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An Investigation of the Effects of Instantaneous Jockeying in Queues by Simulation

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AN INVESTIGATION OF THE EFFECTS OF INSTANTANEOUS JOCKEYING IN QUEUES BY SIMULATION

BY

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AN INVESTIGATION OF THE EFFECTS OF INSTANTANEOUS JOCKEYING IN QUEUES BY SIMULATION

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser   Date

Head, Mechanical Engineering Department   Date

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GLOSSARY OF TERMS

λ = Arrival rate

μ_i = Service rate offered by i^{th} server

$\rho = \frac{\lambda}{\sum \mu_i}$ (utilization factor)

Q_{nm} = Probability that 'n' units are at first queue and 'm' units are at second queue.

P_o^i = Probability that i^{th} server is idle.

L_i = Number of units at i^{th} server.

L_{qi} = Number of units waiting for i^{th} server.

γ_i = Fraction of customers served by i^{th} server.

P_n = Probability that 'n' units are in the system.

M = Number of channels.

CHAPTER I

INTRODUCTION

In today's world, waiting lines formed by people, machines and other types of units are commonplace. Queues form whenever the current demand for a service exceeds the current capacity to provide that service. In many cases, the interarrival times or service times are not constants, but involve distributions from which the values occur randomly.

A waiting line causes concern when there is an economic value associated with it. A waiting line may have a negative monetary value if units leave without service, or if units have costs as they wait. Idle service facilities may also have a negative monetary value if they imply that the service facility is operating below the optimum level. The system analyst attempts to find an optimum solution by balancing the various costs associated with waiting lines and idle facilities.

Many different types of waiting situations have been formulated and studied [10]. A specific queuing model is a function of

1. Type of population
2. Service station configuration
3. Allowable queue length
4. Queue discipline
5. Interarrival time distribution
6. Service time distribution

Figure 1.1 indicates some of the various subclassifications of the above major classifications.

There are two ways that such systems of service stations and queues can be studied. The first is to define the system through a set of mathematical relations and is called queuing theory. The second is to define the system through logic relations and simulate the operation of the system. In queuing theory, the differential difference method, developed by A. K. Erlang, may be used to develop equations for statistics such as the average length of waiting line, the average waiting time, the average number of units in the system, and the average time in the system. This method uses Markov-chain procedures and the equations are written in terms of the transition matrix. The time-independent steady state solution is obtained by reducing the set of equations to a time-dependent differential equation and setting the rate of change to zero. Such equations, when there is a single channel, a poisson arrival rate distribution, an exponential service time distribution, and a first come first serve queue discipline, can be written and solved without much difficulty. Other simple systems can also be solved in this manner.

In practical problems, the interarrival time distribution and service time distribution may not be those which are directly and easily handled by queuing theory. Moreover, when the system has a complex pattern of parallel and sequential service stations, compounded by exotic queue disciplines, the steady state relations are difficult

