Model Development for Predicting Soil Moisture by Thermography

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MODEL DEVELOPMENT FOR PREDICTING
SOIL MOISTURE BY THERMOGRAPHY

BY

GERARD ALAN BEUTLER

A thesis submitted
in partial fulfillment of the requirements for the
degree Master of Science, Major in
Physics, South Dakota
State University

1980
MODEL DEVELOPMENT FOR

PREDICTING SOIL MOISTURE

BY THERMOGRAPHY

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

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INTRODUCTION

Recent launchings of satellites with thermal infrared imagery sensors, together with those planned in the near future, hold great promise for application of thermal emittance data as a tool for resource management and development. These earth resources satellites allow time-sequential monitoring of land-surface emittance over large areas of the earth at relatively low cost allowing data to be readily available on a routine basis for use by the resource specialist in making management decisions.

The use of thermography for monitoring soil moisture is based on the sensitivity of thermal emittance to surface temperature. Any factor which causes a variation in surface temperature may thus be measured by thermography. Near-surface soil moisture is such a factor. Its presence causes large changes in the specific heat and thermal conductivity of the soil. Phase transformation of water during evaporation or freezing also have large thermal effects on the energy budget of land surface and thus affect the land-surface temperature.

Complications with this method arise because soil temperature and surface emittance depend on a multitude of other physical factors. Plant growth, aspect of slope, water table, wind velocity and other variables alter soil temperatures and thermal emittance. The isolation of emittance variations caused by the presence of soil moisture is very difficult. Therefore, models describing emittance variations associated with various physical features must be developed to isolate their effects and to understand their interdependence. This may
allow one to compensate for their effect during data analysis or to schedule the collection of data when their effect is small.

Another complication arises when the thermal infrared (TIR) image is obtained from satellite-borne sensors. The image obtained includes components of radiation emitted and reflected by the surface modified by both absorption and emission from the intervening atmosphere. To determine exact values of surface temperature, corrections must be made which depend very heavily on atmospheric conditions and thus change with time. Corrections must also be made for surface emissivity and reflections which also change with surface conditions. Thus, considerable difficulty is involved in converting a satellite image into a surface temperature map.

Temperature differences between two points on the earth may be much easier to obtain with reasonable accuracy from a TIR satellite image than exact temperatures. For example, absorption by the atmosphere will decrease the apparent temperature of two points, but the apparent temperature difference between the two points will remain very nearly constant if the absorption is the same over both points. Emissivity and reflectivity difference can also be minimized by making the comparisons between points which have the same plant cover, such as two wheat fields.

In order to measure soil moistures over large areas using differences in thermal emittance, soil moisture must be measured at one reference site. If a model can be used to calculate the soil moisture difference between this site and a second site from the thermal emittance data, soil moisture can be calculated for this
second site. This procedure can be repeated for any group of sites and thus a soil moisture map may be constructed. The general objective of this project was to develop and test a model which could be used in this manner to relate thermal emittance data to soil moisture.

OBJECTIVES

Specific objectives of this project pertaining to the development of thermal infrared emittance (thermography) as a measurement of soil moisture were:

1. To modify an existing heat flow model to accept soil moisture as an input for calculating soil temperature.

2. To collect soil temperature and thermal emittance data to test and modify the model for accuracy of predicting soil moisture from thermal emittance.

3. To apply this model to the prediction of near-surface moisture conditions using thermal emittance data collected both at the site and by satellite-mounted sensors.

BACKGROUND LITERATURE

Literature related to this study includes research oriented toward the mapping of near-surface groundwater as well as that with the specific goal of measurement of soil moisture. Investigators have used IR images to locate springs and wells (Myers and Moore, 1972) by the use of predawn images. This together with a former study by Myers and Heilman (1969) showed that predawn images exhibited a
higher surface temperature for bare soil with higher moisture content in the top 50 cm. Myers and Moore (1972) evaluated the use of airborne thermography for mapping shallow aquifers using emittance patterns of predawn thermography. They obtained statistically significant results for predicting the thickness of the saturated sands and gravels for an August, predawn flight over shallow aquifers in Eastern South Dakota. In a further study, Moore and Myers (1972) illustrated the thermal responses to climatic variables for diurnal and seasonal thermography. Land use, soil moisture, and other sources of thermal differences were easily observed for daytime thermography with their effects diminishing for predawn thermography. They concluded predawn August data were the most useful for identifying shallow aquifers in South Dakota.

Several investigators have studied the relationship between thermal emittance measured from aircraft altitudes and soil temperatures. Schmugge (1978) and Reginato (1976) have shown agreement between such TIR temperatures and those measured by thermocouple on the ground. A study by Tunheim (1977) also found a positive correlation between aircraft TIR imagery and soil temperature fields caused by near-surface water tables associated with saline seeps. Results of this project showed the need for modifying a pure conduction model used to include the effect of soil moisture.

The first evaluation of satellite thermography as an indicator of soil moisture was performed by Moore, et al. (1975). Analysis of Skylab data in this study showed a positive correlation between soil
moisture and thermal emittance. It was concluded that thermal data from satellite altitudes had good potential for use in monitoring soil moisture and for irrigation scheduling.

Quantitative measurements of soil moisture using thermal emittance data, however, require a model relating the effects of subsurface soil moisture on the surface temperature. No such model has been developed, although several similar types of models exist. A model proposed by Kahle, et al. (1975) relates the change in land surface temperature during the diurnal cycle to the thermal inertia of subsurface geological materials. This model, however, does not allow for effects of groundwater or crop cover.

Two other models have been developed by Meyer (1972) for relating surface thermal emittance to the presence of shallow aquifers. These models begin with the assumption that a shallow aquifer would cause the soil temperature at a 50-cm. depth to vary 1°C to 3°C from that of a non-aquifer region. The ability of this subsurface thermal anomaly to produce a corresponding surface thermal anomaly was then investigated by use of these models. The first model simulated the development of a surface thermal anomaly during a single night, and the second simulated the behavior of the thermal anomaly during several successive days.

Each model considered heat transfer in two identical soil layers of 50-cm. thickness. Since daily variations in the temperature are small at 50 cm. (Cartwright, 1968; Carson, 1961), the lower boundary temperature of each soil layer was held constant. The subsurface thermal anomaly was presented by letting these fixed temperatures...
differ by an amount $\Delta T$.

The first model assumed a constant heat flux due to radiation. Using a finite integral transform, the heat transfer problem was analytically solved. Results predicted that a surface temperature difference ranging from 20% to 40% of that assumed at a 50-cm. depth would develop in nine hours. The rate of development depended only on the thermal diffusivity. Values of diffusivity for the calculations were chosen according to Sutton (1953). A somewhat surprising result of this calculation was the prediction that the development of a surface thermal anomaly does not depend on the magnitude of the heat flux radiated from the surface.

The second model assumed a surface heat flux approximated by a rectified sine wave and a terrestrial radiation term as suggested by Smith (1969). No analytic solution was possible in this sense, and thus a finite-difference technique was used in a numerical solution by computer. Calculated temperature profiles showed good qualitative agreement with data taken by Carson (1961).

One significant result predicted from the model was that a maximum value for the thermal anomaly would occur at 0700 hours. This result has recently received support experimentally for the case of groundwater associated with saline seeps (Aaron, et al., 1976).

The finite-difference model by Meyer is the one which was modified and applied to this research.
THEORETICAL MODEL

The finite difference heat flow model developed by Meyer (1972) uses homogeneous soil profiles, each 50 cm. in thickness. The 50-cm. depth was chosen since daily variations in soil temperature are small at this depth (Cartwright, 1968; Carson, 1961). The 50-cm. soil profile is divided into 50 one-cm. layers with 50 equally spaced reference nodes as shown in Figure 1. The m reference points are usually referred to as nodal points. Nodal point 1 coincides with the upper surface of the slab at \( x = 0 \) has been denoted as \( q_s \), while the heat flux out of the lower surface at \( x = L \) is denoted as \( q_L \).

