factor \( \frac{Z_2}{Z_1} \) can be replaced by a term involving only the propagation constants \( \gamma_1 \) and \( \gamma_2 \) of the media. This term is derived by noting that

\[ \frac{Z_2}{Z_1} = \sqrt{\frac{\varepsilon_r r_1 \varepsilon_0}{\varepsilon_r r_2 \varepsilon_0}} = \frac{\gamma_1}{\gamma_2} \quad (2-31) \]

where \( Z_1 \) and \( Z_2 \) are given in equation (2-5) which are reproduced here,

\[ Z_1 = \sqrt{\frac{\mu_0}{\varepsilon_r r_1 \varepsilon_0}}, \quad Z_2 = \sqrt{\frac{\mu_0}{\varepsilon_r r_2 \varepsilon_0}} \quad (2-32) \]
and $\gamma_1$, $\gamma_2$ are the propagation constants for the two media as they occur in the wave functions which are solutions to Maxwell's equations,

$$\gamma_1 = j \omega \sqrt{\mu_0 \varepsilon_{r1} \varepsilon_0}$$  \hspace{1cm} (2-33)$$

$$\gamma_2 = j \omega \sqrt{\mu_0 \varepsilon_{r2} \varepsilon_0}$$ \hspace{1cm} (2-34)$$

Using (2-31) in equation (2-30)

$$\frac{\tanh \gamma_2 d}{\gamma_2} = \frac{1}{\gamma_1} \frac{1}{\text{VSWR}} - j \frac{1}{\text{VSWR}} \tan \frac{2\pi\varepsilon_0}{\lambda_1}$$ \hspace{1cm} (2-35)$$

Equation (2-35) is an expression relating the dielectric parameters of region 2 to the same parameters of region 1 along with measurable characteristics of the standing wave in region 1. To make the left-hand term more suitable for quantitative analysis both sides are divided by $d$ (the thickness of medium 2), making the left-hand side of the form $\tanh \frac{X}{X}$. The altered form of (2-35) then is

$$\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{1}{\gamma_1 d} \frac{1}{\text{VSWR}} - j \frac{1}{\text{VSWR}} \tan \frac{2\pi\varepsilon_0}{\lambda_1}$$ \hspace{1cm} (2-36)$$

Equation (2-36) is the desired result of the derivation, giving indirectly the complex dielectric constant of medium 2 from measurements in medium 1 and the parameters of medium 1. In
principle, then, this relation can provide the answers sought. In practice, however, it is found preferable to reformulate and further reduce (2-36).

Conversion for Bounded Propagation

The most common method of making measurements on standing waves is by using a probe in a transmission line in which the standing wave exists. Accurate, well-designed equipment is widely available for making such measurements. The unbounded-space configuration would require in most cases fabrication of a special test jig. For tests on materials of granular nature, such as the grains which were of primary interest in this research, the unbounded-space set-up is impractical. For these reasons, the tests on dielectrics performed during this investigation were done by the "short-circuited wave guide" method. Figure 2-3 depicts the arrangement of the dielectric material, the wave guide, and the shorting plate which comprise the sample and the sample holder.

Converting (2-36) to conform to the wave guide environment merely requires changing the applicable terms in this expression to indicate that they are peculiar to wave guide transmission. The terms affected are the propagation constants $\gamma_1$ and $\gamma_2$, and the wavelength $\lambda_1$. They are changed to reflect wave guide values by adding the subscript $g$ to them, thus becoming $\gamma_{1g}$, $\gamma_{2g}$, and $\lambda_{1g}$, respectively. Nothing need be said as yet as to the relation between the unbounded and bounded (guided) factors. (In any case, only the
relation between the intrinsic factor $\gamma_2$ and the guided factor $\gamma_{2g}$ is important since the other factors can be determined from measurements.) It is sufficient to express (2-36) in terms of these guide parameters, namely,

$$
\frac{\tanh \gamma_{2g}^d}{\gamma_{2g}^d} = \frac{1}{\frac{1}{\text{VSWR}} - j \tan \frac{2\pi \nu_0}{\lambda_{1g}}} \cdot (2-37)
$$

The procedure will be to determine $\gamma_{2g}$ from this expression, then convert it to the intrinsic value from which the dielectric constant can be obtained for region 2. The details of this process will be covered in the ensuing presentation. At this point it is necessary to consider (2-37) further.

One simplification of the expression is to designate region 1 as being lossless. Since the medium is air at atmospheric pressure, it has in fact negligible loss as discussed in Chapter I. This means that a wave traveling through air (assuming it is relatively dry air) suffers very little attenuation, so that $\gamma_{1g} = \alpha_{1g} + j\beta_{1g} \approx j\beta_{1g}$. Now, $\beta_{1g} = \frac{2\pi}{\lambda_{1g}}$, so

$$
\gamma_{1g} = j \frac{2\pi}{\lambda_{1g}}. \quad (2-38)
$$

Putting this in (2-37) gives

$$
\frac{\tanh \gamma_{2g}^d}{\gamma_{2g}^d} = -j \frac{\lambda_{1g}}{2\pi \text{VSWR}} \frac{1}{\frac{1}{\text{VSWR}} - j \tan \frac{2\pi \nu_0}{\lambda_{1g}}} \cdot (2-39)
$$
Figure 2-3. Wave guide sample holder cross-section

Figure 2-4. Schematic representation of standing waves with and without sample
Equation (2-39) is a modified form of (2-36) to fit the particular conditions of the sample abutting a metal plate and within an otherwise air-filled rectangular wave guide.

**Lossless Dielectric**

The dielectric sample will be considered lossless at this point because of simplicity in calculations. Equation (2-39) involves the hyperbolic tangent of a complex argument. Such a function makes calculation of the variable, \( \gamma_{2g} \), rather difficult; a plot of the left-hand function of (2-39) can be found, however, in references such as *Dielectric Materials and Applications*, reference 9 of the bibliography.

