Correlation of Soil Moisture with Soil Surface Temperature Obtained by Thermography

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CORRELATION OF SOIL MOISTURE WITH SOIL SURFACE TEMPERATURE OBTAINED BY THERMOGRAPHY

BY

DANIEL PAUL ROESLER

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science Major in Engineering South Dakota State University 1984
CORRELATION OF SOIL MOISTURE WITH SOIL SURFACE
TEMPERATURE OBTAINED BY THERMOGRAPHY

This thesis is approved as a creditable and independent
investigation by a candidate for the degree, Master of Science,
and is acceptable as meeting the thesis requirements for this
degree, but without implying that the conclusions reached by
the candidate are necessarily the conclusions of the major
department.

Dr. Jerald A. Tunheim
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ACKNOWLEDGEMENTS

The author wishes to thank Dr. Jerald A. Tunheim for his help and guidance during this study. The author also wishes to thank the whole SDSU physics department for their support and friendship and for the smooth transition from Augustana to SDSU.

The work upon which this thesis is based was supported in part by funds provided by the Office of Water Research and Technology Project No. 14-34-0001-1263.
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INTRODUCTION

Recent developments in satellite technology have made thermal infrared imagery readily available to resource scientists for research purposes. Since thermal infrared radiation emitted by any surface depends on the temperature of that surface, any factors which affect the temperature of the earth's surface can in theory be measured by satellite. Soil moisture is such a factor. The larger heat capacity and thermal conductivity of moisture laden soils together with evaporation from moist surfaces will affect the surface temperature to a considerable degree. These effects cause a bare or canopy covered wet soil area to be cooler during the day than a corresponding dry soil area. If the relationship between soil moisture and surface temperature can be quantified, satellite thermal infrared imagery should allow the monitoring of soil moisture over large areas of the earth.

Since the atmosphere attenuates the radiation emitted by the earth, the actual surface temperature is difficult to measure accurately from satellite heights. However, the temperature difference between two locations on the earth can be measured to a better approximation if the atmosphere attenuates the radiation equally over both surfaces. Thus, the temperature difference between two locations is more accessible than the actual surface temperature of each surface. If surface temperature difference does correlate to soil moisture difference, then relative moisture
content can be calculated. Consequently, if a reference moisture is available for one location, the moisture content of the soil at the other location can be obtained.

The difference in surface temperature may depend on many variables other than soil moisture. Soil type, climatic conditions, plant canopies, topography and other variables can alter the surface temperatures. By controlling these variables for two soil plots and varying soil moisture, a correlation between moisture difference and surface temperature difference can be investigated and analyzed.

A finite difference heat flow model has been developed which will predict soil temperatures as a function of time throughout the soil profile from the surface to a depth of 50 cm (Meyer, 1972). Experimental tests of this model have shown it to qualitatively agree with field conditions indicating that its equations reasonably simulate actual physical processes that occur in the soil (Beutler, 1980). Many parameters associated with soil and solar radiation as well as plant canopy characteristics have been incorporated into the model. A series of calculations utilizing the model have predicted a quadratic relationship between soil moisture difference and surface temperature difference (Ness, 1982).

The primary goal of this research is to explore the experimental relationship between these variables utilizing soil plots of differing soil moisture profiles and comparing the results with model predictions.
OBJECTIVES

Specific objectives of this project pertaining to the prediction of soil moisture by acquired surface temperature differences were:

1. To experimentally collect surface temperature data with varying soil moisture profiles along with other pertinent variables controlled or measured.
2. To statistically investigate the relationship between surface temperature differences and soil moisture profiles.
3. To compare results of this analysis to those predicted by the computer models.
BACKGROUND LITERATURE

The first evaluation of satellite thermography as an indicator of soil moisture was performed by Moore, et al (1975). This evaluation of SKYLAB data showed a positive correlation between soil moisture and thermal emittance. It was concluded that thermal data from satellite altitudes had good potential for use in monitoring soil moisture and for irrigation scheduling. Schmugge (1978) and Reginato (1976) have shown agreement between thermal infrared temperatures and those measured by thermocouples in contact with the soil. A study by Tunheim (1977) found significant correlation between aircraft thermal infrared imagery and soil temperature in fields caused by near-surface water tables associated with saline seeps. A considerable number of studies resulted from the Heat Capacity Mapping Mission (HCMM) launched by NASA in April 1978. The satellite carried a radiometer sensitive to the 10.5-12.5 micron range. A number of these correlation studies of the relationship between thermal infrared imagery and surface characteristics have been summarized in a final report. A general conclusion of this report is that more work needs to be done before thermal infrared measurements may be used for any operational use to measure soil moisture (Short, 1982).

By constructing models to simulate the interaction of solar radiation with the surface of the earth, surface temperature characteristics can be understood and related to thermal emittance. Such models are generally complex and intricate. These models use
soil physics and complex mathematics to simulate solar heat flux into the surface of soils.

Meyer (1972) developed a finite difference heat transfer model to simulate the flow of heat through a soil medium as a function of soil moisture. This model would produce a temperature profile for a medium of soil at a constant moisture content in a diurnal period.

Price (1980) developed a model to use remotely sensed thermal data to infer 24 hour mean evaporation rates and the diurnal heat capacity as controlled by soil moisture. This model calculates mean surface temperature by establishing relationships for absorbed and emitted longwave and shortwave radiation. Albedo and surface temperatures are obtained by satellite measurements, while other variables are obtained from meteorological data and knowledge of surface characteristics.

