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AN INTEGRATED CIRCUIT ANALOG COMPUTER

BY

STHAPORN UDOMSIN

A thesis submitted in partial fulfillment of the requirements for the degree Master of Science, Department of Electrical Engineering, South Dakota State University

1972

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S. U.
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A. An Application of Computers

Traditionally, engineers apply both mathematical analysis and experimentation to the solution of engineering problems. Physical systems are frequently expressed in terms of mathematical models for which the solution may be obtained. In general the problem can be described by a set of differential equations, the solution of which gives the dynamic response of the system. Without the use of a computer, solving these equations may be tedious and exhaustive. A digital computer accomplishes this task with great accuracy; theoretically the accuracy of a digital computer solution is unlimited. But an analog computer, although less accurate, can be used to handle the task far more rapidly and easily. Many engineering problems do not require accuracy beyond three significant figures, and therefore do not require the employment of a digital computer. Even in the cases where high accuracy is required, it is often desirable to obtain an analog solution before seeking the final solution on a digital computer. The chief advantage of analog computation is that the operator can gain insight into the problem. Analog computer results are normally presented graphically. The coefficients in the problems can be varied by corresponding parameter changes and the block of computing elements can be visualized as a mathematical operation. Considering the cost, an analog computer is less expensive, but with
a specified component accuracy of better than 0.1 percent, its cost rises sharply for even small accuracy improvement. Figure 1 shows a quantitative comparison of the cost between an analog and a digital computer.

Besides the accuracy, other disadvantages of the analog computer are its inability to store the results and to make logical decisions. Hybrid computers have been developed to realize the advantages of both types of computers and proved to be the most convenient for engineering design applications.

B. Analog Computer

Analog computers used today may be divided into two broad categories, general purpose and special purpose. The general purpose computers can be further subdivided into two classes: the direct or physical analog computer and the indirect or mathematical analog computer. Since the mathematical analog computer simulates the system behavior, it may be referred to as an analog simulator, but more commonly as an analog computer. Here any reference to analog computer will mean in particular the electronic mathematical analog computer.

The electronic analog computer consists of basic elements which perform specific mathematical operations.

1. Summation and subtraction of two or more variables.
2. Multiplication and division of a variable by a constant.
3. Integration and differentiation of a variable with respect to time.
Figure 1. Relative cost versus accuracy of analog and digital computer for different sizes of installations.
4. Multiplication and division of a variable by a variable.
5. Generation of a function of a variable or variables.

A very wide range of problems can be solved by application of these computing elements.

The first three operations can be performed by the use of a high gain dc operational amplifier with suitable feedback elements. This is discussed in various books of analog computation, such as References 2 and 6. Many methods have been employed to do multiplication and division of two variables, discussed in References 2, 6 and 13. Many functions can be generated in the analog computer as solutions of a differential equation, some generated by means of nonlinear elements, while some need special circuitry. Function generating is discussed in detail in References 2 and 3.

C. Educational Analog Computer

Without severe reduction in the usage, the cost of an analog computer can be dramatically reduced by eliminating expensive units such as function generators and using lower quality components. For educational purposes where economic considerations are an important factor, many of the problems need not be complicated and accuracy is not critical. An educational analog computer may consist of a few dc operational amplifiers, dc sources, coefficient potentiometers, a panel meter and some computing elements. This type of analog computer should be capable of solving ordinary linear and nonlinear differential equations with reasonable accuracy.
D. Purpose of the Study

The Heathkit Model EC-1 Educational Analog Computer, priced at $199.95, is the least expensive general purpose analog computer on the market. This computer contains nine unstabilized, vacuum tube dc amplifiers with dc open loop voltage gain of about 1000. The low amplifier gain and amplifier drift limit the accuracy of the solution. Results obtained are sometimes confusing.

Many integrated circuit operational amplifiers available on the market seem to give much better performance, priced from a few dollars in monolithic form up to hundreds of dollars in hybrid form. The low cost operational amplifiers, designed for various applications, are usually not intended for use in an analog computer.

The purpose of this study is to select and evaluate low cost integrated circuit operational amplifiers which are capable of analog computer application. If such operational amplifiers, which meet desired specifications, can be found they are to be incorporated into the analog computer to replace the vacuum tube operational amplifiers. It is expected that these modifications will result in more accurate and more reliable operation with fewer maintenance problems. Another advantage of this modification is its low operating voltage, ±10 volts, thus enabling the use of lower voltage devices and interconnection to other forms of integrated circuits. In the fast growing field of integrated circuits, digital logic circuits and some function generators are available at reasonable prices. This renders further improvements and hybridization of the analog computer more attractive.
CHAPTER II

OPERATIONAL AMPLIFIER

A. Ideal Operational Amplifier

The operational amplifier was originally conceived to perform linear mathematical operations in analog computation. Ideally, its voltage transfer ratio is solely dependent upon the input and feedback elements, $Z_I$ and $Z_F$. To implement the ideal operational amplifier the following requirements must be met.

1. High open-loop voltage gain.
2. High input impedance.
3. Low output impedance.
4. 180 Degree phase shift between input and output voltage.
5. Low dc voltage and current offset.
6. Low offset voltage and offset current drift.
7. Low noise.
8. Stable when operating at the desired closed-loop gain.
9. Wide bandwidth.
10. High slew rate.
11. Linearity over the desired operating voltage range.

B. $\mu$A741 Operational Amplifier

The $\mu$A741 is a monolithic operational amplifier manufactured by Fairchild Semiconductor. It is intended for a wide range of applications to replace the popular $\mu$A709 which is now obsolete. It is
designed for high performance and ease of operation. The $\mu A741$ amplifier is frequency-compensated and short circuit-protected and, in addition, it has a provision for nulling. It is essentially a two-stage amplifier comprised of a high gain differential amplifier input stage followed by a high gain driver stage with a class AB output.\textsuperscript{19} The circuit diagram and electrical characteristics for the $\mu A741$ are shown in Appendix A. These data, supplied by the manufacturer, were used to compare its characteristics with those of the ideal operational amplifier described in the previous section.

Several other monolithic operational amplifiers were considered, including the $\mu A709$, CA3033 and $\mu A725$, as well as a combination of the $\mu A727$ and $\mu A741$.

The $\mu A709$ and CA3033 amplifiers are also suitable for use as a dc amplifier in an analog computer. However, both need an external RC network for frequency compensation and their respective performances are not as good as the $\mu A741$.

The $\mu A725$ has very good characteristics, including high open-loop voltage gain, low noise, low offset and low drift. However, when operating as a unity gain inverter, its slew rate, 0.001 volt per microsecond, is very poor. Slew rate as measured for an EAI380 amplifier, connected as a unity gain inverter, is about 0.1 volt per microsecond.

The $\mu A727$ is a temperature-controlled preamplifier with a gain of 100. Since all components of the circuit are kept at a constant temperature it has a very low drift. When used as the input stage of a $\mu A741$, the drift due to the $\mu A741$ is suppressed by a factor of 100, the gain
of the preamplifier. Thus, the drift of the combination, due largely to the \( \mu A727 \), is very low. This combination has low input offset current (2.5 nA), high open loop voltage gain (2x10^7) and very low drift (5 \( \mu V/\text{week} \)). Its performance is comparable to a high performance discrete component operational amplifier with chopper stabilization. The frequency compensation network connected to the \( \mu A727 \) must be selected to obtain stability at the desired closed-loop gain. As tested, the combination can be used when \( Z_1 \) is limited at 100 k\( \Omega \) and the closed-loop gain is between 1 and 10. By choosing a proper compensation network, these constraints may be relaxed. This combination was not used in this computer because of its cost, about $23 compared to $5.93 for the \( \mu A741 \). The combination of \( \mu A727 \) and \( \mu A741 \) amplifiers would be desirable for a high performance analog computer with high accuracy computing elements.

C. Load Impedance, Input Impedance, Output Impedance and Open-Loop Voltage Gain

The equivalent circuit of the operational amplifier with an inverting feedback configuration is shown in Figure 2. The load resistance is assumed to be large enough so its effect on the transfer characteristic is negligible. An analysis of the circuit follows.

\[
V_{o1} = -A_0(\omega) (V_e - V_1) \tag{2-1}
\]

where

\[
V_e = \frac{V_{1n} \left[ (Z_F + Z_{o1})//\left( Z_1 + R \right) \right]}{Z_1 + \left[ \frac{(Z_F + Z_{o1})//\left( Z_1 + R \right)}{Z_{o1}+Z_F} \right]} + \frac{V_{o1} \left[ Z_1//\left( Z_1 + R \right) \right]}{Z_{o1}+Z_F + \left[ Z_1//\left( Z_1 + R \right) \right]} \tag{2-2}
\]
\[ V_1 = V_e \frac{R_R}{Z_1 + R_R} \]  

(2-3)

Substitute \( V_e \) and \( V_1 \) into Equation (2-1) and solve for \( V_{oi} \)

\[ V_{oi} = \frac{-A_0(\omega)Z_1(Z_F + Z_{oi})V_{in}}{(Z_F + Z_{oi})(Z_1 + R_R) + Z_I(Z_F + Z_{oi} + Z_1 + R_R) + A_0(\omega)Z_1Z_I} \]  

(2-4)

