A Study of Ferroresonant Conditions on Distribution Circuits Involving Cable-connected Power Transformers

Ronald H. Miller
A STUDY OF FEPRORESONANT CONDITIONS ON DISTRIBUTION CIRCUITS
ININVOLVING CABLE-CONNECTED POWER TRANSFORMERS

BY

RONALD H. MILLER

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This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable for meeting the thesis requirements for this degree. Acceptance of this thesis does not imply that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Wayne E. Knobach
Thesis Advisor

F. C. Fitchen, Head
Electrical Engineering Department
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CHAPTER I
INTRODUCTION

Many, if not most of the authors who have written on the subject of ferroresonance have a tendency to describe it as a "phenomenon." This might be a poor word to use, since it could indicate to some readers that a circuit composed of relatively normal circuit elements is, for some reason, acting mysteriously. However, an analysis of the ferroresonant circuit shows that ferroresonance isn't necessarily an unnatural occurrence, but is somewhat complicated, difficult to understand, and difficult to predict.

As far as electric utilities are concerned, ferroresonance is undesirable on distribution or transmission systems because it may result in either transient or sustained overvoltages under certain conditions. Lightning arrestor flashover, excessive transformer hum, transformer damage, cable damage, and phase reversals have been attributed to ferroresonance. The probability of ferroresonant conditions resulting in damage to equipment has only relatively recently risen to an appreciable level. Increased use of underground cable, with an inherent increase in system capacitance, significantly increases the chances for ferroresonant occurrences.

Basically, the ferroresonant circuit resembles a simple series R-L-C circuit as shown in Fig. 1-1. The difference lies in the fact that the ferroresonant circuit is nonlinear due to the inductive component. This component is a saturable core reactance, which has its impedance value dictated by the saturation characteristic of the iron core.
Fig. 1-1. A simple, series, R-L-C circuit.
This nonlinear element accounts for the fact that three modes of irregular voltage may be possible on the same circuit. One mode is an unstable, transient overvoltage that quickly disappears. A second mode is a sustained voltage of nearly normal value. The third mode is a sustained overvoltage that can reach destructive values under certain conditions, and this high mode represents a ferroresonant overvoltage condition.

If a ferroresonant circuit appears within a power system, the inductive component will in most cases be the winding of an unloaded transformer. The capacitance involved will usually be that of the transformer supply circuit (overhead lines, underground cable, or both).

A review of the literature indicates that the problem is actually a very old one. An early report of 60 years ago attributes overvoltages on a test circuit to ferroresonance. The subject was apparently not of great concern until after World War II as evidenced by the amount of attention it has received within the past 25 to 30 years. Several tests have been conducted on ferroresonance, including network analyzer studies, actual field tests on existing equipment, and extensive laboratory tests. Numerous articles and chapters have been written on the subject, and a limited number of mathematical methods for the calculation of ferroresonant overvoltages have been presented. Although the subject has been covered in quantity, there are at least two good reasons for researching ferroresonance.

The first reason is supported by the complexity of the subject. Ferroresonance is affected by several factors, many of which are not readily apparent in most of the available literature. Most authors are
incomplete in certain areas and vague in others. One goal of this thesis will be to present a discussion of ferroresonance that is more clear and complete.

The second reason lies in the fact that the prediction of ferroresonance, using system parameters, does not appear to be entirely satisfactory. The second goal of this thesis will be to present a fresh approach to this problem via a computer solution. The computer program will be an attempt to solve for ferroresonant overvoltages given information about the primary cable and transformer involved. A solution will be presented for one situation only. This will consist of an unloaded, ungrounded wye-connected transformer bank supplied by a length of underground cable with only one phase energized. If the solution is accepted for this case, it could be applied to other situations.
CHAPTER II
FERRORESONANT CIRCUITS

The ferroresonant circuit occurring within a power system, as stated earlier, basically resembles the simple series R-L-C circuit since it contains a resistance, an inductance, and a capacitance in series with a voltage supply. More attention will be given to the similarities (and dissimilarities) of these circuits later.

When one or two conductors of a three-phase circuit supplying an ungrounded, unloaded transformer bank are open, current paths exist from the closed conductors through the exciting impedance of the transformer bank, and thence to ground through the shunt capacitance of the one or two open conductors. If the voltage supply is ungrounded, the current path may be completed through the capacitance-to-ground on the supply side of the phase opening. (Refer to Fig. 2-1.) The series configuration may occur as a result of conductor breakage, blown fuses, or single-phase switching. The resistance of the circuit will be conductor and transformer ohmic resistance. The capacitance can be that associated with overhead lines, underground cables, and capacitor banks. Certain instances of ferroresonance due to the inherent capacitance of the transformer itself have been reported, although it appears that internal transformer capacitances can be neglected at voltage levels below 19.9/34.5 kV.\textsuperscript{11} As mentioned earlier, the inductance of the circuit will be that of a transformer winding on a core, and will be nonlinear in nature.

Single-phase switching is probably the most frequent cause of
Fig. 2-1. A possible ferroresonant circuit with an ungrounded voltage source and ungrounded transformer bank.
ferroresonant concern because of the widespread use of this switching procedure on distribution systems. Single-phase fused cutout switches are often used to connect an overhead line to a section of underground cable, which in turn is connected to the primary windings of a power transformer. Fig. 2-2 shows a diagram of an ungrounded, wye-connected transformer (only the primary windings are shown) supplied by a length of three-phase underground cable which originates at single-phase switches. One switch is closed, as might be the case during energization of the transformer. The components of the ferroresonant circuit are clearly seen. The inductive component is actually two primary phase windings in series for a wye-connected primary circuit. The cable capacitance is shown as a lumped parameter for each phase, although this is not actually the case. Fig. 2-2 shows that two possible current paths are available. Fig. 2-3 shows the same circuit with one phase open, as might be the case during de-energization. Here again, two current paths are available.

Ungrounded, wye-connected transformer primaries are not alone in their susceptibility to ferroresonant overvoltages. Any connection that provides a transformer winding (or windings) as the inductive component in a series R-L-C circuit is vulnerable. Even single-phase transformers connected line-to-ground may contribute to a ferroresonant circuit, as shown in Fig. 2-4. Fig. 2-5 and Fig. 2-6 illustrate possible ferroresonant circuits containing delta and Tee connected transformers, respectively. A ferroresonant circuit could also exist for two energized phases with either of these transformer connections.
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Fig. 2-6. A possible ferroresonant circuit composed of a Tee connected transformer primary with one phase energized.
CHAPTER III
REVIEW OF LITERATURE

Although it is a fairly common condition, ferroresonance is an unusual problem because the possibility of its occurrence on a given circuit is not readily apparent. Furthermore, it is a complex problem due to the nonlinearity involved. It is also an important problem in terms of equipment damage and system reliability as far as power systems are concerned. For these reasons, dozens of investigators have written on the subject over the last 65 years. This chapter will present briefly the contents of some of the papers. An emphasis will be placed on the most recent literature available, however.

One of the first accounts of ferroresonance was cited by H. B. Dwight and C. W. Baker in the year 1911.1 These men reported "double voltages" occurring while a 2400-volt generator was being tested for grounds. A voltage transformer was connected between one terminal of the generator and ground during the test. A voltmeter was connected across the transformer secondary. Erratic voltages of either 1400 or 3600 volts were reported during repeated switching operations. The ferroresonant circuit which existed consisted of a current path through the magnetizing reactance of the transformer to ground, returning to the source through the capacitance-to-ground of the generator windings.

Dwight and Baker also reported the prediction of the measured voltage as impossible, since the value seemed to depend on which part of the voltage wave the switch was closed. The existence of three possible voltage states was recognized with one state being unstable. H. B. Dwight
writes of similar double voltage occurrences in a later article.\textsuperscript{2}

In 1941, Edith Clarke, H. A. Peterson, and P. H. Light published the results of an extensive transient network analyzer (TNA) study.\textsuperscript{3}

The equivalent circuit of the system studied is shown in Fig. 3-1. The parameters \( C_1' \) and \( C_0' \) represent the positive- and zero-sequence capacitances of the supply circuit on the system side of the single-pole switches. The parameters \( C_1 \) and \( C_0 \) are the positive- and zero-sequence capacitances of the supply circuit on the load side of the switches.\textsuperscript{4}

Several different system conditions were studied, including fault and no-fault conditions. The effects of different transformer connections were also taken into account. The results of the tests are presented as curves of steady state, open-phase voltages plotted against the ratio of \( X_{c1} \) to \( X_m \), where \( X_{c1} \) is the positive-sequence capacitive reactance of the supply circuit on the load side of the switches, and \( X_m \) is the magnetizing reactance of the transformer bank at normal voltage. Calculated values are shown along with the experimental ones for comparison.