In the model a rectified sine wave is used to approximate heat flux \( q_s \) (Smith, 1969). Since it is difficult to measure, it is treated as a parameter composed of a sinusoidal solar term and a blackbody radiation term. Its functional form is described as

\[
q_s = A \sin \frac{\pi t}{L} - R,
\]

where \( A \), the amplitude of the solar term, is the maximum solar radiation occurring during the day. The variable \( t \) is the time of day measured from sunrise and \( L \) is the number of hours of daylight. The second term, \( R \), is the terrestrial radiation term as suggested by Smith (1969). The radiation term results in a negative surface heat flux during the night as has been observed experimentally (Lettau and Davidson, 1957). The terrestrial radiation equation used is

\[
R = \sigma \left[ T^4 - (T')^4 \right],
\]
where \( R \) is the net outward radiation flux, \( \kappa \) is the Stefan-Boltzmann constant, \( T \) is the surface temperature in degrees Kelvin, and \( T' \) is the effective temperature of surrounding node \( n \). The volume of the material surrounding node \( n \) is the surface area and \( \Delta x \) is the distance between nodes \( n \) and \( n+1 \), and the amount of heat transferred from node \( n-1 \) to node \( n \) and the amount of heat stored within \( A_{n-1} \) is denoted by \( q_{n-1,n} \). The heat rate at each node is written in finite difference form as:

\[
\frac{E_{n+1} - E_n}{\Delta x} = -q_{n,n+1} - \frac{k}{A_{n+1}} (T - T_{n+1}) - \frac{k}{A_n} (T - T_n) - q_{n-1,n} + q_{n,n+1}
\]

Figure 1. Assignment of Nodal Points and Heat Flux Terms for the Finite Difference Model.

Figure 2. Energy Balance for Node \( n \).
where \( R \) is the net outgoing radiation, \( \sigma \) is the Stefan-Boltzmann constant, \( T \) is the surface temperature in degrees Kelvin, and \( T' \) is the effective atmospheric temperature in degrees Kelvin (Fleagle, 1950).

Now consider a volume of material surrounding node \( n \) \((n = 2, 3, \ldots, m-1)\) as shown in Figure 2. The volume of the material surrounding node \( n \) is \( A\Delta x \), where \( A \) is a unit surface area and \( \Delta x \) is the distance between nodal points. The amount of heat transferred from node \( n-1 \) to node \( n \) is denoted by \( q_{n-1,n} \), and the amount of heat transferred from node \( n \) to node \( n+1 \) is denoted by \( q_{n,n+1} \). The heat stored within the volume is given by \( E_{sn} \).

For one-dimensional heat transfer, the law of conservation of energy applied to node \( n \) results in the equation

\[
q_{n-1,n} = q_{n,n+1} + E_{sn}.
\]

The rate at which heat is transferred between nodal points is written in finite difference form as

\[
q_{n-1,n} = -kA \frac{T_n - T_{n-1}}{\Delta x}
\]

\[
q_{n,n+1} = -kA \frac{T_{n+1} - T_n}{\Delta x},
\]

where \( T_{n-1} \) is the temperature of node \( n-1 \), \( T_n \) is the temperature of node \( n \), \( T_{n+1} \) is the temperature of node \( n+1 \), and \( k \) is the thermal conductivity of the material between the nodal points. If the conductivity of the volume element surrounding each nodal point is different, the conductivity between nodal points may be written as the average of the volume elements.
Thus, for equation (5)

$$k = \frac{k_{n-1} + k_n}{2}$$  \hspace{1cm} (6)

Equation (4) may then be written as

$$q_{n-1,n} = -\left(\frac{k_{n-1} + k_n}{2}\right) A \frac{T_n - T_{n-1}}{\Delta x}.$$  \hspace{1cm} (7)

Similarly, equation (5) becomes

$$q_{n,n+1} = -\left(\frac{k_n + k_{n+1}}{2}\right) A \frac{T_{n+1} - T_n}{\Delta x}.$$  \hspace{1cm} (8)

The energy storage term expresses the rate at which the temperature of the volume changes. This term may be written in finite difference form as

$$\dot{E}_{sn} = (\rho c)_n A \Delta x \frac{T'_{n} - T_n}{\Delta t}$$ \hspace{1cm} (9)

where $\rho$ is the density, $c$ is the heat capacity, $\Delta t$ is the time increment, $T_n$ is the temperature of node $n$ at time $t$, and $T'_n$ is the temperature of node $n$ at time $t+\Delta t$.

Substituting equations (7), (8), and (9) into equation (3) and rearranging terms yields

$$\frac{T_{n}' - T_n}{\Delta t} = \frac{1}{2(\rho c)_n(\Delta x)^2} \left[ (k_{n-1} + k_n)T_{n-1} - (k_{n-1} + 2k_n + k_{n+1})T_n + (k_n + k_{n+1})T_{n+1}\right].$$  \hspace{1cm} (10)
Solving for the temperature at time $t+\Delta t$ results in the equation

$$T_n' = \frac{(k_{n-1} + k_n)\Delta t}{2(pc)_n(\Delta x)^2} T_{n-1} + \left[1 - \frac{(k_{n-1} + 2k_n + k_{n+1})\Delta t}{2(pc)_n(\Delta x)^2}\right] T_n$$

$$+ \frac{(k_n + k_{n+1})\Delta t}{2(pc)_n(\Delta x)^2} T_{n+1}.$$  \hspace{1cm} (11)

Now consider the transfer of heat at the surface $x = 0$. Figure 3 shows the volume element for node 1.

![Figure 3. Energy Balance for Node 1.](image)

The energy balance can be written as

$$q_s = q_{1,2} + \dot{E}_{s1}.$$ \hspace{1cm} (12)

The rate of heat transfer from node 1 to node 2 is

$$q_{1,2} = -\left(\frac{k_1 + k_2}{2}\right) A \frac{T_2 - T_1}{\Delta x}.$$ \hspace{1cm} (13)
Now since node 1 is at the surface, the volume of material surrounding node 1 is $\frac{\Delta x}{2} A$. The energy storage term is then

$$
\dot{E}_{sl} = (\rho c)_1 A \frac{\Delta x}{2} \frac{T_1' - T_1}{\Delta t}.
$$

(14)

Substituting equations (12) and (13) into equation (14) and rearranging yields

$$
\frac{T_1' - T_1}{\Delta t} = \frac{2q_s}{A(\rho c)_1 \Delta x} + \frac{k_1 + k_2}{(\rho c)_1 (\Delta x)^2} [T_2 - T_1].
$$

(15)

Solving for the new temperature $T_1'$ gives

$$
T_1' = \frac{2q_s \Delta t}{A(\rho c)_1 \Delta x} + \frac{[1 - \frac{(k_1 + k_2)\Delta t}{(\rho c)_1 (\Delta x)^2}]}{T_1} + \frac{(k_1 + k_2)\Delta t}{(\rho c)_1 (\Delta x)^2} T_2.
$$

(16)

Finally, consider the node at the lower boundary $x = L$. Figure 4 shows the volume element for node $m$.

![Figure 4: Energy Balance for Node m.](image-url)
The energy balance equation for node \( m \) is

\[
q_{m-1,m} = q_L + E_{sm}. \tag{17}
\]

The rate of heat transfer from node \( m-1 \) to node \( m \) is

\[
q_{m-1,m} = -\left(\frac{k_{m-1} + k_m}{2}\right) A \frac{T_m - T_{m-1}}{\Delta x}. \tag{18}
\]

Again, since the volume element surrounding node \( m \) is only \( \frac{\Delta x}{2} \) \( A \), the energy storage term can be written as

\[
E_{sm} = (\rho c)_m A \frac{\Delta x}{2} \frac{T'_m - T_m}{\Delta t}. \tag{19}
\]

Substituting equations (17) and (18) into equation (19) and rearranging results in the equation

\[
\frac{T'_m - T_m}{\Delta t} = \frac{k_{m-1} + k_m}{(\rho c)_m \Delta x^2} T_{m-1} - T_m - \frac{2q_L}{A(\rho c)_m \Delta x}. \tag{20}
\]

Solving for the new temperature \( T'_m \) gives

\[
T'_m = \frac{(k_{m-1} + k_m)\Delta t}{(\rho c)_m \Delta x^2} T_{m-1} + \left[1 - \frac{(k_{m-1} + k_m)\Delta t}{(\rho c)_m \Delta x^2}\right] T_m - \frac{2q_L\Delta t}{(\rho c)_m A \Delta x}. \tag{21}
\]

The finite difference equations have now been derived. These equations are equations (11), (16), and (21). To solve a heat transfer problem, the initial temperature of each of the \( m \) nodal points must be specified. This is identical to the specification of an initial condition for an analytically solved problem. To calculate the
new temperature at time $\Delta t$, the heat flux terms $q_s$ and $q_L$ must be specified. Equations (16) and (21) can then be used to determine the new boundary temperatures. The new temperature of each of the interior nodal points can be determined by solving equation (11) for each node. The resultant temperatures obtained for the $m$ nodal points can then be used to calculate the temperature at time $2\Delta t$. The iteration process is then continued to obtain the temperature at any desired future time.

