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A STUDY OF SCATTERING OF ELECTROMAGNETIC WAVES
BY DIELECTRIC CYLINDERS

BY

ALBERT SHIHYUAN WANG

A thesis submitted
in partial fulfillment of the requirements for the
degree Master of Science, Department of
Electrical Engineering, South Dakota
State University

1969
A STUDY OF SCATTERING OF ELECTROMAGNETIC WAVES
BY DIELECTRIC CYLINDERS

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser /Date

Head, Electrical Engineering / Date
Department
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A.S.W.
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CHAPTER I

INTRODUCTION

The formal solution of the scattering of a plane electromagnetic wave by a system of two co-axial cylinders was solved by Adey\textsuperscript{1}; experiments and calculations have been reported for the case of a conducting core surrounded by a polystyrene sleeve. The high frequency forward scattering properties were investigated by Kodis and Wu\textsuperscript{7}. The back scattering properties of a dielectric coated cylinder were studied by Tang\textsuperscript{11}. Sureau\textsuperscript{10} made an investigation on the reduction of scattering crosssection of dielectric cylinder by metallic core loading.

With the coming importance of fluid material, its tensor permittivity or its tensor permeability makes the investigation of its interaction with an externally applied field more complicated. The solutions of Maxwell's equations in such a medium have been intensively studied by a number of authors\textsuperscript{3}. Scattering of electromagnetic waves from an infinitely long magnetic cylindrical plasma were studied by Platzman and Ozaki\textsuperscript{8}. Other work has been done for the case of oblique incidence and irregular surface by several authors. Since all experimental investigations of scattering properties of a fluid material must be made with the fluid material contained in a dielectric sleeve, the effects of the presence of the
sleeve on the overall scattering properties turn out to be an interesting problem.

The object of this dissertation is to try to predict the characteristics of a cylindrical core of fluid material without the outside sleeve by analyzing the scattered fields with a sleeve. An attempt is made to investigate the change of the total scattered fields as a function of sleeve thickness, and to obtain an analytic method to predict the characteristics of the core material. Firstly, a theory will be developed to describe the interaction of electromagnetic waves with the composite dielectric cylinders. Then numerical data obtained through computer calculations will be studied by Fourier analysis to predict the scattering by the fluid material alone. The calculations presented apply to scattering by infinitely long co-axial cylinders immersed in a homogeneous and isotropic medium. All units of theoretical work throughout this dissertation are in the M.K.S. system for convenience.
CHAPTER II

THEORETICAL ANALYSIS OF SCATTERING BY CYLINDERS

A. General Solutions of Wave Equations in Cylindrical Coordinates

Electric and magnetic fields that vary with time are governed by physical laws described by a set of equations known collectively as Maxwell's equations. In summary, the four equations that describe electromagnetic phenomena in any dielectric media are

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial T} \]  \hspace{1cm} (2-1)

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial T} + \vec{J} + \rho \vec{V} \]  \hspace{1cm} (2-2)

\[ \nabla \cdot \vec{D} = \rho \]  \hspace{1cm} (2-3)

\[ \nabla \cdot \vec{B} = 0 \]  \hspace{1cm} (2-4)

\( \vec{E} \) and \( \vec{B} \) are electric field and magnetic field respectively, \( \vec{V} \) is the velocity of charged particles. Where \( \vec{J} \), the conduction current density, and \( \rho \), the charge density, are related by the continuity equation,

\[ \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial T} = 0 \]  \hspace{1cm} (2-5)

The electric displacement \( \vec{D} \) and the magnetic intensity \( \vec{H} \) are related to \( \vec{E} \) and \( \vec{B} \) through the electric and magnetic polarization of material.
media. The general matrix form of these relations in Cartesian coordinate system can be expressed as

\[
\begin{align*}
\begin{pmatrix}
D_x \\
D_y \\
D_z
\end{pmatrix} &=
\begin{pmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\
\varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix} \\
&= \begin{pmatrix}
E_{xx} & E_{xy} & E_{xz} \\
E_{yx} & E_{yy} & E_{yz} \\
E_{zx} & E_{zy} & E_{zz}
\end{pmatrix} \quad (2-6)
\end{align*}
\]

and

\[
\begin{align*}
\begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix} &=
\begin{pmatrix}
\mu_{xx} & \mu_{xy} & \mu_{xz} \\
\mu_{yx} & \mu_{yy} & \mu_{yz} \\
\mu_{zx} & \mu_{zy} & \mu_{zz}
\end{pmatrix}
\begin{pmatrix}
H_x \\
H_y \\
H_z
\end{pmatrix} \\
&= \begin{pmatrix}
H_{xx} & H_{xy} & H_{xz} \\
H_{yx} & H_{yy} & H_{yz} \\
H_{zx} & H_{zy} & H_{zz}
\end{pmatrix} \quad (2-7)
\end{align*}
\]

In a collisionless ionized medium with a steady magnetic field impressed in the + Z direction, the permittivity \(\varepsilon\) is a tensor of the form given in equation (2-8) and the permeability \(\mu = \mu_0\), the permeability of free space.

\[
\varepsilon = \begin{pmatrix}
1 & j\varepsilon_2 & 0 \\
-j\varepsilon_2 & \varepsilon_1 & 0 \\
0 & 0 & \varepsilon_3
\end{pmatrix} \quad (2-8)
\]

The permeability \(\bar{\mu}\) takes the form of equation (2-7) for ferrite type media. The permittivity of such media may usually be taken to be a scalar, where

\[
\begin{align*}
\bar{\mu} &=
\begin{pmatrix}
\mu_1 & j\mu_2 & 0 \\
-j\mu_2 & \mu_1 & 0 \\
0 & 0 & \mu_3
\end{pmatrix} \\
&= \begin{pmatrix}
\mu_1 & j\mu_2 & 0 \\
-j\mu_2 & \mu_1 & 0 \\
0 & 0 & \mu_3
\end{pmatrix} \quad (2-9)
\end{align*}
\]
Derivation of $\varepsilon'$s and $\mu'$s have been studied by several authors. For a homogeneous and isotropic medium, the permittivity and the permeability become scalar quantities. In this case, we can obtain the so-called Helmholtz equation, or the reduced wave equation, from Maxwell's equations for harmonic time dependence. The wave equations for electric and magnetic fields derived from equations (2-1) to (2-4) assuming a time dependence of $e^{-j\omega T}$ are,

$$\nabla^2 \bar{E} + K^2 \bar{E} = 0 \quad (2-10)$$

and

$$\nabla^2 \bar{H} + K^2 \bar{H} = 0 \quad (2-11)$$

The constant $K$ is called wave number and may be expressed in the form

$$K = \omega \sqrt{\frac{\varepsilon}{\mu}} = \frac{2\pi}{\lambda} \quad (2-12)$$

The Helmholtz equation in cylindrical coordinates for each component of electric or magnetic fields, represented by $\Psi$, is shown to be

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \frac{\partial \Psi}{\partial \rho}) + \frac{1}{\rho^2} \frac{\partial^2 \Psi}{\partial \theta^2} + \frac{\partial^2 \Psi}{\partial z^2} + K^2 \Psi = 0 \quad (2-13)$$

The corresponding geometry is shown in Fig. 2-1.
Using the method of separation of variables, the solutions of Helmholtz equations can be expressed as

\[
\Psi = \sum_n \sum_{K_z} C_{n,K_z} B_n(K \rho \rho) \ h(n\phi) \ h(K_z z)
\]  

where

\[
K_\rho^2 + K_z^2 = k^2
\]

and \(B_n(K \rho \rho)\) is the solution to the Bessel's equation of order \(n\), \(n\) must be an integer if \(\Psi\) is to be single-valued.