Clarke, Peterson and Light also present the methods used to calculate the sustained overvoltages on the open conductors. General expressions for the voltage to ground of the open conductors are given for different transformer connections in terms of \( X_t \), where \( X_t \) is defined as the effective exciting reactance of the transformer bank to the current flowing in the open conductors. The reactance \( X_t \) varies with the type of transformer connection according to the voltages existing across the windings, and is expressed in terms of the per-unit effective transformer exciting reactances of the windings. Clarke and Peterson discuss these calculations further in other references.\textsuperscript{5,6}
Fig. 3-1. The transient network analyzer equivalent circuit of the system studied by Clarke, Peterson, and Light.
The authors conclude as a result of the study that the ratio of \( X_{cl} \) to \( X_m \) is an important factor in avoiding ferroresonant overvoltages. More specifically, this ratio should be greater than three for one conductor open and greater than six for two conductors open in order to keep open conductor voltages below 1.73 times normal line-to-ground voltage on the load side of the opening. Furthermore, with a knowledge of \( X_{cl}/X_m \), the authors present a formula to calculate \( \ell \) in miles, an allowable length of line between fuses or single-pole switches and the load transformer bank. The formula is

\[
\ell = \frac{\frac{I_m \text{ KVA} \times 10^3}{KV^2}}{\frac{X_{cl}}{X_m} \left(2\pi f C_1\right)}
\]

(3-1)

where
- \( KVA \) = rated kilovolt-amperes of the transformer bank.
- \( KV \) = normal line-to-line voltage of the circuit in kilovolts.
- \( I_m \) = average per-unit rms exciting current at rated voltage.
- \( 2\pi f C_1 \) = positive-sequence capacitive susceptance of the circuit in micro-mhos per mile.

The authors conclude that the \( X_{cl}/X_m \) criterion presented for the grounded system is also adequate to prevent overvoltages on an isolated neutral system.

A transient network analyzer study financed by the Rural Electrification Administration is discussed by G. G. Auer and A. J. Schultz.\(^7\) The test appears similar to that conducted by Clark, Peterson, and Light in many respects, although it was specifically set up to study 14.4/24.9 KV grounded-wye systems. The test was conducted because of
evidence of serious overvoltage problems on certain REA systems. Instances of damaged lightning arresters, automatic circuit reclosers and distribution transformers, along with three-phase motor reversals and high secondary voltages, were reported.

All systems studied involved overhead distribution lines connected to a solidly grounded source. All three-phase load transformer bank windings were connected ungrounded-wye on the primary side. Several single-phase transformers connected line-to-ground were distributed along the line. The TNA equivalent circuit shown in Fig. 3-2 represents the system studied, where all three-phase transformer banks are paralleled and combined into one equivalent unit, and all single-phase transformers are combined and shown as a wye-wye grounded neutral bank. Furthermore, because of equipment limitations, it was assumed that the sum of the KVA ratings of all the single-phase transformers would be equal to the total KVA of the three-phase transformers.

The first part of the study was taken with the single-phase transformers disconnected. The results are presented as per-unit line-to-neutral voltages plotted against miles of line. Curves are shown for three different values of total three-phase ungrounded transformer KVA. Curves are also shown for the two circuit conditions consisting of one phase energized-two phases de-energized, and two phases energized-one de-energized. Two modes of voltage are shown for each set of curves, representing the two stable conditions.

The second part of the study involved both the grounded and ungrounded transformers shown in Fig. 3-2. Results are again presented
Fig. 3-2. Transient network analyzer equivalent circuit used by Auer and Schultz.
as curves. The highest sustained overvoltages are shown plotted against the ratio of the line charging KVA and the total transformer KVA. Effect of various single-phase saturation curves was also studied.

The TNA results of Auer and Schultz are substantiated by field tests discussed by L. B. Crann and R. B. Flickinger. These tests, made in 1952, were on distribution circuits that had previously experienced equipment damage. Approximately 125 sets of readings were obtained on four different distribution systems. Crann and Flickinger report that a high mode overvoltage usually occurred about once in every three switch openings. Whenever the overvoltage was not established, a low mode sine wave of about 15 per cent of normal peak voltage was measured. Higher voltages were consistently reported with two phases rather than one phase opened.

Additional tests were made in 1953 for the purpose of evaluating suggested remedial measures to reduce overvoltage occurrence and magnitude. Use of the grounded-wye-wye transformer connection, use of a neutral resistor, and circuit loading were tested in the field to substantiate TNA results.

R. H. Hopkinson has also written extensively on several TNA studies. Two of the studies reported deal with specific transformer connections, namely, the delta-wye and the wye-delta. Hopkinson's expressed purpose of the TNA tests was to determine the lengths of lateral (cable or overhead line) which could be safely used between a transformer and a single-phase switching device. Both papers included results obtained from using three-phase, three-legged, core-type banks,
and banks of three single-phase transformers. The tests were extended in each case to determine the amount of resistive secondary load needed to prevent ferroresonant overvoltages. In the wye-delta connected system, the test was also extended to determine neutral resistor requirements to prevent overvoltages. The three-phase equivalent circuit used by Hopkinson closely resembles that of Fig. 3-1.

Results of the tests are presented in curve form. Times normal winding voltages and times normal open-phase voltages are plotted against a ratio of $X_{co}$ to $X_m$, where $X_{co}$ is the zero-sequence capacitive reactance of the supply circuit, and $X_m$ is the transformer magnetizing reactance at rated voltage. Curves are presented for conditions of both one and two open phases. Minimum resistive load requirements to stabilize ferroresonant overvoltages are plotted as percentage of full load against $X_{co}/X_m$, and also as $R/X_m$ versus $X_{co}/X_m$, where $R$ is the secondary resistive load required to control ferroresonance.

A highlight of these papers is Hopkinson's method of converting a known value of $X_{co}/X_m$ to a length of lateral, although the method appears to be an extension of a formula presented by Clarke, Peterson, and Light.\(^3\) The formula expresses the feet of cable or overhead line in terms of transformer KVA, KV, percent exciting current, zero-sequence capacitance per foot, and $X_{co}/X_m$. The formula is

$$l = \frac{10 I_{exc}^2 \text{KVA}}{2\pi f C_0/\text{ft} K V^2 X_{co}/X_m} \quad (3-2)$$

where $l$ is the allowable lateral length.

A formula is also presented for the calculation of secondary load
required to control ferroresonant overvoltage. Methods for overvoltage calculation are also presented in the "Delta-Wye" paper. Expressions for the voltages to ground of the open conductors are presented in terms of $X_m$, the transformer magnetizing reactance. The magnetizing reactance depends on an assumed value of $V_m$, the voltage across the transformer winding. The assumed values of $V_m$ are saturation curve values, and a corresponding value of $X_m$ can be found for each voltage assumed over the range of the curve.

An earlier paper of Hopkinson deals with yet another TNA study. This study was made on a ferroresonant circuit that involves no cable or overhead lines. The capacitance of this circuit is that of the transformer internal capacitance network and bushings. This study was prompted by reports of overvoltages and transformer failure as a result of switching at the transformer location. The TNA test was done to correlate field study results on a specific 19.9/34.5 kV distribution transformer (ungrounded wye-delta) owned by Pacific Power and Light Company. The study showed that for two energized phases, overvoltages as high as 4.1 times normal could result. Several measures to prevent ferroresonance were evaluated, including the use of a grounded neutral, the use of resistive load, the use of shunt capacitors from each phase to ground, and the use of a neutral resistor.

A substantial part of this paper was devoted to a discussion of an equivalent circuit to represent a transformer internal capacitance network.

F. S. Young, R. L. Schmid, and P. I. Fergestad report the results of a laboratory investigation of ferroresonance. The tests were
performed on laboratory equipment which simulated actual operating conditions. Three 15 KV, 100 ampere load break cutouts were used to energize test transformers from a three-phase 13 KV grounded source. Different values of capacitance were connected line-to-ground to simulate different lengths of cable. Three-phase transformer banks of 13 KV and either 150 or 300 KVA were tested with delta, wye-grounded, wye-ungrounded, and Tee primary winding connections used.

The tests consisted of a series of single-phase energizing and de-energizing operations. Each switching sequence was performed from 25 to 50 times for cable lengths ranging from 100 to 5000 feet.

Results of the tests are presented as oscillograms of transformer primary terminal voltages. Waveshapes are shown before and after the instant of switching. Transient and sustained overvoltages are clearly seen. Probability data of transient voltage occurrence is shown plotted as a function of line-to-ground cable capacitance. Attention is also given to the remedial measures of secondary loading and three-phase switching.