Choice of values for $\Delta x$ and $\Delta t$ depends on the thermal properties of the soil considered and the thickness of the soil layer. For the 50-cm. layer considered and the thermal properties of soil used, the values $\Delta t = 60$ seconds and $\Delta x = 1$ cm. were found to be sufficient.

The model was modified during the study by considering two soil profiles having different soil moistures, but identical in other respects. When the percent soil moisture $(\theta_v)$ and percent soil $(\theta_s)$ are known, the percent air (aeration porosity, $E_a$) can be found. With these values known, the heat capacity and conductivity can be calculated in the following manner.

Heat capacity for a volume of soil is found by the following equation:

$$C = \theta_v C_w + \theta_s C_s + E_a C_a,$$

where $C_w$, $C_s$ and $C_a$ are the heat capacities of water, soil and air, respectively.

The values used for heat capacities are:

$$C_w = 1.00 \text{ cal/cm}^3/\text{°C}$$
\[ C_s = 0.48 \text{ cal/cm}^3/\text{°C} \]

\[ C_a = 0.00030 \text{ cal/cm}^3/\text{°C} \]

Since \( C_a \) is a small part of the heat capacity, it is neglected in model calculations. The model in its original form was only applicable to a homogeneous soil layer; thus, it was modified so each one-cm. soil layer could have a different moisture. If the soil profile's moisture varies with depth, variation in heat capacity and thermal conductivities occurs. To adapt to non-homogeneous conditions, the model was modified to accept experimental soil moisture values at depths of 1, 8, 25 and 42 cm. Values are then calculated by the model by interpolation and extrapolation over the 50-cm. profile. Thermal conductivity for each soil volume is then calculated by the method developed by DeVries (1963). This method generates an apparent thermal conductivity which approximates heat transfer due to mass movement of water, phase change of water, convection and conduction.

This equation is given by:

\[ \lambda = \frac{\Sigma x_i \lambda_i}{\Sigma x_i} \]

(23)

where \( \lambda \) is the apparent thermal conductivity of a granular material, \( \lambda_i \) is the thermal conductivity of the soil's individual components, \( x_i \) is the volume fraction occupied by each soil fraction, and \( \kappa_i \) is the ratio of the average temperature gradient in the granules across the medium.
The value of \( k \) can be calculated using the following equation:

\[
\kappa_i = \frac{1}{3\gamma_\lambda} \left[ 1 + \left( \frac{\lambda_i}{\lambda_o} - 1 \right) g_a \right]^{-1}.
\]  

(24)

The \( g_a \) value is found using an unsaturated soil, with water as a continuous medium:

\[
g_a = 0.333 - \frac{E_a}{\Sigma} (0.333 - 0.035)
\]

(25)

where \( \Sigma \) is the soil porosity.

The conductivities of the various soil constituents, \( \lambda \), are given these values:

- \( \lambda_s \) - conductivity of soil 0.00525 cal/cm sec °C
- \( \lambda_w \) - conductivity of water 0.00142 cal/cm sec °C
- \( \lambda_a \) - conductivity of air \( 0.0000615 + 0.00196 x_w \) cal/cm sec °C

The finite-difference model (Figure 5) has the following inputs:

1. soil heat flux,
2. soil moisture profile,
3. dry soil conductivity,
4. physical properties of the soil which include (a) bulk density, \( \rho \), (b) amount of soil by volume,
5. initial temperature profile and
6. effective air temperature.

Outputs from the model calculations are soil temperature profiles for the two sites and a surface temperature difference as a function of time.
Figure 5. Schematic representation of the finite-difference model in its present format.
DATA COLLECTION

Sites chosen for this study were located at the South Dakota State University Agricultural Engineering Farm, which is near Brookings, South Dakota. Since soils vary considerably in this area, soil texture by hydrometer method, bulk density and porosity were analyzed through a depth of 50 cm. Results of the analysis are shown in Table 1 and Figures 6, 7 and 8. The percent of volume occupied by soil particles for a dry soil condition is shown in Figure 6. Since the percent soil particles increases with depth, the porosity decreases with depth. Figure 7 shows the variation in soil components with depth. The percent of silt per volume increases comparably. The bulk density increases with depth as shown in Figure 8. Classification of the soil according to USDA standards is a silt loam for the entire profile.

TABLE 1
Physical Properties of Soil Used

<table>
<thead>
<tr>
<th>Sample Depth (cm)</th>
<th>Particle Size</th>
<th>Bulk Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sand (%)</td>
<td>Silt (%)</td>
</tr>
<tr>
<td>0.0 - 7.6</td>
<td>27.7</td>
<td>61.5</td>
</tr>
<tr>
<td>15.2 - 22.9</td>
<td>25.0</td>
<td>65.5</td>
</tr>
<tr>
<td>30.5 - 38.1</td>
<td>18.6</td>
<td>74.3</td>
</tr>
<tr>
<td>45.7 - 53.3</td>
<td>15.6</td>
<td>78.4</td>
</tr>
</tbody>
</table>

The data collection site was divided into two plots each approximately 10 m². To prevent water movement from one plot to the other, a trench was excavated to a depth of 100 cm., and a plastic vapor
Figure 6. Percent soil by volume of soil profile as a function of depth.
Figure 7. Percent of the soil particles composed of sand, silt and clay as a function of depth.
Figure 8. Bulk density of soil profile as a function of depth.
barrier was buried. This barrier allowed one plot to remain dry while the other was irrigated to a desired soil moisture.

To measure soil temperatures, thermocouples were implanted in each plot at depths of 1, 5, 10, 25, 50 and 100 cm. Thermal emittance (apparent surface temperature) was measured utilizing a Barnes PRT-5 mounted on an apparatus, as shown in Figure 9 and Figure 10, which scanned each plot every 15 minutes during data collection.

Soil and air temperatures, together with relative humidity, were collected every hour while solar radiation and net radiation data were collected every 15 minutes.

Soil moisture data was acquired by the gravimetric method with collection of soil samples an hour before solar noon (Jackson et al., 1976) to best represent the average moisture content. The gravimetric method of soil moisture gives a value of soil moisture by weight, $\theta_m$,

$$\theta_m = \frac{\text{mass water}}{\text{mass dry soil}}. \tag{26}$$

In the model, soil moisture by volume is required. Thus, $\theta_m$ is multiplied by the soil bulk density to give the model input, or,

$$\theta_v = \rho \theta_m. \tag{27}$$

During the study, data were collected for several diurnal cycles for both barley and rye crop covers, together with the bare soil conditions after the canopies were removed. Figure 11 and Figure 12 show the barley crop canopy and bare soil, respectively, for which detailed data are shown in this report. Data for the rye crop were qualitatively similar and will not be shown in detail.
Figure 9. Photograph of Barnes PRT-5 on scanning apparatus.
Figure 10. Photograph showing the scanning apparatus used to move the Barnes PRT-5 across the experimental plots and other experimental instrumentation.
Figure 11. Barley crop canopy present during the data collection on August 5 and 6, 1978.
Figure 12. Bare soil present during data collection on August 7, 8 and 9, 1978.
RESULTS

Data were collected for bare surface conditions for a 52-hour period starting at 1000 hours, August 7, 1978. Two adjacent plots were prepared as previously described so that one plot would have a somewhat higher soil moisture profile than the other. Results of gravimetric soil moisture measurements for these plots are shown in Figures 13, 14 and 15 as a function of depth. Experimental values are fitted to a smooth profile curve by the use of a cubic spline, as described by Kimball (1976). The decrease in soil moisture with increasing depth shown by these graphs is due to a decreasing holding capacity of the soil near the 50-cm. depth.