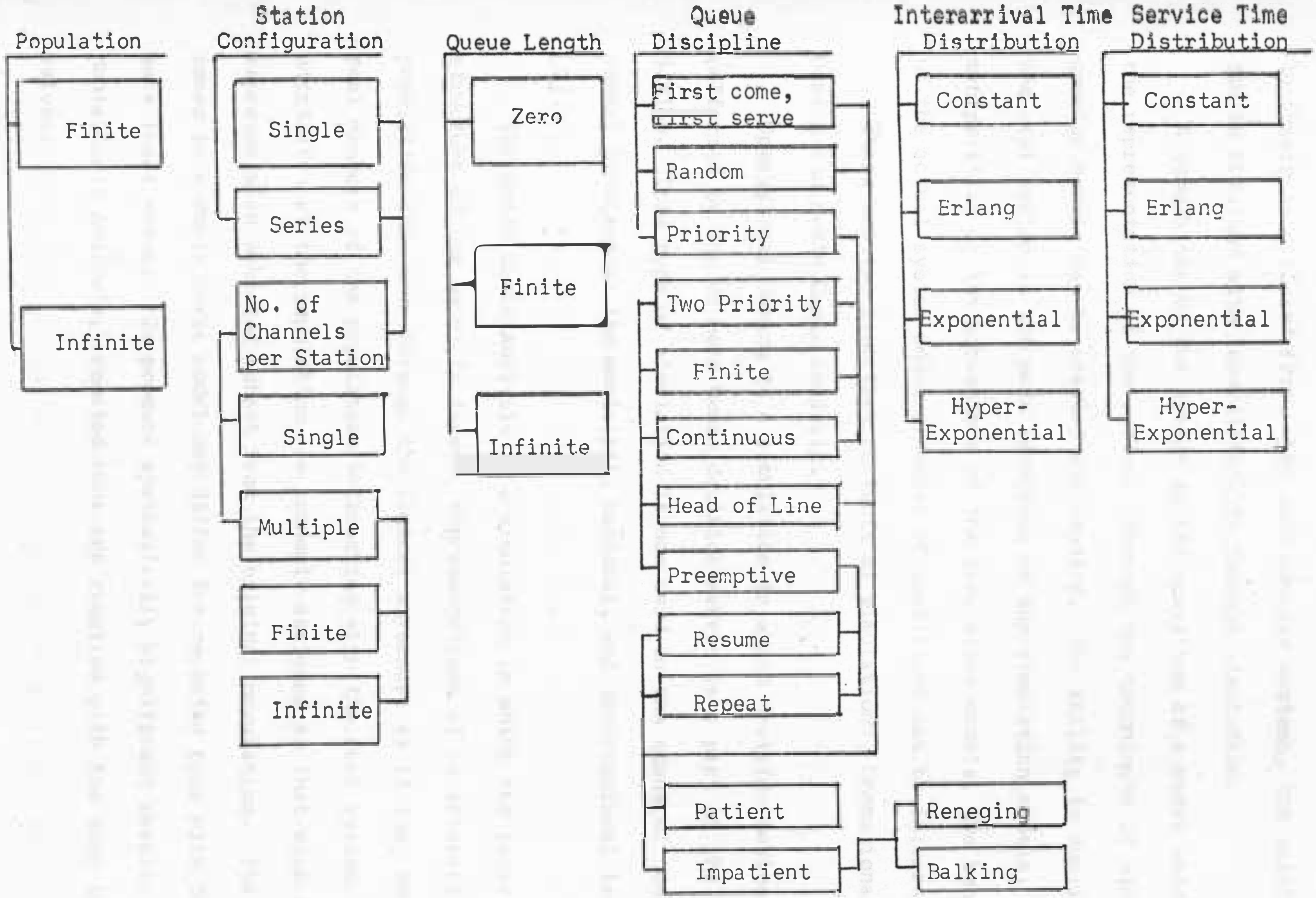

Figure 1.1. Variables in Queueing Situations [1].

to obtain in a closed form. For such complex systems, the solution can be obtained with less difficulty through simulation.

A simulation of the system is the operation of a model which is the representation of the system. Through the techniques of simulation, complex models may be created and studied. The ability to duplicate the real system is the main advantage of the simulation models. By manipulation of the parameters of the simulation models, the behaviour of the actual system under a number of conditions can be inferred.

There are two important variants of simulation: Operational Gaming and Monte Carlo Analysis.

Operational Gaming is a simulation in which decision making is performed by one or more human decision-makers as a part of the simulation. This type of simulation is now used in the study of governmental problems at the municipal, national, and international levels [2].

The Monte Carlo Analysis is a simulation in which the logical structure of the model is fixed. Representations of an interacting population then move through the logical structure as if they were real members of the population interacting with the real system. The attributes of the population are randomly assigned so that each sample represents an unbiased subset from the original population. The outcomes in a Monte Carlo model may differ for repeated runs with the same input values. To produce statistically significant results in a Monte Carlo Analysis, repeated runs are required with the same input values.

Essentially a computer Monte Carlo simulation involves the following elements:

1. The objectives of the simulation must be determined and a criterion must be established for evaluating the degree to which the objectives are fulfilled by the experiment.
2. A logical model (flow charts) must be constructed to simulate the system.
3. A computer program must be formulated that converts the model into an operable simulation program.
4. The program must be used as an experimental device to study the system that the program represents.

The program can be written in general purpose languages such as FORTRAN, ALGOL, COBOL, PL/I or in special purpose macro-languages such as GPSS, SIMSCRIPT, GASP, SIMPAC, DYNAMO or PROGRAM SIMULATE [8]. Among these languages, GASP is not a true macro-language but consists of a number of FORTRAN subroutines. Some of the macro-languages and their procedures have been discussed and compared by K. D. Tocher [11].