\( V_{out} \) expressed in terms of \( V_{oi} \) and \( V_{in} \), assuming \( R_L \) approaches \( \infty \) is

\[ V_{out} = \frac{V_{oi} \left[ Z_F + Z_I/(Z_1 + R_R) \right]}{Z_{oi} + \left[ Z_F + Z_I/(Z_1 + R_R) \right]} + \frac{V_{in} \left[ Z_{oi}(Z_F + Z_{oi})/(Z_1 + R_R) \right]}{(Z_F + Z_{oi}) \left[ Z_I + (Z_F + Z_{oi})/(Z_1 + R_R) \right]} \]  

(2-5)

Substitute \( V_{oi} \) into Equation (2-5)

\[ \frac{V_{out}}{V_{in}} = \frac{Z_{oi}(Z_1 + R_R) - A_0(\omega)Z_1Z_F}{(Z_F + Z_{oi})(Z_1 + R_R) + Z_I(Z_F + Z_{oi} + Z_1 + R_R) + A_0(\omega)Z_1Z_I} \]  

(2-6)

Assuming \( Z_1 \gg Z_I/(Z_F + Z_{oi}) \)

and \( Z_{oi} \ll Z_F \)

\[ \frac{V_{out}}{V_{in}} = \frac{-A_0(\omega)Z_F}{Z_F + Z_I + A_0(\omega)Z_I} \]  

(2-7)

If \( A_0(\omega)Z_I \gg Z_F + Z_I \)

\[ \frac{V_{out}}{V_{in}} = \frac{-Z_F}{Z_I} \]  

(2-8)

Details are given in Reference 8.
Figure 2. Operational amplifier with inverting feedback configuration.
In deriving this principal equation we have made four assumptions as follows.

1. \( R_L \to \infty \)

Loads which may be connected to the output of the operational amplifier include a coefficient potentiometer, the input of the next operational amplifier and the output meter. Consider the worst case when all possible loads are connected and each resistance has its minimum possible value. The connection is shown in Figure 3. Total load resistance is about 4 kΩ.

\[
V_{\text{out}} = \text{output voltage without load.}
\]

\[
V^*_{\text{out}} = \text{output voltage with load connected.}
\]

The relation is given in Reference 7.

\[
V^*_{\text{out}} = V_{\text{out}} \frac{R_L}{Z_{\text{of}} + R_L} \quad (2-9)
\]

\[
R_L = \text{total load resistance}
\]

\[
Z_{\text{of}} = \text{output impedance of the closed-loop amplifier}
\]

\[
Z_{\text{of}} = \frac{Z_{\text{of}}}{1 + \frac{Z_L}{Z_F} A_0(\omega)} \quad (2-10)
\]

the percent error of the output voltage

\[
e = \frac{V_{\text{out}} - V^*_{\text{out}}}{V_{\text{out}}} \times 100\% \quad (2-11)
\]
Figure 3. Maximum possible load of an operational amplifier.

Total $R_L = 10 \, k\Omega \parallel 10 \, k\Omega \parallel 20 \, k\Omega = 4 \, k\Omega$
\[
e = \frac{Z_{oi}}{Z_{oi} + R_L} \quad \times 100\% \tag{2-12}
\]

\[
e = \frac{Z_{oi}}{Z_{oi} + \left[1 + \frac{Z_I}{Z_F} A_o(\omega)\right] R_L} \quad \times 100\% \tag{2-13}
\]

For the \(\mu A741\) operational amplifier using \(A_o(\omega)\) and \(Z_{oi}\) as given in the specification, Appendix A,

\[A_o(\omega) = 1.5 \times 10^5\]

\[Z_{oi} = 75 \Omega\]

\[Z_I = Z_F = 100 \, k\Omega\]

\[R_L = 4 \, k\Omega\]

e, percent error \[
\leq \frac{75}{75 + \left[1 + 1.5 \times 10^5 \times 4 \times 10^3\right]} \times 100 \rightarrow 0
\]

So the assumption that \(R_L \rightarrow \infty\) is proved to be valid. The specification that \(R_L \geq 2 \, k\Omega\) ensures that the transfer function will not be affected by the load resistance, \(4 \, k\Omega\).

2. \(Z_i \gg Z_I/ (Z_F + Z_{oi})\)

\[Z_i = 2 \, M\Omega\]

\[Z_F = Z_I = 100 \, k\Omega\]

\[Z_{oi} = 75 \Omega\]

\[Z_I/ (Z_F + Z_{oi}) \leq 50 \, k\Omega \ll 2 \, M\Omega\]

3. \(Z_{oi} \ll Z_F\)

\[Z_{oi} = 75 \Omega\]

\[Z_F = 100 \, k\Omega\]
4. \[ \frac{A_o(\omega)Z_I}{Z_F + Z_I + A_o(\omega)Z_I} \rightarrow \frac{Z_F}{Z_I} \]

requiring that \( A_o(\omega)Z_I \gg Z_F + Z_I \).

For the example note that
\[ A_o(\omega)Z_I = 1.5 \times 10^5 \times 100 \, k\Omega \gg 200 \, k\Omega. \]

D. Offset, Drift and Noise

For a transistorized operational amplifier the input connections of the first stage differential amplifier are made directly to the transistor bases, or the gates and grids in case of FET and tube amplifier. Thus, it is unavoidable to have small dc bias currents. The average of the two dc input currents is termed the input bias current. The difference in the currents into the two inputs with output voltage equal to zero is termed the input offset current. The voltage which must be applied between the input terminals to obtain zero output voltage is termed the input offset voltage.

The total offset at the input terminals is equal to the sum of the input offset voltage and the voltage drop across the input impedance due to the input offset current. Although this offset is rather small, its effect may be significant when the operational amplifier is used as an integrator. In monolithic operational amplifiers the offset is often compensated by inducing a small difference in the collector current of the two channels of the first stage differential amplifier.

A fixed input offset is usually not a problem since it can be compensated, but both input offset voltage and input offset current
Drift is usually specified as the coefficient of input offset change. Drift, itself, may be considered as a combination of thermal noise and excess noise (1/f noise). Three main factors that affect drift are temperature, time and power supply voltage. Drift in a dc amplifier is a serious problem since the offset cannot be distinguished from a change in signal. Due to drift, the operational amplifiers used in analog computers need to be balanced after some period of operation.

Figure 4 is the equivalent circuit of operational amplifier for studying the effect of bias current and offset.

Total offset voltage at the input = $V_E$
Maximum total offset voltage at the input = $V_{EM}$

$$V_E = V_Os - (I_{os} + I_{B2}) (R_I/ R_F) + I_{B1} R_R$$  \hspace{1cm} (2-14)

Set $R_R$ equal to $R_I/ R_F$ to cancel the effect of bias current. So

$$V_E = V_{os} + I_{os} R_R$$ \hspace{1cm} (2-15)
$$V_{EM} = V_{os} + I_{os} R_R$$ \hspace{1cm} (2-16)

For $\mu A741$ operational amplifier, from Appendix A

$V_{os} = 1 \text{ mV typical}$
$I_{os} = 20 \text{ nA typical}$
$R_R = 50 \text{ k} \Omega$

Expected value of the maximum total offset at the input of $\mu A741$ operational amplifier is

$$\overline{V_{EM}} = 10^{-3} + 50 \times 10^{-3} \times 20 \times 10^{-9} \text{ V}$$
$$\overline{V_{EM}} = 2 \text{ mV}$$
Figure 4. Equivalent circuit for studying effect of bias current, offset current and offset voltage.
For the μA741 there was no available data about time drift, so a measurement was made as shown by the schematic in Figure 5 to investigate this effect.

A 50-0-50 microammeter in series with 490Ω resistance was used to detect the null error. The offset was amplified 8.3/0.101 times. After the null adjustment was made, the offset was so small that the time integral of the offset was measured instead of the offset itself.

The recorded curve of integrated drift is shown in Figure 6.

$$E_0 = \frac{1}{RC} e_{os} t$$

(2-17)

where

$$E_0 = \text{output voltage at the end of time } t$$
$$e_{os} = \text{offset voltage at the input}$$
$$\frac{1}{RC} = \frac{1}{0.1 \times 1} = 10 \text{ seconds}$$
$$t = \text{integrating time} = 1 \text{ minute}$$
$$e_{os} = \frac{E_0}{10 \times 60}$$

The record was made during the period July 23 to July 26, 1971. The maximum $E_0$ obtained was 0.19 volt on July 24, at 3:00 p.m.

$$e_{os} = \frac{0.19}{10 \times 60} = 0.32 \text{ mV}$$

Thus, within a four-day period, the offset drifted to a maximum value of 0.32 mV. The approximate time drift is 0.1 mV/day.
Figure 5. Schematic diagram for measuring amplifier drift.
Figure 6. Recorded results for determining the drift.
In the \( \mu \text{A741} \) the offset is compensated by a 10 k\( \Omega \) potentiometer connected between pins 1 and 5. This technique is capable of compensating up to 15 mV of the offset at the input and avoids the use of a 1 M\( \Omega \) resistance for \( R_I \).

\( Z_I \) and \( Z_F \) are selected to perform the mathematical operation required. Thus the equivalent resistance connected to the inverting input, \( R_I//R_F \), is not fixed at 50 k\( \Omega \) as assumed, and so the idea that the effect of the bias current is cancelled by setting \( R_R = R_I//R_F \) can not always be achieved because of varying gain requirements. \( R_I//R_F \) can take a minimum value of 4.76 k\( \Omega \) (when both inputs are connected to 10 k\( \Omega \) and \( Z_F \) is 100 k\( \Omega \)) to a maximum value of 100 k\( \Omega \) (when one input is connected to 100 k\( \Omega \) and \( Z_F \) is a capacitance). The error voltage at the input depends upon the equivalent resistance connected to the inverting input and could be as high as 5 mV. \( R_R \) was chosen to be 50 k\( \Omega \) and the nulling circuit set as \( (R_I//R_F) = (100 \text{ k}\Omega//8.3 \text{ M}\Omega) \). This is proposed to minimize the effect of bias current when the operational amplifier is used as an integrator.