The justification for assuming the lossless case is the practical consideration that many dielectrics have very low loss factors so that \( \gamma_{2g} \) can be treated as a simple quantity instead of a complex one with very little introduction of error. Moreover, the simple expression can be exploited even for materials with appreciable loss if a correction is subsequently made, as will be demonstrated in the following.

The assumption that the dielectric in region 2 is lossless is contained in the relation

\[
\gamma_{2g} = j \beta_{2g} . \tag{2-40}
\]

Also, note that the VSWR becomes very large because \( E_{\text{min}} \) approaches zero. This simplifies (2-39) to
\[
\frac{\tanh j\beta_2g^d}{j\beta_2g^d} = -\frac{\lambda_{lg}}{\lambda_{lg}} \tan \frac{2\pi x_0}{\lambda_{lg}}.
\]

(2-41)

The hyperbolic function can be converted to a circular function by noting that \(\tanh jX = j \tan X\). Then (2-41) can be written as

\[
\frac{\tan X}{X} = -\frac{\lambda_{lg}}{\lambda_{lg}} \tan \frac{2\pi x_0}{\lambda_{lg}},
\]

(2-42)

where \(X = \beta_2g^d\), or, alternatively,

\[
\frac{\tan X}{X} = \frac{\lambda_{lg}}{\lambda_{lg}} \tan (-\frac{2\pi x_0}{\lambda_{lg}}).
\]

(2-43)

Equation (2-43) still requires knowledge of the distance of the first minimum \((x_0)\) from the boundary of the dielectric. With commonly available slotted sections the first minimum cannot be directly located. It is necessary therefore to derive an equivalent factor in terms of the location of nulls which are accessible on the ordinary wave guide slotted section. Figure 2-4 shows possible standing wave patterns with and without the dielectric sample, and related dimensions. From this figure, \(D_a = n(\frac{\lambda_{lg}}{2})\), where \(n\) is an integer. Also, \(D_s = m(\frac{\lambda_{lg}}{2}) + x_0 + d\), where \(m\) is an integer. Note also that \(D_a = D_s + \Delta l\), so

\[
n(\frac{\lambda_{lg}}{2}) = m(\frac{\lambda_{lg}}{2}) + x_0 + d + \Delta l.
\]
Transposing gives

\[ x_0 = \frac{\lambda_{1g}}{2} (n - m) - \Delta l - d. \]  

(2-44)

This is the substitute factor sought for \( x_0 \); when it is inserted into (2-43), this expression becomes

\[ \frac{\tan X}{X} = \frac{\lambda_{1g}}{2\pi d} \tan \left[ -\frac{2\pi}{\lambda_{1g}} \left( \frac{\lambda_{1g}}{2} (n - m) - \Delta l - d \right) \right] \]

\[ = \frac{\lambda_{1g}}{2\pi d} \tan \left[ +\pi (m-n) + \frac{2\pi}{\lambda_{1g}} (\Delta l + d) \right] \]  

(2-45)

The term \( \pi (m - n) \) in the argument contributes nothing because \( \tan(\pi \pm \eta) = \tan \eta \). Therefore (2-45) can be put into the form finally used in this investigation: \[ \sqrt{10} \]

\[ \frac{\tan X}{X} = \frac{\lambda_{1g}}{2\pi d} \tan \frac{2\pi}{\lambda_{1g}} (\Delta l + d). \]  

(2-46)

To summarize what has been done, the basic expression, equation (2-36), has been modified for use in the shorted wave guide test set and subsequently to facilitate computation by assuming a lossless region 2, resulting in equation (2-46). It is now possible to determine \( X(\beta_{2g} d) \) by simply measuring \( \Delta l \) (the shift in null toward the short when the dielectric sample is inserted) and \( \lambda_{1g} \) (twice the distance between nulls) on a slotted section connected ahead of the test chamber. \( X \) is determined from tables of the function \( \frac{\tan X}{X} \) or by trial and error. Unfortunately, there are an
infinite number of values of $X$ which will satisfy a particular numerical value of the right-hand side of (2-46). The correct value can be estimated by knowing the approximate value of $\varepsilon_{r2}'$ (dielectric constant of region 2), but this is rather improbable. The only other way of isolating the desired value is to measure $\Delta l$ for two different sample lengths, evaluate expression (2-46) for both, and calculate $\varepsilon_{r2}'$ for a number of the possible values of $X$ for both samples. The true dielectric constant should remain relatively constant for the two trials whereas the incorrect values should vary significantly. Obviously, the amount of calculation required in this process prohibits doing it by manual methods; a computer program is required to reduce the time and work involved. The program will be discussed later.

Relation of Dielectric Constant to Measured Quantities

At this point the derivation of the relation between the dielectric constant and the factor $\beta_{2g}d$ must be undertaken. The starting point toward this end is the expression for the propagation constant in the dielectric-filled wave guide,

$\gamma_{2g} = \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + \gamma_2^2}$, \hspace{1cm} (2-47)

where $\gamma_{2g}$ and $\gamma_2$ are as previously defined, $m$ and $n$ are the mode numbers, and $a$ and $b$ are the width and height of the wave guide, respectively. Many books contain the derivation of this useful expression for rectangular wave guides [11]. This equation provides
the relation needed between the intrinsic propagation constant $\gamma_2$ (from which the dielectric constant will later be derived) and the bounded propagation constant $\gamma_{2g}$. For the assumed lossless condition

$$\gamma_{2g} = j\beta_{2g}$$

and

$$\gamma_2 = j\beta_2,$$  \hspace{1cm} (2-48)

thus, in place of (2-47), after squaring both sides,

$$-\beta_{2g}^2 = \left(\frac{mn}{a}\right)^2 + \left(\frac{mn}{b}\right)^2 - \beta_2^2$$

or

$$\beta_{2g}^2 = \beta_2^2 - \left[\left(\frac{mn}{a}\right)^2 + \left(\frac{mn}{b}\right)^2\right].$$  \hspace{1cm} (2-49)