Young, Schmid, and Førgestad also present a basic analysis of ferroresonance. A description of the sustained overvoltage is included, along with a description of residual flux and moment of voltage application.

A discrepancy exists between the findings of Young, Schmid, and Førgestad, and those of Hopkinson, as pointed out by R. F. Lawrence and R. L. Schmid. Lawrence and Schmid state that according to the laboratory tests, 200 feet of cable could be connected to a 13 KV, 150 KVA
transformer before ferroresonant overvoltages were observed. They show that application of Hopkinson's formula allows only 16.8 feet of cable. Similarly, these men report that a 2% secondary load was adequate to eliminate the overvoltages while one of Hopkinson's formulas shows that a 20% secondary load was needed. Hopkinson answers the inquiries about the discrepancies by suggesting that something was wrong with the laboratory tests.

F. C. Van Wormer presents a unique equivalent circuit along with an explanation of ferroresonance. Equations derived from an analysis of this circuit are also presented. The value of transformer air core reactance, $X_t'$, is shown to be an important reference point in ferroresonant conditions. Van Wormer states that the air core reactance is known to be independent of the iron used in the core, and is related to the area and length of the coil, and the number of turns. Furthermore, as a transformer core becomes saturated, its magnetizing reactance approaches the air core reactance.

Van Wormer's voltage versus capacitive reactance curves show that when $X_c$, the capacitive reactance of the circuit, is less than $X_t'$, the voltage has only one possible value near a positive 1 per-unit. When $X_c$ is greater than $X_t'$ but less than $5X_t'$, the voltage may have three possible values, one of which will be unstable. For $X_c$ greater than $5X_t'$, the voltage has only one value which is negative.

A model transformer was tested with no load to verify the calculated values. The process was repeated for a new set of equations which take into consideration a load on the transformer.
Van Wormer also relates the importance of switching procedures in avoiding ferroresonant conditions.\textsuperscript{14,15} It is suggested that switching be done at the transformer location for three-phase banks of 15 KV and below. A discussion of ferroresonant possibilities due to internal transformer capacitance is presented along with remedial measures for larger KV transformers.

R. Rudenberg uses a graphical approach to analyze the occurrence of ferroresonant voltages.\textsuperscript{16} The existence of three possible voltage states is clearly shown. It is pointed out that one state represents a transient condition which quickly disappears. This graphical method is based on the assumption that the saturation curve which defines the reactor shows the rms voltage across the reactor as a function of rms current through the reactor, an assumption which is not strictly true. Rudenberg's approach will be presented in more detail later in this thesis.
Comparison of True and Ferroresonance

A resonant condition is known to exist on the simple R-L-C circuit shown in Fig. 1-1 if the magnitude of the inductive and capacitive reactances are equal. The expression for $I$, the current in this circuit, is

$$ I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (4-1) $$

where $E$ is the applied voltage, $R$ is the resistance, $X_L$ is the inductive reactance, and $X_C$ is the capacitive reactance. It can be seen that the current will be limited only by the resistance of the circuit if a resonant condition exists. For small values of resistance, the current is capable of reaching very high values, which results in high voltages across the capacitance and the inductance. If $L$ and $C$ are constants, the circuit will resonate at one value of source frequency, $f_0$, where

$$ f_0 = \frac{1}{2\pi \sqrt{LC}} \quad (4-2) $$

If the source frequency is adjusted until it equals $f_0$, the current magnitude will begin increasing as $f_0$ is approached. According to an article written by Line Material Industries, increased current amplitudes resulting from a change in system frequency cannot exist instantaneously. Rather, current amplitudes increase over a number of successive cycles. An explanation of this transient condition is not presented by the above reference. The nature of the current over the
transient interval is evidently linked to the time constant of the R-L-C circuit. Current amplitudes increase to a steady-state value defined by the reduced value of circuit impedance.

In a ferroresonant circuit the constant inductance component is replaced by a nonlinear element consisting of an unloaded transformer core, as shown in Fig. 4-1. The inductive reactance, $X_L$, of this element, and therefore the inductance, $L$, is known to be a function of current because of the nonlinearities due to saturation of the core. A typical transformer saturation curve, shown in Fig. 4-2, shows exciting current, $I_m$, plotted against excitation voltage, $V_m$. Any point on the nonlinear portion of the curve represents a different value of $X_L$, which is given by the slope of the curve at that point.

Essentially, the transformer saturation curve consists of three sections. The first section is the beginning of the curve which is a near-vertical straight line. Inductive reactance has a large, nearly constant value along this segment. The second section of the curve is the knee, in which the slope changes from near vertical to near horizontal. The inductive reactance magnitude drastically decreases in this section. The third section includes the straight line portion from the knee to the tail of the curve. Here again the slope is nearly constant. Because of saturation, the inductive reactance is much less in this area than in the first section of the curve, and the inductive reactance approaches an air core value.

The properties of the saturation curve account for the fact that the ferroresonant circuit responds differently over each portion of a
Fig. 4-1. A series ferroresonant R-L-C circuit.
Fig. 4-2. A typical, transformer saturation curve.
voltage cycle. The rms value of operating voltage of a distribution transformer usually falls on the knee of its saturation curve, where \( X_L \) is at an intermediate value. The crest value of transformer operating voltage therefore falls on the near-horizontal segment of the curve, where \( X_L \) is somewhat lower. It can be seen that inductive reactance varies significantly throughout the voltage cycle. The probability, therefore, of \( X_L \) matching up with a value of circuit capacitive reactance to produce a resonant condition at some instant is increased.

The variable inductance feature of an iron-core reactance accounts for the fact that ferroresonant circuits result in lower level oscillations than do true resonant circuits.\(^{17}\) If a series R-L-C circuit containing a nonlinear inductor was brought to a resonant condition by adjustment of source frequency, current amplitude would increase with each cycle. However, increased current means decreased inductive reactance, and decreased inductance. Therefore the resonant frequency, \( f_0 \), given by Eq. (4-2), changes, and voltage and current amplitudes level off.

**Graphical Analysis**

It has been mentioned earlier that three voltage "modes" may exist on a ferroresonant circuit, depending upon which part of the voltage cycle the switch is closed. That three modes, or values of voltage, may exist is perhaps most clearly seen by an analysis of the graphical approach, as presented by Rudenberg,\(^{16}\) and also by Johnson.\(^{19}\)

Shown in Fig. 4-3 as functions of current are three voltage components present in a lossless, series L-C circuit. The voltage equation
Fig. 4-3. The graphical representation of the voltage components of a lossless series L-C circuit.
of such a circuit is

$$E_S = E_L + E_C$$  \hspace{1cm} (4-3)$$

where $E_S$ is the source voltage, $E_L$ is the voltage across the inductance, and $E_C$ is the voltage across the capacitance. The inductance voltage curve is seen to be the already familiar magnetic saturation characteristic. The voltage across the capacitance is known to be $180^\circ$ out of phase with $E_L$, and

$$E_C = -\frac{I}{\omega C}$$  \hspace{1cm} (4-4)$$

where $I$ is current and $\omega$ is system angular frequency. Rearrangement of Eq. (4-3) and (4-4) gives

$$E_L = E_S + I/\omega C$$  \hspace{1cm} (4-5)$$

Eq. (4-5) is shown graphically in Fig. 4-3. The source voltage $E_S$ is shown as a straight horizontal line, indicating a constant value. The capacitance voltage is shown as a plot of $I/\omega C$, a straight line superimposed on $E_S$, and having a slope equal to $1/\omega C$. This line is identified as $E_S + I/\omega C$ on Fig. 4-3, and will be referred to as a capacitance voltage line for simplicity. It can be seen that the slope of the capacitance voltage line changes inversely as the capacitance value is changed. For a given value of capacitance, Eq. (4-5) is satisfied only at the intersection or intersections of the capacitance voltage line and the inductive voltage magnetization characteristic.

Fig. 4-3 also indicates that one, two, or no intersections may exist, depending on the capacitance value. The capacitance voltage line intersects the magnetic characteristic at two points in Fig. 4-3. The
magnitude of $E_L$ is only slightly larger than the magnitude of $E_S$ at the lower point of intersection. Decreasing values of capacitance shifts this intersection point to higher voltages. For a certain value of capacitance, the capacitance voltage line will be tangent to the magnetic characteristic, representing one intersection point. A further decrease in capacitance will not result in an intersection point on the curve shown in Fig. 4-3.

Fig. 4-4 shows the inductive magnetization characteristic extended into quadrant three. According to Rudenberg, this is done to account for the negative voltage and current values which exist when $X_C$ is greater than $X_L$, as shown by the capacitance voltage line $E_{CA}$ in Fig. 4-4. The source voltage in this instance will essentially be in phase with $E_C$. Furthermore, the current values indicated in quadrant three will lead the source voltage, instead of lag as they do in quadrant one. The intersection shown as point A in Fig. 4-4 represents the operating condition for this case. The capacitor voltage line $E_{CA}$ has a large magnitude with a polarity as shown. A large voltage $E_{LA}$ exists across the inductor as shown.