Camillo and Schmugge (1981) have developed a very intricate model simulating moisture and heat flow through soils. This model gives wetness and temperature profiles as a function of time. Simulations of heat and moisture flow qualitatively agree with actual field conditions. This qualitative agreement between simulated and measured profiles is an indication that the model's equations are reasonably accurate representation of physical processes involved.

An empirical model has been developed by Idso (1975) that empirically calculates soil moisture from the maximum diurnal
temperature change for bare soil surface. Zhang Ren-hua (1980) has modified this model utilizing energy budget equations for the ground surface. This model attempts to simulate surface interaction between soil and solar flux and from this empirically predict soil moisture. Also using empirical methods, Heilman and Moore (1981) have shown surface temperatures for soils can be estimated from remote measurements of canopy temperatures if minimum air temperature and percent canopy cover are known. They also show good correlations between soil moisture and surface temperature at the 0 to 4 cm depth for barley.

Predicting soil moisture from surface temperature obtained by thermography lead to the modification of Meyer's finite difference model. Tunheim (1977) modified this model to produce temperature profiles for an irrigated and dry plot of soil with identical homogeneous soil profiles. From these two temperature profiles, differences could be obtained for a 24 hour period. This was done to relate soil moisture difference to surface temperature difference. Beutler (1980) modified the program to simulate any soil profile types for the two plots considered. He has also shown that the computer model correctly simulates the functional dependency of surface temperature difference throughout the diurnal cycle for bare soils. Ness (1982) further modified the model to accept crop parameters. He found the computer model predicts a quadratic relationship between moisture difference and surface temperature difference for time independent soil moistures. The
advantage of this model relating soil moisture difference to surface temperature difference bypasses many calibration problems faced by models depending on accurate surface temperatures, to predict soil moisture. With the availability of daily thermal infrared data for areas of interest the soil moisture difference should be available from the relationships predicted by this model for surface temperature differences.
GENERAL RADIATION THEORY

Thermal infrared radiation is a small part of the total electromagnetic spectrum. Understanding electromagnetic radiation (EMR) in general is essential in understanding thermal infrared radiation. Electromagnetic radiation is a dynamic form of energy manifested only by its interaction with matter. The propagation of EMR is given by

\[ v = f \lambda \]

where \( f \) is frequency, \( \lambda \) is wavelength, and \( v \) is the wave propagation velocity which is 300 million meters per second in free space.

Heat energy is the kinetic energy of the random motion of particles of matter, and the quantity of heat energy present in a substance is measured by its temperature. The molecules excited by the heat energy move to higher energy states and jump back to lower energy states with the emission of EMR. Thus the heat energy is changed to thermal infrared radiation.

An ideal emitter, called a blackbody, is one which transforms heat energy into radiant energy with the maximum rate permitted by thermodynamic laws. This establishes the maximum radiation rate possible when emission is due to the conversion of heat energy. Any real material at the same temperature cannot emit thermal radiation at a rate in excess of that of a blackbody. The reverse process of absorption must likewise be maximum so that a blackbody must absorb and convert all incident radiation energy into heat energy regardless of the wavelength of radiation. Planck has
derived a formula for the spectral radiation which a blackbody should have based upon theoretical thermodynamic reasoning. Planck's formula for blackbody spectral radiation is given by

$$R(\lambda) = \frac{B}{A^\lambda} \frac{\lambda^5}{(e^{\frac{\lambda}{T}} - 1)^4}$$

where $A$ is $3.75 \times 10^{-16}$ watt meters$^2$, $B$ is $1.44 \times 10^{-2}$ meter$^2$°K and $T$ is the absolute temperature in degrees Kelvin.

The radiation wavelength spectrum emitted depends on the temperature of the blackbody. For very large or very small wavelengths the amount of radiation emitted is small. For some wavelength between these extremes, the radiation reaches a maximum value depending upon the temperature of the blackbody. That wavelength for which the spectral radiation has its maximum value is given by the Wien Displacement Law

$$\lambda = \frac{2.893 \times 10^{-3} °K}{T}$$

where $\lambda$ is the wavelength of maximum radiation and $T$ is the temperature in degrees Kelvin.

The radiation of a blackbody for the spectral band of all wavelengths is given by the Stefan-Boltzmann law

$$R = \sigma T^4$$

where $\sigma$ is the Stefan-Boltzmann constant and $T$ is the absolute temperature of the body. Any body of material at a specific temperature emits radiation in accordance with its own characteristics. It is convenient and customary to express the potential to emit
radiation due to thermal energy conversion as the ratio of spectral radiation of a material to the spectral radiation of a blackbody at the same temperature. This ratio is the spectral emissivity of that material and is given by

\[ \varepsilon = \frac{R_m}{R_b} \]

where \( R_m \) is the radiation emitted by the material and \( R_b \) is the radiation emitted by a blackbody at the same temperature. The spectral emissivity is nearly independent of temperature for most common materials in the terrestrial environment. Significant changes in spectral emissivity can be expected whenever the material undergoes a change of state such as melting, vaporization, oxidation or any other change which alters the fundamental arrangement of the atomic and molecular components.

Kirchhoff's law states that under conditions of thermal equilibrium, the spectral emissivity of a material must be equal to the spectral absorption. One can expect this law to apply to most terrestrial conditions. It is common to determine the spectral emissivity of a substance by measuring its spectral absorption.