It is seen from (2-49) that $\beta_{2g}$ goes to zero whenever $\beta_2^2$ equals $(\frac{mn}{a})^2 + (\frac{mn}{b})^2$. This means that the wave doesn't propagate for this condition (because the phase shift constant is zero), so it is called the cutoff condition of the particular mode designated by $m$ and $n$. The associated cutoff wavelength $\lambda_c$ is determined from

$$\beta_{2g}^2\bigg|_{cutoff} = \left(\frac{2m}{\lambda_c}\right)^2 = \left(\frac{mn}{a}\right)^2 + \left(\frac{mn}{b}\right)^2.$$  \hspace{1cm} (2-50)

Using this in (2-49) $\beta_{2g}^2$ becomes

$$\beta_{2g}^2 = \beta_2^2 - \left(\frac{2m}{\lambda_c}\right)^2.$$
or, transposing,

\[ \beta_2^2 = \beta_{2g}^2 + \left( \frac{2n}{\lambda_c} \right)^2 \]  \hspace{1cm} (2-51)

It remains to derive the connection between \( \varepsilon_{r2} \) and the phase shift constant \( \beta_2 \). This is accomplished by referring to the definition of equation (2-5) which is repeated here,

\[ \gamma_2 = j \omega \sqrt{\mu_0 \varepsilon_{r2} \varepsilon_0} \cdot \]

From this, for the assumption of a lossless dielectric,

\[ j \beta_2 = j \omega \sqrt{\mu_0 \varepsilon_{r2} \varepsilon_0} \]  \hspace{1cm} (2-52)

since \( \varepsilon'' \) is assumed negligible. Squaring both sides of (2-52) gives

\[ \beta_2^2 = \omega^2 \mu_0 \varepsilon_{r2}' \varepsilon_0 \]  \hspace{1cm} (2-53)

from which the desired relation between \( \varepsilon_{r2}' \) and \( \beta_2 \) is obtained:

\[ \varepsilon_{r2}' = \frac{\beta_2^2}{\omega^2 \mu_0 \varepsilon_0} \]  \hspace{1cm} (2-54)

Further simplification of this is possible considering that

\[ \frac{1}{\omega^2 \mu_0 \varepsilon_0} = \frac{\varepsilon^2}{(2\pi f)^2} = \left( \frac{\lambda_0 f}{2\pi f} \right)^2 = \left( \frac{\lambda_0}{2\pi f} \right)^2 \]
making (2-54) read

$$\varepsilon'_r = \left( \frac{\beta_2 \lambda_0}{2\pi} \right)^2, \quad (2-55)$$

where $\lambda_0$ denotes the wavelength in free space at the frequency of interest, and $c$ is the velocity of light in free space.

The expression for $\beta_2$ from (2-51) may be substituted in (2-55), which makes it

$$\varepsilon'_r = \left[ \beta_2^2 \lambda_0^2 + \left( \frac{2\pi}{\lambda_c} \right)^2 \right] \left( \frac{\lambda_0}{2\pi} \right)^2$$

$$= \left( \frac{\beta_2 \lambda_0}{2\pi} \right)^2 + \left( \frac{\lambda_0}{\lambda_c} \right)^2. \quad (2-56)$$

The last term, $\left( \frac{\lambda_0}{\lambda_c} \right)^2$, can be written in terms of the directly measured wave guide wavelength $\lambda_{1g}$ by use of another well-known expression from the wave guide theory:

$$\lambda_{1g} = \frac{\lambda_0}{\sqrt{1 - \left( \frac{\lambda_0}{\lambda_c} \right)^2}}. \quad (2-57)$$

This can be rewritten as

$$1 - \left( \frac{\lambda_0}{\lambda_c} \right)^2 = \left( \frac{\lambda_0}{\lambda_{1g}} \right)^2.$$
so

\[
\left( \frac{\lambda_0}{\lambda_c} \right)^2 = 1 - \left( \frac{\lambda_0}{\lambda_{1g}} \right)^2 \tag{2-58}
\]

And (2-56) becomes, alternatively,

\[
\epsilon_{r2} = \left( \frac{\beta_{2g} \lambda_0}{2\pi} \right)^2 + 1 - \left( \frac{\lambda_0}{\lambda_{1g}} \right)^2 \tag{2-59}
\]

Finally, the first term on the right-hand side is multiplied in numerator and denominator by \( d \) so that the factor \( \beta_{2g} d = \chi \) from equation (2-46) can be used directly for calculation. This makes (2-59) take the form

\[
\epsilon_{r2} = \left( \frac{\chi \lambda_0}{2\pi d} \right)^2 + 1 - \left( \frac{\lambda_0}{\lambda_{1g}} \right)^2 \tag{2-60}
\]

**Correction of Dielectric Constant due to Losses**

Equations (2-46) and (2-60) are the pair of expressions needed for calculating the dielectric constant of a material by the short-circuited wave guide method. It is recalled, however, that the equations were derived under the simplifying assumption that the loss factor of the material was zero. For materials with loss factors not approximating this criteria, calculation by use of (2-46) and (2-60) results in a value for the dielectric constant that is only a first-order approximation. Although it is not the aim in this investigation to obtain highly accurate results but
rather to acquire a reasonable estimate of the dielectric constant, it is desirable to make an adjustment to the first-order approximation whenever the correction is a significant fraction of the value computed from the two equations cited. It is the purpose in the concluding portion of this chapter to define what is significant and to show how the correction is made.