As the capacitance value gets smaller, the intersection will move upward on the negative magnetic characteristic. For a very small value of capacitance, the inductor voltage will become very small as the capacitance approaches an open circuit.

The capacitance voltage line $E_{CB}$ is shown in Fig. 4-4 to intersect the magnetic characteristic curve at three points, identified as B1, B2, and B3. This indicates that three operating points exist for a
Fig. 4-4. A graphical representation of ferroresonance on a lossless, series L-C circuit.
particular capacitance value. However, only intersections B1 and B2 represent stable operating points. Point B1 represents a condition of relatively low voltage and lagging current. Point B2 represents a condition of high voltage and leading current. Point B3 is known to represent an unstable, transient situation which quickly reverts to intersection B1. An explanation of the unstable nature of point B3 is given by Rudenberg and will not be repeated here. Which condition occurs on a circuit is reported to be dependent on the instant of switching. This is evidently not obvious from an analysis of the graphical approach.

Certain assumptions have been incorporated into the presentation of the graphical approach thus far. Voltage and current quantities have both been assumed as sinusoidal in the analysis, which is not the case when the circuit contains an iron core. Secondly, obvious inaccuracies will arise because no losses were considered. Rudenberg extends his graphical approach to include the effects of resistance, although an understanding of this extension is not necessary to establish the existence of the three voltage modes.

**Sustained Overvoltages**

Although three voltage modes are known to be possible on a particular ferroresonant circuit, the graphical approach shows that only two of the modes are actually serious overvoltages. The unstable mode in the first quadrant consists of an overvoltage, and although this voltage may reach damaging magnitudes, it evidently does not cause great concern because of its short duration. The stable voltage mode in
quadrant one is not considered dangerous because the magnitude of this voltage is only slightly above that of the applied voltage. The overvoltages which exist in quadrant three are sustained quantities of high magnitude, and are given the greatest attention in literature due to their destructive nature.

The occurrence of trapped charge on ferroresonant circuit capacitance is evidently the key to understanding the basis for the sustained overvoltages. The trapped charge characteristic of ferroresonance is due to the variable inductance nature of a saturable core reactance. If circuit capacitance exceeds a critical value, high current flows when the voltage cycle falls on the tail of the saturation curve, where the inductive reactance is quite low. During this current, the capacitance will accumulate a charge. The current flow in the reverse direction is largely blocked by the relatively high inductive reactance of the unsaturated region of the magnetization curve. During the negative half-cycle of the applied voltage wave, the trapped charge on the capacitance acts in accordance with the applied voltage to raise the voltage across the inductance. Therefore the negative saturation region is reached in less time over the negative half-cycle of the applied voltage wave than was the positive saturation region over the positive half-cycle of the applied voltage wave. Negative current is allowed to exist until the unsaturated region of the curve is reached. The capacitance becomes oppositely charged to a higher voltage than during the previous half-cycle. This charge is also trapped, and is not released until the positive saturation region is reached. Again,
the voltage on the capacitance caused by the trapped charge acts in the direction of the applied voltage wave to increase the inductance voltage, thereby driving it to saturation in still less time than before. In this manner, the overvoltages increase in magnitude and frequency until a maximum amplitude level is reached. For low loss systems, the capacitance voltage waveshape will approximate a square wave which will eventually fall into phase with the applied voltage.\textsuperscript{12} The square waveshape is a result of the trapped charge existing on the capacitance during each half cycle. The voltage remains constant during the interval of no current, and is proportional to the amount of charge. The inductance voltage also approximates a square wave to some extent and is 180° out of phase with the applied voltage.

If the capacitance is below a second critical value, ferroresonance will not occur.\textsuperscript{19} What little charge accumulates on the capacitance during the positive half-cycle is not trapped, but is bled off by the small reverse current which flows during the unsaturated region of the magnetization curve. If no charge remains, no voltage component will supplement the negative half-cycle of the applied voltage wave; therefore the voltage across the inductance is not increased. The magnetic core is not pushed deeper into saturation, and the capacitance voltage does not increase.

Operation without the occurrence of ferroresonance is possible for large capacitance values.\textsuperscript{19} Evidently an appreciable charge may be trapped on the capacitance, but because of the large capacitance value, the voltage produced by the charge is small. This voltage, which
supplements the negative half-cycle of applied voltage wave, is not large enough to drive the core into saturation. However, this operating condition may result in ferroresonant overvoltages if a transient disturbs the system. Effects of either residual flux or inrush current may cause saturation to occur, and ferroresonance will begin.

Part of the above discussion is referenced to Young, Schmid, and Fergestad. These authors present a description of sustained overvoltage which is based on the following assumptions:

1. There is no residual flux in the core.
2. There is no initial charge on the capacitor.
3. The circuit involved is energized by a sinusoidal voltage applied at the instant it passes through zero.

An idealized saturation curve is also assumed to facilitate the analysis.

The authors discuss two of these assumptions to a certain extent. The existence of residual flux in the core is stated to have an effect on the length of time the sustained overvoltages require to reach their final level. The moment of applied voltage has the same effect, according to these authors, who state that for switching times other than voltage zero, the capacitor voltage will remain constant for an indefinite number of cycles of applied voltage. Nothing is said about the existence of another mode of voltage due to the instant of switching, which seems strange compared to information presented by others. Young, Schmid, and Fergestad instead report instances of a high frequency transient overvoltage existing on their equipment. The
author's explanation of this transient is not particularly clear.

S. Q. Turley states that trapped charge is responsible for a subharmonic condition which may be possible on ferroresonant circuits. An overvoltage may exist across the inductive component due to this condition. The frequency of this overvoltage is noted to be lower than that of the applied voltage. Subharmonic responses in a nonlinear R-L-C circuit are discussed more diligently elsewhere.

Remedial Measures

Field and laboratory tests, as well as transient network analyzer studies indicate that several measures are effective in the reduction and/or elimination of ferroresonant overvoltages. Most of the solutions presented in the literature are relatively uncomplicated. No single measure represents a perfect solution, however, since the desirable effects from a ferroresonant viewpoint are usually accompanied by disadvantages from other viewpoints.

Three-phase switching is considered an effective method of controlling ferroresonance, provided that all three-phases are energized or de-energized at essentially the same instant. This is logical since the R-L-C circuit needed to develop overvoltages is avoided by simultaneous pole operation. However, equipment cost places restrictions on this measure as far as distribution circuits are concerned. Hopkinson states that all three phases must be switched within one-half cycle of fundamental frequency in order to avoid ferroresonant overvoltages, although it is not apparent how he arrives at the half-cycle restriction.

This limitation undoubtedly does severely increase the price of needed
switchgear. Furthermore, three-phase switching would not eliminate the problem completely, since the possibility of conductor breakage or blown fuses always exists.

The series R-L-C circuit can also be avoided by using grounded wye transformer primary connections. In most cases, the presence of a grounded neutral point "shorts out" all or most of the capacitance in ferroresonant circuits. For long overhead lines however, the use of grounded wye transformer primaries is not a guarantee against ferroresonant overvoltages. Crann and Flickinger report an overvoltage of 140 percent of normal peak voltage obtained during test conditions on 9 miles of open-circuited overhead line connected to a grounded wye-wye 14.4/24.9 KV, 75 KVA transformer bank. A ferroresonant R-L-C circuit was established between the line-to-line capacitance of the over-head conductors and the magnetizing reactance of the open circuit transformer. In cable circuits the line-to-line capacitance is negligible, and ferroresonant overvoltages can in general be avoided by connecting the transformer primary grounded wye.

The haphazard grounding of wye-connected transformer primaries for the purpose of avoiding ferroresonance may introduce new problems, however. The use of a grounded wye-delta transformer bank has generally been avoided in the past. During a line-to-ground fault this connection acts as a grounding bank under system ground faults, and possible transformer damage can result.\textsuperscript{10} Hopkinson states that this difficulty can be avoided where a large number of wye-delta banks are used. By grounding the neutrals of all banks, the grounding bank problem will be
shared to such an extent that no one bank will seriously overheat.

The use of grounded wye-wye transformers may also present some disadvantages such as transformer tank heating due to load unbalance in three-phase transformers with three-legged cores. Van Wormer points out that at smaller sizes, transformers can be made with four- or five-legged cores to alleviate tank heating. For larger transformer sizes where cost may be prohibitive to core manufacture, Van Wormer suggests the use of a low resistance metallic liner just inside the transformer tank, thereby reducing the $I^2R$ losses to a minimum.