The exact solution of the Helmholtz equation can be obtained by applying suitable boundary conditions. Harrington\(^5\) has presented a complete description of cylindrical wave functions in his book.

**B. Scattering of Plane Waves by Co-axial Dielectric Cylinders**

(Parallel Polarization)

Most of the early works of scattering by co-axial cylinders were studied by Adey\(^1\). A plane wave of unit amplitude propagates in the +X direction in a medium of permittivity \(\varepsilon_1\) and permeability \(\mu_1\). It is incident on a cylinder with a core radius \(a\) and a co-axial sleeve of radius \(b\). The wave has its electric vector polarized parallel to the cylinder axis. Fig. 2-2 illustrates this arrangement. The medium constants of the core cylinder are \(\varepsilon_3\) and \(\mu_3\) while those of the sleeve are assumed to be \(\varepsilon_2\) and \(\mu_2\).
The incident plane wave is represented in cylindrical coordinates as a Fourier-Bessel series as shown in Eq. (2-16).

\[ E_z^i = e^{jK_l x} = e^{jK_l \rho \cos \phi} \]

\[ = \sum_{n=0}^{\infty} (j)^n J_n(K_l \rho) e^{jn\phi} \]

\[ = \sum_{n=0}^{\infty} e_n(j)^n J_n(K_l \rho) \cos n\phi \quad \text{(2-16)} \]

where

\[ e_n = 1 \quad \text{for} \quad n = 0 \]
\[ e_n = 2 \quad \text{for} \quad n \neq 0 \]

The fields in the three regions can be represented by the following forms:

Region 1,

\[ E_1 = \sum_{n=0}^{\infty} e_n A_n H_n^{(1)}(K_l \rho) \cos n\phi \quad \text{(2-17)} \]

Region 1: \( \varepsilon = \varepsilon_1 \); \( \mu = \mu_1 \) \( a \leq \rho < \infty \)
Region 2: \( \varepsilon = \varepsilon_2 \); \( \mu = \mu_2 \) \( a \leq \rho \leq b \)
Region 3: \( \varepsilon = \varepsilon_3 \); \( \mu = \mu_3 \) \( \rho \leq a \)

Fig. 2-2. Geometry of scattering by co-axial cylinders
Region 2,
\[ E_2^{T} = \sum_{0}^{\infty} e_n [B_n J_n(K_2\rho) + C_n Y_n(K_2\rho)] \cos n\phi \] (2-18)

Region 3,
\[ E_3^{T} = \sum_{0}^{\infty} e_n D_n J_n(K_3\rho) \cos n\phi \] (2-19)

Where the \( J_n(x) \) and the \( Y_n(x) \) are the Bessel functions of the first and second kinds respectively, the \( H_n^{(1)}(x) \) are the Hankel functions of the first kind, and the \( K \)'s are the wave numbers appropriate to the various regions (\( K = \frac{2\pi}{\text{wave length}} \)).

From the Maxwell's equations in cylindrical coordinates, we have the magnetic field
\[ H_\phi = \frac{1}{\omega \mu} (\nabla \times \hat{a}_z E_2^\phi) \] (2-20)

where \( \hat{a}_z \) is the unit vector in Z direction.

In applying the boundary conditions of continuity of the tangential electric and magnetic fields at the interfaces between the three regions, we have

\[ H_n^{(1)}(K_{1b}) A_n - J_n(K_{2b}) B_n - Y_n(K_{2b}) C_n \]
\[ = - (j)^n J_n(K_{1b}) \] (2-21)

\[ J_n(K_{2a}) B_n + Y_n(K_{2a}) C_n - J_n(K_{3a}) D_n = 0 \] (2-22)
\[ \begin{align*}
\dot{H}_n^{(1)} (K_{1b}) \frac{K_1}{\mu_1} A_n - J_n^{(K_{2b})} \frac{K_2}{\mu_2} B_n - Y_n^{(K_{2b})} \frac{K_2}{\mu_2} C_n \\
&= -(j)^n J_n^{(K_{1b})} \frac{K_1}{\mu_1} \\
&= J_n^{(K_{2a})} \frac{K_2}{\mu_2} B_n + Y_n^{(K_{2a})} \frac{K_2}{\mu_2} C_n - J_n^{(K_{3a})} \frac{K_3}{\mu_3} D_n \\
&= 0
\end{align*} \tag{2-23} \]

From (2-21) to (2-24), \( A_n \) can be solved as

\[ \frac{-A_n}{(j)^n} = \frac{\nabla_a}{\Delta} \tag{2-25} \]

In Eq. (2-25), \( \nabla_a \) is given by

\[
\nabla_a = \begin{vmatrix}
J_n(p) & -J_n(s) & -Y_n(s) & 0 \\
\frac{K_1}{\mu_1} J_n^{(p)} & -\frac{K_2}{\mu_2} J_n^{(s)} & -\frac{K_2}{\mu_2} Y_n^{(s)} & 0 \\
0 & J_n(m) & Y_n(m) & -J_n(q) \\
0 & \frac{K_2}{\mu_2} J_n^{(m)} & \frac{K_2}{\mu_2} Y_n^{(m)} & -\frac{K_3}{\mu_3} J_n^{(q)}
\end{vmatrix} \tag{2-26}
\]
Where \( q = K_2 a \), \( m = K_2 a \), \( s = K_2 b \), \( p = K_1 b \), and prime indicates a differentiation with respect to the argument. \( \Delta \) is obtained from the determinant of (2-26) by the following changes in the first column:

\[
J_n(p) \rightarrow H_n^{(1)}(p)
\]

\[
J_n'(p) \rightarrow H_n'(1)(p)
\]

(2-27)

If the sleeve is thin, i.e., \( t = b - a < a \), one can simplify (2-26) further. From Taylor expansion for the Bessel function expressions and the Wronskian of the Bessel function (Appendix A), relation (2-26) becomes

\[
- \frac{A_n}{(j)^n} = \frac{J_n(p) F + J_n'(p) G}{H_n^{(1)}(p) F + H_n'(1) G}
\]