The infrared radiant emission from any surface is given by

\[ R = \varepsilon \sigma T^4 \]

where \( \varepsilon \) is emissivity, \( \sigma \) is the Stefan-Boltzmann constant and \( T \) is the absolute temperature of the surface. A lower emissivity gives the appearance of a cooler temperature since temperature differences are detected as emitted differences in \( R \). Since terrestrial materials
have a spectral emissivity between 0.85 and 0.95 (Suits, 1983), the remotely sensed temperature from thermal radiation measurements will generally be lower than the real surface temperature. However, since the temperatures measured at two different locations will be lower than the actual surface temperatures by approximately the same amount, the difference in temperatures between two locations can be measured to good approximation.

The atmosphere also absorbs radiation emitted by the earth's surface and emits its own radiation. This effect leads to an attenuation of thermal infrared radiation as it travels through the atmosphere, and causes a remotely sensed temperature of the earth's surface to be lower than the actual surface temperature. However, the difference in temperature between two locations can still be accurately measured if the absorption by the atmosphere is the same over both locations.
FINITE DIFFERENCE MODEL

The finite difference heat-flow model used in this project was originally developed by Meyer (1972). The model's primary objective was to simulate the transfer of heat through a homogeneous medium. Modifications had to be made to make it apply to a composite material such as soil. If each constituent of soil is known it is possible to calculate the average thermal conductivity of the soil. DeVries (1963) has developed an equation that will calculate an average thermal conductivity for all constituents in any soil type. Some of these constituents in soil are water, air, quartz and other soil particles. Beutler (1980) modified the model by adding the DeVries calculations making the model adaptable to any soil type.

The volumetric heat capacity for each of these constituents is also an important value necessary to calculate the changes in soil temperature as a function of time. It is determined by multiplying the density of each constituent by its specific heat. This produces the heat capacity per unit volume for any medium. Knowing the thermal conductivity K, and the temperature gradient dT/dx for the soil medium, one can calculate the soil heat flux S from the relationship

\[ S = K \frac{dT}{dx}, \]

where S has units of watts per meter\(^2\) or calorie per sec meter\(^2\). This relationship makes possible the calculation of the heat flow from the soil surface down to a predetermined depth. A volume of
soil will only increase in temperature if the energy leaving the volume is less than the energy flowing into the volume. The energy needed to cause this increase will depend on the volumetric heat capacity.

The total energy transferred into a soil element must equal the sum of the energy transferred out of the element and the amount of energy left in the soil. This is the conservation of energy law. By using this law in a finite difference form and the volumetric heat capacity of the soil element, the temperature of finite increments of soil can be calculated.

The model employs a finite element calculus method such that small elements, \( \Delta x/\Delta t \), can be substituted for \( dx/dt \). The increments used are one centimeter in thickness. Since fifty centimeters is considered to be a boundary of constant temperature for a diurnal cycle, the model considers fifty such elements. The temperature of each increment is found by first solving, for the temperature of the surface element, and then for each successive increment down to a fifty centimeter depth.

The solar radiation absorbed by the soil surface is the primary factor in determining the temperature profile of the soil. Since solar radiation is a function of the time of year and latitude, the maximum amount of solar radiation for any given day of the year can be calculated for an ideal day at a given location. Net radiation is a linear function of solar radiation and can also be obtainable.
for any given day (Beutler, 1980). During the night time hours, radiation is emitted by the soil surface and represents a negative heat flow. The total radiation can then be represented by a rectified sinusoidal function with a maximum at solar noon.

The model has a time interval of 1 second which allows good accuracy for an acceptable amount of computer calculation time. By knowing initial boundary conditions such as initial soil temperature profile, soil constituent profile, soil moisture profile, incoming solar radiation and soil surface characteristics the model can calculate the rate of heat flow through the soil, the energy stored in each increment of soil and the temperature for each increment. A schematic representation of the model is shown in Figure 1.

By performing a calculation for a dry plot and another for an irrigated plot of soil, it is possible to calculate surface temperature difference between the two plots throughout the diurnal cycle. By keeping the dry plot at a constant soil moisture and varying the moisture content of the irrigated plot, several calculations can be performed and a graph can be constructed portraying soil moisture as a function of surface temperature difference. This graph displays a predicted quadratic relationship between moisture difference and surface temperature difference as shown in Figure 2 (Ness, 1980).

The model can predict the relationship between moisture difference and surface temperature difference for any soil type or
Figure 1. Schematic representation of the finite-difference model in its present form.
Figure 2. Quadratic relationship between surface temperature difference and soil moisture difference as predicted by finite-difference heat flow model.
plant canopy. If these resulting relationships can be experimentally verified and indexed for soil types and crop canopies, soil moisture difference for vast areas should be obtainable using surface temperature differences measured by satellite thermal infrared detectors.
EXPERIMENTAL METHODS

The thermal infrared detector used for this project was a Telatemp Model AG-42 Infrared Thermometer. This detector has a spectral range of 8 to 14 microns and an accuracy of +.5°C. This thermal infrared thermometer (TIT) was calibrated so that its output in millivolts was identical to apparent temperature in degrees Celsius.

The detector was mounted at a height of three meters as shown in Figure 3. The detector has a twenty degree angle of observation. The target area covered by the detector was approximately one square meter. This method of detection is somewhat analogous to a satellite system which would be utilized to monitor soil moisture.