The operational amplifier with FET input stage has lower input bias current and lower input offset current. The error due to input offset current may be reduced by the use of the operational amplifier with FET input stage. However, the input offset of the FET input operational amplifier is more sensitive to temperature change. It may not be very satisfactory unless operating in a temperature-controlled environment.
Higher frequency changes in the input current and voltage are referred to as noise, which is a combination of thermal noise and shot noise. Noise model of the operational amplifier for studying the effect of noise is shown in Figure 7.

\[
\begin{align*}
E_{\text{sig}} & = \text{signal voltage} \\
E_{\text{ns}} & = \text{thermal noise of source resistance} \\
& = 4KTR_s \Delta f \quad (2-18) \\
E_n & = \text{equivalent noise voltage generator} \\
& = \sqrt{\int_{f_1}^{f_2} e_n^2 df} \quad (2-19) \\
I_n & = \text{equivalent noise current generator} \\
& = \sqrt{\int_{f_1}^{f_2} i_n^2 df} \quad (2-20) \\
E_{\text{ni}} & = \text{total equivalent noise voltage at the input} \\
& = \sqrt{E_{\text{ns}}^2 + E_n^2 + I_n^2 Z_s + 2CE_n I_n Z_s} \quad (2-21)
\end{align*}
\]

Note that the last term approaches zero because \( C \), correlation coefficient, is zero over most of the frequency range. \( Z_s \) = source impedance

\[
Z_s = \frac{Z_i}{Z_f}
\]

let \( Z_i = Z_f = 100 \text{ k}\Omega \)

\[
Z_s = 50 \text{ k}\Omega \\
E_{\text{ns}} = 4KTR_s \Delta f = 1.61 \times 10^{-20} \times 5 \times 10^4 \times 10^6 \text{ V} = 8.05 \times 10^{-10} \text{ V}
\]
Figure 7. Noise model of the operational amplifier.\textsuperscript{9}
$E_{ni}$ is given in the data sheet, Appendix A, for the frequency range 10 Hz - 100 k Hz. Curve 15 on Page 84 indicates $12 \mu V$ rms for $Z_s = 50 k\Omega$.

From the curves 13 and 14, Page 84, $e_n^2$ and $i_n^2$ appear to be constant in the region $100 k\text{ Hz}$ to $1 M\text{ Hz}$.

\[ e_n^2 = 4 \times 10^{-16} \text{ V}^2/\text{Hz} \]
\[ i_n^2 = 3 \times 10^{-25} \text{ A}^2/\text{Hz} \]

For the total noise at the input in the frequency range 10 Hz - 1 MHz, taken as the bandwidth of the operational amplifier, we obtain:

\[
E_{ni} = \sqrt{49 \times 10^{-20} + 144 \times 10^{-12} + 4 \times 10^{-16} \times 9 \times 10^5 + 3 \times 10^{-25} \times (50 \times 10^3)^2 \times 9 \times 10^5}
\]

\[ = \sqrt{(144 + 360 + 675) \times 10^{-12}} \]

\[ = \sqrt{11.79 \times 10^{-10}} = 34.3 \mu V_{rms} \]

Total noise at the input is $34.3 \mu V_{rms}$ at a temperature of $25^\circ C$.

E. Stability, Bandwidth and Slew Rate

The gain of an ideal operational amplifier is constant, independent of frequency. But due to stray capacitance in the circuit and high frequency limitation of transistors the gain of an operational amplifier decreases and the phase shift between output and input voltages increases as the frequency increases. The attenuation of the amplifier
gain and the increase of negative phase shift with frequency creates the possibility of self-oscillation and increases the output error.

The voltage transfer function of an inverting feedback amplifier may be written:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-A_0(\omega)}{1+A_0(\omega)\beta} = -\frac{1}{\beta} \frac{1}{1 + \frac{1}{A_0(\omega)\beta}}
\]  

(2-22)

where \(A_0(\omega)\) is the open-loop voltage gain, a function of frequency, and \(\beta\) is the voltage transfer function of the feedback path.

As the open-loop voltage gain, \(A_0(\omega)\), decreases, \(1/A_0(\omega)\beta\) increases and so increases the output error. Consider the situation when \(A_0(\omega)\beta = -1\), \(V_{\text{out}} = -\infty\), and the system is unstable causing self-oscillation. For stable operation the loop gain, \(A_0(\omega)\beta\), of an amplifier must not be greater than unity at the frequency where phase shift is 180°. A conventional design criterion for unconditional stability is the requirement that the attenuation rate be at least -6 dB/octave at the closed loop gain crossing. For analog computer application the smallest required closed loop gain is usually unity so the operational amplifier must be frequency-compensated to obtain stability at unity closed loop gain.

Frequency compensation in a monolithic operational amplifier is usually achieved by the method of pole cancellation. An RC network is connected to the output stage to introduce a pole at a lower frequency and a zero to cancel the original pole.
This compensation network significantly reduces the slew rate. Slew rate is defined as the time rate of change of closed-loop amplifier output voltage under large signal conditions. It appears that the slew rate is usually related to the bandwidth of the open-loop amplifier but not the bandwidth of the closed-loop amplifier nor the closed-loop gain of the amplifier.

Bandwidth is defined as the frequency range over which the gain is approximately constant. The negative feedback extends the higher cutoff frequency by the factor \( 1 + A_0(\omega)\beta \). The bandwidth of an amplifier is a function of its closed-loop gain. The product of bandwidth and the closed-loop gain of an amplifier with a fixed compensation network is nearly a constant, usually referred to as the gain-bandwidth product.

The \( \mu A741 \) operational amplifier is internally compensated. Its gain-bandwidth product is \( 10^6 \), that is, the bandwidth is 1 MHz when the closed-loop gain is unity and 100 kHz when the closed-loop gain is ten. Its slew rate is 0.5 V/μsec.

F. Linearity

Output and input of an operational amplifier should be linearly related, that is \( V_{out}/V_{in} = -Z_F/Z_I \). But the output voltage of any realizable amplifier is limited, and the relationship between input and output voltage is not linear throughout the entire range of output voltage. At the higher limits of output voltage the amplifier is operating in the saturation region. To ensure that the closed-loop gain is linear, operating the amplifier with the output voltage near
the higher limits should be avoided. To avoid operating the operational amplifier in its saturation region which leads to inaccurate results and possible damage to the amplifier, overvoltage indicators are used to warn the operator whenever the output voltage is beyond the desired range.

When the \( \mu A741 \) operational amplifier is supplied by \( \pm 15 \) V power supply its output voltage range is \( \pm 14 \) V. This assures that it will operate linearly in the required range of \( \pm 10 \) V. Although operating in the saturation region will not damage the \( \mu A741 \) operational amplifier, it leads to incorrect results. The overvoltage indicators were not included in this computer. The approximate maximum value for each amplifier should be determined during magnitude scaling. When a problem is run on the computer, any critical value should be checked by the output readout device to ensure that the maximum voltage at the output of any amplifier does not exceed \( \pm 12 \) V.
CHAPTER III

DESIGN

A. General Description

The schematic diagram of the analog computer is shown in Figure 8. It consists of:

Nine operational amplifiers manually balanced by screwdriver adjustments on the front panel.

One four quadrant analog multiplier.

±15 Volts regulated power supply for all active components, capable of supplying current higher than 150 mA.

10 Volts ungrounded, regulated power supply for coefficient potentiometer setting and initial condition voltage supply.

50-0-50 Microammeter, 2 percent full scale accuracy, calibrated to read ±1 V, ±3 V, and ±10 V. Meter is also used for output voltage reading, coefficient potentiometer setting and offset nulling.

One four contact relay for manual operation, repetitive operation and reset. In the repetitive operation mode the rate of repetition can be varied from 1/15 Hz up to 35 Hz.

Five 10 kΩ coefficient potentiometers.
Figure 8. Schematic diagram for the analog computer
These components are enclosed in the case for the EC-1 Heathkit Analog Computer with original vacuum tube components removed. Computer programming is done on the front panel using plug-in computing elements and patch cords.

B. Operational Amplifier

Detailed characteristics of the operational amplifier are given in Chapter 2. Table 1 shows its characteristics in comparison to three other operational amplifiers used in EC-1, EASE and EAI380. These characteristics are as given in specifications released by the manufacturer. Some parameters, however, are not available. It may not be convenient or necessary in all cases to make the measurements, so estimations will be made instead from the circuit diagrams and available data.

EC-1 Heathkit's amplifier is the most simple operational amplifier, consisting of a pentode cascaded to a triode. This gives an unstabilized amplifier with open-loop voltage gain of $10^3$.

EASE's amplifier consists of a high gain dc amplifier of gain $10^5$ with stabilizing amplifier of gain $10^3$. This gives a high performance operational amplifier of gain $10^8$ and very low drift. Its deficiencies include poor characteristics of the tube; lack of ruggedness, short life-time, large physical size, and lengthy warm-up period.

The EAI380 amplifier consists of a high gain dc amplifier and a chopper stabilizer. This transistor operational amplifier gives dc gain of $3 \times 10^7$, operating in the range of $\pm 10$ V. The computing elements are connected internally.
Table 1. Characteristics of $\mu$A741 in comparison to three other operational amplifiers.