(2-28)

where

\[
G = \frac{2 J_n(q)}{\mu_1^2} + \frac{2 t^j(q)}{\mu_2^2}
\]

(2-29)

and

\[
F = \left( \frac{K_2}{\mu_2} \right)^2 \left[ 2 t \left( \frac{b(m^2 - n^2) - nt}{\mu_1 \mu_2} \right) \right] J_n(q)
\]

\[
+ \frac{K_2 K_3}{\mu_2^2 \mu_3} \left[ \frac{2(n^2 - a^2 + nt^2)}{\mu_1 \mu_2 \mu_3} \right] J_n'(q)
\]

(2-30)
For

\[ H_n^{(1)}(p) = J_n(p) + j Y_n(p) \]

we have

\[ \frac{-A_n}{(j)^n} = \frac{A^2 - j AB}{A^2 - B^2} \]

\[ A = J_n(p) \cdot F + J_n'(p) \cdot G \]

\[ B = Y_n(p) \cdot F + Y_n'(p) \cdot G \]

The scattered field at point \( P(\rho, \phi) \) is then obtained by substituting (2-32) in (2-17). The total electric field vector at point \( P(\rho, \phi) \) is then given by Eq. (2-35).

\[ E_T = e^{jK_1 \rho} \cos \phi + \sum_{n=0}^{\infty} e_n \frac{jAB - A^2}{A^2 - B^2} (j)^n H_n^{(1)}(K_1 \rho) \cos n\phi \]  

(2-35)

For far field approximation, \( H_n^{(1)}(K_1 \rho) \) can be expressed as

\[ H_n^{(1)}(K_1 \rho) = (\frac{2}{j n K_1 \rho})^{1/2}(j)^{-n} e^{jK_1 \rho} \]

(2-36)

\((K_1 \rho \text{ large})\)

Using Eq. (2-36), the total electric field at point \( P(\rho, \phi) \) becomes

\[ E_T = e^{jK_1 \rho} \cos \phi + \sum_{n=0}^{\infty} e_n \frac{1 - \frac{1}{(n K_1 \rho)^{1/2}}} {A^2 - B^2} \cdot \frac{jAB - A^2}{A^2 - B^2} e^{jK_1 \rho} \cos n\phi \]

\[ = R e \left\{ E_T \right\} + j I m \left\{ E_T \right\} \]

(2-37)
The $Z$ component of the total electric field ($E^T_z$) is a function of the geometry and dimensions of the co-axial cylinders. Most of the measuring devices now available, such as diode detectors, are power measuring devices. The information obtained is proportional to the square of the magnitude of the electric field. Eq. (2-38) gives $|E^T_z|$ at point $P(\rho,\phi)$ which can be measured by modern detecting devices.

C. Perpendicular Polarization

If the magnetic vector of the incident plane wave is oriented parallel to the cylinder axis and the amplitude of the incident magnetic vector is $Y_o (Y_o = \omega \varepsilon_0 / K_o)$, Eq. (2-37) applies for far field approximation if everywhere $\mu$ is replaced by $\varepsilon$. 

$$|E^T_z| = \left[ (Re \{E^T_z\})^2 + (Im \{E^T_z\})^2 \right]^{1/2}$$ (2-38)
CHAPTER III

THE NUMERICAL RESULTS

The preceding chapter presented the theory supporting the discussion throughout this chapter. The magnitude of total incident and scattered electric field as a function of $\phi$ and $t$ will be studied numerically by computer calculations, and a dielectric core of radius $5\text{ cm}$ will be used.

The total $|E_z^T|$ as a function of $\phi$ is calculated at one degree intervals, and the distance $\rho = R$ is chosen such that far field approximation is always valid. In order to investigate the field pattern analytically, the function $|E_z^T(\phi)|$ is expanded into a Fourier Series\textsuperscript{12}. This can be done due to the fact that $|E_z(\phi)|$ is periodic over $2\pi$ radians. Thus,

$$|E_z^T(\phi)| = a_0 + \sum_{n=1}^{\infty} a_n \cos n\phi + \sum_{n=1}^{\infty} b_n \sin n\phi$$  (3-1)

The $b_n$ coefficients are all zero because $|E_z(\phi)|$ is an even function, therefore,

$$|E_z^T(\phi)| = a_0 + \sum_{n=1}^{\infty} a_n \cos n\phi$$  (3-2)

All the value of $a_n$'s can be calculated by approximations suggested by Tolstov\textsuperscript{12}. That is

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx \, dx \quad a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x) \, dx$$

$(n = 1, 2, 3, \ldots)$
\[ a_n \sim \frac{2}{M} \sum_{k=0}^{M-1} f(x_k) \cos \frac{2k\pi}{M} n; \quad a_0 \sim \frac{1}{M} \sum_{k=0}^{M-1} f(x_k) \]  

(3-3)

Where the interval \([0, 2\pi]\) is divided into \(M\) equal parts and \(f(x_k)\) corresponds to the value of \(f(x)\) at \(x = x_k\).

If \(a_n < \delta\) (\(\delta\) sufficiently small), for \(n > N\), all \(a_n\)'s of which \(n\) is greater than \(N\) can be neglected. Therefore,

\[ |E_z^T(\phi)| = a_0 + \sum_{n=1}^{N} a_n \cos n\phi \]  

(3-4)

A. Lossless Dielectric Core

For a lossless dielectric core, the permittivity of the material is a real number and also a scalar quantity. Assuming a dielectric core of dielectric constant \(\varepsilon_r/\varepsilon_0 = 2.25\) with a sleeve of dielectric constant \(\varepsilon_2/\varepsilon_0 = 4\), the function \(|E_z^T(\phi)|\), at an operating frequency 0.957 KMHz \((K_1 = K_0 = 20)\), is computed using Eq. (2-38). Fig. 3-1 shows a plot of \(|E_z^T(\phi)|\) with respect to \(\phi\).

From Fig. 3-1, the Fourier coefficients of \(|E_z^T(\phi)|\) are evaluated and then tabulated in Table 3-1. If \(\delta = 0.01\), all \(a_n\)'s of which \(n\) is greater than 22 can be neglected, i.e., \(N = 22\) in this case.