The correction for the dielectric constant can be derived from the relation given in (2-5), which is, \( \gamma_2 = \alpha_2 + j\beta_2 = j\omega \sqrt{\mu_0 \hat{\varepsilon}_{r2} \varepsilon_0} \). Now from equation (1-5), \( \hat{\varepsilon}_{r2} = \varepsilon'_{r2} - j\varepsilon''_{r2} \). Putting this into the foregoing and squaring both sides yields

\[
(\alpha_2 + j\beta_2)^2 = -\omega^2 \mu_0 \varepsilon_0 \left( \varepsilon'_{r2} - j\varepsilon''_{r2} \right)
\]

\[
= \omega^2 \mu_0 \varepsilon_0 \varepsilon'_{r2} (1 - j\tan \delta) \tag{2-61}
\]

by equation (1-14). Expanding this and equating real and imaginary parts gives

\[
\alpha_2^2 - \beta_2^2 = -\omega^2 \mu_0 \varepsilon_0 \varepsilon'_{r2}
\]

\[
\alpha_2 \beta_2 = \frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_{r2} \tan \delta}{2} \tag{2-62}
\]

Solving the lower equation for \( \alpha_2 \) and inserting it into the upper equation results in the quadratic equation in \( \beta_2^2 \),

\[
(\beta_2^2)^2 - \omega^2 \mu_0 \varepsilon_0 \varepsilon'_{r2} \beta_2^2 - \frac{(\omega^2 \mu_0 \varepsilon_0 \varepsilon'_{r2} \tan \delta)^2}{4} = 0 \tag{2-63}
\]
for which the solution is

\[
\beta_2 = \frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r}{2} + \sqrt{\left(\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r}{4}\right)^2 + \left(\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r \tan \delta}{4}\right)^2}
\]

\[
= \frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r}{2} (1 \pm \sqrt{1 + \tan^2 \delta})
\]  \hspace{1cm} (2-64)

Taking the square root of (2-64) produces

\[
\beta_2 = \pm \sqrt{\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r}{2}} (1 \pm \sqrt{1 + \tan^2 \delta})^{\frac{1}{2}} .
\]  \hspace{1cm} (2-65)

In order to avoid a negative \(\alpha_2\) (corresponding to negative attenuation or power gain, an impossible condition), the positive sign is selected on both radicals. Thus (2-65) is rewritten

\[
\beta_2 = \sqrt{\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon'_r}{2}} (1 + (1 + \tan^2 \delta)^{\frac{1}{2}})^{\frac{1}{2}} .
\]  \hspace{1cm} (2-66)

Applying the special binomial theorem to the inner radical and using only the first two terms gives the good approximation:

\[
(1 + \tan^2 \delta)^{\frac{1}{2}} \approx 1 + \frac{\tan^2 \delta}{2}
\]  \hspace{1cm} (2-67)

(The next term in the expansion would be \(-\frac{\tan^4 \delta}{8}\), which is obviously insignificant if \(\tan \delta \ll 1\).) Using (2-67) simplifies (2-66) to
\[
\beta_2 = \sqrt{\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon_{r2}'}{2} \left(2 + \frac{\tan^2 \delta}{2}\right)^{\frac{1}{3}}}
\]

\[
= \sqrt{\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon_{r2}'}{2} \left(1 + \frac{\tan^2 \delta}{4}\right)^{\frac{1}{3}}}. \tag{2-68}
\]

The special binomial expansion is again employed on the second radical, thus,

\[
(1 + \frac{\tan^2 \delta}{4})^{\frac{1}{3}} \approx 1 + \frac{\tan^2 \delta}{8}, \tag{2-69}
\]

making (2-68) read

\[
\beta_2 = \sqrt{\frac{\omega^2 \mu_0 \varepsilon_0 \varepsilon_{r2}'}{2} \left(1 + \frac{\tan^2 \delta}{8}\right)}
\]

\[
= \beta_2(\text{lossless})(1 + \frac{\tan^2 \delta}{8}). \tag{2-70}
\]

Equation (2-70) demonstrates that the phase-shift constant depends not only upon the real part of the complex dielectric constant but also upon the imaginary part or loss factor \(\varepsilon_{r2}'\) through the loss tangent. Thus the dielectric constant \(\varepsilon_{r2}'\) theoretically cannot be determined by measurement of null shift alone as was supposed in (2-46). For greatest accuracy the general expression of (2-39) would have to be utilized. However, as previously stated and as shown by (2-70), when the material has low loss \((\tan \delta < 0.1)\) the theoretical \(\beta_2\) and the \(\beta_2\) calculated from (2-46) and (2-51), for the lossless case, will, for practical purposes, be equal. For example, for \(\tan \delta = 0.1\), \(\beta_2 = \beta_2(\text{lossless})(1 + 0.00125),\)
a difference of only 0.125 per cent. When this \( \beta_2 \) is put in (2-55) to compute the corrected value of the dielectric constant \( |\varepsilon'_r|_c \), the change is

\[
|\varepsilon'_r|_c = \varepsilon'_r(1 + 0.00125)^2 = \varepsilon'_r(1.0025),
\]

still much less than one per cent. Corrections to the dielectric constant of less than five percent will be deemed insignificant here. This means that for loss tangents less than about 0.45 no correction of the value obtained from (2-60) is necessary. For loss tangents between 0.45 and 0.75 the correction can be made in one step as given by

\[
|\varepsilon'_r|_c = \varepsilon'_r(1 + \frac{\tan^2 \delta}{8})^2. \tag{2-71}
\]

For loss tangents above 0.75 higher order terms in the expansion must be used.

**Determination of Loss Tangent and Loss Factor**

It has been noted that the corrected dielectric constant is in general greater than the first-order approximation and depends on the magnitude of \( \tan \delta \). Nothing, as yet, has been said of how \( \tan \delta \) is to be determined. An expression for \( \alpha_2 \) can be derived from the two equations of (2-62) in the same way that (2-70) was obtained. The result is

\[
\alpha_2 = \frac{\beta_2 \tan \delta}{2} \tag{2-72}
\]

so,

\[
\tan \delta = \frac{2\alpha_2}{\beta_2}. \tag{2-73}
\]
An independent measurement of attenuation caused by the material can be made and $\alpha_2$ determined from it. The measurement set-up and auxiliary equations for converting attenuation to $\tan \delta$ will be presented in the next chapter.