<table>
<thead>
<tr>
<th></th>
<th>EC-1 Heathkit</th>
<th>Mod. 1480 op-amp EASE</th>
<th>Mod. 6,614-2 dual dc amp EAI380</th>
<th>$\mu$A741</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic element</td>
<td>Tubes</td>
<td>Tubes</td>
<td>Transistors</td>
<td>Integrated circuit</td>
</tr>
<tr>
<td>Power supply</td>
<td>+300 V</td>
<td>+250 V</td>
<td>$\pm$15 V</td>
<td>$\pm$15 V</td>
</tr>
<tr>
<td></td>
<td>-150 V</td>
<td>-465 V</td>
<td>$\pm$30 V</td>
<td>$\pm$30 V</td>
</tr>
<tr>
<td>Power consumption</td>
<td>1.5 W</td>
<td>4.8 W dc</td>
<td>11.97 W ac</td>
<td>50 mW</td>
</tr>
<tr>
<td>(stand by)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating range</td>
<td>$\pm$60 V</td>
<td>$\pm$100 V</td>
<td>$\pm$10 V</td>
<td>$\pm$10 V</td>
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<tr>
<td>DC voltage gain</td>
<td>$10^3$</td>
<td>$10^8$</td>
<td>$3 \times 10^7$</td>
<td>$2 \times 10^5$</td>
</tr>
<tr>
<td>$Z_{in}$</td>
<td>High</td>
<td>$5 \times 10^6 \Omega$</td>
<td>$10^4 \Omega$</td>
<td>$2 \times 10^6 \Omega$</td>
</tr>
<tr>
<td>$Z_{out}$</td>
<td>400 $\Omega$</td>
<td>0.01 $\Omega$</td>
<td>50 $\Omega$</td>
<td>75 $\Omega$</td>
</tr>
<tr>
<td>Input offset after</td>
<td>75 mV</td>
<td>100 $\mu$V</td>
<td>20 $\mu$V</td>
<td>0.1 mV</td>
</tr>
<tr>
<td>balancing</td>
<td>**</td>
<td>*</td>
<td>**</td>
<td>**</td>
</tr>
<tr>
<td>Drift</td>
<td>$\pm$5 mV</td>
<td>100 $\mu$V/day</td>
<td>Very low</td>
<td>100$\mu$V/day</td>
</tr>
<tr>
<td></td>
<td>short term</td>
<td></td>
<td></td>
<td>**</td>
</tr>
<tr>
<td>Input noise</td>
<td>4 mV$\text{rms}$</td>
<td>5 mV$\text{rms}$</td>
<td>1.06 $\mu$V$\text{rms}$</td>
<td>34.3 $\mu$V$\text{rms}$</td>
</tr>
<tr>
<td>Bandwidth of unity</td>
<td>500 Hz</td>
<td>10 kHz</td>
<td>125 kHz</td>
<td>1 MHz</td>
</tr>
<tr>
<td>gain inverter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slew rate (V/sec)</td>
<td>$5 \times 10^4$</td>
<td>**</td>
<td>$2 \times 10^5$</td>
<td>$6 \times 10^5$</td>
</tr>
</tbody>
</table>

* Estimated from available data
** Measured value

Otherwise obtained from References 13, 14 and 15.
The $\mu$A741 integrated circuit operational amplifier which was selected to replace the EC-1 Heathkit's operational amplifier is extremely small and has very low power consumption. The main advantages of an integrated circuit operational amplifier are its circuit simplicity and its low cost. Its disadvantages are due to limited performance caused by price versus performance considerations in the integrated circuit manufacturing. The $\mu$A741, although not considered as a high performance computing amplifier in terms of drift and voltage gain, satisfactorily fulfills the requirements of a computing amplifier. The circuit diagram of the operational amplifier is shown in Figure 9.

C. Power Supply

Each operational amplifier requires a power supply of ± 15 volts with supply currents of 1.7 mA up to a maximum of 2.8 mA. The power supply must provide ± 15 volts with sufficient current for these nine operational amplifiers, the voltage follower and the multiplier. Total required current is about 35 mA. The power supply must be line-regulated to prevent the variation of output voltages due to line voltage changes and load current-regulated to prevent interference between various operational amplifiers. The specifications for regulated power supplies depend on the size and precision of the computer, generally regulated to within 0.1 to 1.0 percent of their nominal values for anticipated variations in the line voltage and load current. The power supply should also have a low temperature coefficient (0.01 percent/°C to 0.05 percent/°C) and low ripple (3mVpp to 100 mVpp). For an
To front panel amplifier connection

Figure 9. Circuit diagram for the operational amplifier, μA741.
integrated circuit operational amplifier the power supply sensitivity is always specified, usually in terms of the ratio of the change in input offset voltage to the change in the supply voltage. Specifications of the power supply may be based on this characteristic of the operational amplifier.

μA723 integrated circuit voltage regulators are used. The circuit diagram is shown in Figure 10. This connection is as given by the manufacturer, Fairchild Semiconductor. The temperature coefficient is 0.002 percent/°C, ripple reduction is 74 dB, line regulation is 0.02 percent and load regulation is 0.03 percent.

Both positive and negative output voltages change less than 0.5 V when the input voltage changes from † 18 V to † 30 V and the load current changes from zero to 150 mA. Ripple cannot be detected on an oscilloscope with a scale sensitivity of 50 mV/cm.

D. 10 V Regulated Power Supply

The computer is equipped with a 10 V regulated power supply to be used as a reference voltage in coefficient potentiometer setting and as an initial condition voltage supply. The circuit diagram of the power supply is shown in Figure 11. The Darlington pair Q₁, Q₂ functions as a series control element. Q₃ performs as a comparator, comparing a sample of the output voltage to the reference voltage drop across the Zener diode.

The output voltage remains constant at 10 V as the input voltage changes from 18 V to 30 V and the load current changes from 0 to 50 mA. Ripple cannot be detected on an oscilloscope with a scale sensitivity of 50 mV/cm.
Figure 10. Circuit diagram for power supply voltage regulator, \( \mu A723 \).
Figure 11. Circuit diagram for 10 V power supply regulator.
E. Panel Meter

A 50-0-50 microammeter is the only readout device furnished in the computer. An XY recorder, oscilloscope, or strip chart recorder may be used to display the output by connecting to the amplifier output terminals. The meter is also used to read the voltage applied to the meter input terminals and to detect the amplifier nulling. Instead of using exact resistance, resistances in series with potentiometers are used to provide meter calibration. The meter connection is shown in Figure 12. Current through the meter is limited at about 140 µA. The voltage drop across the 5 kΩ series resistance is limited by the breakdown voltage of the diode 1N251, 0.7 V.

F. Amplifier Nulling

Offset of the operational amplifiers can be compensated without removing the problem setup. A nulling switch on the front panel connects the amplifier to ground through 0.101 MΩ input resistance and 8.3 MΩ feedback resistance. The input offset is thus amplified 83 times and then detected by the microammeter in series with 490Ω resistance. This gives a very sensitive detector. The input offset of the operational amplifier can be reduced to less than 0.1 mV. Two 1N251 diodes are connected in parallel with the meter to protect it from any transient high voltage that may occur while depressing the nulling switch. The simplified diagram for amplifier nulling is shown in Figure 13.
From meter function switch

Output no. 1
Output no. 2

Meter range switch

Figure 12. Panel meter connection.
Figure 13. Simplified diagram for amplifier nulling.
G. Coefficient Potentiometer Setting

A 10 V regulated power supply incorporated with the meter is used for setting coefficient potentiometers.

The main error in coefficient potentiometer setting is due to the additional voltage drop in \((1-\alpha) R_p\) caused by the current flowing to the load resistance, \(R_L\). Figure 14 shows coefficient-setting potentiometer with connecting load. The effect is described by an expression for the effective potentiometer setting.

\[
k = \frac{E_k}{E_I} = \frac{\alpha}{1 + \alpha(1-\alpha) \frac{R_p}{R_L}}
\]

Note that \(k\), the voltage attenuation, is a function of the potentiometer setting, \(\alpha\), and the ratio of the potentiometer resistance to the load resistance, \(\frac{R_p}{R_L}\). Error correction by using the given equation is tedious. But with a high input resistance meter, accurate potentiometer setting is easily accomplished by a direct measurement of the voltage with the load resistance connected.

The meter input binding posts are connected to the meter through a voltage follower which provides a high input resistance of 400 MΩ. The \(\mu A725\) integrated circuit operational amplifier is connected as a voltage follower. The frequency compensated network shown in Figure 15 is connected to prevent oscillation. The \(\mu A725\) was selected because of its low drift and low offset. With the voltage follower configuration its offset and drift are negligible. The input offset is 0.6 mV and the drift is 1.5 \(\mu V/\)day. With no connection to the meter input binding post, the output of the voltage follower is -13.0 V.
Figure 14. Coefficient-setting potentiometer with load.
From meter input binding post

\[ C_2 = 0.02 \ \mu F \]
\[ R_2 = 10.5 \ \Omega \]
\[ C_2 = 0.05 \ \mu F \]

Figure 15. \( \mu A725 \) connected as a voltage follower.
H. Multplier

Among various types of electronic multipliers on the market, the time division multiplier gives the best accuracy and the best performance attainable, while the transconductance multiplier gives moderate accuracy at lowest cost. The AD530K is a monolithic transconductance multiplier manufactured by Analog Devices Inc., priced at $45.00. The scale factor is externally adjustable. The input and output offsets are externally trimmed. Its total error is about one percent. Detailed specifications and the pin configuration as given by the manufacturer are in Appendix B.