The variations of the Fourier coefficients \(a_n\) of \(|E_z^T(\phi)|\) as the thickness \(t\) is varied are shown in Figs. 3-2 to 3-5. From the figures shown, a parabolic approximation can be made to describe the characteristics of these variations, provided that \(K_2 t\) is kept small.
Table 3-1. Comparison of Fourier coefficients

<table>
<thead>
<tr>
<th>TRUE VALUE</th>
<th>EXTRAPOLATED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀</td>
<td>0.989507</td>
</tr>
<tr>
<td>a₁</td>
<td>0.003268</td>
</tr>
<tr>
<td>a₂</td>
<td>-0.025329</td>
</tr>
<tr>
<td>a₃</td>
<td>0.005332</td>
</tr>
<tr>
<td>a₄</td>
<td>-0.015872</td>
</tr>
<tr>
<td>a₅</td>
<td>0.013549</td>
</tr>
<tr>
<td>a₆</td>
<td>0.003563</td>
</tr>
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<td>a₇</td>
<td>0.021785</td>
</tr>
<tr>
<td>a₈</td>
<td>0.029303</td>
</tr>
<tr>
<td>a₉</td>
<td>0.020402</td>
</tr>
<tr>
<td>a₁₀</td>
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</tr>
<tr>
<td>a₁₁</td>
<td>0.000260</td>
</tr>
<tr>
<td>a₁₂</td>
<td>0.013698</td>
</tr>
<tr>
<td>a₁₃</td>
<td>-0.023589</td>
</tr>
<tr>
<td>a₁₄</td>
<td>-0.039301</td>
</tr>
<tr>
<td>a₁₅</td>
<td>-0.007490</td>
</tr>
<tr>
<td>a₁₆</td>
<td>-0.020908</td>
</tr>
<tr>
<td>a₁₇</td>
<td>0.026831</td>
</tr>
<tr>
<td>a₁₈</td>
<td>0.051742</td>
</tr>
<tr>
<td>a₁₉</td>
<td>-0.021590</td>
</tr>
<tr>
<td>a₂₀</td>
<td>-0.038774</td>
</tr>
<tr>
<td>a₂₁</td>
<td>0.010163</td>
</tr>
<tr>
<td>a₂₂</td>
<td>0.017611</td>
</tr>
<tr>
<td>a₂₃</td>
<td>-0.002014</td>
</tr>
</tbody>
</table>
In order to predict the value of $a_n$ at $t = 0$, the $n^{th}$ Fourier coefficients of $|E_z|^2(\emptyset)$ corresponding to three different $t$'s are needed to extrapolate the value of $a_n$ at $t = 0$ because the variation of $a_n$ is apparently parabolic. The extrapolation can be done by the method of a difference table\(^9\), i.e.,

Suppose

$$
\begin{align*}
\Delta f(x) &= f(x + h) - f(x) \\
\Delta^2 f(x) &= \Delta \left[ \Delta f(x) \right] = \Delta f(x + h) - \Delta f(x) \\
&\vdots \\
\Delta^n f(x) &= \Delta \left[ \Delta^{n-1} f(x) \right] = \Delta^{n-1} f(x + h) - \Delta^{n-1} f(x)
\end{align*}
$$

and

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$\Delta f(x)$</th>
<th>$\Delta^2 f(x)$</th>
<th>$\Delta^3 f(x)$</th>
</tr>
</thead>
</table>
| $x$ | $f(x)$ | $\Delta_1 f(x)$ | $\Delta_2^2 f(x)$ | $
\Delta_3^3 f(x)$ |
| $x+h$ | $f(x+h)$ | $\Delta_1^2 f(x)$ | $\Delta_2^2 f(x)$ | $
\Delta_3 f(x)$ |
| $x+2h$ | $f(x+2h)$ | $\Delta_1^2 f(x)$ | $\Delta_2^2 f(x)$ | $
\Delta_3 f(x)$ |
| $x+3h$ | $f(x+3h)$ | $\Delta_1^2 f(x)$ | $\Delta_2^2 f(x)$ | $
\Delta_3 f(x)$ |

If $f(x)$ is parabolic, then $\Delta_1^2 f(x) = \Delta_3^3 f(x) - \Delta_2^2 f(x) = \Delta_2^2 f(x)$. For $h = \text{constant}$, then at $x = 0$

$$
f(0) = 3 \cdot \left[ f(h) - f(2h) \right] + f(3h) \quad (3-5)
$$

All the $a_n$'s at $t = 0$ are calculated by Eq. (3-5) assuming $h = 0.001 \text{ meter}$ and the results are listed in Table 3-1. The
coefficients \( a_n \) are shown to three place accuracy. The dotted points in Fig. 3-1 represent the maxima and the minima of the reconstructed field pattern for zero thickness of sleeve calculated by using twenty three Fourier coefficients, and are compared to the field pattern derived from Eq. (2-38) by putting \( t = 0 \). There is very little deviation in the occurrences of the maxima and minima with respect to \( \phi \). The errors in \( \left| E_z^T(\phi) \right| \) are less than one percent. The results for \( \varepsilon_3/\varepsilon_0 \approx 4 \) and \( \varepsilon_3/\varepsilon_0 > 4 \) are illustrated in Fig. 3-6 and Fig. 3-7 respectively. If the operating frequency is doubled and the magnitudes of \( E_z^T \) calculated at same distance \( R \), the variations in \( a_n \) as functions of thickness \( t \) are shown in Figs. 3-8 to 3-15. Using Eq. (3-5), the calculated \( a_n \)'s are good for at least two significant figures and the reconstructed field patterns for zero thickness sleeve (dots) are compared to the original patterns (solid lines) in Figs. 3-16 to 3-18. The errors in \( \left| E_z^T \right| \) are still within one percent and the deviation in the occurrences of maxima and minima with respect to \( \phi \) are less than one degree.

For the above examples, more than forty \( a_n \)'s were needed for the magnitude of the largest neglected \( a_n \) to be smaller than 0.01. The number of \( a_n \)'s required to reconstruct the field pattern depends on the highest fluctuation frequency, or the smallest period \( (T) \) between two adjacent peaks. For example, \( T \) in Fig. 3-18 is eight degrees, then

\[
N \sim \frac{360^\circ}{T} = 45
\]  

(3-6)
Three terms are added to allow acceptable tolerance, i.e.,

\[ N \sim \frac{360^\circ}{\tau} + 3 \quad (3-7) \]

B. The Lossy Dielectric Core

All the above calculations are based on the lossless dielectric material. For a lossy dielectric core with a complex permittivity \( \varepsilon' - j\varepsilon'' \), the total electric field pattern is shown in Fig. 3-19. There are changes in the field magnitudes and there are shifts of the maximum and minimum points also, but this information does not yield a general rule to differentiate between a lossy dielectric core from a lossless one.