Emissivity calculations were made on the surfaces used in previous research (Beutler, 1980). The emissivity values were close to 1.0 so the detector was adjusted accordingly. Thus, for this project, soil and plant canopies were treated as near perfect blackbody radiators.

Data was collected during the three summers of 1981-1983. Data was recorded throughout the diurnal cycle on an hourly basis. Periods usually lasted from twenty-four to seventy-two hours. The data recorded consisted of thermal infrared temperatures, thermocouple temperatures, wet and dry bulb air temperatures, solar and net radiometers and soil moisture at four depths. The thermocouples
Figure 3. Thermal infrared thermometer (TIT) as used in data collection.
were buried at surface, 5 cm, 10 cm, 25 cm, and 50 cm. This data was recorded for both a dry and irrigated plot of soil so that the temperature difference could be compared to soil moisture differences.

The soil moisture was monitored by taking soil samples every day at 1200 hours (Jackson, 1976) for depths of 1-15 cm, 15-30 cm, 30-50 cm, and a surface sample. The soil probe left a hole about one inch in diameter. The soil samples were taken outside of the square meter target area since the holes left by the probe would alter the value displayed by the TIT. Soil moisture by weight was acquired using the gravimetric method. The wet and dry plots were separated by a four foot deep plastic sheet to maintain a well defined moisture difference. The plots were approximately two meters wide and four meters long. Each plot would be wetted down and its counterpart left dry. The TIT would be set on the edge of each plot and rotated 180 degrees taking five readings. An average of the five readings was then used in the analysis.

During the data collection period, data for seventy-four diurnal cycles were recorded. These data were divided into their respective canopies. As many as six different plots were used simultaneously with three different canopies or different moisture contrasts. Canopies included spring wheat, oats, barley and bare surfaces. The canopies were planted with six inch row widths.
The data for this study were collected on a site at the South Dakota State University Agricultural Engineering Farm, near Brookings, South Dakota. The soil is classified, according to USDA standards as a silt loam (Beutler, 1980).
ANALYSIS TECHNIQUES

The statistical indicator used in determining significant correlation between soil moisture difference and surface temperature difference is called the correlation coefficient of multiple determination. This value, called $R^2$, is defined to be

$$R^2 = \frac{SSR}{SST}$$

where SSR is the regression sum of square error and SST is the total sum of square error. This quantity $R^2$, indicates what proportion of total variation in the response $Y$ is explained by the fitted model. $R^2 \times 100\%$ can be interpreted as the percentage of variation explained by the postulated model. To explain why this is true an investigation of linear regression must be made.

Let $(x_1, y_i), (i=1,2,\ldots,n)$, denote a random sample of data of size $n$. If additional samples were taken using exactly the same $x$ values, the $y$ values would be expected to vary. Thus the value $y_i$ in the ordered pair $(x_i, y_i)$ is a value of some random variable $Y_i$. $Y/x$ is defined as the random variable $Y$ corresponding to a fixed value of $x$. The term linear regression implies that the mean of $Y/x$ is linearly related to $x$ in the usual slope intercept form

$$\mu_{Y/x} = \alpha + \beta x$$

where $\mu_{Y/x}$ represents the mean response to several samples of fixed values and $\alpha$ and $\beta$ are parameters to be estimated from the sample data. Denoting the estimates of $\alpha$ and $\beta$ by $a$ and $b$ respectively,
the estimated response \( y \) is obtained from the sample regression line

\[ y = a + bx. \]

Figure 4 illustrates the difference between the true response and the estimated response. The response \( y_i \) can be written as

\[ Y_i = \mu_{Y/x} + E_i \]

where \( E_i \) is the difference between \( y_i \) and \( \mu_{Y/x} \). Each observation satisfies the relation

\[ y_i = \alpha + \beta x_i + e_i \]

where \( e_i \) is the value assumed by \( E_i \) and \( Y_i \) takes on the value of \( y_i \). Similarly, using the estimated regression line

\[ y = a + bx \]

each pair of observations satisfies the relation

\[ y_i = a + bx_i + e_i \]

where \( e_i \), the residual, denotes the difference between the sample response and the estimated regression equation. The difference between \( e_i \) and \( e_i \) is also shown in Figure 4.

Parameters \( a \) and \( b \), the estimates of \( \alpha \) and \( \beta \), will be constructed such that the sum of the squares of the residuals is a minimum. The residual sum of squares is often called the sum of square errors about the regression line and is denoted by \( SSE \). The minimization method of estimating parameters \( a \) and \( b \) is called the method of least squares. Solving for \( e_i \), squaring, \( SSE \) appears as

\[ SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2. \]
Figure 4. Comparing unknown hypothetical line with sample estimated line and comparing error $\varepsilon_i$ with residual $e_i$. 

$y = a + bx$

$\mu_y/x = \alpha + \beta x$
Taking the partial derivative of SSE with respect to \(a\) and \(b\) yields

\[
\frac{\partial (SSE)}{\partial a} = -2 \sum_{i=1}^{n} (y_i - a - bx_i)
\]

and

\[
\frac{\partial (SSE)}{\partial b} = -2 \sum_{i=1}^{n} (y_i - a - bx_i)x_i
\]

Setting the partial derivatives equal to zero and rearranging the terms, a set of equations called normal equations can be obtained.