Figure 16 shows circuit connection of the multiplier. When used as a multiplier the \( Z_{in} \) terminal is connected to the output terminal, and when used as a divider the \( Y_{in} \) terminal is connected to the output terminal. Squaring and square root operations are accomplished by connecting \( X_{in} \) to \( Y_{in} \) when the AD530K is connected as a multiplier and a divider, respectively. When connected as a divider a negative value of \( X \), the divisor, is forbidden.

I. Repetitive Operation

Sometimes it is desirable to investigate the different behaviors resulting from a change in the parameters and/or initial conditions of the system. By automatically switching the computer between OPERATE and RESET modes while simultaneously changing the parameters or initial conditions, the effects of the change can be immediately observed on an oscilloscope.\(^1\) This is very convenient when using simulation to optimize the design of a system.
Figure 16. Circuit connection of the AD530K multiplier.
A multivibrator is used to operate the relay for repetitive operation. The rate of repetition can be varied from 1/15 Hz up to 35 Hz. The duration of the RESET period is about one millisecond which is long enough to let the capacitor discharge through the closed switch of the relay.

The circuit diagram of the multivibrator is shown in Figure 17. Q1 is a unijunction transistor operating as an astable multivibrator whose period of oscillation can be adjusted by a 40 kΩ variable resistor. The output from the multivibrator is fed to a Darlington pair Q2, Q3 which handles the high current required in operating the relay.

Photographs of the computer front panel and the computer chassis are shown in Figures 18 and 19.
Figure 17. Circuit diagram of the multivibrator for repetitive operation.
Figure 18. The front panel of the integrated circuit analog computer.

Figure 19. Component layout on the chassis of the integrated circuit analog computer.
CHAPTER IV

APPLICATIONS

The complexity of the problems that can be solved is limited because this analog computer contains only nine operational amplifiers and each amplifier has only two inputs. These restrictions are due to the front panel layout on the EC-1 Analog Computer and are not restrictions imposed by the use of integrated circuits. The analog computer will be used to solve a few simple equations that are often encountered in the study of electrical circuits and control system engineering. The solutions obtained from the analog computer will then be compared to the solutions obtained from hand calculation. A discussion follows concerning the accuracy of analog computations in general and an investigation of the sources of error which may arise.

A. Accuracy in Analog Computation

The accuracy of a solution obtained from analog computation depends upon the nature of the problem and the ability of the operator as well as the accuracy of the instruments. It is not possible to specify the accuracy of an analog computer without considering the precision of the computing components. Even if the precision of the computing components are known, it still may not be possible to specify the accuracy of the analog computer in a given problem. The precision of these components, such as input and feedback elements, limits the accuracy of solutions obtainable from an analog computer.
Instrumental error is due to individual errors in each of the instruments combined in some unspecified fashion. The instrumental error can be minimized by selecting suitable magnitude and time scales which make optimum use of an accurate readout device which has sufficient frequency response. Magnitude of the signal should be maximum without overloading the amplifiers while the computing time should be minimum provided that the results can be followed by the readout device.

Finite input impedance, finite open-loop voltage gain and nonzero output impedance of the operational amplifier introduce static error. But in this case the error is dominantly that due to inaccurate values of the computing elements, potentiometer settings and initial condition settings. All carbon film and deposited carbon resistors used for the computation were matched to within one percent. For economic reasons, five percent mylar capacitors (0.1 \( \mu \)F and 1.0 \( \mu \)F) and polystyrene capacitors (10 \( \mu \)F) were used. The coefficient potentiometers were inexpensive one-turn potentiometers without dial calibration. This necessitated setting potentiometers by means of the reference voltage and panel meter. This method gave accurate results to within two percent provided that the load remained connected in the circuit while setting the potentiometers.

Drift in the operational amplifiers is the major error when computation extends over a long period of time. The operational amplifiers should be nulled before each computer setup and the null verified whenever a high precision result is desired.
Frequency response and slew rate establish an upper frequency limit for the signal. The gain-bandwidth product of the \( \mu A 741 \) is \( 10^6 \). Thus the operational amplifier is capable of operating at a frequency above 10 k Hz. Its slew rate is \( 0.5 \) V/\( \mu \)sec.

The voltage transfer function of an analog integrator is approximately equal to \(-1/\text{RCS}\). Due to \( R_1 \), leakage resistance associated with the feedback capacitance, the voltage transfer function of an integrator is \( \frac{-R_1}{R} \frac{1}{R_1\text{CS} + 1} \). The low frequency gain of an integrator is limited by either open-loop voltage gain of the operational amplifier or the ratio of leakage resistance and input resistance, \( R_1/R \). This nonideal frequency response of the integrator may have significant effect upon some problems.

Sources of noise in the computer, including amplifier noise, pickup noise and noise in the readout device establish a threshold of usable signal at about 0.5 mV. The linear operating region of the operational amplifier establishes a maximum amplitude for the signal so the amplifier output voltage must not exceed \( \pm 12 \) V.

Accuracy of the solution is often limited by the readability and the accuracy of the readout device. Typical accuracies for these readout devices are: panel meter; 2.0\%, XY plotter; 0.1\% - 0.25\%, strip chart recorder; 0.5\% - 1.0\%, and oscilloscope; 5.0\% - 10.0\%, when they are properly calibrated and operating within their specified frequency ranges.
Accuracy of the computing components in this analog computer is limited by tolerances of the computing elements used; 1% in summation and 5% in integration. In general the accuracy of the solutions obtained from this computer can not be better than these limits.

B. Example 1, Simultaneous Algebraic Equations

\[2X_1 + X_2 + X_3 = 0 \quad (4-1)\]
\[X_1 + 2X_2 + X_3 = 7 \quad (4-2)\]
\[X_1 + X_2 + 2X_3 = 5 \quad (4-3)\]

To ensure stability of the computer setup the machine equations must be chosen so that the self-loop gain for each variable is less than unity.\(^4\)

Solving for the variable in Equations (4-1), (4-2) and (4-3)

\[2X_1 = 0 - X_2 - X_3 \quad (4-4)\]
\[2X_2 = 7 - X_1 - X_3 \quad (4-5)\]
\[2X_3 = 5 - X_1 - X_2 \quad (4-6)\]

The programming diagram for this problem is shown in Figure 20 with the loop gain for each variable equal to 1/12.

Computer results are \[X_1 = -2.9\]
\[X_2 = 4.0\]
\[X_3 = 1.9\]

Calculated results are \[X_1 = -3.0\]
\[X_2 = 4.0\]
\[X_3 = 2.0\]

Accuracy of the computed results is about 5 percent.
Figure 20. Programming diagram for Example 1.
C. Example 2, Second Order Differential Equation

The second order differential equation can be written in standard form as follows.

\[
\frac{d^2x}{dt^2} + 2 \xi \omega_n \frac{dx}{dt} + \omega_n^2 x = 0
\]  \hspace{1cm} (4-7)

where

\[\omega_n = \text{undamped natural frequency in radians per second}\]
\[\xi = \text{damping ratio}\]

Solution to this equation is

\[
x = e^{-\omega_n t} \left[ K_1 e^{\omega_n \sqrt{\xi^2 - 1} t} + K_2 e^{-\omega_n \sqrt{\xi^2 - 1} t} \right]
\]  \hspace{1cm} (4-8)

Waveshape of \( x \) depends on the damping ratio, \( \xi \):

- \( \xi = 0 \), \( x = x_0 \cos \omega_n t \) \hspace{1cm} Undamped
- \( \xi < 1 \), \( x = x_0 e^{-\xi \omega_n t} \cos \omega_n \sqrt{1-\xi^2} t \) \hspace{1cm} Underdamped
- \( \xi = 1 \), \( x = \frac{x_0}{2} (1+t) e^{-\omega_n t} \) \hspace{1cm} Critically damped
- \( \xi > 1 \), \( x = x_0 e^{-\xi \omega_n t} \cosh \omega_n \sqrt{1-\xi^2} t \) \hspace{1cm} Overdamped

The machine equation is

\[
\frac{1}{100} \frac{d^2x}{dt^2} + \frac{\beta}{10} \frac{dx}{dt} + x = 0
\]  \hspace{1cm} (4-9)

The corresponding \( \omega_n = 10 \) radians per second and \( \xi = \beta / 2 \)

The initial conditions are \( \frac{dx}{dt} (0) = 0 \)

and

\( x (0) = 8 \)
The programming diagram is shown in Figure 21. Waveforms of \( x, \) 
\[ \frac{1}{10} \frac{dx}{dt} \text{ and } -x \text{ versus } \frac{1}{10} \frac{dx}{dt} \] 
for various values of \( \beta \) are shown in Figures 22 and 23. From the recorded results, frequencies of oscillation and percent overshoot of \( x \) were measured and compared to the calculated results.

<table>
<thead>
<tr>
<th>( \zeta = \beta/2 )</th>
<th>0.0</th>
<th>0.10</th>
<th>0.25</th>
<th>0.50</th>
<th>1.0</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of oscillation</td>
<td>measured</td>
<td>9.8</td>
<td>9.9</td>
<td>9.7</td>
<td>7.0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>calculated</td>
<td>10.00</td>
<td>9.95</td>
<td>9.70</td>
<td>8.66</td>
<td>0</td>
</tr>
<tr>
<td>Percent overshoot of ( x )</td>
<td>measured</td>
<td>100</td>
<td>81</td>
<td>50</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>calculated</td>
<td>100</td>
<td>72</td>
<td>44</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

The accuracy of this solution is about 20 percent. Resolution of the oscilloscope trace severely limited the accuracy for this problem. Higher accuracy can be obtained by scaling down the problem time and using a more accurate recording device, such as X-Y recorder.