C. Shifts of Maximum and Minimum Points

Another interesting aspect of the total electric field pattern is that there is a constant shift of the positions of the maximum and minimum points when the dielectric constant of the core is varied. For some fluid material, such as plasma, the dielectric constant of the plasma depends on the plasma oscillation frequency which is controlled by the gas pressure. Thus, the shifts of the positions of these maxima or minima may enable us to tell the change in the core dielectric constant. Fig. 3-20 gives the positions (\( \phi \)) of the first maximum as a function of the square-root of the core dielectric constant; three curves are given corresponding to three different core dimensions, but with \( K_o a = 1 \) and sleeve thickness
t = 0.003 meter. Fig. 3-21 gives the magnitudes of the first maximum as a function of \((\varepsilon_3/\varepsilon_0)^{1/2}\). The dielectric constant of the core can be obtained by comparing Fig. 3-20 to Fig. 3-21; both figures are needed because there is a reversal of the shift of the first maximum which occurs at \(\varepsilon_3/\varepsilon_0 = 14.44\).
Fig. 3-1. The field pattern with $K_0R = 20$, and $\varepsilon_2/\varepsilon_0 = 4$. 

$E_z^T$ vs. $\phi$ in degrees 

$K_0R = 20$, $\varepsilon_2/\varepsilon_0 = 4$
Fig. 3-2. The variations of Fourier coefficients with $K_0R = 20$, $\epsilon_2/\epsilon_0 = 4$, and $\epsilon_3/\epsilon_0 = 2.25$
Fig. 3-3. The variations of Fourier coefficients with $K_0R = 20$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-4. The variations of Fourier coefficients with $K_0R = 20$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-5. The variations of Fourier coefficients with $K_0 R = 20$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-6. The field pattern with $K_0 R = 20$, and $\varepsilon_3/\varepsilon_0 = 4.41$
Fig. 3-7. The field pattern with $K_0R = 20$, and $\varepsilon_2/\varepsilon_0 = 9.0$.
Fig. 3-8. The variations of Fourier coefficients with $K_0R = 40$, $\epsilon_2/\epsilon_0 = 4$, and $\epsilon_3/\epsilon_0 = 2.25$.
Fig. 3-9. The variations of Fourier coefficients with \( K_0R = 40, \frac{\varepsilon_2}{\varepsilon_0} = 4, \) and \( \frac{\varepsilon_3}{\varepsilon_0} = 2.25 \)
Fig. 3-10. The variations of Fourier coefficients with $K_0R = 40$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-11. The variations of Fourier coefficients with $K_0^R = 40$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-12. The variations of Fourier coefficients with $K_0R = 40$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-13. The variations of Fourier coefficients with $K_0R = 40$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$. 
Fig. 3-14. The variations of Fourier coefficients with $K_0R = 40$, $\epsilon_2/\epsilon_0 = 4$, and $\epsilon_3/\epsilon_0 = 2.25$.
Fig. 3-15. The variations of Fourier coefficients with $K_0R = 40$, $\varepsilon_2/\varepsilon_0 = 4$, and $\varepsilon_3/\varepsilon_0 = 2.25$
Fig. 3-16. The field pattern with $K_0R = 40$, and $\varepsilon_2/\varepsilon_0 = 4$

$\varepsilon_2/\varepsilon_0 = 2.25$
Fig. 3-17. The field pattern with $K_0R = 40$, and $\varepsilon_2/\varepsilon_0 = 4.41$.
Fig. 3-18. The field pattern with $K_0R = 40$, and $\epsilon_3/\epsilon_0 = 9.0$
\[ \frac{\varepsilon_3^{'}}{\varepsilon_0} + j \frac{\varepsilon_3^{''}}{\varepsilon_0} = 2 + j1.5 \]

\[ K_0R = 20 \]

\[ \varepsilon_2/\varepsilon_0 = 4 \]

Fig. 3-19. The field pattern with lossy dielectric core material
The shift of first maximum

\[ \phi \text{ (degrees)} \]

\[ a = 5 \text{ cm} \]
\[ a = 4 \text{ cm} \]
\[ a = 3 \text{ cm} \]

\[ E_2/E_0 = 4 \]

\[ \varepsilon_2/\varepsilon_0 = 4 \]

\[ K_0a = 1 \]
\[ t = 3 \text{ mm} \]

Fig. 3-20. The shift of first maximum
Fig. 3-21. The variations of the magnitudes of the first maximum

\[ K_0 a = 1 \]
\[ t = 3 \text{ mm} \]
\[ \varepsilon_2/\varepsilon_0 = 4 \]
CHAPTER IV

EXPERIMENTS AND CONCLUSIONS

A. Experimental Results

In chapter III, it was found that if the total electric field plots $|E_z^T|$ were available for scattering by co-axial dielectric cylinders for several sleeve thicknesses $t$, then the pattern of $|E_z^T|$ for zero sleeve thickness can be predicted using Fourier analysis and extrapolation of the resulting Fourier coefficients. In order to use parabolic curve fitting techniques, it was found that the sleeve thickness be small, or $K_2t$ be small.

Experiments were set up to validate the theory discussed in chapter III. The experimental arrangement is shown in Fig. 4-1. A HP-686A oscillator was used as a signal source and a plane wave was radiated from a X-band horn using standard microwave transmission equipment. The detecting system was an open-ended wave guide and a wave guide crystal detector (HP-X424A). The magnitude of the incident and scattered field from a composite cylindrical scatterer was recorded on a Moseley 3S X-Y recorder. The incident wave was modulated by a 1 KHz signal so that a tuned amplifier such as HP-415B standing wave indicator could be used to process the detected signal before applying it to the recorder. The incident electric field in this arrangement was oriented parallel to the axis of the composite cylinders. Several core and sleeve materials were used for these experiments.
Three total electric field patterns corresponding to decreasing thickness of plexiglass tubes ($\varepsilon_2/\varepsilon_0 = 2.59$) as the cylindrical sleeves and air ($\varepsilon_3/\varepsilon_0 = 1.0$) as a dielectric core, were obtained and are shown in Figs. 4-2 to 4-4. A pattern for zero sleeve thickness was also obtained to check the validity of the reconstructed pattern for zero thickness by extrapolation technique using patterns obtained in Figs. 4-2 to 4-4. The experimental pattern for $t = 0$ is shown in Fig. 4-5. Fourier coefficients were obtained for the patterns of Figs. 4-2 to 4-4 as outlined in chapter III. Several of these coefficients are shown in Figs. 4-6 to 4-10, as functions of the sleeve thickness $t$, and the extrapolated $a_n$'s at $t = 0$ are compared with the experimental ones in Table 4-1.

![Fig. 4-1. Experimental arrangement](image)
Table 4-1. List of the extrapolated Fourier coefficients and the experimental ones for field pattern of zero sleeve thickness.