The imaginary part of the complex dielectric constant, or loss factor, can now be extracted by

$$\varepsilon''_{r2} = \left| \varepsilon'_{r2} \right| c \tan \delta' . \quad (2-74)$$

Thus the desired plot of $\varepsilon'_{r2}$ and $\varepsilon''_{r2}$ with frequency can be made and the frequency range for maximum absorption can be found (if it exists in the frequency range selected for measurement).

The frequency at which the loss factor peaks isn't necessarily the optimum frequency for microwave heating of a particular material. Consideration also must be given to penetration of the wave into the dielectric body. The depth of penetration is defined as the distance within the material at which the energy of the wave is reduced to $1/e$ of its value at the boundary of the material. The depth of penetration decreases as the frequency increases, in general, and is given by

$$\text{Depth of penetration} = \frac{\lambda_0}{2\pi \tan \delta \sqrt{\varepsilon'_{r}}} . \quad (2-75)$$

---

$^2$From a paper to be published by Professor H. D. Gorakhpurwalla, College of Engineering, South Dakota State University.
The dimensions of the body to be heated are therefore important in determining the frequency for obtaining uniform treatment and satisfactory efficiency. Greater penetration is a prime advantage of heating with microwaves over infrared heating. (The larger absorption at microwave frequencies for dipolar substances was mentioned in chapter one.)

In summary it should be noted that the important results obtained are contained in three equations: (2-46), (2-60), and (2-71). These three expressions are the primary ones used during this research.
CHAPTER III

EXPERIMENTATION AND RESULTS

Frequency Range and Properties of Water

The preceding chapter presented the theory supporting the experimental work to be discussed in the present chapter. All null-shift measurements were taken by the short-circuited wave guide technique already considered at some length. The method for obtaining loss tangent data will be described later in this chapter. The frequency range used was roughly the "X-band" of microwave frequencies, 8.5 to 12 GHz.

The choice of frequency range was influenced by two factors:
1. Equipment designed for operation at X-band was available.
2. Information indicated that the absorption maximum for mixtures of a dielectric containing water may occur in the X-band.

The second factor needs to be further explained. Results of measurements by others on water put its absorption frequency (that is, its dipole relaxation frequency) at about 17 GHz. Further, it was determined that bound water (water absorbed in another material) had a lower absorption frequency than free water or pure water. How much lower depends upon how tightly bound and to what kind of molecules. Thus, it is probable that an absorption maximum would be found for a mixture of grain and water (newly harvested grains often contain moisture in excess of 10%) at a frequency somewhere between 8.5 and 17 GHz.
As has been alluded to previously, the materials of primary interest in this research are various grains such as corn, soybeans, wheat, and barley. To the knowledge of the writer, the dielectric characteristics at microwave frequencies of these common agricultural products have not previously been determined, although measurements at frequencies below 100 mHz have been reported [12].

The obvious consequence of choosing to make measurements on such materials is that the carefully-developed theory in chapter two is not exactly applicable. The materials are by nature unhomogeneous, having kernels of different sizes and shapes with empty spaces between. Thus the volume as well as the boundary of a sample of any grain would be inherently uneven and multi-directional reflection of an electromagnetic wave would be caused by the sample. The grains could be ground and otherwise processed to give them a more homogeneous composition, but this would defeat the purpose here---to obtain an estimate of the dielectric properties of the grains in their natural state.

This investigation then is undertaken with the knowledge that the results are only reasonable approximations because of the deviation of the tested sample from the theoretically-assumed homogeneous sample. The deviation from the ideal would be expected to be most pronounced for grains with large kernels such as corn and soybeans. The experience gained from this research, however, indicates that the deviation from ideal may not result in as great an error in application of the theoretical expressions as might be
anticipated. In many cases, even with samples of corn, the change in null locations with several different samples were very small. This would not occur if the spacing and juxtaposition of the kernels with respect to each other and the wave guide surfaces were extremely critical factors. Apparently the grain occupies a great enough percentage volume of the wave guide (compared to the volume of air) that the assumption of homogeneity is not far from true, and the results are fairly good approximations, especially when results of several trials are averaged.

Method for Obtaining Null-Shift Data

The arrangement of the test equipment for obtaining the null-shift information to be used in equation (2-46) is shown in block-diagram form in Figure 3-1. The set-up is standard for making standing wave measurements, with the sample holder comprising the load. The sample holder is simply a section of WR-90 wave guide with a brass plate attached to one flange. In the test set-up it is oriented in the vertical plane to permit easy insertion of the grain sample.

Nulls or minima of the standing wave pattern were located in the normal manner, by moving the probe to a position where the standing wave indicator dipped and then reading the carriage scale. Additional accuracy was obtained by attaching a dial indicator with graduations of 0.001 inch to the carriage. For some samples which produced low VSWR it was necessary to locate points on both sides
Figure 3-1. Block diagram of test set-up for null-shift measurements
of the minimum corresponding to the same voltage level and then compute the median.

Measurements to acquire the data for the empty sample holder condition were done only once since the configuration was the same throughout the investigation. Before each test on a grain sample the location of a null for one frequency was rechecked with the empty sample holder. Data recorded for the empty holder configuration consisted of the locations of two adjacent minima of the standing wave pattern; which pair of minima selected was arbitrary. As seen from Figure 2-4 and equation (2-46), it is the shift in the null location ($\Delta l$) when the sample is introduced that is a measure of the dielectric constant. The null recorded with the sample in the holder will be between the two recorded with the holder empty, and $\Delta l$ will be calculated from the two sets of data. The waveguide wavelength $\lambda_{lg}$ is simply twice the distance between the empty nulls. The results of empty holder measurements and calculations are given in Table 3-1 and comprise a tabulation of reference data that was used throughout the remainder of the experimentation and calculation.