The principal advantage of using a special purpose language is that it requires less programming time. These languages have been written to facilitate the programming of certain types of systems. For example, PROGRAM SIMULATE was designed primarily for simulating large-scale economic systems, whereas GPSS, SIMSCRIPT, and GASP are well suited for scheduling and waiting line problems. In general, the computer running time is increased by using a macro-language over that for a general purpose language. Another advantage of macro-languages

is that they have error-checking techniques that can detect logical errors such as detecting if units are not terminated after service completion, as well as rule violations and capacity violations. The general purpose languages do not possess the ability to check such logical errors. The general purpose languages are relatively more flexible than the special purpose languages. Another consideration is the kind of output report needed to give required information about the simulated system. If a general purpose language is used, there will be a minimum number of restrictions imposed on the format of the output reports. In case of a special purpose language, the output format requirements of the language must be adhered to. All the macro-languages except GASP require, as a minimum, IBM 360/50 computers or their equivalent.

The choice of the language used in the simulation depends on the type of the computer available and the programmer's familiarity with the various languages. FORTRAN will be used in this simulation program because macro-languages are not available at the SDSU computer center.

With the development of simulation techniques, it is increasingly commonplace to simulate a complicated system for which mathematical solutions are hard to obtain. For example, the basic Queuing Theory for a multichannel system assumes either one queue, or multiple queues, in which the units are not allowed to switch lines. However, in many practical situations, each service channel has a separate and distinct waiting line and units do change from one line to another.

CHAPTER II

LITERATURE REVIEW

Although the idea of simulating systems on a computer is less than twenty years old, there is extensive literature on simulation studies. With the development of high-speed computers, there has been a tremendous growth in the use of simulation techniques. Since this study is oriented towards a waiting line model, only models pertaining to the queuing systems will be reported.

A service station may have one of the following forms:

1. one queue, one server
2. one queue, multiple servers
3. sequential channels, with or without queues
4. multiple queues, multiple servers

The total facility may involve combinations of these stations.

Mathematical solutions have been obtained for one queue, one server and one queue, multiple servers models with various combinations of the other variables [10,1,7]. Therefore these models are simulated only to verify a particular simulator.

In the sequential channel model, mathematical equations have been developed for the two stations-in-series case with no queue allowed before either station. In this, the arrival rate distribution is a poisson distribution and the service time distribution is the exponential distribution [1]. Equations have also been developed for many other situations [10]. These could be simulated and verified.

The multiple queues, multiple servers model using infinite populations and infinite queues will be considered in depth as it involves customer strategies other than balking and reneging. They are:

1. Preference for a particular channel when queues are of equal length or when all channels are empty.
2. Joining the shortest queue if the queues are different lengths.
3. Changing from one queue to another.

The first theoretical multiple queues, multiple servers model was derived as a number of individual single queue, single server models [7]. For example, in a two queues, two servers model, if arriving units choose between the queues at random, the system is really just two independent single channel systems, each with an arrival rate of $\frac{1}{2}\lambda$ and an identical service rate of μ.

The strategy of preference was considered by Krishnamoorthy [5] in his one queue, two servers model. Here, the units will use their preference only when both servers are free. This model assumed a poisson arrival rate and exponential service time for both servers. The service rate of the servers need not necessarily be the same. First come, first serve queue discipline was used.

The strategy of joining the shortest queue was considered by Koenigsberg [3] in his two queues, two servers model, with a poisson arrival rate and exponential service time for both servers. The service rate of the servers need not be the same.

He also showed that preference has no effect when the strategy of joining the shortest queue is also considered.

The strategy of switching queues (changing from one queue to another) is called jockeying. A waiting customer jockeys in anticipation of shorter delay. There are two types of jockeying:

1. Probabilistic jockeying
2. Instantaneous jockeying

In the probabilistic jockeying, the customers leave the longer queue at some rate proportional to the difference in lengths of the waiting lines.

In the instantaneous jockeying, the customer in the longer queue jockeys to the shorter queue, when the difference in queues exceeds one.

Koenigsberg [3] derived equations for a two channel model for both types of jockeying. He assumed a poisson arrival rate and exponential service time for both servers. The service rate of the servers could be different. He allowed an infinite queue before each channel. In the probabilistic jockeying model, he showed that the results are the same as the two queues, two servers model with no jockeying allowed when the arriving units always join the shorter queue.

A final derivation was for a two queues, two servers model with preferences and jockeying allowed. In this model the arriving unit will join the shortest queue. If the queues are of equal length then the unit will join a preferred queue. Whenever the difference in line lengths exceed one, the unit from the longer line jockeys to the shorter line.