<table>
<thead>
<tr>
<th></th>
<th>EXPERIMENTAL VALUE</th>
<th>EXTRAPOLATED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.52445</td>
<td>0.56828</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.33937</td>
<td>0.39285</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.17383</td>
<td>0.16641</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.04215</td>
<td>0.01394</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-0.00196</td>
<td>-0.00485</td>
</tr>
<tr>
<td>$a_5$</td>
<td>0.00375</td>
<td>0.02674</td>
</tr>
<tr>
<td>$a_6$</td>
<td>0.00799</td>
<td>0.03328</td>
</tr>
<tr>
<td>$a_7$</td>
<td>0.00177</td>
<td>0.02279</td>
</tr>
<tr>
<td>$a_8$</td>
<td>-0.00168</td>
<td>0.02154</td>
</tr>
<tr>
<td>$a_9$</td>
<td>-0.00068</td>
<td>0.01956</td>
</tr>
<tr>
<td>$a_{10}$</td>
<td>-0.00134</td>
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</tr>
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</tr>
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<td>$a_{12}$</td>
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<td>0.00259</td>
</tr>
<tr>
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<td>$a_{14}$</td>
<td>-0.00659</td>
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</tr>
<tr>
<td>$a_{15}$</td>
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<td>$a_{16}$</td>
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<tr>
<td>$a_{17}$</td>
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<td>$a_{18}$</td>
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<tr>
<td>$a_{19}$</td>
<td>-0.00193</td>
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</tr>
<tr>
<td>$a_{20}$</td>
<td>0.00107</td>
<td>-0.01305</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>0.00602</td>
<td>-0.00772</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>0.00595</td>
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</tr>
<tr>
<td>$a_{23}$</td>
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</tr>
<tr>
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<tr>
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<td>0.00134</td>
<td>-0.02063</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>$a_{29}$</td>
<td>-0.00176</td>
<td>0.02431</td>
</tr>
</tbody>
</table>
If all the Fourier coefficients are examined closely, the nonparabolic nature of the variations of these coefficients smaller than 0.007 can be easily perceived. These nonparabolic coefficients cannot be taken into account using extrapolation technique outlined in chapter III. More than three data points would be needed to extrapolate these coefficients to zero thickness case using higher order variations. Fortunately, the nonparabolic variations occur in the coefficients which are small and therefore may be neglected in the reconstruction of the total electric field pattern with zero sleeve thickness. It has been shown that using only the first thirty Fourier coefficients calculated by Eq. (3-5), the reconstructed pattern plotted in Fig. 4-11 as dotted line, fits well with the experimental one. A better fit is obtained by using only those coefficients of Figs. 4-6 to 4-10 whose magnitudes are greater than 0.007. This is shown in Fig. 4-12. The slight nonparabolic nature of $a_0$ is evident in Figs. 4-11 and 4-12 as an increase in the average value of the $|E_z|^2$ curve over the experimental one. This shift in the reconstructed pattern with respect to the experimental one also may be predicted by comparing $a_0$ obtained from Fig. 4-5 and its numerically extrapolated value in Table 4-1. It may be pointed out here, and also illustrated by further experimental results which follow, that if the plexiglass tubes were thinner, the reconstructed electric field pattern for zero thickness extrapolation would have been more accurate.

The assumed need for small thickness of the sleeve ($K_2t$ small) for use of parabolic extrapolation to zero thickness is made more
obvious by experiments conducted in the same manner as the one just described, but with a highly reflective copper core cylinder.

The experimental patterns of the total electric field for decreasing sleeve thickness, using plexiglass sleeve, and copper core, are shown in Figs. 4-13 to 4-15. The experimental field pattern for zero dielectric sleeve is shown in Fig. 4-16. A horn detector antenna was used in these measurements to make the detector system more sensitive. The measured field strength shows an overall antenna gain pattern impressed on the total field. This directional property of the receiving antenna was also perceived for open-ended waveguide detector used in the experimental results described in the beginning of this chapter.

The Fourier coefficients for the plots shown in Figs. 4-13 to 4-15 were obtained using Eq. (3-3) through computer calculations. Plots of some of the coefficients as a function of t are shown in Fig. 4-17. These coefficients, as can be seen from the plots, exhibit parabolic variations for thickness of the dielectric sleeve \( t \leq 0.25 \) inch of plexiglass for an operating frequency of 8.5 KMHz. Small thickness approximation here is therefore \( K_2 t \leq 1.83 \).

Plexiglass tubes were replaced by Pyrex (corning 7740) glass tubes \( (\varepsilon_2/\varepsilon_0 = 4.5)^{16} \), and the total electric field patterns corresponding to various thicknesses t were obtained. These plots are reproduced in Figs. 4-18 to 4-20. A plot of the electric field pattern with zero sleeve thickness was also obtained in Fig. 4-21.
The variations of the Fourier coefficients as functions of $t$ are shown in Fig. 4-22. Although the total sleeve thickness of the glass is thinner in absolute terms ($6$ mm of glass as compared to $3/8"$, or $9.54$ mm of plexiglass), the product $K_2t$ is approximately the same as that due to plexiglass. The foregoing analysis implied that, if a simple parabolic extrapolation technique were to be used to predict the pattern of the core material alone, then the thickness of the dielectric sleeve in terms of $K_2t$ must be kept small.

B. Conclusions

Two conclusions can be drawn from the foregoing numerical analysis and experimental results. First, the change in the dielectric constant of the core material can be perceived by watching the shift of the first maximum of the total electric field; second, prediction of total electric field of scattering by a fluid material alone, without a sleeve, can be obtained by the following procedures:

1. Put the dielectric core material in a thin dielectric sleeve cylinder with $K_2t < K_3a$; measure the total electric field by a detector.

2. Put another two dielectric sleeves of same $K_2t$ outside of the previous one consecutively; measure each electric field separately.

3. Calculate the Fourier coefficients of each measured field by Eq. (3-3).
4. If the thickness \( t \) of the three sleeves is same, all \( a_n \) 's for zero sleeve thickness can be obtained by Eq. (3-5).

5. Reconstruct the total electric field of scattering by the dielectric material alone by \( a_n \)'s obtained from item 4.

Since conclusions made above are based on the study of isotropic dielectric core, an anisotropic material with a tensor permittivity requires further investigation. Furthermore, to separate the antenna characteristics from the measured field strength is another problem worthy of study.
Fig. 4-2. The total electric field pattern. Core-air. Sleeve-plexiglass.

Fig. 4-3. The total electric field pattern. Core-air. Sleeve-plexiglass.
Fig. 4-4. The total electric field pattern. Core-air. Sleeve-plexiglass.

Fig. 4-5. The total electric field pattern. Core-air. Sleeve-plexiglass.
Fig. 4-6. The variations of Fourier coefficients as a function of sleeve thickness (inches). Core-air, Sleeve-plexiglass.
Fig. 4-7. The variations of Fourier coefficients as a function of sleeve thickness (inches). Core-air. Sleeve-plexiglass.
Fig. 4-8. The variations of Fourier coefficients as a function of sleeve thickness (inches). Core-air. Sleeve-plexiglass.
Fig. 4-9. The variations of Fourier coefficients as a function of sleeve thickness (inches). Core-air. Sleeve-plexiglass.
Fig. 4-10. The variations of Fourier coefficients as a function of sleeve thickness (inches). Core-air. Sleeve-plexiglass.
Fig. 4-11. The experimental field pattern and the reconstructed field pattern.
Fig. 4-12. The experimental field pattern and the reconstructed field pattern.
Fig. 4-13. The total electric field pattern. Core-copper. Sleeve-plexiglass.