The use of open-wye/open-delta connected transformers has proven to be a generally ferroresonant-free practice. A disadvantage lies in the fact that the KVA bank rating of this connection is only 86.6 percent of the two transformers making up the bank.

Many of the ferroresonant disturbances caused by switching operations can be avoided by switching procedure, provided that switches are located at the transformer. Van Wormer suggests that for a cable-connected transformer, the cable should be initially energized alone by closing a set of disconnect switches. Then the transformer should be energized by closing a second set of disconnect switches at the transformer location. This switching order prevents a series R-L-C connection. Van Wormer goes on to say that the fuses at the transformer should be selected to clear transformer and secondary faults, while a second set of fuses situated at the source end of the cable should respond only to primary cable faults. A condition favorable to ferroresonance can be avoided by fuse coordination of this nature.
Almost all of the field and laboratory tests and TNA studies have included observations of transformer secondary loading as a ferroresonant overvoltage control measure. The resistive load, which is connected to the transformer secondary before switching, is reflected back into the primary circuit, and provides a damping effect on the ferro non-linear oscillations. Secondary loading is unanimously reported to be effective, but the question of how much load to be used in a particular situation causes disagreement, as mentioned in a previous chapter.

Hopkinson presents a formula to calculate secondary resistance per phase needed to prevent ferroresonant overvoltages, according to the results of his TNA studies.\(^9\),\(^{10}\) The formula, manipulated to give secondary load in terms of percent rated current is

\[
\& I_R = \frac{I_{exc}}{R/X_m} \quad (4-6)
\]

where

- \(I_{exc}\) is transformer percent exciting current.
- \(R\) is the secondary resistance per phase.
- \(X_m\) is the transformer magnetizing reactance.

The ratio \(R/X_m\) in Eq. (4-6) is presented by Hopkinson as data, shown plotted against \(X_{co}/X_m\). Calculated secondary load values presented in table form by Hopkinson for one and two miles of cable and for several transformer sizes indicate that secondary load requirements may fall within the range of 1 to 60 percent of full load.

Young, Schmid, and Fergastad investigated the effects of resistive secondary loading up to 4 percent of the transformer rating.\(^{12}\) They concluded that secondary loads reduced the probability of obtaining
both transient and sustained overvoltages. However, the sustained overvoltages, when they did occur, had the same magnitudes as those appearing during no load.

The use of secondary resistance loading involves a definite cost disadvantage in terms of energy consumption. This cost could be significant if a large number of transformers were involved. For this reason utility companies might not be enthusiastic towards secondary loading as a remedial measure.

The use of neutral resistors is also reported as a successful remedial measure. Hopkinson concluded as a result of his TNA study that the neutral resistor value should not exceed 0.05 \( X_m \) with no load on the secondary, if ferroresonant overvoltages are to be avoided.\(^{10}\) In equation form, the neutral resistor value \( R_N \) is presented as

\[
R_N = \frac{5000 \, KV^2}{I_{\text{exc}} \, KVA}
\]

where \( I_{\text{exc}} \) is transformer exciting current.

KV is rated transformer voltage.

KVA is rated transformer kilovolt-amperes.

Additional studies were also performed to determine the minimum value to be permitted during unbalanced load conditions. Hopkinson made his tests on the basis of a line-to-ground fault at the transformer bank primary terminal. Neutral resistor values were determined for several transformer bank sizes and voltage ratings. With a range of resistor values to choose from (between minimum and maximum values), further analysis was made to determine the correct resistor value based on
minimum power loss. Hopkinson reported that minimum power loss required a resistor of as large an ohmic value as possible.

Crann and Flickinger report the successful use of neutral resistors during their field tests of 1952-53. Three times normal peak voltage was measured on an open phase on a floating wye-delta bank. A neutral resistor of 60,000 ohms prevented the overvoltages from occurring. With a neutral resistor of 35,000 ohms, transients as high as twice normal peak voltage were obtained for durations up to 10 cycles. However, the steady-state overvoltage is reported to have been completely suppressed, with only 10 percent of normal peak voltage measured on the open phase.

One obvious disadvantage of the use of neutral resistors is that they cannot be used with a delta transformer primary connection.

Other measures that have been attempted in an effort to reduce ferroresonant overvoltages have been largely unsuccessful. Among these are the use of shunt reactors to neutralize circuit capacitance, and the use of shunt capacitors to "swamp" it out. It appears, therefore, that from among the solutions discussed in literature, the use of grounded-wye transformer primaries is the most desirable in preventing the occurrence of ferroresonant overvoltages, although they may be undesirable in other respects.
CHAPTER V

COMPUTER ANALYSIS OF A SPECIFIC FERRORESONANT CIRCUIT

The involved mathematics which accompany an analysis of a nonlinear circuit immediately suggest the use of a digital computer. As part of this thesis project, a computer program was written to analyze the circuit of Fig. 5-1 from a ferroresonant viewpoint. More specifically, the program was written to solve the circuit by using the distributed parameter equations for a long transmission line. As presented by Stevenson,\(^2\) these long line equations are

\[ V_S = \frac{V_R + I_R Z_C}{2} e^{\gamma x} + \frac{V_R - I_R Z_C}{2} e^{-\gamma x} \]  
(5-1)

and

\[ I_S = \frac{V_R/Z_C + I_R}{2} e^{\gamma x} - \frac{V_R/Z_C - I_R}{2} e^{-\gamma x} \]  
(5-2)

where \( V_S \) in Eq. (5-1) is sending end voltage, and \( I_S \) in Eq. (5-2) is sending end current. The voltage \( V_R \) is the receiving end voltage, \( I_R \) is the receiving end current, \( Z_C \) is the characteristic impedance of the line, \( \gamma \) is the propagation constant of the line, and \( x \) is the length of the line.

The diagram shown in Fig. 5-1 represents a circuit consisting of an ungrounded wye connected transformer bank connected to a length of underground cable. Only one phase is energized, as is indicated by the voltage source \( V_{S1} \), which represents one phase of a grounded-wye supply. The line-to-ground capacitance, along with the series impedance, is shown as a distributed parameter. Of main importance will be the voltages across the transformers \( V_A \) and \( V_B \), the voltage \( V_{S2} \).
Fig. 5-1. The circuit used for computer analysis of ferroresonance.
existing at the terminal of transformer A to ground, and the voltage \( V_{R2} \) existing on the transformer side of the open switch. It should be noted that although two open phases exist in the circuit of Fig. 5-1 the same conditions exist on both, and therefore a description of only one phase is sufficient.

An analysis of any circuit necessarily begins with the circuit parameters. Since the program was written, for the most part, to analyze a general circuit, provisions have been made in the program for most of the circuit information to be supplied to the computer as input data. Input data includes transformer KV and KVA, transformer saturation curve values, source voltage, and several cable parameters. Cable resistance per unit length, inductance per unit length, insulation dielectric constant, the cable diameter to the outside edge of the insulation, and the diameter of the inner conductor are used to calculate \( z \), the series impedance per unit length, and \( y \), the shunt admittance per unit length. These values are then used to calculate \( Z_C \), the characteristic impedance, which is

\[
Z_C = \sqrt{z/y}
\]  

and \( \gamma \), the propagation constant, which is

\[
\gamma = \sqrt{z \cdot y}
\]  

The series impedance is easily set equal to the combination of the resistance and the inductive reactance components. The capacitance of the cable is calculated within the program according to
where \( C \) is the capacitance in farads per 1000 feet of cable, \( K \) is the dielectric constant of the insulation, \( D \) is the diameter of the cable and insulation, and \( d \) is the diameter of the conductor only. The shunt admittance of the cable is set equal to the combination of the shunt conductance, which will be considered negligible, and the capacitive susceptance component.

Transformer saturation curve values of current and voltage enable transformer voltages to be calculated for a given value of current through the transformer. The current in transformer B in Fig. 5-1 will be twice that in transformer A. The program is written so that a voltage across a transformer will be calculated if the considered current magnitude is greater than the smallest curve current value. If the considered current value is bracketed by two curve current values, the voltage will be found by a linear interpolation technique. If the current magnitude is larger than the largest curve current value, a method of extrapolation is used to extend the curve. The curve extension by this method is a straight line, which passes through the fourteenth and fifteenth coordinates of the curve, the last two values. The effect of this assumption on the results will be discussed later.

The major purpose of the program is to arrive at a solution for the circuit to see if overvoltages do exist. In effect, this involves arriving at an appropriate value of current, along with an associated phase angle with the source voltage considered as a reference. This
value of current can be checked for "correctness" by an analysis of
certain voltages existing on the circuit.