These two equations,

\[
na + b\sum x_i = \sum y_i
\]

\[
a\sum x_i^2 + b\sum x_i^2 = \sum x_i y_i
\]

yield

\[
b = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2}
\]

and

\[
a = \bar{y} - bx.
\]

Thus by knowing the \(x_i\) and \(y_i\) values it is now possible to calculate an estimated regression line with slope \(b\) and intercept \(a\).

In order to construct the \(R^2\) value, it is necessary to partition the sum of square errors, SSE, into various quantities.

Since

\[
SSE = \sum_{i=1}^{n} (y_i - a - bx_i)^2
\]

and

\[
a = \bar{y} - bx
\]

then it can be shown (Walpole, 1978) that
SSSE = \sum_{i=1}^{n} (y_i - \overline{y})^2 - b \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) .

By rearranging terms and substituting for b, SSE can be split into two quantities

SST = \sum_{i=1}^{n} (y_i - \overline{y})^2

and

SSR = b \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})

such that

SSE = SST - SSR

where SST indicates the total variation about the hypothetical regression equation

\mu_{Y/X} = \alpha + \beta x

and SSR, called the regression sum of square error indicates how much of this variation is due to the regression parameters. Thus SSE indicates the amount of variation in the response not explained by the estimated regression equation. Hence an \( R^2 \) value of one,

\[ R^2 = \frac{SSR}{SST} = 1, \]

implies that the SSR value equals the SST value constituting zero error. An \( R^2 \) value of zero indicates SSR = 0 leaving

SSE = SST

indicating that no correlation exists between the independent variable \( x_i \) and the response \( y_i \).

In constructing multiple linear regression relationships, the same procedures are used as in simple linear regression. The
purpose of multiple linear regression relationships is to find other variables that can account for the response. For the case of \( K \) independent variables, \( x_1, x_2, \ldots, x_k \), the mean of \( Y/x_1, x_2, \ldots, x_k \), is given by the multiple linear regression model

\[
\mu_{Y/x_1, x_2, \ldots, x_k}
\]

and the estimated response is obtained from the sample regression equation

\[
y_i = b_0 + b_1 x_1 + \ldots + b_k x_k
\]

where each coefficient \( \beta \) is estimated by \( b_i \), \( (i=1\ldots n) \), from \( n \) sample data using the method of least squares. In constructing the multiple linear regression equation, the error or residual term can again be added such that

\[
y_i = b_0 + b_1 x_1 + \ldots + b_k x_k + e_i
\]

where \( y_i \) is the response due to \( x_i \) and \( e_i \) is the residual associated with the response \( y_i \). In using the least squares method to arrive at the estimates \( b_0, b_1, \ldots, b_k \), the equation

\[
SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_{1i} - \ldots - b_k x_{ki})^2
\]

can be minimized by differentiating SSE with respect to \( b_0, b_1, \ldots, b_k \), and generating \( K + 1 \) normal equations. Each coefficient \( b_k \) can be solved for as in the simple linear regression. However the more variables \( x_k \) that are added to the estimated regression model, the more complex it is to solve the normal equations simultaneously.
By using matrices, it is possible for high speed computers to quickly give estimates of the regression coefficients. Once the coefficients are solved, SSE can again be partitioned into various quantities such that

$$\text{SSE} = \text{SST} - \text{SSR}$$

where

$$\text{SST} = \sum y_i^2,$$

$$\text{SSR} = \sum_{j=0}^{k} b_j g_j - \left( \frac{\sum y_i}{n} \right)^2,$$

$$j = (0, 1, \ldots, k) \text{ and } i = (1, 2, \ldots, n),$$

$$g = \begin{bmatrix}
    g_0 = \sum_{i=1}^{n} y_i \\
    g_1 = \sum_{i=1}^{n} x_{i1} y_i \\
    \vdots \\
    g_k = \sum_{i=1}^{n} x_{i1} y_i
\end{bmatrix},$$

$$b = \begin{bmatrix}
    b_0 \\
    b_1 \\
    \vdots \\
    b_k
\end{bmatrix},$$

such that

$$\text{SSE} = \sum_{i=1}^{n} y_i^2 - b_0 \sum_{i=1}^{n} y_i - b_1 \sum_{i=1}^{n} x_{i1} y_i - \ldots - b_k \sum_{i=1}^{n} x_{ik} y_i.$$
and

\[ \frac{SSR}{SST} = R^2, \]

called the multiple correlation coefficient, indicates the proportion of total variation in the response, \( y_i \), that is explained by the fitted model. In the case of multiple linear regression

\[ R^2 = \sum_{j=0}^{k} \sum_{i=1}^{n} b_j x_i - \frac{\left( \sum_{i=1}^{n} y_i \right)^2}{n} \]

Thus, \( R^2 \) can indicate the adequacy of a postulated model given a set of sample data, \( x_{ki}, y_i \). In this project \( R^2 \) was used to indicate correlations between soil moisture difference, the response \( y_i \), and soil surface temperature difference which is the independent variable \( x_{ki} \). A high \( R^2 \) value indicates the proportion of variation explained by the independent variables \( x_k \) is high and that the model is a good predictor of the response with the given sample data. However, \( R^2 \) values can not give an absolute indication as to the content of the postulated model since there may be other independent variables that could be used in the model. It is also possible that some of the variables in the postulated model may not be necessary.