Fig. 4-14. The total electric field pattern. Core-copper. Sleeve-plexiglass.

$t = 3/8''$

$f = 8.5 \text{ KMHz}$

$t = 1/4''$

$f = 8.5 \text{ KMHz}$
Fig. 4-15. The total electric field pattern. Core-copper. Sleeve-plexiglass.

Fig. 4-16. The total electric field pattern. Core-copper. Sleeve-plexiglass.
Fig. 4-17. The variations of Fourier coefficients as a function of sleeve thickness (inches). Core-copper. Sleeve-plexiglass.
Fig. 4-18. The total electric field pattern. Core-copper. Sleeve-pyrex.

Fig. 4-19. The total electric field. Core-copper. Sleeve-pyrex.
Fig. 4-20. The total electric field pattern. Core-copper. Sleeve-pyrex.

Fig. 4-21. The total electric field pattern. Core-copper. Sleeve-pyrex.
Fig. 4-22. The variations of Fourier coefficients as a function of sleeve thickness (mm). Core-copper. Sleeve-pyrex.
REFERENCES


APPENDIX A

SIMPLIFICATION OF DETERMINANT OF EQUATION (2-10)
The determinant $\Delta$ in Eq. (2-10) can be expanded as

$$
\Delta = H_n^{(1)}(p) \left[ J_n(q) \left( \frac{k_2^2}{\mu_2^2} \right) ^2 \left( J_n'(m) J_n'(s) - Y_n'(m) J_n'(s) \right) 
+ J_n'(q) \left( \frac{k_2 k_2}{\mu_2^2} \right) \left( J_n'(s) Y_n(m) - Y_n'(s) J_n(m) \right) \right] 
+ H_n^{(1)}(p) \left[ J_n(q) \left( \frac{k_2 k_1}{\mu_2 \mu_1} \right) \left( J_n(s) Y_n'(m) - Y_n(s) J_n'(m) \right) 
+ J_n'(q) \left( \frac{k_1 k_2}{\mu_1 \mu_2} \right) \left( Y_n(s) J_n(m) - J_n(s) Y_n(m) \right) \right]
$$

(A-1)

For small $t$, one can simplify Bessel functions from the first two terms of the Taylor's expansion, i.e.,

$$
J_n(k_2 b) = J_n(k_2 a) + k_2 t \ J_n'(k_2 a)
$$
or

$$
J_n(s) = J_n(m) + k_2 t \ J_n'(m)
$$
and

$$
J_n'(s) = -J_{n+1}(s) + \frac{n}{s} J_n(s) 
= -J_{n+1}(m) - k_2 t \ J_{n+1}'(m) 
+ \frac{n}{s} \left( J_n(m) + k_2 t \ J_n'(m) \right)
$$

(A-3)

Similarly,

$$
Y_n(s) = Y_n(m) + k_2 t \ Y_n'(m)
$$

(A-4)
and

\[ Y_n'(s) = - Y_{n+1}(m) - K_2 t Y_{n+1}'(m) \]
\[ + \frac{n}{s} \left( Y_n(m) + K_2 t Y_n'(m) \right) \]  \hspace{1cm} (A-5)

Combine the above five equations,

\[
\Delta = H_n^{(1)}(p) J_n(q) \left( \frac{K_2}{\mu_2} \right)^2 W(w_1, w_2) \left[ \frac{n}{m} - \frac{n}{s} - \frac{n^2 + n - m^2}{m^2} \right] \\
+ H_n^{(1)}(p) J_n'(q) \frac{K_2 K_3}{\mu_2^2 \mu_3} W(w_1, w_2) \left[ \frac{n^2 + n}{m} - \frac{n}{m} - \frac{K_2 t n}{s} \right] \\
+ H_n^{(1)}(p) \frac{K_1}{\mu_1} W(w_1, w_2) \left[ \frac{K_2}{\mu_2} J_n(q) + K_2 t J_n'(q) \right] \]  \hspace{1cm} (A-6)

where

\[
W(w_1, w_2) = J_n(m) Y_n'(m) - Y_n(m) J_n'(m) \\
= J_n(m) Y_{n-1}(m) - Y_n(m) J_{n-1}(m) \\
= J_{n+1}(m) Y_n(m) - Y_{n+1}(m) J_n(m) \\
= \frac{2}{\nu m}
\]