Note that noise seen in the pictures was noise in the oscilloscope rather than noise from the computer. This could be eliminated by using another oscilloscope.

D. Example 3, Simulation of an Electrical Network.

Kirchhoff's voltage equations of the circuit diagram shown in Figure 24 can be written as follows.

\[ e_i = L_1 \frac{di_1}{dt} + \frac{1}{C} \int i_1 \, dt - \frac{1}{C} \int i_2 \, dt \]  \hspace{1cm} (4-10)
Figure 21. Programming diagram for Example 2.
Figure 22. Computer solution for second order differential equation, Example 2.
Figure 23. Computer solution for second order differential equation, Example 2.
\[ \begin{align*}
0 &= R i_2 + L_2 \frac{d i_2}{d t} + \frac{1}{C} \int i_2 \, dt - \frac{1}{C} \int i_1 \, dt \\
\text{let } q_1 &= \int i_1 \, dt \text{ and } q_2 = \int i_2 \, dt \\
\dot{q}_1 &= -\frac{1}{L_1 C} q_1 + \frac{1}{L_1 C} q_2 + \frac{e_1}{L_1} \\
\dot{q}_2 &= -\frac{R}{L_2} \dot{q}_2 - \frac{1}{L_2 C} q_2 + \frac{1}{L_2 C} q_1
\end{align*} \] (4-11)

For the values given we can write the machine equations
\[ \begin{align*}
\dot{q}_1 &= -100 q_1 + 100 q_2 + (0.5 \sin 20t) \times 20 \\
\dot{q}_2 &= -10 \dot{q}_2 - 100 q_2 + 100 q_1
\end{align*} \] (4-12)

Programming diagram is shown in Figure 25. Waveforms of 10e, -10i1, -10i2, 10e1 versus -10i1 and 10e1 versus -10i2 are shown on Figure 26. The factors of ten were necessary for magnitude scaling of the equations. It must be noted that photographing reversed the pictures from left to right. Time scale must be read toward the left hand. Results obtained from the pictures are
\[ \begin{align*}
i_1 &= 1.45 \angle -90^\circ e_1 \\
i_2 &= 0.40 \angle 120^\circ e_1
\end{align*} \]

Results from hand calculation are
\[ \begin{align*}
i_1 &= 1.44 \angle 86.9^\circ e_1 \\
i_2 &= 0.40 \angle 126.8^\circ e_1
\end{align*} \]

The solution to this problem has an accuracy within 5 percent.

E. Example 4, Simulation of a Nonlinear Control System.

A block diagram for general switching controlled plant is shown in Figure 27.
$e_1 = 0.5 \sin 20t$

$R = 0.5 \Omega$

$L_1 = 0.05$ henry

$L_2 = 0.05$ henry

$C = 0.20$ farad

Figure 24. Circuit diagram of the electrical network simulated in Example 3.
Figure 25. Programming diagram for simulation of an electrical network of Example 3.
Figure 26. Computer solution of Example 3.
Specially consider the case of an ideal relay having control

effect $N$ with the governing function as $F(t) = k_1 e(t) + k_2 de(t)/dt$.
The plant transfer function is $1/(s^2 + 100)$. The block diagram of the

system to be simulated is shown in Figure 28. Let the driving

function $r(t)$ be equal to zero and investigate the characteristic

response to a set of initial conditions. As $r(t)$ is equal to zero,

e(t) is equal to $-c(t)$, the computer setup can be simplified as shown

in Figure 29.

The control equation of the system may be written as

$$e(t) + \omega_0^2 e(t) = -N \text{ sgn } F(t)$$ \hspace{1cm} \text{(4-16)}

Amplifiers #6 and #7, used for changing signs of $-e$ and $-e$,

were disconnected for no sign change. All resistances are 100 k\( \Omega \)

and capacitances are 1 \( \mu F \). Amplifier #1 with two zener diodes was

used to simulate the ideal relay with two possible outputs of +4.5 V

and -4.5 V.

The phase-plane plots of the system with various governing

functions are shown in Figure 30. The response of the system depends

upon the governing function $F(t)$.

For $k_1 = 0$, $k_2 = -1$ and $k_1$, $k_2 = 0$ the system response is
divergent.

For $k_1 = -1$, $k_2 = 0$ and $k_1 = 1$, $k_2 = 0$ the system response is

oscillatory.

For $k_1 = 0$, $k_2 = 1$ the response is undesirable for $e$ has a

finite value when input is zero.

For the case $k_1 = k_2 = 1$ the system seems to be satisfactory.
Figure 27. Block diagram of general switching controlled plant.
Figure 28. Block diagram of the system to be simulated in Example 4.
Figure 29. Computer setup for simulation of the switching controlled system, Example 4.
Figure 30. Phase-plane plot of the switching controlled system, Example 4.
Details may be found in Reference 10. Mathematical analysis of this system is rather involved so the accuracy of the computed result was not determined.

Similar switching control system with various nonlinear parameters can be simulated on this analog computer. The analog computer is useful in nonlinear system analysis because mathematical methods may be rather complicated and may not give very accurate results due to approximations made in the solutions.

F. Example 5, Second Degree Differential Equations.

The example to be simulated was Volterra's competition equation with the general form

\[
\frac{dx}{dt} = \alpha x - \beta xy
\]

\[
\frac{dy}{dt} = -\gamma y + \delta xy
\]

the equilibrium condition is at \( x = \gamma / \delta \) and \( y = \alpha / \beta \). The phase-plane plot of the system can be obtained from the equation

\[
\frac{dy}{dx} = -\frac{\gamma}{x} \frac{\delta x - \gamma}{\beta y - \alpha}
\]

the equations for computer setup are

\[
\frac{dx}{dt} = 2x - xy
\]

\[
\frac{dy}{dt} = -5y + 2xy
\]

the programming diagram is shown in Figure 31 and the phase-plane plot obtained from the analog computer, recorded by an XY plotter is as shown in Figure 32. From the recorded result the equilibrium of the
Figure 31. Programming diagram for Volterra's equation.
Figure 32. Computer solution for Volterra's competition equation, Example 5.
system is at $x = 2.4$, $y = 1.9$. From hand calculation the equilibrium of the system is at $x = 2.5$, $y = 2.0$. The accuracy of the computed result is about 5 percent.

G. Example 6, Narrowband Frequency Modulation.

The equation for phase modulation of a sinusoidal signal may be written as

$$f_c(t) = \cos (\omega_c t + \beta \sin \omega_m t)$$

$$= \cos \omega_c t \cos (\beta \sin \omega_m t) - \sin \omega_c t \sin (\beta \sin \omega_m t)$$

where

$f_c(t)$ is the narrowband FM output signal.

$\omega_c$ is the angular frequency of the carrier signal.

$\omega_m$ is the angular frequency of the modulating signal.

$\beta$ is the maximum phaseshift of the FM output.

for $\beta \ll \frac{\pi}{2}$ or $\beta < 0.2$ radian

$$f_c(t) \approx \cos \omega_c t - \beta \sin \omega_m t \sin \omega_c t$$

From this approximation, a narrowband frequency modulated signal can be generated by the system whose block diagram is shown in Figure 33. Diagram of the computer setup is shown in Figure 34. Waveforms of the modulating signal, carrier, output of the multiplier and the narrowband FM output are shown in Figure 35.

Because the maximum phaseshift, $\beta$, is very small, the frequency deviation of the output can not be observed. Also there was some amplitude modulation in the output. The output signal looks more like
Figure 33. Block diagram for narrowband FM system of Example 6.
$E_m \sin 2\pi \times 1.25 \times 10^2 t$

$E_c \sin 2\pi \times 10^4 t$

Figure 34. Computer setup for narrowband FM system of Example 6.
Figure 35. Recorded results for narrowband FM system, Example 6.
low modulation index AM signal than FM signal. In order to show that the output really contains frequency change, the output was fed to a voltage limiter and a frequency discriminator. The voltage limiter constrained the amplitude of the signal to $\pm 4.7$ volts, thus eliminating the amplitude modulation. The output from the voltage limiter was fed to a frequency discriminator which converted the frequency deviation to amplitude deviation. The output of the frequency discriminator contained a measurable but small amplitude change. The result was not recorded.

The computer setup for the voltage limiter and the frequency discriminator is shown in Figure 36. The voltage transfer function of the frequency discriminator was similar to an LC tank circuit with a resonant frequency at $0.81 \times 10^3$ Hz.

Modulation and demodulation are nonlinear processes, often requiring multiplication. This is usually accomplished by using devices whose characteristics are nonlinear or linear time-varying. The analog computer, designed for this research, contains one analog multiplier and nine operational amplifiers which can perform a wide variety of operations. This computer is thus a convenient instrument to simulate many systems of modulation and demodulation.
Figure 36. Operational amplifiers connected as voltage limiter and frequency discriminator for narrowband frequency modulation detection.
CHAPTER V

CONCLUSION

The purpose of this study has been fulfilled. The vacuum-tube amplifiers in the Heathkit EC-1 Analog Computer have been replaced by the integrated circuit operational amplifiers, μA741. All computing facilities and control circuits which were contained in the EC-1 Analog Computer were redesigned to operate with the μA741. One analog multiplier was included to extend the use of the computer to second degree equations. The computer was tested and used to solve several representative problems. Every unit functioned perfectly and the results obtained were satisfactory. Over-all performance of the analog computer is considerabily better than the EC-1 Analog Computer.