The primary objective of this project was to investigate the relationship between surface temperature difference and soil moisture difference. Linear regression and multiple linear regression analysis were the primary methods used to obtain correlations existing between the two. Simple linear regression was
first used to find the direct relationship between these two variables. Then multiple linear regression was used to check the quadratic relationships between the same variables. Other variables were added in the multiple regression calculations to investigate possible atmospheric effects on soil moisture difference predictions. All statistical models were constructed such that soil moisture was the dependent variable or the predicted value. The independent variables used to predict soil moisture difference were surface temperature difference, the square of surface temperature difference, relative humidity, pressure deficit, and air temperature.

Statistical Analysis System (SAS, 1982) a statistical package available on the IBM 370-3031 at the SDSU campus, was employed to run statistical analysis for this project. The procedures used were Proc Rsquare, Proc Stepwise and Proc Reg. Each of these procedures would calculate statistical indicators for the various constructed models. The indicator used for this project was the Rsquare value. This value indicates the amount of data predicted by the model. If the $R^2$ value is .50 then half of the actual values are predicted by the model using sample data.

The first process in the analysis consisted of constructing the statistical model. The moisture data available consisted of average values for the depths of 0-15 cm, 15-30 cm, 30-50 cm and a surface sample. The models constructed were used to predict
the moisture at these various depths as well as certain average depths. Proc Rsquare was used to obtain the $R^2$ value for each model. The general format for each of these models appeared in the form: Moisture Difference=Surface Temperature Difference + Other Variables. When the best model was found Proc Stepwise was employed to indicate the Rsquare effect each variable had on the model thus indicating the relative importance of each variable. After removing any unimportant variables final analysis was performed using Proc Reg. This procedure will plot the predicted values as well as the actual values for visual model inspection. The final model indicates the best correlation between soil moisture difference and surface temperature difference. Since data were collected for various crop canopies; spring wheat, barley, oats and bare dirt, statistical analysis was performed on each crop type.
RESULTS

The adequacy of various postulated models was investigated using SAS for regression calculations and the $R^2$ value as the statistical indicator. The analysis was divided into three categories consisting of bare soil, small grain canopy and a canopy of specific grain type. The postulated models were constructed such that the response is soil moisture difference at various depths and the predictors were the various independent variables such as surface temperature difference, air temperature and other climatic conditions.

Soil moisture samples were taken one hour before solar noon each day to obtain a moisture profile to a depth of 50 cm. Soil samples were collected at the surface, 1-15 cm, 15-30 cm, and 30-50 cm. The resulting values of soil moisture by weight were denoted as $S_1$, $S_2$, $S_3$, and $S_4$ respectively. The moisture content of each sample was determined experimentally by weighing the sample, drying the sample by microwave oven then weighing the sample again. The moisture to soil ratio is then found by dividing the difference between original weight and dried weight by the dried weight.

The soil moisture difference was obtained by calculating the difference between moisture to soil ratios for both wet and dry plots. The soil moisture values for samples taken from dry plots were denoted as $SD_1$, $SD_2$, $SD_3$, and $SD_4$ for $S_1$, $S_2$, $S_3$ and
S4 respectively and the values for irrigated plots are denoted as SW1, SW2, SW3, and SW4 respectively.

The difference in moisture for each sample depth was obtained by calculating the difference between wet and dry samples of the same depths. These various moisture values are denoted as DM1=SW1-SD1, DM2=SW2-SD2, DM3=SW3-SD3 and DM4=SW4-SD4. Since soil moisture at one particular depth may not have the best possible correlation with surface temperature difference, averages of the moisture difference values at different depths were also constructed. The average difference in moisture for DM1 and DM2 was denoted as

$$ADM_{12} = (DM_1 + DM_2)/2,$$

and the average between DM2 and DM3 was denoted as

$$ADM_{23} = (DM_2 + DM_3)/2.$$

A weighted average moisture difference with DM2 given twice the weight as DM1 and DM3 was also constructed by

$$WAM = (ADM_{12} + ADM_{23})/2.$$

Three more moisture difference values were constructed to consider average moisture differences for the total profile between wet and dry plots. The first is calculated by the equation

$$DM = (DM_1 + DM_2 + DM_3 + DM_4)/4.$$ 

The second is calculated by

$$DMA = (DM_1 + DM_2 + DM_3)/3.$$ 

The third is calculated by

$$DMC = (DM_2 + DM_3 + DM_4)/3.$$
Thus, ten possible moisture difference values were used in constructing models correlating soil moisture difference to surface temperature difference.

Apparent surface temperatures recorded throughout the diurnal cycle were sometimes erratic due to occasional clouds passing overhead, bare spots of soil in an otherwise canopy-covered target area, and various other factors. By plotting this temperature data as a function of time, a smoothing curve was fitted to the original data by use of a cubic spline (Kimball, 1976) as shown in Figure 5. The maximum temperature difference between a wet and dry plot could be obtained from the fitted temperature curves by locating the maximum temperature difference from a plot as shown in Figure 6. This maximum temperature difference usually occurred at about 1400 hours (CDT). The temperature difference found in this manner was denoted as FT. The temperature difference from the original data was also used in the statistics and was denoted as RT.

Psychrometer readings were also taken throughout the diurnal cycle. These readings consisted of wet bulb and dry bulb temperatures. From these temperatures it was possible to calculate several climatic conditions such as vapor pressure, saturation pressure, relative humidity, and vapor pressure deficit. The dry bulb temperature reading indicates the outside air temperature and was denoted as DB. The pressure deficit was denoted by PD and the
Figure 5. Surface temperature for irrigated and nonirrigated wheat canopy plots fitted to a smoothing curve. The higher surface temperature values correspond to the nonirrigated plot.
Figure 6. Temperature difference curve for wet and dry wheat plots.
relative humidity as RH. These climatic conditions were used in
the statistical models and tested for significance.