**Isolating the True Dielectric Constant**

As pointed out in chapter two, the value of $X$ satisfying (2-46) is not unique. This corresponds to the fact that the measured null shift can be caused by any number of dielectric constants; the shift observed may actually be several wavelengths
Table 3-1. Empty sample holder null locations and wave guide wavelengths

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Null 1 Location (cm)</th>
<th>Null 2 Location (cm)</th>
<th>$\lambda_{lg}$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>11.917</td>
<td>9.119</td>
<td>5.596</td>
</tr>
<tr>
<td>9.0</td>
<td>12.841</td>
<td>10.398</td>
<td>4.886</td>
</tr>
<tr>
<td>9.5</td>
<td>11.872</td>
<td>9.678</td>
<td>4.388</td>
</tr>
<tr>
<td>10.0</td>
<td>11.408</td>
<td>9.415</td>
<td>3.986</td>
</tr>
<tr>
<td>10.5</td>
<td>11.252</td>
<td>9.416</td>
<td>3.672</td>
</tr>
<tr>
<td>11.0</td>
<td>11.200</td>
<td>9.499</td>
<td>3.402</td>
</tr>
<tr>
<td>11.5</td>
<td>11.295</td>
<td>9.704</td>
<td>3.182</td>
</tr>
<tr>
<td>12.0</td>
<td>11.488</td>
<td>10.000</td>
<td>2.976</td>
</tr>
</tbody>
</table>
plus $\Delta l$ as would be produced by a very high dielectric constant, or just $\Delta l$ as caused by a low dielectric constant. (Even dielectric constants less than one will arise in the course of computation, but these are immediately rejected as being impossible.) This ambiguity or lack of knowledge of the true shift of the null is overcome by taking null-shift data for two sample lengths and performing the calculations. The results when plotted for the frequency range given will appear typically as illustrated in Figure 3-2. From the figure it is evident which curves represent the true dielectric constant variation with frequency. The true value should not be affected by the length of sample whereas the parasitic values should. Some variation between the two curves (and any subsequent curves plotted from samples of the same grain) is to be expected because of several factors: the unhomogeneity already considered, inaccuracy in measuring sample lengths, and inaccuracy in measuring null locations. In general it is more difficult to determine null locations for samples of greater length because of the higher loss and consequent lower VSWR. The nulls become quite shallow and broad, especially at the lower frequencies. From this standpoint it would be desirable to use shorter sample lengths overall, however, samples that are too short aggravate the effects of unhomogeneity. A suitable compromise has to be made. Sample lengths of five and three centimeters were used extensively in this investigation.

The procedure then for taking data with the grain samples in the holder was to perform two tests across the band of frequencies
Figure 3-2. Typical results of tests and calculations for two samples of five and three centimeters in length.
identified using different sample lengths. From the results of these the approximate true dielectric constant variation with frequency was obtained. Repeat tests with the shorter sample length were then conducted in order to arrive at a representative average. Five tests on each grain were normally adequate.

It is noted that the repeat tests required less computer time because the true value of dielectric constant had been isolated by the first two tests, so that the parasitic values could be ignored.

The computer program used to perform the calculations represented by equations (2-46) and (2-70) is shown in Appendix I. Essentially, the program makes the computer evaluate the right-hand side of (2-46) for the given input data (Δl, frequency, λlg, and d), try all values of X from 0 to 20 in 0.01 steps (or less, as will be described later), and calculate $\varepsilon_r^1$ by (2-70) for all X values that satisfy (2-46). Because of the great change in slope of the tan X function for values of X from 0 to $\frac{\pi}{2}$, it was necessary to reduce the trial graduations of X for the steep-slope regions to less than 0.01 to avoid missing values satisfying (2-46). As seen from the program, the graduations are 0.000625 for the region near $X = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$, and so forth. Missed values of X were still encountered infrequently for large values of the right-hand side of (2-46). The program could have been further improved to eliminate this, but the frequency of the occurrence did not justify it. Hand
calculation was employed whenever the value missed was important to the analysis of the material (not a parasitic value).

The upper limit on $X$ (20.0) was selected because it was judged from measurements made at lower frequencies on similar grains by another investigator \cite{12} that no higher values of dielectric constant than that given by $X = 20$ would be encountered.

The results of the computer runs were plotted in the manner shown in Figure 3-2, using only the true values. The results for each particular grain are given in Appendix II. From these plots a single representative curve for each grain was obtained by simply taking the average of the values at each frequency. This curve is the first approximation characteristic of $\varepsilon_{r2}^\prime$ for the grain in question.

Next, it was necessary to calculate the corrected dielectric constant, using (2-71), at each frequency where the correction was significant (as defined in chapter two).

**Method for Obtaining Loss Tangent and Loss Factor**

The loss tangent of the grains was found by making an attenuation measurement on samples at the same range of frequencies \cite{13}. This was done by using a power meter connected to a wave guide switch as shown in the block diagram of Figure 3-3. The difference in the calibrated attenuator settings to maintain a constant reading on the power meter when the thermistor mount is moved from one position to the other is the attenuation of the sample. $\alpha_{2g}$ is related to
Figure 3-3. Block diagram of test set-up for obtaining attenuation data.
attenuation through a proportionality constant. Tan δ may be calculated by using this and the wave guide phase-shift constant $\beta_{2g}$ in equation (2-73). Substituting (2-51) and (2-55) into that expression produces

$$\tan \delta = \frac{\lambda_{0}^2 \gamma_{2g}}{n \sqrt{\varepsilon_{r2}^\prime}} \left( \frac{1}{1 - \left( \frac{\lambda_{0}}{\lambda_{c}} \right)^2 \frac{1}{\varepsilon_{r2}}} \right).$$  

(3-1)

This is the first approximation of the loss tangent. The loss factor is then computed by

$$\varepsilon_{r2}^\prime = \varepsilon_{r2}^\prime \tan \delta.$$  

(3-2)

The results of applying the correction factor, when required, in accordance with equation (2-71) are presented in the curves of Appendix III. On the same graphs are shown the curves for the loss factors as found by equation (3-2) or (2-74), as applicable. These two curves for each grain sample represent the end product of the testing described in this chapter.