Therefore, the basic mathematical solutions have been developed in the multiple queues, multiple servers with various customer strategies considered. They are limited to a two channel case with a poisson arrival rate and exponential service times. It does not appear practical to extend these models to more channels, particularly when it may be desirable to consider other distributions of interarrival times and service times. A simulation model is therefore proposed that will accommodate the situations outlined in Table 2-1.

TABLE 2-1.

Capability of the Simulation Model

1. Number of channels	Minimum of 2 Maximum of 10
2. Type of queue	Multiple queues with infinite or finite number in each queue. (If finite, the maximum number of queue allowed before each channel is same).
3. Interarrival time distribution	a. exponential b. constant c. ten-celled histogram
4. Service time distribution	a. exponential b. constant c. ten-celled histogram
5. Queue discipline	first come, first serve with instantaneous jockeying
6. Customer preference is allowed when queues are equal or missing.	

CHAPTER III

MODEL AND PROCEDURE

The multiple queues, multiple servers model with jockeying is a queuing situation for which inadequate models have been developed. It is desirable to fabricate a simulation model that will allow such systems to be better understood.

In this model, three customer strategies are considered:

1. Joining the shortest queue.
2. Joining a preferred queue, when the queues are of equal length.
3. Switching to the shorter queue when one exists.

Jockeying will occur immediately whenever the difference between any two queue lengths is greater than one. The simulation program will accommodate as many as ten parallel channels with either an infinite or finite queue for each channel. Each channel may have a different mean service time and service time distribution. The interarrival time distribution and service time distribution need not necessarily be the same. The interarrival and service time will each have one of the following distributions:

1. constant
2. exponential
3. a histogram with a maximum of ten cells

The generalized flow chart for this computer simulation is shown in Figure 3.1. The variable time increment procedure is used. Jockeying takes place after a service is completed, but before the clock changes.

It is worthwhile to follow through the computer logic. At a given clock time, the computer first must determine if the time is for an arrival or a service completion time. If it is an arrival then the next arrival time is generated by computing a pseudo-random number. In case of infinite queue situation, the arrival is placed in the shortest queue, if a difference exists, or in a selected channel if the queues are of equal length, including zero. The selection process will make use of another pseudo-random number. In finite queue situation, if a unit can be accommodated in any one of the queues, then the unit is placed in that queue; otherwise the unit is rejected. When a unit is placed in a queue, that service facility is tested for availability. If it is empty, the unit is placed in that facility, a pseudo-random number is computed and the service time is generated. The statistics are collected before advancing the clock to the next time.

When the clock time is for a service completion, the unit is removed from the correct station. If a queue exists, a new unit is inserted and the service time is generated. Next, the length of the queue is compared with the other queues. If one or more other queues are two units longer than the queue for the channel where the service has been completed, the last unit in one of these queues is moved to the completed service channel. The longer queue is arbitrarily selected as the queue that has the smallest designator number. Again the statistics are collected before advancing the clock to the next time.