Apparent surface temperatures of these plots are shown in Figures 16 and 17 for a 52-hour period. These measurements were made utilizing the Barnes-PRT 5 mounted on the scanning apparatus shown in Figure 10. The points shown are the experimental values, while the continuous curve is the result of smoothing this data by use of a cubic spline. Note the amplitude of the temperature variation during the diurnal cycle is less for the higher moisture plot. This is consistent with the results reported previously by Idso et al. (1975).

The apparent surface temperature difference between the two plots is shown as a function of time in Figure 18. To obtain values for this plot, temperature differences were calculated by using values from the cubic spline curves of Figures 16 and 17. A feature of this plot is its close similarity in functional form to the
Figure 13. Soil moisture profile (percent by volume) for August 7, 1978. Profile (A) is for the dry plot and Profile (B) is for the irrigated plot.
Figure 14. Soil moisture profile (percent by volume) for August 8, 1978. Profile (A) is for the dry plot and Profile (B) is for the irrigated plot.
Figure 15. Soil moisture profile (percent by volume) for August 9, 1978. Profile (A) is for the dry plot and Profile (B) is for the irrigated plot.
Figure 16. Apparent temperature for bare soil surface of the dryland plot soil profile beginning 1000 hours, August 7, 1978 and continuing for 52 hours thereafter.

Figure 17. Apparent temperature for bare soil surface of the irrigated soil profile beginning 1000 hours, August 7, 1978 and continuing for 52 hours thereafter.
Figure 18. Apparent temperature difference between the dry and irrigated plots for bare soil. Data is shown for a 52-hour period starting at 1000 hours, August 7, 1978.
individual apparent temperature curves from which it was derived. This similarity is particularly significant since the diurnal amplitude of the surface temperature has previously been related to near-surface soil moisture (Idso, et al., 1975; Idso and Ehler, 1976; Schmugge et al., 1978). Since temperature difference shows the same functional form as surface temperatures, it seems very likely that the amplitude of the temperature difference can be related to soil moisture differences. This type of technique for remotely measuring soil moistures would have the advantage of bypassing the calibration problems inherent in thermal emittance measurements.

Results of theoretical model calculations for August 8, 1978 are shown in Figure 19. Input to the calculation are the bulk densities of the soil, the measured net radiation and percent soil moisture by volume (See Appendix A). Comparing Figure 19 with the experimental plot of Figure 18 shows the functional dependence of the theoretical curve to agree very well with the experiment, particularly during the daylight hours. The magnitude of the calculated daylight temperature difference is smaller with a maximum calculated temperature difference of about 6°C, compared to a measured difference of about 10°C. A possible explanation of this difference is the additional cooling of the irrigated plot due to water evaporation from the surface. Increasing the amplitude for net radiation allows the model to simulate the daytime surface temperature differences accurately. However, the calculated soil profiles become much warmer than those measured.
Figure 19. Theoretical model calculation of surface temperature difference between the dry and irrigated plots for August 8, 1978.
Figure 20. Soil temperature profiles calculated by the theoretical model for the dry plot (A) and the irrigated plot (B) for August 7, 1978 at 0400 hours. Experimental temperatures are represented by (•).
Figure 21. Soil temperature profiles calculated by the theoretical model for the dry plot (A) and the irrigated plot (B) for August 7, 1978 at 0800 hours. Experimental temperatures are represented by (*).
Figure 22. Soil temperature profiles calculated by the theoretical model for the dry plot (A) and the irrigated plot (B) for August 7, 1978 at 1600 hours. Experimental temperatures are represented by (×).
Figure 23. Soil temperature profiles calculated by the theoretical model for the dry plot (A) and the irrigated plot (B) for August 7, 1978 at 2200 hours. Experimental temperatures are represented by (•).
Figure 24. Soil moisture profile (percent by volume) for August 5, 1978. Profile (A) is for the dry plot and Profile (B) is for the irrigated plot.
Figure 25. Soil moisture profile (percent by volume) for August 6, 1978. Profile (A) is for the dry plot and Profile (B) is for the irrigated plot.
Figure 26. Apparent temperature measured over the barley canopy of the dryland plot. Data collection begins at 1100 hours, August 5, 1978 and continues for 32 hours thereafter.
Figure 27. Apparent temperature measured over the barley canopy of the irrigated plot. Data collection begins at 1100 hours, August 5, 1978, and continues for 32 hours thereafter.
Apparent temperature difference between the dry and the irrigated plots with a crop canopy. Data is shown for a 32-hour period starting at 1100 hours August 5, 1978.
Experimental soil temperatures are compared in Figures 20-23, with theoretical temperature profiles calculated by the model using measured inputs. The functional form of the calculated temperature profiles are very similar to the measured profiles. The theoretical values, however, tend to be warmer during the day and cooler during the night. This result also implies that evaporation from the surface cannot be ignored in model calculations and must be accounted for with a parameter which effectively reduces the net radiation term to obtain the soil heat flux.

Soil moisture profiles for barley plots used to determine the effects of a plant canopy are shown in Figures 24 and 25. These plots were prepared in the same general manner as the bare plots and the same type of data collected. Figures 26 and 27 show the apparent surface temperatures of the two plots for a 32-hour period beginning at 1100 hours, August 5, 1978. Figure 28 shows the apparent surface temperature obtained by subtracting corresponding apparent temperatures from the cubic spline graphs of Figures 26 and 27.

Comparison of Figures 26 and 27 with Figures 16 and 17 shows a drastic effect of the plant canopy on apparent surface temperatures. Both the functional dependence and the actual apparent temperature values are quite different for the barley canopy. However, the functional dependence for the apparent temperature differences as shown in Figures 18 and 28 are considerably more alike in functional form than the apparent surface temperatures. During the daylight hours the barley plots exhibit approximately one-half the temperature
difference of the bare plots, but the only difference in functional form is a slightly slower rate of decrease in temperature difference late in the afternoon. The crop canopy essentially eliminates the observed temperature difference for the nighttime hours. These results suggest that apparent temperature differences during the middle of the day may be the most likely indicator of soil moisture differences in the case of a plant canopy.

A theoretical calculation of the surface temperature difference for the barley plots is shown in Figure 29 for August 5, 1978. Again the calculated temperature difference is smaller than the measured apparent temperature difference, but the ratio of the two is approximately the same as for the bare soil discussed previously. Since no apparent temperature difference is observed for the barley canopy during the night, the model obviously is not valid in its present form for that time period. Since the surface temperature difference predicted by the model is due to the difference in blackbody radiation, the model must be modified to emit equal blackbody radiation for that time period.

Several calculations were carried out to determine the dependence of the surface temperature difference on surface soil heat flux. In these calculations two plots were considered with soil properties identical to the experimental plots used for this study. Soil moisture by volume was assumed to be 10% in one plot and 20% in the second. Soil heat flux values were chosen to span a range which would include most experimental situations for a clear day. The maximum temperature
Figure 29. Theoretical model calculation of surface temperature difference between the dry and irrigated plots for August 5, 1978.
Figure 30. Calculated surface temperature difference between two plots as a function of net radiation amplitude. One plot is considered to have a constant soil moisture profile of 10% by volume while the other has a constant profile of 20%.
Figure 31. Calculated surface temperature difference between two soil plots as a function of moisture difference. One plot is considered to have a fixed soil moisture profile of 10% by volume while the other is varied from that value.
difference predicted during the day was plotted as a function of the maximum soil heat flux. Results shown in Figure 30 display a linear relationship. If further theoretical and experimental results show this linear relationship to be valid, differences in daily solar radiation which exist during the satellite overpass could be easily accounted for during analysis of data.