If the correct value of current existing in the transformer was
known, the voltages across the transformers could be calculated, as
related earlier. Two assumptions concerning the phase angle of these
voltages must be made at this point because of lack of sufficient trans-
former test information. The currents through both transformers are
assumed to be in phase, as are the voltages across each transformer.
Secondly, the current and voltage in each transformer are assumed to
have a constant angular displacement, and, as an approximation, the
voltage across each transformer is said to lead its current by 80 degrees.
This assumption allows the magnitude of the voltage across both trans-
formers to be expressed as the sum of the voltages across transformers A
and B. The total transformer voltage, $V_R$, may be expressed as a complex
number, since its angle is known. The voltage $V_{R1}$ can be found by
solving Eq. (5-1) for $V_R$, the receiving end voltage. The sending end
voltage, $V_S$, in the resulting equation will be $V_{S1}$ shown in Fig. (5-1),
and the receiving end current $I_R$ will be equal to twice $I_{S2}$, the current
through transformer B. As a next step, the voltage $V_{S2}$, existing on the
terminal of transformer A, can be found by

$$V_{S2} = V_{R1} - V_R$$  (5-6)

The voltage existing on the transformer side of the open switch, $V_{R2}$, can
be found by the use of Eq. (5-2). Since $I_{R2}$, the receiving end current
at the open switch is zero, the equation for $V_{R2}$ becomes
\[ v_{R2} = \frac{2 I_{S2} Z_C}{e^{\gamma x} - e^{-\gamma x}} \]  

(5-7)

As a check on the value of current used throughout, the value of voltage \( V_{S2} \) could be calculated again using Eq. (5-1). Since \( I_{R2} \) is zero, this equation reduces to

\[ V_{S2} = \frac{1}{2} V_{R2} (e^{\gamma x} + e^{-\gamma x}) \]  

(5-8)

If the correct value of current, which includes both magnitude and angle, was used, the values of \( V_{S2} \) calculated by Eq. (5-6) and Eq. (5-8) will be equal.

Theoretically, a current value should exist which will act as a solution to the circuit in the manner presented above. The search for this solution is complicated by the fact that two variables are involved, the magnitude and the phase angle. Therefore, an iterative type approach to the problem is suggested. A Gauss-Siedel iteration technique was employed in the program to converge upon a solution. Two voltage difference equations were used as a basis for the iteration method.

Minimization of the error voltage between \( V_{S2} \), as calculated by Eq. (5-6), and \( V_{S2} \), as calculated by Eq. (5-8) was desired. This error voltage, \( W \), in equation form is

\[ W = \frac{2V_{S1}}{e^{\gamma x} + e^{-\gamma x}} - \frac{2I_{S2}(e^{\gamma x} - e^{-\gamma x})Z_C}{(e^{\gamma x} + e^{-\gamma x})} - V_R - \frac{V_{R2}}{2} (e^{\gamma x} + e^{-\gamma x}) \]  

(5-9)

The error voltage \( W \) was broken down into its real and imaginary components. These components were obtained as functions of real and imaginary components of the current in transformer A. The real part of \( W \), when set equal to zero, can be solved for the imaginary current component. This
gives one of the equations used in the iterative process. A second equation was obtained by setting the imaginary part of $W$ equal to zero, and solving for the real part of the current. Since the imaginary current is expressed in terms of the real part of the current, and the real current is expressed in terms of the imaginary current, the choice of a Gauss-Siedel type iterative method was logical.

Shown in Fig. 5-2 is a simplified flow diagram of the computer program. The program itself is given in Appendix 2. The iterative section of the program begins with initial values of current components that are read into the computer as data. The magnitude of current and the angles on current and transformer voltage are calculated. The transformer voltages are found using the current magnitude and the saturation curve. A new value of imaginary current is then calculated using the assumed value of the real current component. This new value of imaginary current is in turn used to calculate a new value of real current. The process returns to the beginning of the iterative process, and current magnitude and transformer voltages are re-calculated. New values of real and imaginary components of current are again calculated. This process is repeated until components of current are found to converge. Starting on the second iteration, the current values are tested for convergence. If the absolute value of the differences between the $n$th and $n-1$ iterative values of both real and imaginary components reduces to 0.0001 or less, the iterative process stops. Transformer voltages are calculated once more using the last current value. The two values of $V_{S2}$ are calculated, along with $V_{R2}$, the voltage on the open
M = 0
KT = 0

CALCULATE Z, Y, ZC

READ RIS2, UIS2

MY = 0

Fig. 5-2. Flow diagram of computer program.
Fig. 5-2 (Continued)
Fig. 5-2 (Continued)
switch, and $I_{SL}$, the current at the source.

The fact that the program did converge to a solution for cable lengths of 200 and 500 feet indicates that the theory behind the program is essentially sound. The results of the program, which will be presented in the next chapter, are certainly affected by the several assumptions and approximations necessary. The program could be improved considerably, especially if more definite knowledge was available concerning phase relationships between saturation curve values.
A computer solution was successfully run for two different cable
lengths connected to an unloaded, three-phase transformer bank. Each
transformer of the bank was rated at 7200 volts and 25 kVA. The trans-
former exciting current was assumed to be 2% at no load, meaning that
0.0694 amperes on the curve corresponds to a value of 7200 volts. The
cable involved is a 15 KV, single conductor number 2 solid aluminum
underground cable, with a full size concentric wire neutral. The cable
data used in the program is as follows:

- Resistance--0.311 ohms per 1000 feet
- Inductance--0.0
- Dielectric Constant--2.3
- Diameter over conductor--0.258 inches
- Diameter over insulation--0.65 inches

Fig. (6-1) shows a part of the computer print-out of the solution
to the circuit of 500 feet of cable. Values are given for the series
impedance, shunt admittance, and characteristic impedance per 1000 feet
of cable. These values do not change with a change in cable length. The
source voltage was read in as 7200 + j0.0 volts.

For 500 feet of cable, the program converged to a solution on the
36th iteration. At this point the absolute value of the difference
between the nth and n-1 imaginary current values had reduced to 7.748 x
10^-6 amperes. The difference of the real currents had reduced to 8.29 x
10^-5 amperes. Both of these values are less than the requirement for
FERRORESONANCE COMPUTER STUDY BASED ON DISTRIBUTED LINE CONSTANTS

TRANSFORMER KV EQUALS 7.20
TRANSFORMER KVA EQUALS 25.00
RATED CURRENT EQUALS 3.4722 AMPS

THE SERIES IMPEDANCE OF THE CABLE EQUALS 3.11E-01 & J 0.0 OHMS PER 1000 FEET
THE SHUNT ADMITTANCE OF THE CABLE EQUALS 0.0 & J 1.589E-05 MHOS PER 1000 FEET

THE LENGTH OF THE CABLE IS 500.0 FEET

Fig. 6-1. Transformer and cable data print-out for the computer solution of the circuit shown in Fig. 5-1.
convergence in the program. The magnitude of $I_{S2}$ equals 0.2284 amperes or 6.577 percent rated current on this iteration, and beta, the angle on the current, equals 1.6453 radians, or 94.3 degrees. Alpha, the angle on the transformer voltages, equals 3.0421 radians. The magnitudes of transformer voltages corresponding to this exciting current are 9354.5 volts and 12,209 volts on transformers A and B, respectively. As shown in Fig. 6-2, the component parts of the two values of $V_{S2}$ are nearly equal in magnitude, although a sign difference does exist between the imaginary components of the two values. The reason for the sign difference is not apparent. A magnitude of 28,737 volts has been calculated for $V_{R2}$. The value of $I_{S1}$, the current at the source is also given in Fig. 6-2.

For 200 feet of cable, the conditions for convergence were met after only 22 iterations. The absolute value of the difference between the $n$th and the $(n-1)$ imaginary current values had reduced to $7.56 \times 10^{-6}$ amperes. The difference of the real currents had reduced to $7.67 \times 10^{-5}$ amperes. The magnitude of $I_{S2}$ on this iteration equals 0.0723 amperes or 2.08 percent rated current, and beta, the angle on the current is 1.642 radians, or 94 degrees. Alpha, the angle on the transformer voltages equals 3.0363 radians. The magnitudes of transformer voltages corresponding to this exciting current are 7250. volts and 8306.7 volts on transformers A and B, respectively. The two values of $V_{S2}$ are $22,674 - j 1604.4$ volts as calculated by Eq. (5-6), and $22,672 + j 1620.8$ volts as calculated by Eq. (5-8). As before, there exists a sign difference between the imaginary components. A magnitude of 22,730 volts has been calculated for $V_{R2}$. In real and imaginary form this is
THE MAGNITUDE OF VR2 IS EQUAL TO 2.8737E 04 VOLTS

IN REAL AND IMAGINARY FORM, VR2 = 2.8656E 04 & J 2.1505E 03 VOLTS


VB, THE VOLTAGE ACROSS TRANSFORMER B, EQUALS-1.2149E 04 & J 1.2132E 03 VOLTS

VS2, CALCULATED BY KIRCHHOFFS LAW, EQUALS 2.8657E 04 & J -2.1428E 03 VOLTS

VS2, CALCULATED BY LINE FORMULAS, EQUALS 2.8656E 04 & J 2.1505E 03 VOLTS

THE CURRENT AT THE SOURCE IS -3.42E-02 & J 5.13E-01 AMPS

Fig. 6-2. Voltage and current output for the computer solution of the circuit shown in Fig. 5-1 for 500 feet of cable.
22,672 + j 1620.8 volts. The current at the source is -0.0103 + j 0.167 amperes, or 4.819 percent rated transformer current.