The first models constructed for analysis were based on
bare soil plots. These plots were prepared by removing all vege-
tation and smoothing the surface. One plot was irrigated and the
other left dry. For this case the temperatures recorded were not
fitted to a smoothing curve and only original temperature dif-
ferences were used in the analysis.

The first statistical model constructed in the analysis
of bare soil plots yielded the equation

\[ DM_1 = 0.85RT - 2.93 \]

This particular model produced an \( R^2 \) value of 0.62 indicating that
62% of the moisture difference response is explained by surface
temperature difference. This highly significant correlation for
the surface sample is shown in Figure 7.

The second model constructed yielded the equation

\[ DM_2 = 0.09RT + 1.97. \]

This model produced an \( R^2 \) value equal to 0.07 indicating virtually
no correlation between moisture difference DM2 and surface tempera-
ture difference RT. The resultant graph is shown in Figure 8.

The third model constructed yielded the equation

\[ ADM_{12} = 0.47RT - 0.47 \]

where ADM_{12} is the average percent moisture difference between DM1
and DM2. This model produced an \( R^2 \) value equal to 0.63 indicating
a significant correlation. The resultant graph is shown in Figure 9.
Figure 7. Linear regression between surface temperature difference RT and soil moisture difference DMI for bare soil.
Figure 8. Linear regression between surface temperature difference RT and soil moisture difference DM2 for bare soil.
Figure 9. Linear regression between surface temperature difference RT and soil moisture difference ADM12 for bare soil.

\[ ADM12 = 0.47RT + 0.47 \]

\[ R^2 = 0.63 \]
Attempts to use other available variables did not yield a significant increase in $R^2$ values. These variables include the surface temperature difference squared, relative humidity and air temperature. Pressure deficit was not available. A summary of $R^2$ values produced by the various models is shown in Table 1.

The second part of the analysis is to construct models based on the data from canopy covered soils. These plots contained canopies of various small grains. All crop types were considered to be indistinguishable.

The first statistical model yielded the equation

$$DM1 = 0.31RT + 7.41$$

This model produced an $R^2$ value equal to .08 indicating virtually no correlation. The resultant graph is shown in Figure 10. No improvement in $R^2$ value was noted when fitted temperature differences were used.

The second model yielded the equation

$$DM2 = 0.49RT + 2.86$$

This model produced an $R^2$ value of .39 which is low but significant. The resultant graph is shown in Figure 11. Replacing the temperature difference with the fitted temperature difference yielded the equation

$$DM2 = 0.58FT + 3.11$$

producing an $R^2$ value equal to .38. This result is shown in Figure 12.
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<th>DM3</th>
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Figure 10. Linear regression between surface temperature difference RT and soil moisture difference DMI for small grain canopies.
Figure 11. Linear regression between surface temperature difference RT and soil moisture difference DM2 for small grain canopies.
Figure 12. Linear regression between surface temperature difference FT and soil moisture difference DM2 for small grain canopies.
The next models investigated used ADM12 and yielded the equation

\[ ADM12 = 0.40RT + 5.15 \]

and

\[ ADM12 = 0.40FT + 5.5 \]

produced \( R^2 \) values of .26 and .19 respectively. These results are shown in Figure 13 and 14.

The next models constructed yielded the equations

\[ ADM23 = 0.47RT + 2.61 \]

producing an \( R^2 \) value of .41 and

\[ ADM23 = 0.57RT + 2.76 \]

producing an \( R^2 \) value equal to .42. These two resultant graphs are shown in Figure 15 and 16. A table of all models with significant results are shown in Table 2. The air temperature, DB, was the only significant climatic variable.

The third and final part of the analysis investigates each canopy type. The canopy types consist of oats, spring wheat and barley. For each crop, models were constructed as done previously for bare and canopy covered soils. The resulting \( R^2 \) values appear in Tables 3, 4 and 5 for each crop.

Models constructed from oats canopies yielded the equations

\[ DM2 = 0.48RT + 2.91 \]

producing an \( R^2 \) value of .55 and

\[ DM2 = 0.60FT + 3.09 \]
Figure 13. Linear regression between surface temperature difference $RT$ and soil moisture difference $ADM12$ for small grain canopies.
Figure 14. Linear regression between surface temperature difference FT and soil moisture difference ADM12 for small grain canopies.

\[ \text{ADM12} = 0.40 \text{FT} + 5.5 \]

\[ R^2 = 0.19 \]
Figure 15. Linear regression between surface temperature difference RT and soil moisture difference ADM23 for small grain canopies.

\[ \text{ADM23} = 0.47 \text{RT} + 2.61 \]

\[ R^2 = 0.41 \]
Figure 16. Linear regression between surface temperature difference \( FT \) and soil moisture difference ADM23 for small grain canopies.

\[
\text{ADM23} = 0.57FT + 2.76
\]

\[
R^2 = 0.42
\]
### TABLE 2

\( R^2 \) for Canopy Models

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**TABLE 3**

$R^2$ for Oats Models

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TABLE 4

\( R^2 \) for Spring Wheat Models

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$DM2 = 0.48RT + 2.91$

$R^2 = 0.55$

Figure 17. Linear regression between surface temperature difference RT and soil moisture difference DM2 for oats.
Figure 18. Linear regression between surface temperature difference FT and soil moisture difference DM2 for oats.
producing an $R^2$ value of .59. Plots for oats are shown in Figures 17 and 18. Models constructed for spring wheat and barley did not yield highly significant $R^2$ values and are not plotted. Air temperature was again a significant climatic variable.