Preliminary calculations were also carried out to determine the relationship which might exist between moisture difference and maximum temperature difference observed during the day. Values for net radiation and physical properties of the soil were again chosen to correspond to the experimental plots of this study. The reference plot was chosen to have a soil moisture of 10% by volume, and the soil moisture of the other was varied to a maximum of 21%. Results shown in Figure 31 display an approximately linear relationship. If this also proves true in further theoretical and experimental investigations, the development of a practical technique for utilizing apparent surface temperature differences to measure soil moisture will be greatly simplified.

Data was also collected utilizing two fields about seven miles apart which were planted to corn. These were chosen so that the expected resolution of the HCMM satellite would be sufficient to allow a value of thermal emittance to be assigned to each field by imagery from this satellite. Thermocouples were buried on each field site at depths of 1 cm., 5 cm., 10 cm., 25 cm., 50 cm. and 100 cm. Soil temperature measurements were then taken several times during the
summer at the HCMM overpass times. Attempts to use the theoretical model to simulate the soil temperature in terms of soil moisture and percent crop cover have been quite successful. HCMM data, however, were not received until after the termination of this study.

Preliminary analysis gives some support to results previously discussed in this report. HCMM satellite data showed an apparent temperature difference of approximately $0 \, ^\circ C$ at 0200 hours and a $0.3 \, ^\circ C$ difference at 1400 hours on July 13, 1978. The $0 \, ^\circ C$ result at 0200 hours is consistent with the observed results for the barley plots previously discussed since they showed no apparent temperature differences during the nighttime hours. Calculations previously carried out with the model predicted a temperature difference of about $0.5 \, ^\circ C$ at 1400 hours. This was before the model was modified to accept the bulk density and other soil physical properties. Thus, further calculations with the model in its present form must be carried out to test its reliability. One problem with the data acquired was the small difference in soil moisture between these two sites at the time of overpass. Thus, only very small temperature differences were involved, and the percent of uncertainty was large.
CONCLUSIONS

1. Two bare soil plots with different soil moistures differ in surface temperature in a well defined functional manner during the diurnal cycle. This functional dependence is similar to the time dependence of surface temperature for each plot.

2. A thick crop canopy destroys the apparent surface temperature difference during the nighttime hours.

3. The functional form of the apparent surface temperature difference with time due to a soil moisture difference is changed considerably less by a thick crop canopy than individual surface temperatures. Thus, temperature differences may be a much more accurate measurement of soil moisture than the variation in individual diurnal surface temperatures as has been proposed previously by other investigators.

4. The theoretical model used in this study predicts a functional form for the apparent surface temperature difference very similar to that observed for the daylight hours. The magnitudes of the theoretical temperature differences are smaller than the experimental values for both a bare soil and a crop canopy. The ratio of calculated temperature difference to that measured is approximately the same in both cases.

5. Since the observed nighttime surface temperature differences vary considerably in functional form for the bare soil situation and are zero during the night for the crop canopy, nighttime thermal emittance data does not seem promising for use in
measuring soil moisture.

6. Model calculations predict a linear relationship between soil heat flux and the surface temperature difference arising from soil moisture variations.

7. Model calculations predict an approximate linear relationship between surface temperature difference and soil moisture difference. If conclusions 6 and 7 prove true in further theoretical and experimental studies, the development of a practical technique for utilizing apparent surface temperature to measure soil moisture will be greatly simplified.

8. The overall results of this study reveal promise for the development of a method to monitor soil moisture by satellite in the following manner. Using points on the curve comparing apparent surface temperature differences, one could calculate soil moisture differences for a group of chosen sites. If soil moisture is then measured at one site, soil moisture may be calculated for the other sites.
SUGGESTIONS FOR FURTHER STUDY

During the course of this project, the need for further investigation in several areas became obvious. One of these areas is the effect of a crop canopy on the amount of solar radiation that actually reaches the soil surface. Such a study would involve collection of soil temperature, solar radiation and leaf area index (LAI) data for a series of crop canopies. Model calculations would then be carried out to find the surface heat flux which would best simulate the observed soil temperature profiles. With the aid of a light penetration model (Mann, et al., 1980), a correlation between surface soil heat flux and solar radiation would be attempted using LAI as a parameter.

A better knowledge is also needed of the effect relative humidity (RH) has on the effective air temperature used in the model. The effective air temperature in the model was found to have considerable effect on the radiative loss during the night but little effect during the daylight hours. Therefore, a relationship of effective air temperature and RH is desirable during nighttime hours.

Temperature difference between an irrigated and a non-irrigated plot is greater under field conditions than those predicted by the model. One possible cause of this difference is a greater evaporation cooling of the irrigated plot's surface. Thus, additional knowledge of the evaporative process at the soil-air interface would be helpful. Very well monitored soil moisture would be required, together with additional theoretical calculations, to relate the
additional temperature difference to soil moisture profile.

With these two modifications to the soil model, theoretical calculations can be carried out for typical field and climatic conditions. A multiple linear regression curve can then be obtained which relates soil moisture of a soil plot, solar radiation, a reference soil moisture, surface temperature difference measured remotely, crop canopy and soil type. Using the linear regression curve a soil moisture at any location can be calculated from remotely sensed thermography data, soil moisture from a reference site and solar radiation data obtained from a weather station.
LITERATURE CITED


Appendix A

HPL PROGRAM FOR FINITE-DIFFERENCE HEAT FLOW SIMULATION MODEL

The following program listing is written in a language used by Hewlett-Packard in the 9835A mini-computer furnished by the Water Resources Institute at South Dakota State University.

A[*] Temperature of Profile A
B[*] Temperature of Profile B
C[*] Conductivity of Profile A
D[*] Conductivity of Profile B
E[*] Heat capacity of Profile A
F[*] Heat capacity of Profile B
H[*] Time of day (Hour)
G[0] Term used by DeVries in calculation of conductivity
G[1] Heat capacity of water
H[1] Initial starting hour
H[2] Ending hour
I[0:50] Thermal inertia for site A
J[0:50] Thermal inertia for site B
K[0] Conductivity of air
K[1] Conductivity of water
K[2] Conductivity of soil
M[1] Initial starting minute
M[2] Minute when calculation is to end
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>O[*]</td>
<td>Soil moisture for site A</td>
</tr>
<tr>
<td>P[*]</td>
<td>Soil moisture for site B</td>
</tr>
<tr>
<td>Q[*]</td>
<td>Thermal diffusivity for site A</td>
</tr>
<tr>
<td>R[*]</td>
<td>Allocation to store old temperature for site A</td>
</tr>
<tr>
<td>S[*]</td>
<td>Allocation to store old temperature for site B</td>
</tr>
<tr>
<td>T[*]</td>
<td>Thermal diffusivity for site B</td>
</tr>
<tr>
<td>U[0:50]</td>
<td>Aeration porosity for site A</td>
</tr>
<tr>
<td>V[0:50]</td>
<td>Aeration porosity for site B</td>
</tr>
<tr>
<td>W[1]</td>
<td>Effective air temperature</td>
</tr>
<tr>
<td>W[2]</td>
<td>Amount of soil by volume at 1-cm. depth</td>
</tr>
<tr>
<td>W[3]</td>
<td>Amount of soil by volume at 8-cm. depth</td>
</tr>
<tr>
<td>W[4]</td>
<td>Amount of soil by volume at 24-cm. depth</td>
</tr>
<tr>
<td>X[1]</td>
<td>Distance between nodal points</td>
</tr>
<tr>
<td>Y[1]</td>
<td>Ending day</td>
</tr>
<tr>
<td>Z[I,J,K]</td>
<td>Real data</td>
</tr>
<tr>
<td>A</td>
<td>Soil heat flux for site A</td>
</tr>
<tr>
<td>B</td>
<td>Soil heat flux for site B</td>
</tr>
<tr>
<td>C</td>
<td>Time from sunrise to solar noon</td>
</tr>
<tr>
<td>D,E,F,G</td>
<td>Soil bulk density at 1, 8, 24 and 42 centimeters</td>
</tr>
<tr>
<td>1</td>
<td>Counter</td>
</tr>
<tr>
<td>J</td>
<td>Counter</td>
</tr>
<tr>
<td>K</td>
<td>Counter</td>
</tr>
<tr>
<td>L</td>
<td>Day Length</td>
</tr>
<tr>
<td>M</td>
<td>Amplitude of soil heat flux for site A</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>N</td>
<td>Number of equally spaced nodal points</td>
</tr>
<tr>
<td>P</td>
<td>Time between printouts</td>
</tr>
<tr>
<td>Q</td>
<td>Amplitude of soil heat flux for site B</td>
</tr>
<tr>
<td>R</td>
<td>Dummy variable</td>
</tr>
<tr>
<td>T</td>
<td>Time interval between calculation in seconds</td>
</tr>
<tr>
<td>X</td>
<td>Dummy variable</td>
</tr>
<tr>
<td>Z</td>
<td>Real data file</td>
</tr>
</tbody>
</table>
PROGRAM LISTING (HPL)