The voltages calculated for both cable lengths appear to be high. A large part of the blame for this can certainly be placed on the transformer saturation curve approximations incorporated into the program. For current values above the last curve current value, the curve has to be extended. As stated in Chapter V, the extension is a straight line passing through the fourteenth and fifteenth curve coordinates, which are the last two points on the curve. Voltages are then calculated according to the formula for a straight line,

\[ y = mx + b \]  

(6-1)

where \( y \) is the transformer voltage, \( m \) is the slope of the line, \( x \) is the current value, and \( b \) is the \( y \)-intercept voltage value. For currents only slightly above the last curve current value, the approximation is accurate. For larger currents, however, the saturation curve should be represented by a near-horizontal line, and the voltage across the transformer would tend to approach a constant value for increasing current values.

The curve current values used in the program ranged from 0.006 amperes at 900 volts to 0.108 amperes at 7750 volts. For the 200 foot cable solution, the current through transformer B, equal to twice \( I_{S2} \), was 0.1445 amperes. This value is considerably larger than 0.108 amperes, and it is very conceivable that the voltage of 8306 volts across this transformer is unrealistic. For the 500 foot cable solution, the current in both transformers at the iteration of convergence is on
the extended part of the curve. The current in transformer A, equal to $I_{s2}$, is 0.2284 amperes or 6.57 percent rated current, and therefore the current in transformer B is 0.4567 amperes or 13.15 percent rated current. Logically, by the time a value of 6 percent has been reached on the curve, further increase in current will result only in a small voltage increase. However, the calculations show that the magnitude of $V_B$ is nearly 3000 volts larger than the magnitude of $V_A$. Solution of the program for longer cable lengths would probably attain convergence at larger currents, and still more unrealistic voltages would develop.

The saturation curve problem could be avoided by placing enough curve values into the computer until the slope of the extension is very small. This was not done at the time the program was written because the large current magnitudes were not anticipated. Furthermore, saturation curves for specific transformers are not readily available, and as a result the shape of the curve used was also approximate. The approximation of angular relationships between saturation curve voltages and currents may account for other discrepancies in the results.

The results show that overvoltages do exist on the circuit, although program improvements will reduce the magnitudes of the overvoltages to more realistic results. Hopkinson states that transformer winding voltages should be limited to 110 percent of rating to avoid excessive core heating, and also that transformer terminal to ground voltages should be limited to 125 percent of normal to avoid lightning arrester failure for effectively grounded systems. Program results thus far indicate that these percentages will be exceeded even with program improvements. The type of approach used in the program gives more proof
to support an already well-known fact, that ferroresonance is very
difficult to analyze mathematically.
REFERENCES


APPENDIX 1
H. A. Peterson briefly describes a Transient Network Analyzer in Chapter 1 of *Transients in Power Systems*. The TNA described by this author and that used by himself, Clarke, and Light were probably very similar, if not identical. The TNA used by Auer and Schultz,²⁷ and the one used by Hopkinson⁹,¹⁰ were probably also basically similar to the one described by Peterson. As an aid to better understanding of the TNA tests performed, Peterson's component by component description of a Transient Network Analyzer will be repeated here. The TNA consists of:

1. Tapped inductive reactors for simulating transformer, generator, or other lumped reactances.

2. Capacitors of suitable sizes for simulating the capacitance of transmission lines, cables, or capacitor banks.

3. Resistors of suitable ohmic range to simulate resistance loads, losses in transmission lines, or losses of other kinds.

4. Artificial transmission line units, for simulating transmission line behavior under transient conditions.

5. Single-phase 1:1 turn ratio transformers having special saturation characteristics to simulate those of large power transformers.

6. A three-phase 60 cycle, 110 volt sine wave variable-speed generator for energizing the miniature system.

7. Various recurring synchronous switches (both electronic and mechanical) for performing the desired switching operations in the miniature systems.

8. A cathode-ray oscillograph for observing the transient voltages and currents resulting from an imposed fault or switching condition.
9. A means for photographing the trace appearing on the cathode-ray oscillograph screen when a permanent record is desired.

10. Two ground-fault neutralizer coils designed to have per-unit saturation characteristics closely duplicating those of large full-voltage units.

11. Miniature Thyrite arresters. These are small wafers of Thyrite material about 5/8 inch in diameter which can be stacked in a variety of combinations so that it is possible to simulate an arrester of almost any voltage rating.
APPENDIX 2
COMPUTER PROGRAM FOR THE ANALYSIS OF A SPECIFIC FERROSONANT CIRCUIT

90 FORMAT (2F10.0)
91 FORMAT (1H1,10X,'FERROSONANCE COMPUTER STUDY BASED ON DISTRIBUTE
10 LINE CONSTATS'//)
92 FORMAT (2F7.0)
93 FORMAT (F5.0)
94 FORMAT (2F6.0)
95 FORMAT (5F6.0)
99 FORMAT (1H0,10X,'VA, THE VOLTAGE ACROSS TRANSFORMER A,EQUALS',1PE
111.4,' & J',1PE11.4,' VOLTS'//11X,'VB, THE VOLTAGE ACROSS TRANSFOR
2MER B, EQUALS',1PE11.4,' & J',1PE11.4,' VOLTS'//)
100 FORMAT (1H1,10X,'THE MAGNITUDE OF VR2 IS EQUAL TO',1PE11.4,' VOLTS
1''//11X,'IN REAL AND IMAGINARY FORM, VR2 =',1PE11.4,' & J',1PE11.4,
2' VOLTS'//)
101 FORMAT (1H0,10X,'VS2, CALCULATED BY KIRCHHOFFS LAW, EQUALS',1PE12
1.4,' & J',1PE12.4,' VOLTS'//11X,'VS2, CALCULATED BY LINE FORMULAS,
2 EQUALS',1PE12.4,' & J',1PE12.4,' VOLTS'//)
102 FORMAT (1H0,10X,'THE CURRENT AT THE SOURCE IS',1PE10.2,' & J',1PE
10.2,' AMPS'//)
103 FORMAT (1H0,10X,'THE SERIES IMPEDANCE OF THE CABLE EQUALS',1PE10.2
1,' & J',1PE11.3,' OHMS PER 1000 FEET'//11X,'THE SHUNT ADMITTANCE O
2F THE CABLE EQUALS',1PE10.2,' & J',1PE11.3,' MHOS PER 1000 FEET'//
311X,'THE CHARACTERISTIC IMPEDANCE OF THE CABLE EQUALS',1PE11.3,' &
4 J',1PE11.3,' OHMS'//)
104 FORMAT (1H0,10X,'TRANSFORMER KV EQUALS',F6.2//'11X,'TRANSFORMER KVA
1 EQUALS',F6.2//'11X,'RATED CURRENT EQUALS',F8.4,' AMPS'//)
105 FORMAT (1H0,10X,'THE LENGTH OF THE CABLE IS',F6.1,' FEET'//)
106 FORMAT (1H0,10X,'THE CURVE VALUES ARE',//12X,'AMPS',6X,'VOLTS'//)
107 FORMAT (1H0,10X,'ASSUMED CURRENT EQUALS',F5.2,' & J ',F6.3,' AMPS')
111 FORMAT (1H0,1P3E17.6)
C