Chapter I serves as an introduction to the analog computer in general and specifies the problem of the research.

Chapter II is a study of the operational amplifier, the most essential part in an analog computer. In this chapter it is shown that general purpose integrated circuit operational amplifiers are suitable for the use as a dc amplifier in an analog computer. As a result, an analog computer can be built much less sophisticated and at much lower expense than by the conventional method of using discrete-component amplifiers.

Characteristics of the nonideal operational amplifier and their effects upon computed results were investigated. Chapter II also
discusses which parameters are considered to be important in selecting an operational amplifier. Manufacturer's technical data are often sufficient in evaluation of an operational amplifier. Table 1 shows characteristics of the \( \mu A741 \) operational amplifier in comparison with three other operational amplifiers. One may consult these data as an aid in selecting an integrated circuit operational amplifier.

Chapter III contains circuit diagrams and circuit descriptions of every unit in the analog computer. Design concepts and specifications for each unit are discussed. This chapter answers some questions that may arise in designing an analog computer.

Chapter IV presents typical examples of linear and nonlinear equations solvable on this computer. By this means some measure of the accuracy of the solution from this computer is obtained. This chapter may also be useful in the application of the analog computer to the solution of specific problems.

For a low cost analog computer with component accuracy of 1 percent, one may follow the design in Chapter III, but instead making use of higher precision computing elements, since it has been shown that the precision of this computer is limited by the precision of these computing components. The complexity of the problem solvable on an analog computer is usually limited by the number of operational amplifiers it contains. The front panel of the EC-1 Analog Computer has allowed space for only nine operational amplifiers and each amplifier can have only two inputs. For a more flexible computer, one should
discard the EC-1 case and design a better front panel. The use of internally connected input and feedback elements would result in a compact front panel. A removable patch panel allows the preservation of a computer setup for later reuse and increases the utility of a computer appreciably. A well-designed front panel layout is very important in a large analog and hybrid computer.

The author suggests that the present work be extended by designing a high precision and high performance analog or hybrid computer, using integrated circuits. It has been shown in this work that the major error of the operational amplifier is due to drift and offset current of the operational amplifier. This error can be suppressed by using a low drift and low input offset current operational amplifier. The operational amplifier with FET input has very low offset current. Although it is more sensitive to temperature change, it may be suitable if used in an air conditioned room. The use of a temperature-controlled preamplifier is advisable. The combination of μA727, temperature-controlled preamplifier, and μA741, general purpose operational amplifier, was tested and found to be very satisfactory.

Error due to computing elements such as input and feedback impedances, potentiometers and initial conditions can be overcome by using high precision components. A higher accuracy panel meter is necessary because the accuracy of potentiometer and initial condition setting is limited by the accuracy of the panel meter. A digital voltmeter is expensive but necessary if an accuracy of better
than 0.1 percent is desired. Otherwise potentiometer and initial condition settings must be done by means of a reference voltage, reference potentiometer and a null-meter, which is less reliable. Overload indicators and some operating control systems may be necessary.
REFERENCES


West Long Branch, New Jersey: Electronic Associates Inc., 
1969.

17. Schick, L. L. "Linear Circuit Application of Operational 

No. 4, April, 1971.

19. A New High Performance Monolithic Operational Amplifier, \( \mu \)A741. 
APPENDIX A

TECHNICAL DATA FOR \mu A741,

OPERATIONAL AMPLIFIER
FEATURES:
- NO FREQUENCY COMPENSATION REQUIRED
- SHORT-CIRCUIT PROTECTION
- OFFSET VOLTAGE NULL CAPABILITY
- LARGE COMMON-MODE AND DIFFERENTIAL VOLTAGE RANGES
- LOW POWER CONSUMPTION
- NO LATCH UP

GENERAL DESCRIPTION — The UA743 is a high performance, compensated operational amplifier, constructed on a single silicon chip, using the Fairchild Gatewell® process. It is designed for a wide range of analog applications. High common-mode voltage range of up to 12 ~ 15 volts with high output impedance makes the UA743 ideal for use as a voltage follower, the high gain and ease of operation make it ideal for use in high-speed, linear circuitry applications. The UA743 has the same pin configuration as the Fairchild 741 operational amplifier, but requires no external components for operation in open-loop. The silicon technology allows high-speed operation in closed-loop applications.

ABSOLUTE MAXIMUM RATINGS
Supply Voltage
Internal Power Dissipation
Differentiated Input Voltage
Input Voltage (Max)
Output Voltage (Max)
Voltage Between Input and Ground
Storage Temperature (Max)
Operating Temperature Range
Lead Temperature (Soldering, 10 sec)
Output Short-Circuit Duration (Max)

NOTES:
1. Rating applies for case temperatures to 70°C. Package leads at 6.5°C/°C for ambient temperature above 70°C.
2. For supply voltages less than ±15 V, the absolute maximum input voltage shall be equal to the supply voltage.
3. Short circuit may be by either supply. Supply must be ±15 V, the temperature is 25°C ambient temperature.

* Patented Fairchild process.

ORDER PART NO. US741312

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FAIRCHILD LINEAR INTEGRATED CIRCUITS µA741

ELECTRICAL CHARACTERISTICS (V₉ = ±15 V, Tₐ = 25°C unless otherwise specified)

<table>
<thead>
<tr>
<th>PARAMETERS (see definitions)</th>
<th>CONDITIONS</th>
<th>MIN.</th>
<th>TYP.</th>
<th>MAX.</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Offset Voltage</td>
<td>Rₓ ≤ 10 kΩ</td>
<td></td>
<td></td>
<td></td>
<td>mV</td>
</tr>
<tr>
<td>Input Offset Current</td>
<td></td>
<td>1.0</td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input Bias Current</td>
<td></td>
<td>20</td>
<td>200</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>Input Resistance</td>
<td></td>
<td>80</td>
<td>500</td>
<td></td>
<td>nΩ</td>
</tr>
<tr>
<td>Input Capacitance</td>
<td></td>
<td>2.0</td>
<td>500</td>
<td></td>
<td>µF</td>
</tr>
<tr>
<td>Offset Voltage Adjustment Range</td>
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<td>1.4</td>
<td></td>
<td>±15</td>
<td>mV</td>
</tr>
<tr>
<td>Large-Signal Voltage Gain</td>
<td>Rₛ ≥ 2 kΩ, Vₛ = ±10 V</td>
<td>50,000</td>
<td>200,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output Resistance</td>
<td></td>
<td>75</td>
<td></td>
<td></td>
<td>Ω</td>
</tr>
<tr>
<td>Output Short-Circuit Current</td>
<td></td>
<td>25</td>
<td></td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>Supply Current</td>
<td></td>
<td>1.7</td>
<td>2.8</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>Power Consumption</td>
<td></td>
<td>50</td>
<td>85</td>
<td></td>
<td>mW</td>
</tr>
<tr>
<td>Transient Response (unity gain)</td>
<td>Vₛ = 20 mV, Rₛ = 2 kΩ, Cₛ ≤ 100 pF</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Rise Time</td>
<td></td>
<td>0.3</td>
<td></td>
<td></td>
<td>µs</td>
</tr>
<tr>
<td>Overshoot</td>
<td></td>
<td>5.0</td>
<td></td>
<td></td>
<td>%</td>
</tr>
<tr>
<td>Slew Rate</td>
<td></td>
<td>0.5</td>
<td></td>
<td></td>
<td>V/µs</td>
</tr>
</tbody>
</table>

The following specifications apply for -55°C ≤ Tₐ ≤ +125°C:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Conditions</th>
<th>MIN.</th>
<th>TYP.</th>
<th>MAX.</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Offset Voltage</td>
<td>Rₓ ≤ 10 kΩ</td>
<td>1.0</td>
<td>6.0</td>
<td></td>
<td>mV</td>
</tr>
<tr>
<td>Input Offset Current</td>
<td>Tₓ = +125°C</td>
<td>7.0</td>
<td>200</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>Input Bias Current</td>
<td>Tₓ = +125°C</td>
<td>85</td>
<td>500</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>Input Voltage Range</td>
<td>Tₓ = +125°C</td>
<td>0.03</td>
<td>0.5</td>
<td></td>
<td>µA</td>
</tr>
<tr>
<td>Common Mode Rejection Ratio</td>
<td>Tₓ = +125°C</td>
<td>0.3</td>
<td>1.5</td>
<td></td>
<td>µA</td>
</tr>
<tr>
<td>Supply Voltage Rejection Ratio</td>
<td>Rₛ ≤ 10 kΩ</td>
<td>70</td>
<td>90</td>
<td></td>
<td>dB</td>
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<tr>
<td>Large-Signal Voltage Gain</td>
<td>Rₛ ≥ 2 kΩ, Vₛ = ±10 V</td>
<td>30</td>
<td>150</td>
<td></td>
<td>µV/V</td>
</tr>
<tr>
<td>Output Voltage Swing</td>
<td>Rₛ ≥ 10 kΩ</td>
<td>±12</td>
<td>±13</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>Supply Current</td>
<td>Tₓ = +125°C</td>
<td>1.5</td>
<td>2.5</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>Power Consumption</td>
<td>Tₓ = +125°C</td>
<td>2.0</td>
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<td>mA</td>
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TYPICAL PERFORMANCE CURVES