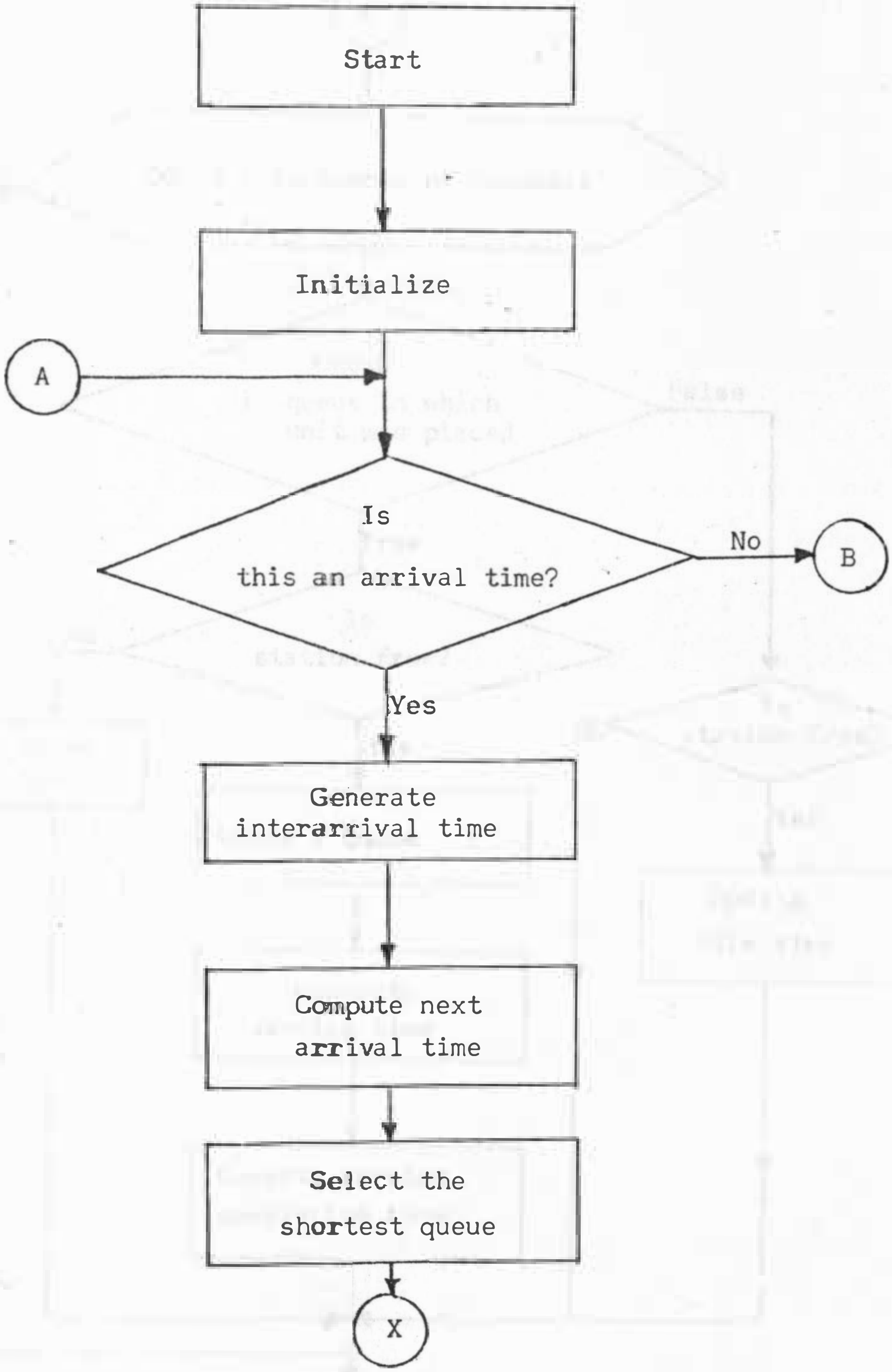

Figure 3.1. Flow Chart for Simulation of Queuing Situation.