DIMENSION C(20), V(20), RIS(100), UIS(100)
REAL LENGTH
COMPLEX AIS2, VR1, VS1, AIS1, VS21, VR2, VS22, VR, Y, Z, ZC, GA, VA, VB, W, E1, E2
C READ IN RATED KV AND KVA OF THE TRANSFORMERS
READ (11,90) TKV, TKVA
TIR=TKVA/TKV
WRITE (12,91)
WRITE (12,104) TKV, TKVA, TIR
C READ CURVE VALUES
DO 3 K=1,15
3 READ (11,92) C(K), V(K)
SLOPE=(V(15)-V(14))/(C(15)-C(14))
YINT=6500.
KT=0
M=0
C READ IN SOURCE VOLTAGE AND LINE LENGTH
READ (11,94) VS1
RVS1=REAL(VS1)
READ (11,93) X
C READ IN CABLE VALUES
READ (11,95) R, AL, AK, D, DD
Z=CMPLX(R, 376.8*AL)
CC=(AK*1.36E-9)/(ALOG10(D/DD))
Y=CMPLX(0.0, (376.8*CC))
C CALCULATE THE CHARACTERISTIC IMPEDANCE AND THE PROPAGATION CONSTANT
ZC=CSQRT(Z/Y)
GA=CSQRT(Y*Z)
WRITE (12,103) Z, Y, ZC
LNGTH=1000.*X
WRITE (12,105) LNGTH
WRITE (12,106)
DD 61 N=1,15
61 WRITE (12,150) C(N), V(N)
C READ ASSUMED REAL AND IMAGINARY COMPONENTS OF CURRENT
READ (11,92) RIS2, UIS2
WRITE (12,107) RIS2, UIS2
N=0
WRITE (12,2000) N
RZC=REAL(ZC)
UZC=AIMAG(ZC)
E1=CEXP(GA*X)&CEXP(-GA*X)
RE1=REAL(E1)
UE1=AIMAG(E1)
WRITE (12,222) RE1, UE1
E2=CEXP(GA*X)-CEXP(-GA*X)
RE2=REAL(E2)
UE2=AIMAG(E2)
WRITE (12,222) RE2, UE2
SE1=(RE1**2.+(UE1**2.)
SE2=(RE2**2.+(UE2**2.)

69
REE = (RE1*RE2*RZC) - (RE1*UE2*UZC) & (UE1*UE2*RZC) & (UE1*RE2*UZC)
UEE = (RE1*UE2*RZC) & (RE1*RE2*UZC) - (UE1*RE2*RZC) & (UE1*UE2*UZC)
REF = (RE2*RZC*RE1) - (RE2*UZC*UE1) & (UE2*UZC*RE1) & (UE2*RZC*UE1)
UEF = (RE2*UZC*RE1) & (RE2*UE1*RZC) - (UE2*RZC*RE1) & (UE2*UZC*UE1)

WRITE (12,777) REE, UEE, REF, UEF, SE1, SE2
WRITE (12,150) SLOPE, YINT

1 MY=0
C CALCULATE CURRENT MAGNITUDE
4 AMIS2 = SQRT((RIS2**2) & (UIS2**2))
WRITE (12,999)
C CALCULATE BETA, THE CURRENT ANGLE
BETA = ATAN2(UIS2,RIS2)
ALPHA = ((80./57.295) & BETA)
WRITE (12,200) AMIS2, BETA, ALPHA
IF (AMIS2 .LT. C(1)) STOP
C CHECK WHETHER CURRENT IS LARGER THAN C(15)
IF (AMIS2 .GT. C(15)) GO TO 20
   J=1
6 IF (AMIS2 - C(J)) 8,10,12
10 J=J+1
12 IF (J .LE. 15) GO TO 6
C AMIS2 EQUALS C(J)
10 VAMAG = V(J)
   GO TO 22
20 VAMAG = SLOPE*AMIS2+C(YINT)
   WRITE (12,500) AMIS2, V(15), SLOPE, VAMAG, YINT
   GO TO 22
C TWO CURVE VALUES BRACKET AMIS2--INTERPOLATE TO FIND VOLTAGE
8 VAMAG = V(J-1) & ((V(J) - V(J-1)) / (C(J) - C(J-1))) * (AMIS2 - C(J-1))
22 BMIS2 = 2.00*AMIS2
   IF (BMIS2 .GE. C(15)) GO TO 21
   L=2
7 IF (BMIS2-C(L)) 14, 16, 18

18 L=L+1
   IF (L.LE. 15) GO TO 7

C BMIS2 EQUALS C(L)

16 VBMAG=V(L)
   GO TO 25

21 VBMAG=SLUPE*BMIS2&YINT
   WRITE (12,150) AMIS2, BMIS2
   GO TO 25

C TWO CURVE VALUES BRACKET BMIS2--INTERPOLATE

14 VBMAG=V(L-1)+(V(L)-V(L-1))/(C(L)-C(L-1))*(BMIS2-C(L-1))

C AMIS2 AND BMIS2 ARE ASSUMED TO BE IN PHASE

25 VRMAG=VAMAG&VBMAG
   WRITE (12,111) VRMAG, VAMAG, VBMAG
   RVR=VRMAG*(COS(ALPHA))
   UVR=VRMAG*(SIN(ALPHA))
   IF (K.T.EQ.1) GO TO 60

C EMPLOY A GAUSS-SIEDEL ITERATIVE METHOD

M=M+1
   WRITE (12,1000) M

C CALCULATE A NEW VALUE OF IMAGINARY CURRENT

UNUM=2.*(((REF*RIS2)-(RVS1*RE1))/SE1)&RVR&((REF*RIS2)/SE2)

UDEN=((2.*UEE)/SE1)&(UEF/SE2)

UIS(M)=UNUM/UDEN
   IF (M.EQ.1) GO TO 72
   WRITE (12,222) UIS(M), UIS(M-1)

UDIF=ABS(UIS(M)-UIS(M-1))
   WRITE (12,666) UDIF

C CHECK THE IMAGINARY CURRENT COMPONENT FOR CONVERGENCE

   IF (UDIF<.0001) 72, 72, 71

71 MY=1

C CALCULATE A NEW VALUE OF REAL CURRENT
72 RNUM=UVR-(2.*((RVSI*UE1)*(REE*UIS2))/SE1)-(REF*UIS2)/SE2
RDEN=((2.*UEE)/SE1)*(UEF/SE2)
RIS(M)=RNUM/RDEN
IF (M.EQ.1) GO TO 49
WRITE (12,222) RIS(M), RIS(M-1)
RDIF=ABS(RIS(M)-RIS(M-1))
WRITE (12,666) RDIF
C CHECK THE REAL CURRENT COMPONENT FOR CONVERGENCE
IF (RDIF-0.0001) 49,49,81
81 MY=1
49 RIS2=RIS(M)
UIS2=UIS(M)
WRITE (12,1000) M
WRITE (12,300) RIS(M), UIS(M), M
IF (M.EQ.1) GO TO 1
IF (M.EQ.50) GO TO 54
IF (MY.EQ.1) GO TO 1
54 WRITE (12,444)
KT=KT+1
GO TO 4
60 VR=CMPLX(RVR,UVR)
AIS2=CMPLX(RIS2,UIS2)
S1=VAMAG*(COS(ALPHA))
S2=VAMAG*(SIN(ALPHA))
S3=VBMAG*(COS(ALPHA))
S4=VBMAG*(SIN(ALPHA))
VA=CMPLX(S1,S2)
VB=CMPLX(S3,S4)
VR1=((2.*VS1)-(2.*AIS2*(CEXP(GA*X)-CEXP(-GA*X))*ZC))/(CEXP(GA*X)+CEXP(-GA*X))
C CALCULATE THE CURRENT AT THE SOURCE
AIS1=((VR1/(2.*ZC)+AIS2)*CEXP(GA*X))-((VR1/(2.*ZC)-AIS2)*CEXP(-GA*X))
C CALCULATE VS2
   VS21=VR1-VR
C CALCULATE THE VOLTAGE ON THE LOAD SIDE OF THE OPEN SWITCH
   VR2=(2.*ZC*AI S2)/(CEXP(GA*X)-CEXP(-GA*X))
   VR2MG=CA BS(VR2)
C CALCULATE VS2 AGAIN
   VS22=(VR2/2.00)*(CEXP(GA*X)+CEXP(-GA*X))
WRITE (12,100) VR2MG, VR2
WRITE (12,99) VA, VB
WRITE (12,101) VS21, VS22
WRITE (12,102) AIS1
WRITE (12,2000) N
WRITE (12,777) VS21, VR, VS22
AAK=REAL(VS21)
AAI=AIMAG(VS21)
TAA=ATAN2(AAI,AAK)
BBR=REAL(VS22)
BBI=AIMAG(VS22)
TBB=ATAN2(BBI,BBR)
AA=CA BS(VS21)
BB=CA BS(VS22)
WRITE (12,777) VR1, AIS1, VR2
WRITE (12,111) AA, VR2MG, BB
WRITE (12,150) TAA, TBB
C CALCULATE THE VS2 ERROR VOLTAGE
   W=VS21-VS22
   WM=CA BS(W)
WRITE (12,111) W, WM
GO TO 59
58 WRITE (12,888)
59 STOP
END