Operational Procedure

0: 706-r0; gto 119
*10302

Subroutine for Soil Heat Flux

1: "QHEAT": H(1)/60*r21
2: H(1)+r21-r22
3: 2-C*r23
4: (H(1)+273.15)/100*r24
5: (A[0]+273.15)/100*r25
6: (B[0]+273.15)/100*r26
7: .000136*r24^4*r27
8: -.000136*r25^4*r27+r11
9: -.000136*r26^4-r27)*r12
10: if r23<=0; gto 15
11: if r23>=L; gto 15
12: rnd
13: r24*A1*r27
14: -(.000136*r25^4-r27)*r12
15: r11=A1; r12=B; ret
*11416

Subroutine for Construction of Profile Plot

16: "Plots":
17: deg:0=R
18: scl 0,10,0,7
19: fxd 0
20: csiz 1.25,1,1,0
21: plt .5,.6,5,1;lbl "SOIL TEMP FOR SITE A"
22: plt 3.7,.6,8,1;lbl "HOUR"
23: plt 4,.6,3,1
24: if H(1)<10; str(H[1])+$;lbl "0"; colt -1,0; lbl ES,"00"; jmp 2
25: cplt -1,0; str(H[1])+ES; lbl ES,"00"
26: plt 5.5,.6,5,1; lbl "SOIL TEMP FOR SITE B"
27: plt 1,.1,1
28: plt 4,.1,2
29: plt 4,.6,2
30: plt 1,.6,2
31: plt 1,.1,2
32: pen
33: csiz 1.1,1,0
34: for I=0 to -50 by -5
35: if I<9; plt .4,.6+I*.1,.1; lbl I; jmp 2
36: plt .55,.6+I*.1,.1; lbl I
37: plt .95,.5+I*.1,.1; lbl "-"; next I
38: csiz 1.25,1,1.93
39: plt 2,.2,6,1,1; lbl "DEPTH(CM)"
40: csiz 1.1,1,0
41: for I=10 to 50 by 5
42: plt -.05+.1*.075,.75,1; lbl I
43: plt .25+.1*.075,.75,1; lbl ";"; next I
44: csiz 1.25,1,1,0
45: plt 2,.5,.1; lbl "TEMP(C)"
46: if 0=R; ofs 5,0; 10-R; gto 27
47: ofs -5,0; ret
*31254
Subroutine for Plot of Theoretical Profile A

48: "plotA":
49: plt .25+A(I)*.075,6+L[I]*.1
50: if I=50;pen
51: ret

Subroutine for Plot of Experimental Profile A

52: "realA":
53: colt -.33,-.25
54: plt .25+Z[J,H[I]/2,1]*.075,6-H[J]*.1,l;lbl "**"
55: ret

Subroutine for Plot of Theoretical Profile B

56: "plotB":
57: plt 5.25+B[I]*.075,6+L[I]*.1
58: if I=50;pen
59: ret

Subroutine for Plot of Experimental Profile B

60: "realB":
61: colt -.33,-.25
62: plt 5.25+Z[J,H[I]/2,2]*.075,6-H[J]*.1,l;lbl "**"
63: ret
Subroutine for Construction of Temperature Difference Plots

64: "Diff plot":
65: def 0
66: set 0.1,0.7
67: csiz 1.25,1,1,270
68: plt 4.5,6.2,1;lbl "SURFACE TEMPERATURE DIFFERENCE"
69: plt 9.5,6,1;lbl "5 CM TEMPERATURE DIFFERENCE"
70: fx d 0
71: plt 1,1,1
72: plt 4,1,2
73: plt 4,6,2
74: plt 1,6,2
75: plt 1,1,2
76: pen
77: csiz 1,1,1,270
78: for I=0 to 24 by 4
79: if I<10;plt .7,5.2-I*.21,1;lbl I;jmp 2
80: plt .7,6.25-I*.21,1;lbl I
81: plt .9,6-I*.21,1;lbl "\";next I
82: csiz 1.25,1,1,270
83: plt .5,3.75,1;lbl "HOUR"
84: csiz 1,1,1,270
85: for I=-6 to 10 by 2
86: if I<9;plt 2.125+I*.1875,6.5,1;lbl I," -";jmp 2
87: plt 2.125+I*.1875,6.6,1;lbl I," -";
88: next I
89: csiz 1.25,1,1,0
90: plt 2.2,6.75,1;lbl "TEMP (C)"
91: line 1,2
92: for I=0 to 24;plt 2.125,6-I*.21;next I;pen
93: fx d 2
94: line
95: if R=0;ofs 5,0;I=-R;gto 70
96: ofs -5,0;ret
*26454

Subroutine for Plot of Surface Temperature Difference

97: "plot3":
98: line
99: plt 2.125+U[I]*.1875,6-O[I]*.21
100: if I=72;pen
101: ret
*17461

Subroutine for Plot of 5 cm Temperature Difference

102: "plotc":
103: line
104: plt 7.125+V[I]*.1375,6-O[I]*.21
105: if I=72;pen
106: ret
*6896
Subroutine for Conductivities in Profile A

107: "CONV: A":
108: .333-V(1)/(1-V(1))*(.333-.035)*G(0)
109: .0000615+.00195*P(1)+K(0)
110: (2/(1+(K(1)/K(1)-1)*G(0)))+1/(1+(K(1)/K(1)-1)*(1-2*G(0))))/3-r1
111: (2/(1+(K(1)/K(1)-1)*G(0)))+1/(1+(K(1)/K(1)-1)*(1-2*G(0))))/3-r2
112: ret
#31843

Subroutine for Conductivities in Profile B

113: "CONDUC B":
114: .333-V(0)/(1-V(1))*(.333-.035)*G(0)
115: .0000615+.00195*P(1)+K(0)
116: (2/(1+(K(1)/K(1)-1)*G(0)))+1/(1+(K(1)/K(1)-1)*(1-2*G(0))))/3-r3
117: (2/(1+(K(1)/K(1)-1)*G(0)))+1/(1+(K(1)/K(1)-1)*(1-2*G(0))))/3-r2
118: ret
#13617

Dimension Statements

119: dim A[0:50],a[0:50],C[0:50],D[0:50],E[0:50],F[0:50],O[0:105],P[0:50]
120: dim X[1],V[0:105],W[5],ES[4]
121: dim Y[0:4],N[3:2],K[0:3],C[0:2],L[0:105],R[0:50],S[0:51],U[0:105]
122: dim Z[5,12,2],W[3],T[0:50],J[0:50],O[0:50],Q[0:50],T[0:50]
#10727

Entering of Calculation Parameters

123: ent X[1],T,H,P,\(N[1],N[2],Y[1],W[1]
124: ent "real data file",Z;1df Z[*]
#32553

Temperature Data Storage

125: 1+U[1];5+U[2];10+U[3];25+U[4];50+U[5]
126: -1+L[1]
127: for I=2 to 50:L[I-1]-L[I];next I
128: 0+M[3]-Y[4]*=1O
129: H[1]*=0+1[I]-M[3]
130: I=r13
#27980