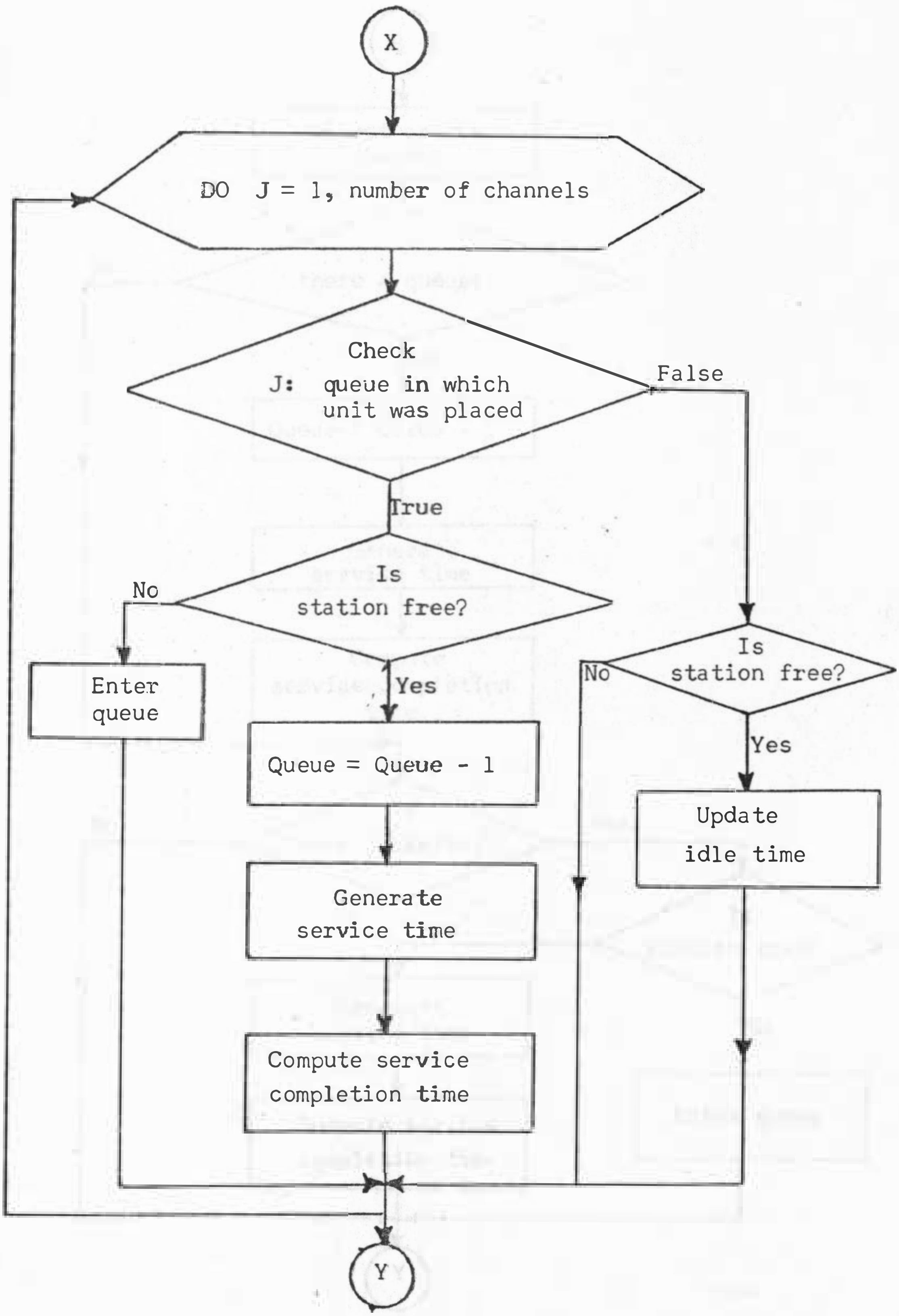

Figure 3.1. (continued)