A brief analysis of the results will be given in the concluding chapter of this thesis. Validation of the null-shift method by tests on "Flexiglas", for which the dielectric constant is known, was performed prior to the tests on grain and is given in Appendix V.
CHAPTER IV
CONCLUSIONS

The curves contained in Appendices II and III comprise the results of the investigation. The dielectric constant curves of Appendix III were obtained by averaging points of the several curves of Appendix II. It was found that no correction due to losses was necessary except for the three highest frequency points of the 22.5% moisture corn; loss tangents at all other points for the grains tested were well below 0.4. The loss tangents were obtained by using attenuation data from reference 13 of the bibliography.

The loss factor characteristics of the higher moisture samples show a tendency toward an absorption peak near 8 GHz. Since loss factor curves may be multi-peaked \( \sqrt{3}, \sqrt{7} \) in heterogeneous materials like grain, there might be another absorption peak above 12 GHz. For high-moisture content grain, therefore, further tests at higher frequencies are indicated. The loss factors for drier samples are much lower in general than for the higher moisture samples and vary less over the frequency range. These samples would be less suitable for treatment with microwaves in this frequency range than the higher moisture variety.

It is noted that the curves are anomalous at several points at the high-frequency end of the band, being especially severe for samples of barley and soybeans. These grains display lower dielectric constants for the higher moisture content samples than for
the lower moisture samples at the high end of the band. This condition is not expected on the basis of work done by others on substances containing water, and also because of the physical consideration that the dielectric constant of water alone is very high. A similar condition exists for the corn and wheat samples at the lowest frequency (8.5 GHz) but this is not as unexpected because of the measurement difficulties encountered there. (Also, the signal generator used was very unstable at that frequency.)

An alternate method for calculating the dielectric constant was tried on the samples of soybeans and barley to see whether or not the null-shift method contributed to the irregularity described. The method involves determination of the distance of the first null of the standing-wave pattern from the boundary of the grain samples and employs the basic expression of equation (2-42) for finding X. Appendix IV contains the details of how $x_0$ is determined from the null locations recorded during the null-shift tests. The dielectric constant curves obtained by using this technique are also included in Appendix IV.

The results of these trials on soybeans and barley indicate an improvement in the curves at the high-frequency end. The irregularity virtually disappears in the case of barley and is reduced for soybeans. In addition the dielectric constant across the band is less for both moisture levels using this method.

It is believed that the results obtained by the first-null method are more nearly the true characteristics than those produced
by the null-shift technique. Apparently the null-shift method introduces greater error in calculation due to any asymmetry in the standing-wave pattern than does the first-null method. Both methods assume, however, that the nulls occur at equally-spaced intervals in the wave guide. This may not be the case here because of the complex nature of the reflection and transmission of the waves in the grain. Additional testing with different test set-ups are needed to obtain more information. The results obtained by this investigation, however, do provide the overall dielectric behavior of the grains in the frequency range investigated.
BIBLIOGRAPHY


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APPENDIX I

COMPUTER PROGRAMS (FORTRAN)
A-I-1 Dielectric Constant Program

100  READ(11,10) FRE, WLG, SHIFT
    FRE=(FRE*1.0E9)
    WRITE (12,13) FREQ
    DL=(6.2832*(3.0 SHIFT))/WLG
    TNXBYX=(WLG*(SIN(DL)/COS(DL)))/18.8495
    WLG=3.0*10/FREQ
    ECOR=(WLG/WLG)*2
    XP=0.0
    DO 25 N=1,2000
        XP=XP+0.01
        CX=cos(XP)
        IF(CX)34.35,34
    34 GO TO 25
    34 TGXP=SIN(XP)/(CX*XP)
    IF(TNXBYX)60,66,70
    60 IF(TGXP)25,66,66
    60 IF(TGXP)66,66,25
    66 DIFF=TNXBYX-TGXP
    DA=ABS(DIFF)
    IF(DA=0.001)44,44,54
    44 EPSLN=(XP-WLG)/18.8495*2 + 1.0-ECOR
    WRITE(12,14) EPSLN, XP, TNXBYX, TGXP
    54 XXP=XP+0.01
    38 CXXP=COS(XXP)
    IF(CXXP)36,35,36
    36 TGXXP=SIN(XXP)/(CXXP*XXP)
    DX=TGXXP-TGXP
    DXS=ABS(DX)
    IF(DXS=0.002)25,25,37
    37 IF(DXS=0.004)47,47,49
    49 IF(DXS=0.008)55,55,51
    51 IF(DXS=0.016)56,56,52
    52 IF(DXS=0.032)57,57,90
    90 IF(DXS=0.064)55,55,91
    91 IF(DXS=0.128)56,56,92
    92 IF(DXS=0.256)57,57,93
    93 IF(DXS=0.512)55,55,94
    94 IF(DXS=1.024)56,56,80
    80 WRITE(12,15)
GO TO 57
47 NG=1
G=0.005
GO TO 101
55 NG=3
G=0.0025
GO TO 101
56  NG=7
G=0.00125
GO TO 101
57  NG=15
G=0.000625
GO TO 101
101  XPP=XP
DO 26 I=1,NG
XPP=XPP G
CXPP=COS(XPP)
IF(CXPP)39,26,39
39  TGXPP=SIN(XPP)/(CXPP*XPP)
IF(TNXBYX)40,58,50
40  IF(TGXPP)58,58,26
50  IF(TGXPP)26,58,58
58  DIFFX=TNXBYX-TGXPP
DXSA=ABS(DIFFX)
IF(DXS-0.032)102,102,81
81  IF(DXS-0.256)103,103,104
102  IF(DXSA-0.001)59,59,26
103  IF(DXSA-0.01)59,59,26
104  IF(DXSA-0.10)59,59,26
59  EPSLN=(XPP/18.8495)-ECOR
WRITE(12,14) EPSLN, XPP, TNXBYX, TGXPP
26  CONTINUE
25  CONTINUE
106  GO TO 100
10  FORMAT (3F12.4)
13  FORMAT (1H, 1E12.4)
14  FORMAT (1H, 1E14.7)
15  FORMAT (10X, ' DL ', 60X)
105  END