Entering of Soil Temperature Profiles

#8924
Interpolation of Soil Temperatures

132: for I=2 to 4; \((\Lambda[5]-\Lambda[1])/4*(I-1)+\Lambda[1]\)\n133: \((\Lambda[5]-\Lambda[1])/4*(I-1)+\Lambda[1];\) next I
134: for I=5 to 9; \((\Lambda[10]-\Lambda[5])/5*(I-5)+\Lambda[5]\)\n135: \((\Lambda[10]-\Lambda[5])/5*(I-1)+\Lambda[5]=B[1];\) next I
136: for I=11 to 24; \((\Lambda[25]-\Lambda[10])/15*(I-1)+\Lambda[10]-\Lambda[1]\)
138: for I=26 to 49; \((\Lambda[50]-\Lambda[25])/25*(I-25)+\Lambda[25]-\Lambda[1]\)
#4089

Surface Temperature Difference

140: \(A[0]-B[0]=S[51]\)
#5957

Determination of Mode for Entering Heat Capacity and Thermal Conductivity

141: J+1=J; if J>1; goto 250
#25970

Entering of Soil Physical Properties

#29564

Calculation of Percent Moisture by Volume

144: \(O[1]*D=O[1]; O[8]*E=O[9]; O[25]*F=O[25]; O[42]*G=O[42]\)
145: \(P[1]*D=P[1]; P[8]*E=P[8]; P[25]*F=P[25]; P[42]*G=P[42]\)
#23434

Calculation of Percent Air

#17723
Specification of Thermal Conductivities

\[ \text{154: } 1;2;X; \text{cfl 'CONDUCT } \lambda' (U[1],O[1],W[X]) \]
\[ \text{156: } 1;2;X; \text{cfl 'CONDUCT } B' (V[1],P[1],W[X]) \]
\[ \text{158: } 1;3;X; \text{cfl 'CONDUCT } A' (U[1],O[1],W[X]) \]

Specification of Heat Capacity


Interpolation of Thermal Conductivities and Heat Capacity

\[ \text{178: for } I=2 \text{ to } 24; C[31-(C[25]-C[0])/17*(8-I)+C[I] \]
\[ \text{179: } D[31-(D[25]-D[0])/17*(8-I)-E[I] \]
\[ \text{180: } E[31-(E[25]-E[0])/17*(8-I)+F[I] \]
\[ \text{181: } P[31-(P[25]-P[0])/17*(8-I)+G[I]; \text{next } I \]
\[ \text{185: } P[25]+P[42]-P[25]+17*(I-25)+G[I]; \text{next } I \]
\[ \text{186: } C[I]+C[0]; D[I]+D[0]; E[I]+E[0]; P[I]+P[0] \]

*4021
Printout of Thermal Conductivity (cal/°C cm)

107: fmt 1.7/,"CONDUCTIVITY";wrt 706.1
108: fmt 1.3/,"DEPTH" SITE A SITE B ";wrt 706.1
109: int(1/3)+1*r2
110: =r3+1*r4
111: for I=0 to r2-1;I=r5+r6
112: if r4>=1;r6+1+r5
113: r6+r2=r7+r8
114: if r4>=2;B=r8+1+r9
115: r8+r2=9
116: fmt 1.5/3x,f3.0,4x,f6.3,3x,f6.3,4x,f3.0,4x,f6.3,3x,f6.3,z
117: wrt 706.1,r5,E[r5],D[r5],C[r7],D[r7]
118: fmt 2.4x,f3.0,3x,f7.5,2x,f7.5
119: wrt 706.2,r9,E[r9],D[r9];next I
*31908

Printout of Volumetric Heat Capacity (cal/cm³ sec °C)

201: fmt 1.8/,"HEAT CAPACITY";wrt 706.1
202: fmt 1.3/4x,3"DEPTH" SITE A SITE B ";wrt 706.1
203: int(1/3)+1*r2
204: r2*r3=r4
205: for I=0 to r2-1;I=r5+r6
206: if r4>=1;r6+1+r5
207: r6+r2=r7+r8
208: if r4>=2;B=r8+1+r9
209: r8+r2=9
210: fmt 1.5/3x,f3.0,4x,f6.3,3x,f6.3,4x,f3.0,4x,f6.3,3x,f6.3,z
211: wrt 706.1,r5,E[r5],F[r5],r7,E[r7],F[r7]
212: fmt 2.4x,f3.0,4x,f6.3,3x,f6.3
213: wrt 706.2,r9,E[r9],F[r9];next I
*16222

Calculation of Thermal Inertia

215: for K=0 to 50
216: (C[K]*E[K])^-5=I[K]
217: (D[K]*E[K])^-5=J[K]
*24481

Calculation of Thermal Diffusivity

218: C[K]/E[K]*-O[K]
219: D[K]/F[K]*T[K]
220: next K
*29332
Printout of Thermal Inertia

221: fmt 1,9/,"THERMAL INERTIA";wrt 706.1
222: fmt 1,3/,"4X,3"DEPTH SITE A SITE B ";wrt 706.1
223: int(4/3)+1-r2
224: r2*3-r3
225: N-r3+1+r4
226: for I=0 to r2-1;I+5+r6
227: if r4>=1;r6+1+r6
228: r6+r2-r7+r8
229: if r4>=2;r3+1+r9
230: r8+r2-r9
231: fmt 1,5x,f3.0,3x,f7.5,2x,f7.5,4x,f3.0,3x,f7.5,2x,f7.5,z
232: wrt 705.1,r5,1[r5],J[r5],r7,1[r7],J[r7]
233: fmt 2,4x,f3.0,3x,f7.5,2x,f7.5
234: wrt 706.2,r9,1[r9],J[r9];next I
*13488

Printout of Thermal Diffusivity

235: fmt 1,9/,"THERMAL DIFFUSIVITY";wrt 706.1
236: fmt 1,3/,"4X,3"DEPTH SITE A SITE B ";wrt 706.1
237: int(4/3)+1-r2
238: r2*3-r3
239: N-r3+1+r4
240: for I=0 to r2-1;I+5+r6
241: if r4>=1;r6+1+r6
242: r6+r2-r7+r8
243: if r4>=2;r3+1+r9
244: r8+r2-r9
245: fmt 1,5x,f3.0,3x,f7.5,2x,f7.5,4x,f3.0,3x,f7.5,2x,f7.5,z
246: wrt 706.1,r5,1[r5],T[r5],r7,O[r7],T[r7]
247: fmt 2,4x,f3.0,3x,f7.5,2x,f7.5
248: wrt 706.2,r9,Q[r9],T[r9];next I
*25225

Time Interval Between Calculations

249: T/(2*X[1]*X[1])+r1;gto 264
*3516

Call for Soil Heat Flux Subroutine

250: c11 'QHEAT'
*11251
Calculation of Nodal Temperatures and Lower Boundary

251: for I=1 to N-1; (F[I]/r1-C[I-1]-2*C[I]-C[I+1])*A[I]+r6
253: (F[I]/r1-D[I-1]-2*D[I]-D[I+1])*A[I]+r7
256: r1/C[1]*R[0]+R[0]
258: r1/F[1]*S[0]+S[0]
259: A[0]—B[0]+S[51]

Reassignment of Nodal Temperatures for Succeeding Iteration

261: for I=0 to N; R[I]+A[I]+S[I]=B[I]; next I
262: A[0]—B[0]+S[51]

Test for Printout Time

263: if '4[4]<P; gto 306

Printout of Pertinent Data

264: r10+1—r10
265: int(r10/2)·r11; r11*2—r12
266: fmt 1,4/; "TEMPERATURE PROFILE AT ",fz2.0,fz2.0," HOURS"
267: wrt 706.1,4[I],4[H[I]
268: fmt 2,4/; "SURFACE TEMPERATURE DIFFERENCE =",f6.3
269: wrt 706.2,5[S[i]
270: fmt 4,4/; "SOIL HEAT FLUX SITE A = ",f12.9, " SITE B = ",f12.9
271: wrt 706.4,A,B
272: fmt 1,2,4,3 "DEPTH TEMP A TEMP B"
273: wrt 706.1
274: int(N/3)+1—r2
275: r2*3—r3
276: r3—r3+1—r4
277: for I=1 to N-1; I=r5+1—r6
278: if r4>=1; r5+1—r6
279: r6+r2—r7—r3
280: if r4>=2; r3+1—r8
281: r8—r7
282: fmt 1,5x,f3.0,4x,f6.3,3x,f6.3,4x,f3.0,4x,f6.3,3x,f6.3,
283: wrt 706.1,5,A[r5],5[A[r7],5[r7]
284: fmt 2,4x,f3.0,4x,f6.3,3x,f6.3,
285: wrt 706.2,9,A[r9],9[A[r9],9[r9]; next I