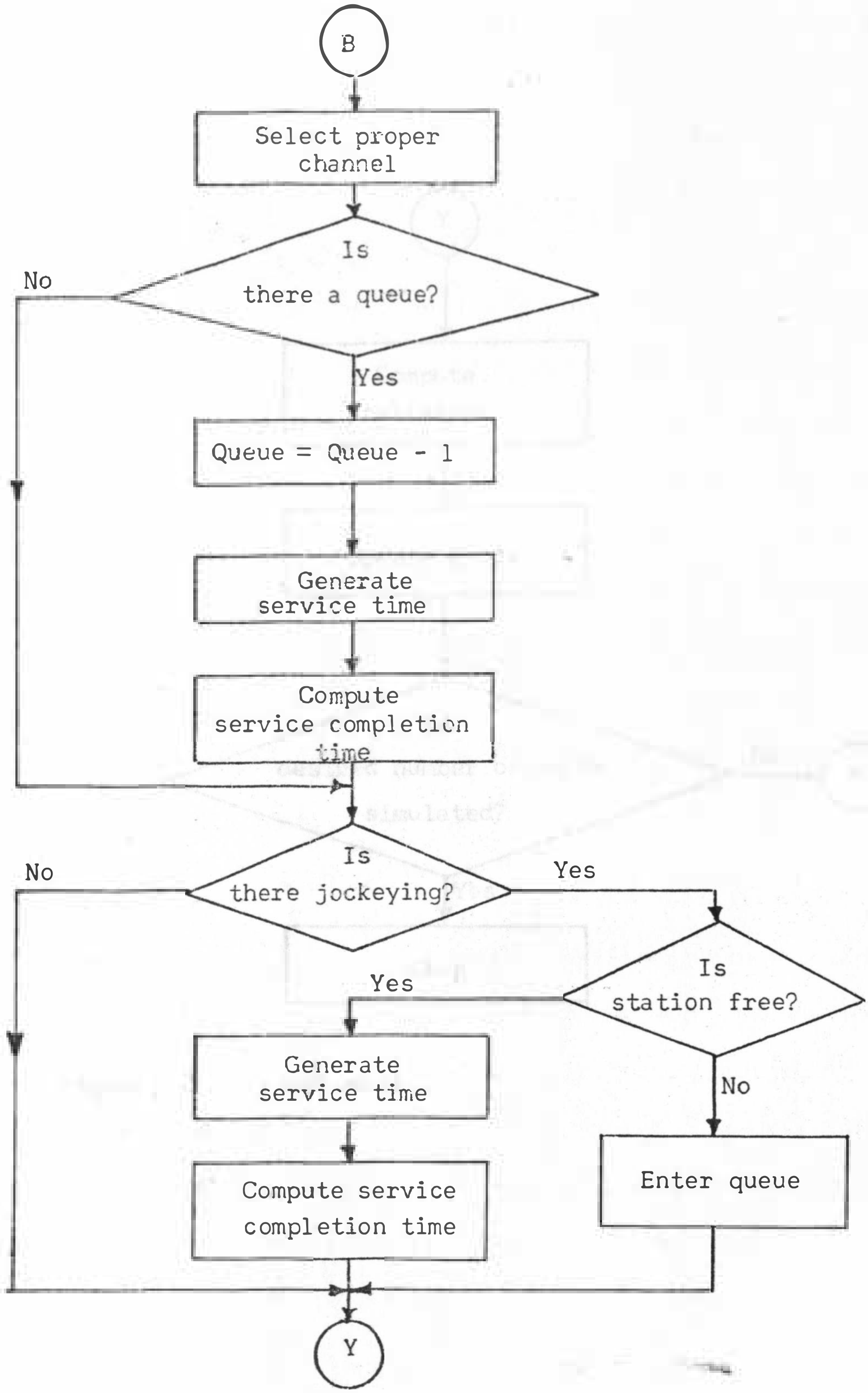

Figure 3.1. (continued)

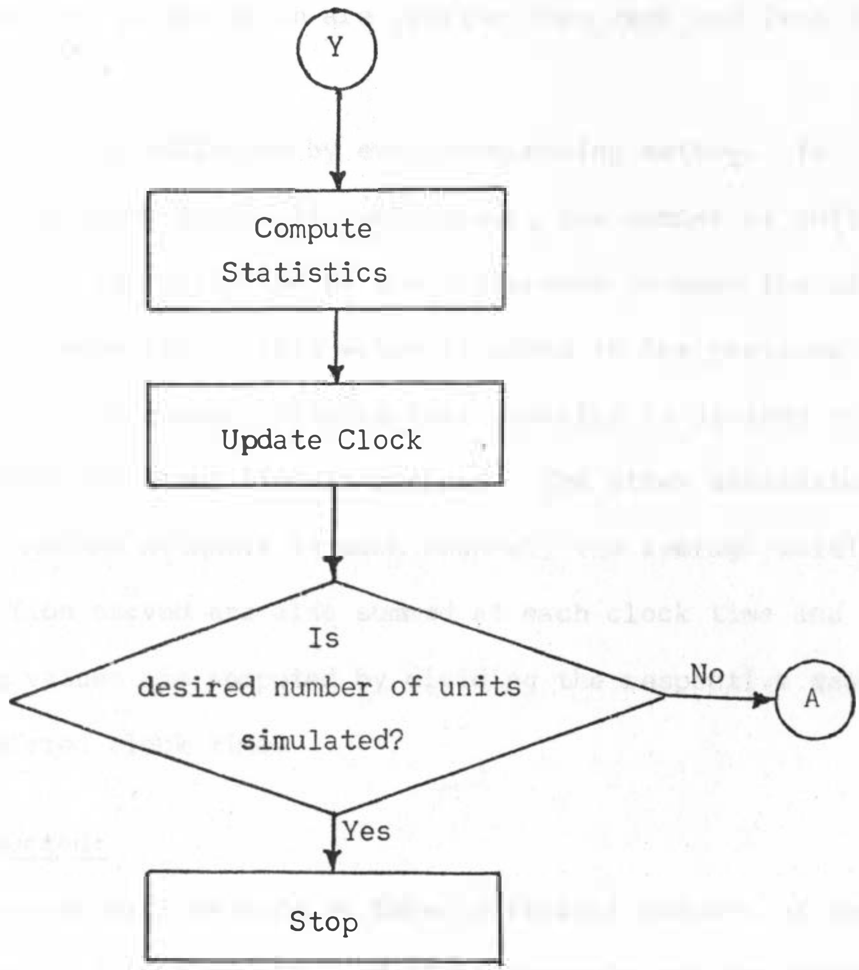

Figure 3.1. (concluded)

For the generation of random numbers, the standard IBM random number generator routine has been modified. The IBM routine will generate a random number between zero and one, inclusively. The modified routine will generate values which are greater than zero and less than or equal to one.