1 OPEN LOOP VOLTAGE GAIN AS A FUNCTION OF SUPPLY VOLTAGE

2 OUTPUT VOLTAGE SWING AS A FUNCTION OF SUPPLY VOLTAGE

3 INPUT COMMON MODE VOLTAGE RANGE AS A FUNCTION OF SUPPLY VOLTAGE
FAIRCHILD LINEAR INTEGRATED CIRCUITS \(\mu A741\)

**TYPICAL PERFORMANCE CURVES**

16 **OPEN LOOP VOLTAGE GAIN AS A FUNCTION OF FREQUENCY**

17 **OPEN LOOP PHASE RESPONSE AS A FUNCTION OF FREQUENCY**

18 **OUTPUT VOLTAGE SWING AS A FUNCTION OF FREQUENCY**

19 **INPUT RESISTANCE AND INPUT CAPACITANCE AS A FUNCTION OF FREQUENCY**

20 **OUTPUT RESISTANCE AS A FUNCTION OF FREQUENCY**

21 **COMMON MODE REJECTION RATIO AS A FUNCTION OF FREQUENCY**

22 **TRANSIENT RESPONSE**

23 **VOLTAGE FOLLOWER LARGE-SIGNAL PULSE RESPONSE**

24 **FREQUENCY CHARACTERISTICS AS A FUNCTION OF SUPPLY VOLTAGE**

25 **FREQUENCY CHARACTERISTICS AS A FUNCTION OF AMBIENT TEMPERATURE**
APPENDIX B

TECHNICAL DATA FOR AD530,

ANALOG MULTIPLIER
GENERAL DESCRIPTION

The Analog Devices AD530J and AD530K perform the functions of multiplying, dividing, squaring, and square rooting by combining a differential input transconductance multiplying element, an operational amplifier, and a zero regulated current source on a single chip of silicon. The devices are essentially self-contained, requiring only four trimming potentiometers and one resistor. Multiplying can be performed in four quadrants and dividing and square rooting in two quadrants. The AD530J and AD530K feature high accuracy, excellent temperature stability and linearity, wide bandwidth, and high output swing, and are available in the hermetically sealed TO-100 package for operation over the 0°C to +70°C temperature range.

ABSOLUTE MAXIMUM RATINGS

Supply Voltage (Vg) ±18V
Power Dissipation 500mW
XYZ Voltages ±Vg
Storage Temperature Range -65°C to +150°C
Operating Temperature Range 0°C to +70°C

ELECTRICAL CHARACTERISTICS (Vg = ±15V, R_L > 2KΩ, T_A = 25°C)

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<th>AD530K</th>
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<tr>
<td>Output Function</td>
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<td>XY/10</td>
</tr>
<tr>
<td>Total Accuracy</td>
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<td>0.5%</td>
</tr>
<tr>
<td>(externally trimmed)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>Adjustable</td>
<td>Adjustable</td>
</tr>
<tr>
<td>Divider Characteristics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output Function</td>
<td>10Z/X</td>
<td>10Z/X</td>
</tr>
<tr>
<td>Total Accuracy</td>
<td>0.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>(externally trimmed)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>Adjustable</td>
<td>Adjustable</td>
</tr>
<tr>
<td>General Characteristics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linearity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X Input (X = 20V p-p, Y = ±10Vdc)</td>
<td>±0.8%</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Y Input (X = 20V p-p, Y = ±10Vdc)</td>
<td>±0.3%</td>
<td>±0.2%</td>
</tr>
<tr>
<td>Feedthrough</td>
<td></td>
<td></td>
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<tr>
<td>(externally trimmed)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X = 0, Y = 20V p-p, f = 50Hz</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Y = 0, X = 20V p-p, f = 50Hz</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Output Offset (adjustable to zero)</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

This information is of a preliminary nature and is subject to change without notice.
AD530 TRIMMING PROCEDURES

A. The suggested procedure for trimming the AD530 for multiplier use is as follows:
1. With $X_{in} = Y_{in} = 0$ volts, adjust $Z_0$ for zero output.
2. With $Y_{in} = 20$ volts p-p (at $f = 50$Hz) and $X_{in} = 0$ volts, adjust $X_0$ for minimum feedthrough.
3. With $X_{in} = 20$ volts p-p (at $f = 50$Hz) and $Y_{in} = 0$ volts, adjust $Y_0$ for minimum feedthrough.
4. Readjust $Z_0$, if necessary.
5. With $X_{in} = 10$ volts dc and $Y_{in} = 20$ volts p-p (at $f = 50$Hz), adjust Gain for Output = $Y_{in}$.

NOTE 1: For best accuracy over limited voltage ranges (e.g. ±5V), gain and feedthrough adjustments should be optimized with the inputs in the desired range. When so optimized, the error may be greater over the specified (±10V) range. However, linearity is considerably better over smaller ranges of input.

B. The suggested procedure for trimming the AD530 for use as a divider is as follows:
1. Set all pots at mid-scale.
2. With $Z_{in} = 0$, trim $Z_0$ to hold the output constant, as $X_{in}$ is varied from -10V to +10V.
3. With $Z_{in} = 0$, trim $Y_0$ for zero, $X_{in} = -10V$.
4. With $Z_{in} = X_{in}$ and/or $Z_{in} = -X_{in}$, trim $X_0$ for the minimum worst-case variation as $X_{in}$ is varied from -10V to -1V.
5. Repeat steps 2 and 3 if step 4 required a large initial adjustment.
6. With $Z_{in} = X_{in}$ and/or $Z_{in} = -X_{in}$, trim the scale factor for the closest average approach to ±10V output as $X_{in}$ is varied from -10V to -3.0V.

APPLICATIONS OF THE AD530

MULTIPLIER

DIVIDER

SQUARE ROOTER

SQUARER
### ELECTRICAL CHARACTERISTICS (Continued)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AD530J</th>
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<th>AD530K</th>
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<th>Units</th>
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<tr>
<td></td>
<td>MIN</td>
<td>TYP</td>
<td>MAX</td>
<td>MIN</td>
<td>TYP</td>
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<tr>
<td>Dynamic Response</td>
<td></td>
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<td>Small Signal</td>
<td>1.0</td>
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<tr>
<td>(−3dB point)</td>
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<td>Full Power</td>
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<td>750</td>
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<td>Slew Rate</td>
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<tr>
<td>1% Amplitude Error</td>
<td>75</td>
<td>75</td>
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<td></td>
<td></td>
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<tr>
<td>1% Vector Error</td>
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<td>5</td>
<td></td>
<td></td>
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<tr>
<td>(0.5° phase shift)</td>
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<tr>
<td>Settling Time (to 2% of final value) (±10V pulse)</td>
<td>1</td>
<td>1</td>
<td>µsec</td>
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<tr>
<td>Overload Recovery (to 2% of final value)</td>
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<td>1</td>
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<td>Output Noise</td>
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<td>0.6</td>
<td>mV rms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5Hz to 5MHz</td>
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<td>mV rms</td>
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<tr>
<td>Input Resistance</td>
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<td>X Input</td>
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<td></td>
<td>MΩ</td>
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<tr>
<td>Y Input</td>
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<td>MΩ</td>
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<td>Z Input</td>
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<td>KΩ</td>
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<td>X Input, Y Input</td>
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<td>2</td>
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<td></td>
<td>µA</td>
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<tr>
<td>Z Input</td>
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<td></td>
<td>µA</td>
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<td>Power Supply Variation</td>
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<td>Multiplier Accuracy</td>
<td>0.2</td>
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<tr>
<td>Output Offset</td>
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<td></td>
<td>MV/V</td>
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<tr>
<td>Scale Factor</td>
<td>0.1</td>
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<td></td>
<td></td>
<td>%/%</td>
</tr>
<tr>
<td>Quiescent Current</td>
<td>3.5</td>
<td>6.0</td>
<td>3.5</td>
<td>6.0</td>
<td>mA</td>
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</tbody>
</table>

### ELECTRICAL CHARACTERISTICS \((V_S = \pm 15V, R_L \geq 2KΩ, T_A = 0^\circ C \text{ to } +70^\circ C)\)

**Multiplier Characteristics**

| Total Accuracy | 3 | 2 | % |

**General Characteristics**

<table>
<thead>
<tr>
<th>Average Temperature Coefficient</th>
<th>of Accuracy</th>
<th>0.06</th>
<th>0.06</th>
<th>%/°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>of Output Offset</td>
<td>0.2</td>
<td>0.2</td>
<td>mV/°C</td>
<td></td>
</tr>
<tr>
<td>of Scale Factor</td>
<td>0.04</td>
<td>0.04</td>
<td>%/°C</td>
<td></td>
</tr>
<tr>
<td>Input Bias Current</td>
<td>X Input, Y Input</td>
<td>0.6</td>
<td>5</td>
<td>µA</td>
</tr>
<tr>
<td></td>
<td>Z Input</td>
<td>1.0</td>
<td>25</td>
<td>µA</td>
</tr>
<tr>
<td>Input Voltage</td>
<td>K, Y, Z for rated accuracy</td>
<td>±10</td>
<td>±10</td>
<td>V</td>
</tr>
<tr>
<td>Output Voltage Swing</td>
<td>R_L \geq 2KΩ,</td>
<td>±10</td>
<td>±10</td>
<td>V</td>
</tr>
<tr>
<td>C_L \leq 1000pF</td>
<td>±10</td>
<td>±10</td>
<td>µA</td>
<td></td>
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