Procedural Step (Reset)

286: 0—4[4]

*3874
Call for Plot Routine

287: if r121r10:goto 306
293: c11 "Plots"
*19455

Plot of Theoretical and Experimental Soil Temperatures

289: for I=0 to 50; c11 'plotA' (A[I],L[I]); next I
290: if H[I]=0:24+H[I]
291: for J=1 to 5; c11 'realA' (Z[J,H[I]/2,1],N[J]); next J
292: for I=0 to 50; c11 'plotI' (B[I],L[I]); next I
293: for J=1 to 5; c11 'realI' (Z[J,H[I]/2,2],N[J]); next J
294: fmt 1,/,9x,*DEPTH TEMP ",wrt 706.1
295: fmt 4,/,"SITE A",z;wrt 706.4
296: fmt 2,3x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2,z
297: fmt 3,4x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2
298: wrt 706.

Printout of Experimental Soil Temperatures for Profile A

294: fmt 1,/,9x,*DEPTH TEMP ",wrt 706.1
295: fmt 4,/,"SITE A",z;wrt 706.4
296: fmt 2,3x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2,z
297: fmt 3,4x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2
298: wrt 706.

Printout of Experimental Soil Temperatures for Profile B

300: fmt 5,/,"SITE B",z;wrt 706.5
301: fmt 2,3x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2,z
302: fmt 3,4x,f3.0,3x,f5.2,4x,f3.0,3x,f5.2
303: wrt 706.3,N[4],Z[4,H[I]/2,2],N[5],Z[5,H[I]/2,2]
304: wrt 706.3,N[4],Z[4,H[I]/2,2],N[5],Z[5,H[I]/2,2]
*24421

Test if Calculation Has Run Desired Time

305: if H[1]=24:00=H[1]
306: if H[1]<H[2];goto 313
307: if H[1]<H[2];goto 310
308: if Y[0]<Y[1];goto 310
309: gto 324
*711
Test if Temperature Difference is to be Calculated

311: if H[3]/(rl3+20)>=1;gto 313
312: qto 317
313: rl3+1+rl3
*30522

Calculation of Surface and 5 cm Temperature Difference

314: A(0)-A(0)*U[M[3]/20]
315: v[5]-v[5]*v[3]/20
*30792

Calculation of New Time

316: M[3]/60+O[3]/20
319: if H[1]<60;gto 323
320: H[1]-60+H[1]
322: H[1]-24+H[1];Y[0]+1+Y[0]
323: gto 141
324: gto
*28526

Plotting of Surface and 5 cm Temperature Differences

325: call 'Diff plot'
326: for l=1 to 72;call 'plot3'(0[I],U[I]);next I
327: for l=1 to 72;call 'plot3'(0[I],v[I]);next I
328: end
*29769
Appendix B

DETERMINATION OF LINEAR REGRESSION FOR NET SOLAR RADIATION AS A FUNCTION OF SOLAR RADIATION

One of the model input parameters is surface soil heat flux. In this study net radiation was used for this input. However, if this technique was to be used on a routine basis, net radiation would not be readily available. Solar radiation would be available since it is recorded on a routine basis at a number of weather stations. Thus, if one can determine net radiation from solar radiation, a model input would be readily available to the resource specialist.

The objective of the project described here is to fit a linear regression equation of net solar radiation as a function of solar radiation for this location. It is therefore necessary to define these two quantities. Solar radiation ($R_S$) is all the radiation from the sun that is reaching the earth at the point measured. Net solar radiation ($R_n$) is the incident solar radiation less all radiation that is reflected (short-wave albedo reflection) or re-emitted (long-wave blackbody re-emission) from the earth. $R_S$ will always be positive or zero, being positive during the day and zero at night. $R_n$, however, can be positive, zero or negative. It was found that $R_n$ was negative from shortly before sunset to shortly after sunrise.

The information gained in this experiment can be used in the soil model program to determine $R_n$ from $R_S$. Robert H. Shaw conducted a similar investigation in late June through November of 1954 at
Iowa State University at Ames, Iowa for both clear and cloudy days, and calculated regressions for both. He found that both the slope and intercept were lower on cloudy days than on clear days and that the correlation was also slightly lower on cloudy days. His methodology, discussed briefly below, was very similar to that employed in this experiment.

**Experimental Procedures**

The data for this experiment were recorded at the Agricultural Engineering Farm at South Dakota State University. The solar radiation flux was recorded with an Eppley pyrheliometer, and the net radiation flux was recorded with a Swissteco net radiometer. Both of these instruments were placed about 1.4 meters from the ground to minimize ground effects and were each connected to potentiometers which registered the data. The data was taken at least once hourly with special attention to the daylight hours. The pyrheliometer and net radiometer were never more than two meters apart. On the first three days of the experiment (5-7 August), the data were taken on a barley cover crop (coverage height approximately 75 cm.) and then on trash-free bare soil the last two days.

This procedure was similar to that used by Shaw in 1954, but there were a few differences. Shaw placed his pyrheliometer on the roof of the Agricultural Engineering Building at Iowa State University and his radiometer in a grassy field three km. away. He also experimented over a period of months on both cloudy and clear days, and on
clipped and unclipped grass. By placing his instruments so far apart, Shaw introduced a potential for error because cloud cover would not always be the same over the roof and the field.

Analysis Techniques

\[ R_n \text{ as a function of } R_s \text{ was plotted as a linear regression for each day tested. } R_n \text{ and } R_s \text{ were always plotted during the daylight hours and appear in Figures 1 and 2.} \]

Linear equations for \( R_n \) as a function of \( R_s \) is desired. The results would be of the form

\[ R_n = m R_s + b, \]

where \( m \) is the slope of the line and \( b \) is the \( R_n \) intercept in cal/cm\(^2\)/min. Table 1 shows the calculated results.

Shaw's results, over a period of months, has \( R_n = 0.87 R_s - 0.06 \) cal/cm\(^2\)/min. with a correlation coefficient of 0.98 for clear days and \( R_n = 0.75 R_s - 0.02 \) cal/cm\(^2\)/min. with a correlation coefficient of 0.97 under cloudy skies.
### Table 1
#### Linear Equations and Correlation Coefficients

<table>
<thead>
<tr>
<th>Date</th>
<th>Equation</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1978)</td>
<td>( R_n = mR_s + b )</td>
<td></td>
</tr>
<tr>
<td>5 August</td>
<td>( R_n = 0.823 R_s - 0.090 )</td>
<td>0.998</td>
</tr>
<tr>
<td>6 August</td>
<td>( R_n = 0.771 R_s - 0.077 )</td>
<td>0.981</td>
</tr>
<tr>
<td>7 August</td>
<td>( R_n = 0.764 R_s - 0.063 )</td>
<td>0.998</td>
</tr>
<tr>
<td>8 August</td>
<td>( R_n = 0.778 R_s - 0.103 )</td>
<td>0.995</td>
</tr>
<tr>
<td>9 August</td>
<td>( R_n = 0.716 R_s - 0.063 )</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Illustrations of the functional dependence of \( R_s \) and \( R_n \) are shown in Figures 1 and 2. The linear relationship between \( R_n \) regressed on \( R_s \) is shown in Figure 3.
Figure 1. Net Radiation vs. Time for 5 August, 1978.
Figure 2. Solar Radiation vs. Time for 5 August, 1978
Figure 3. Net Radiation vs. Solar Radiation for 5 August, 1978.

Net Radiation (Langley/Minute)

Solar Radiation (Langley/Minute)

\[ R_n = 0.823 \times R_s - 0.090 \text{ LY/Min} \]

\[ r = 0.998 \]