Eq. (2-28) can be obtained by using Eq. (A-6) through simplification.
APPENDIX B

COMPUTER PROGRAMS (FORTRAN)
B-1. Theoretical calculation of electric field

```
DIMENSION BS1(99),WB1(99),DBS1(99),DWB1(99),
BS2(99),WB2(99),DBS2(99),DWB2(99)
DIMENSION WW(60)
DIMENSION SUM(55,5)
104 FORMAT (F10.5)
103 FORMAT (3F10.5)
102 FORMAT (2F10.5,I10)
301 FORMAT (1H,F10.0,2F15.6)
901 FORMAT (6F10.6)
READ (11,104) S1
READ (11,102) A,Z,LAX
DO 100 ID=1,4
READ (11,103) C1,C2,C3
READ (11,102) T,Y,NAX
R=2.0
PI=3.14159265
CALL BSWB1 (Z,LAX,BS1,WB1,DBS1,DWB1)
CALL BSWB1 (Y,NAX,BS2,WB2,DBS2,DWB2)
DO 40 J1=3,50
40 SUM(J1,ID)=0.0
DO 20 J=1,181
PH2=J-1
PH1=PH2*PI/180.
SUM1=0.0
SUM2=0.0
N=1
10 CONTINUE
D=S1*A+C2
E=N-1
F=2.0*C2*C2*T*((A+T)*(D-D-E*T)-E*T)*BS1(N)/
(PI*D*D*A*(T+A))
G=2.0*C2*C3*(T*T-A*A+E*T*T)*DBS1(N)/(PI*D*D*A*(T+A))
H=2.0*CM*C2*BS1(N)/(PI*D)+2.0*CM*C3*T*DBS1(N)/(PI*A)
A1=BS2(N)*(F+G)+DBS2(N)*H
A2=WB2(N)*(F+G)+DWB2(N)*H
IF (N-1) 1,1,2
1 SUM1=SUM1+DSUM1
SUM2=SUM2+DSUM2
GO TO 12
2 SUM1=SUM1+2.0*DSUM1
```
SUM2 = SUM2 + 2.0 * DSUM2
TO TO 15
15 IF (LAX-NAX) 701, 701, 702
701 M = LAX-4
GO TO 705
702 M = NAX-4
705 IF (N-M) 12, 12, 13
12 N = N+1
GO TO 10
13 ET = (COS(S1*R*COS(PHI)) + SQRT(1.0/(PI*S1*R)) * (COS(S1*R)*SUM1 - SIN(S1*R)*SUM2) ** 2
FT = (SIN(S1*R*COS(PHI)) + SQRT(1.0/(PI*S1*R)) * (COS(S1*R)*SUM2 + SIN(S1*R)*SUM1) ** 2
GT = ET + FT
GTN = GT
IF (J-1) 77, 77, 78
77 GZ = GT
TO TO 78
78 GTN = GT/GZ
WRITE (12, 301) PH2, GT, GIN
IF (J-1) 3, 3, 5
3 DO 50 J2 = 3, 50
AJ2 = J2
50 SUM(J2, ID) = SUM(J2, ID) + 0.5*GT*COS((AJ2-3.0)*PHI)
GO TO 20
5 IF (J-181) 4, 3, 6
4 DO 60 J3 = 3, 50
AJ3 = J3
60 SUM(J3, ID) = SUM(J3, ID) + GT*COS((AJ3-3.0)*PHI)
6 GO TO 20
20 CONTINUE
DO 70 J4 = 3, 50
70 SUM(J4, ID) = SUM(J4, ID)/90.
DO 1000 J5 = 3, 50
1000 WRITE (12, 501) (SUM(J5, ID), J5 = 3, 50)
GO TO 100
100 CONTINUE
DO 90 J6 = 3, 50
90 WW(J6) = 3.0*(SUM(J6,1)-SUM(J6,2))+SUM(J6,3)
DO 80 J6 = 3, 50, 6
80 WRITE (13, 901) WW(J6), WW(J6+1), WW(J6+2), WW(J6+3), WW(J6+4), WW(J6+5)
END
SUBROUTINE BSHB1 (X, MAX, BS, WB, DSB, DWB)
DIMENSION BS(90), WB(90), DSB(90), DWB(90)
IX = MAX+1
BS(IX) = 0.
BS(IX-1) = 1.0
JX=IX-2
JX=JX+1
DO 10 I=1,JX
I=JX1-I
AK=I-1

10 BS(L)=2.0*(AK+1.0)/(X*BS(I+1)-BS(I+2))

ANORM=BS(1)

DO 15 I=3,IX,2
15 ANORM=ANORM+2.0*BS(I)

DO 20 I=1,IX
20 BS(L)=BS(I)/ANORM

PI=3.14159265
SUM=0.0

DO 30 I=3,IX,2
K=(L-1)/2
AK=K

30 SUM=SUM+(-1.0)**K*BS(L)/AK
WB(1)=2.0/PI*(ALOG(X/2,0)+0.577215664)*BS(1)

-W(2)=BS(2)*WB(1)-2.0/(PI*X)/BS(1)

DO 12 L=3,IX
AK=I-1

12 WB(L)=2.0*(AK-1.0)/X*WB(L-1)-WB(L-2)

DBS(1)=(-1.0)*BS(2)
DWB(1)=(-1.0)*WB(2)

DO 50 I=2,JX
DBS(I)=0.5*(BS(I-1)-BS(I+1))
DWB(I)=0.5*(WB(I-1)-WB(I+1))

50 CONTINUE
RETURN
END

Definition of terms:

\[ S1 = K_0 \]
\[ Y = K_1 b \]
\[ A = a \]

\[ D = K_2 a \]
\[ Z = K_3 a \]

\[ T = t \]

\[ GT = \text{the total electric field} \]

\[ \text{SUM(J5,ID)} = \text{the theoretical Fourier coefficients} \]

\[ WW(J6) = \text{the extrapolated Fourier coefficients} \]
B-2. Fourier coefficients of experimental pattern

DIMENSION W(160)
DIMENSION SUM(160,5)
DIMENSION A(182,5)
101 FORMAT (6F10.5)
102 FORMAT (1H, 6F10.5)
PI=3.14159
DO 100 J=1,4
READ (11,101) (A(I,J),I=1,90,1)
DO 50 12=1,90,1
50 A(I2,J)=SQ.R.T(A(I2,J))
DO 200 I5=91,181
200 A(I5,J)=SQRT(0.1)
DO 10 K1=1,156
10 SUM(K1,JO=0.0
DO 20 I1=1,181,1
PH2=I1-1
PHI=PH2*PI/180.
IF (I1-1) 3, 3, 5
3 DO 30 K2=1,156
AK2=K2
30 SUM(K2,J)=SUM(K2,J)+0.5*A(I1,J)*COS((AK2-1.0)*PHI)
GO TO 20
5 IF (I)-181) 4,3,6
4 DO 60 K3=1,156
AK3=K3
60 SUM(K3,JD=SUM(K3,J)+A(I1,J)*COS((AK3-1.0)*PHI)
6 GO TO 20
20 CONTINUE
DO 70 K4=1,156
70 SUM(K4,J)=SUM(K4,J)/90.0
DO 1000 K7=1,156,6
1000 WRITE (12,102) SUM(K7,J),SUM(K7+1,J),SUM(K7+2,J),
SUM(K7+3,J),SUM(K7+4,J),SUM(K7+5,J)
100 CONTINUE
DO 90 K5=1,156
90 W(K5)-3. *(SUM(K5,1)-SUM(K5,2))+SUM(K5,3)
DO 80 K6=1,156,6
80 WRITE (13,101) W(K6),W(K6+1),W(K6+2),W(K6+3),
W(K6+4),W(K6+5)
END

Definition of terms:

A(I,J) = magnitude of electric field as a function of $\theta$

SUM(K7,J) = Fourier coefficients of experimental pattern
B-3. Reconstruction of field pattern by Fourier coefficients

DIMENSION A(120,3)
201 FORMAT (1H,F10.0,2F10.6)
801 FORMAT (6F10.6)
DO 100 J1=1,3
READ (11T801) (A(J,J1),J=1,30)
PI=3.14159
DO 50 I=1,181
PH2=I=1
PHI=PH2*PI/180.
SUM=0.0
DO 90 J=1,30
BJ=J-1
IF (J-1) 3,3,4
3 SUM=SUM+A(J,J1)/2.0
GO TO 90
4 SUM=SUM+A(J,J1)*COS(BJ*PHI)
90 CONTINUE
IF (I-1) 77,77,78
77 SUM1=SUM
GO TO 78
78 SUM2=SUM/SUM1
50 WRITE(12,201) PH2,SUM,SUM2
100 CONTINUE
END

Definition of terms:

A(J,J1) = Fourier coefficients

SUM = the total electric field