In summarizing the results it must be remembered that the $R^2$ value is an intuitive statistical indication of the adequacy of a particular model. No form of hypothesis testing or confidence estimation can be accomplished without making some assumptions about the distribution of the error $E_i$. However, assuming the error distribution is normal and random, a global test statistic can be employed (Schaeffer, 1982). This test can be expressed as

$$F_n^k(\alpha) = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

where $F$ denotes the $F$ distribution, $k$ is the number of independent variables, $n$ is the number of sample data, $\alpha$ is the level of significance and $R^2$ is the correlation coefficient squared. By knowing the number of sample data and number of independent variables, the $R^2$ value necessary for significance can be solved for a particular model. Employing this test, $R^2$ values will be calculated for the number of sample data and independent variables used for each set of sample data analyzed. $R^2$ values produced by various postulated models that are less then the $R^2$ values calculated a certain significance level will indicate the inadequacy of a particular model.
Bare soil data consisted of 20 sample data. The $R^2$ values determining significance for $\alpha = .05$ and $\alpha = .01$ were .19 and .31 respectively. The models correlating DM1 and ADM12 to RT resulted in $R^2$ values of .61 and .63 respectively. These two models show a level of significance much beyond the 0.01 level. A general conclusion that results from viewing Table 1 is that models correlating surface moisture differences to surface temperature difference have better correlations than subsurface moisture differences. Variables such as RT$^2$, DB, RH and PD produced no significant increase in $R^2$ values when added to the models.

Canopy covered soil data consisted of 53 sample data. The $R^2$ values determining significance for $\alpha = .05$ and $\alpha = .01$ were .07 and .12 respectively. The models correlating DM4 and DM to RT produced $R^2$ values of .45 and .46 respectively. These two models show a level of significance much beyond the 0.01 level. With the addition of DB to the models, a significant increase in $R^2$ was found. A general conclusion that results from viewing Table 2 is that models correlating subsurface moisture difference to surface temperature difference have the best results. Other climatic conditions such as PD and RH had essentially the same effect as DB.

Since DB is much easier to obtain it was used over the other climatic variables. The quadratic term RT$^2$ did not yield a significant increase in $R^2$ values.

Oats canopy data consisted of 23 sample data. The $R^2$ values determining significance for $\alpha = .05$ and $\alpha = .01$ were .17
and .26. All models correlating soil moisture difference to RT and FT were significant at the 0.01 levels except the model using DM1. A significant increase in the $R^2$ value occurred in several models with the addition of DB. DB was again used to represent the climatic variables since PD and RH had essentially the same effect. The addition of the quadratic term did not improve the $R^2$ value in most models except for the model using DM3. This model shows a significant improvement with the addition of $RT^2$ or $FT^2$. Subsurface moisture appears to correlate best to surface temperature difference.

Spring wheat and barley covered soils each had 15 sample data. The $R^2$ values determining significance for $\alpha = .05$ and $\alpha = .01$ were .26 and .39 respectively. It appears from Tables 4 and 5 that these two crops have a very poor correlation between soil moisture difference and surface temperature difference.
CONCLUSIONS

1. Significant correlations between moisture difference and surface temperature difference have been found.

2. Surface soil moisture samples correlate best to surface temperature differences for bare soils. Subsurface soil moisture difference samples correlate best to surface temperature differences for canopy covered soils.

3. Most correlations found were linear. Some models had significant increases in $R^2$ values with the addition of air temperature. In most cases the addition of the quadratic term was not significant in multiple linear regression analysis.
SUGGESTIONS FOR FURTHER STUDY

Determining the soil moisture for a plot of soil is very difficult. The amount of error introduced in soil moisture difference data should be known to a good approximation. An investigation should be made to analyze a moisture profile for an irrigated plot to find out how well one soil moisture sample represents the total soil moisture profile.

The method in determining moisture content by microwave oven should also be investigated. This method should be compared to other methods to see if variations occur for different situations.

A possible method for determining soil moisture difference would be to utilize the thermocouples buried at various depths. From thermocouple temperatures at various depths the thermal diffusivity can be obtained. The difference in thermal diffusivity between irrigated and nonirrigated plots of soil should yield a moisture difference value from the relation

$$D = \frac{K}{C_v}$$

where D is the thermal diffusivity, K is the thermo conductivity and $C_v$ is the volumetric heat capacity. By knowing the percentage of each constituent in the soil, the thermal conductivity and heat capacity can be calculated leaving the percentage of water to be solved for. Soil temperature data recorded for this project would substantiate a large data base for such an investigation.
The crop canopies used in this research were not always a good representation of an average crop canopy. The worst statistical results were produced by data recorded from sparse canopies with a heavy undergrowth of pigeon grass. Poor results were also produced by excessively thick and overly irrigated canopies. Good canopy covers with moderate irrigation produced the best results. The crops planted for this research were drilled in. Broadcasting the crop might be better since the thermal infrared thermometer could sometimes distinguish the area between six inch rows in sparsely planted canopies.
LITERATURE CITED