Definition of terms:

FREQ = frequency
WIG = \lambda_{1g}
WLO = \lambda_{0}
SHIFT = \Delta 0
EPSLN = \varepsilon_{r2}
TNXBYX = \frac{\tan X}{X}
A-I-2 Loss Factor Program

1 READ (11,10) FRE, ATTEN, EPSLN1
   FREQ=FRE*1.0E9
   WLO=3.0E10/FREQ
   ECOR=(WLO/4.572)**2
   ALPHA=ATTEN/46.657
   TAND=WLO*ALPHA/(3.1416*SQRT(EPSLN1))
   TANDD=TAND/SQRT(1.0-ECOR/EPSLN1)
   EPSLN2=EPSLN1*TANDD
   WRITE(12,14) FREQ,TANDD,EPSLN2
   IF(FREQ-0.1240E11)20,21,21
20 GO TO 1
10 FORMAT (3F12.4)
14 FORMAT (1H_,3E12.4)
21 END

Definition of terms:

ATTEN = attenuation
EPSLN1 = \epsilon'_r
WLO = \lambda_0
EPSLN2 = \epsilon''_r
TANDD = tan \delta
FREQ = frequency
APPENDIX II

DIELECTRIC CONSTANT CURVES,
MULTIPLE TESTS
Figure A-II-1. Dielectric constant of South Dakota yellow corn, 22.5% moisture, 4 tests
Figure A-II-2. Dielectric constant of South Dakota yellow corn, 14.39% moisture, 5 tests
Figure A-II-3. Dielectric constant of South Dakota soybeans, 15.88% moisture, 4 tests.
Figure A-II-4. Dielectric constant of South Dakota soybeans, 10.39% moisture, 5 tests
Figure A-II-5. Dielectric constant of South Dakota spring wheat, 17.89% moisture, 5 tests
Figure A-II-6. Dielectric constant of South Dakota spring wheat, 12.97% moisture, 5 tests
Figure A-II-7. Dielectric constant of South Dakota Larker barley, 21.34% moisture, 5 tests
Figure A-II-8. Dielectric constant of South Dakota Larker barley, 9.62% moisture, 5 tests
APPENDIX III

AVERAGED DIELECTRIC CONSTANT AND LOSS FACTOR CURVES
Figure A-III-1. Average dielectric constant and loss factor of South Dakota yellow corn
Figure A-III-2. Average dielectric constant and loss factor of South Dakota soybeans
Figure A-III-3. Average dielectric constant and loss factor of South Dakota Larker barley
Figure A-III-4. Average dielectric constant and loss factor of South Dakota spring wheat
APPENDIX IV

FIRST-NULL TECHNIQUE AND RESULTS
ON SOYBEANS AND BARLEY
**The First-Null Technique**

The location of the first null of the standing-wave pattern with respect to the boundary of the dielectric can be determined approximately if the distance from the boundary of the sample to the null located on the wave guide slotted section is known. Direct measurement is not possible on the test set-up used because of the elbow which forms part of the transmission line. The distance is determined, therefore, by finding a null on the slotted section with the sample holder empty and determining the number of wavelengths at the frequency used that can be present in the total distance from the shorting plate to the null. If the distance from shorting plate to the wave guide slotted section flange is called $W$ and the distance from the flange to the null measured is called $N$, the number of wavelengths $P$ is given by

$$P = \frac{W + N}{\lambda_{lg}}.$$  \hspace{1cm} (A-IV-1)

Now, it is assumed that $P$ is an integral number because a null must exist at the shorting plate, so by using a rough estimate of $W$, $P$ can be determined. This number is then multiplied by $\lambda_{lg}$ and then $N$ is subtracted from it to give $W$ accurately.

With $W$ known the distance of the first null of the standing-wave pattern from the dielectric sample, $x_0$, can be related to the slotted section null location $N_s$ by noting from Figure 2-4 that

$$W + N_s = x_0 + d + Q \lambda_{lg},$$  \hspace{1cm} (A-IV-2)
where $Q$ is assumed integral. Thus,

$$x_0 = W + N_s - d - Q \lambda_{lg}. \quad (A-IV-3)$$

$Q$ is found by dividing the known $\lambda_{lg}$ into $(W + N_s - d)$ and dropping any decimals. Therefore, $x_0$ can be calculated from null locations obtained for the null-shift method, the known sample length $d$, and $\lambda_{lg}$ taken from Table 3-1. This value then is put into equation (2-42) to get $X$ for subsequent calculation of $\xi_{r2}$ in the same way as before.

Application of the method to soybeans and barley data are shown as the average curves of Figure A-IV-1 and Figure A-IV-2.
Figure A-IV-1. Average dielectric constant of South Dakota soybeans by first-null method
Figure A-IV-2. Average dielectric constant of South Dakota Larker barley by first-null method
APPENDIX V

VALIDATION OF THE NULL-SHIFT METHOD
Tests on two samples of "Plexiglas" 5 and 3 centimeters in length by the null-shift method produced the curves shown in Figure A-V-1. The true dielectric constant is clearly near 2.45 and essentially constant across the frequency band. This value may be compared with normally accepted values of 2.55 to 2.60. Thus, the null-shift method produces a set of curves as shown in Figure A-V-1 from which an approximate value of the dielectric constant can be clearly discerned. This result validates the null-shift method used to determine the approximate values of dielectric constant of feed grains in this investigation.
Figure A-V-1. Dielectric constant curves for "Plexiglas" by the null-shift method.