The statistics are collected by event-sequencing method. To compute the average gueue length at each server, the number of units in queue at each server is multiplied by the difference between the clock time and the next event time. This value is added to the previous sum of length time for that queue. Finally that quantity is divided by the clock time at which the simulation is stopped. The other statistics like the average number of units in each channel, the average waiting time and the portion served are also summed at each clock time and finally the mean values are computed by dividing the respective quantities by the simulated clock time.

<u>Study to be Conducted:</u>

A detailed study will be made on three different numbers of channels: two channels, four channels, and eight channels and for utilization factors of 0.2, 0.4, 0.6, and 0.9. The average length of queue before each channel, average number of units in each channel, idle time of each channel and the fraction of customers served by each channel will be determined as output and will be plotted against the utilization factor. The utilization factor is defined as the ratio of the mean

arrival rate to the mean system service rate. In case of a histogram, the mean interarrival time is its expected value and the inverse of the mean interarrival time will give the mean arrival rate. To obtain different utilization factors, the mean arrival rate will be kept at a constant value, and the service rate of all stations will be changed by the necessary factor.

A solution obtained through simulation is expected to have statistical error. To determine the solution with $\pm$ 10 per cent accuracy and 95 per cent confidence, it is required to have five runs of 800 samples each [6]. Statistics will be collected from five runs, each containing 800 samples.

The values for the run at the utilization factor of 0.4 are shown in Table 3-1.

The histogram is written such that the upper row represents the probability corresponding to the interarrival time values in the second row.

TABLE 3-1

Data Table

Utilization factor value is 0.4

No. of channels	Interarrival time distribution: Exponential	Histogram				Mean Service Rate Channel 1	2	3	4	5	6	7	8	Allowable queue
2	0.1667					5	10							∞
4	0.1667					2.5	5	5	2.5					∞
8		0.2 .07	0.5 .08	0.7 .21	1.0 .29	1	1.5	2	2.5	2	2.5	1.5	2	∞
2	0.1667					5	10							6

Length of run = 800

Replications = 5

CHAPTER IV

RESULTS

The different models indicated in Chapter III were simulated and statistics were collected at the different utilization factor values. The tabled results of the simulation runs are shown in Appendix C.

The validity of the simulation model was tested by comparing the output statistics with the same statistics using the relations developed by Koenigsberg [3]. These calculations are found in Appendix B, and the comparisons are shown in Tables 4-1 a,b,c and d. The simulation statistics vary from 0.12 per cent to 7.5 per cent from the calculated values, except for the probability that server 2 is idle (13.41 per cent) in Table 4-1d and several values for the length of queue. The latter was expected since Dyanesh [6] found that 30 replications were needed to reduce the estimation to $\pm$ 10 per cent of the mean value and only 5 replications were carried out.

A study of the output statistics for a number of combinations of channels, service rates and utilization factors indicates that the results in general do not deviate widely from that which might be intuitively expected, and that the various relations are similar to those for simulation models. Nevertheless, it is beneficial to discuss in detail the results that were obtained.

In general, the results were as expected with the faster server having more idle time than the slower servers, even though the average

TABLE 4-1a

COMPARISON OF SIMULATION STATISTICS WITH CALCULATED STATISTICS FOR M=2

$\rho = 0.2$

Channel	Method	L	L_q	P_o^i	γ	μ
1	Mathematical	0.2733	0.0093	0.736	0.44	10
	Simulation	0.2743	0.0103	0.738	0.43	
	Error by Simulation	+0.366%	+10.75%	+0.272%	-2.27%	
2	Mathematical	0.1766	0.0086	0.832	0.56	20
	Simulation	0.1769	0.0080	0.833	0.57	
	Error by Simulation	+0.17%	-6.98%	+0.12%	1.79%	

TABLE 4-1b

COMPARISON OF SIMULATION STATISTICS WITH CALCULATED STATISTICS FOR M=2

$\rho = 0.4$

Channel	Method	L	L_q	P_o^i	γ	μ
1	Mathematical	0.5632	0.0832	0.52	0.40	5
	Simulation	0.5519	0.0742	0.5227	0.394	
	Error by Simulation	-2%	-10.81%	+0.519%	-1.5%	
2	Mathematical	0.4367	0.0767	0.64	0.60	10
	Simulation	0.4289	0.0706	0.642	0.605	
	Error by Simulation	-1.78%	-7.95%	+0.3%	+0.83%	

TABLE 4-1c

COMPARISON OF SIMULATION STATISTICS WITH CALCULATED STATISTICS FOR M = 2

$\rho = 0.6$

Channel	Method	L	L	P_o^i	γ	μ
	Mathematical	1.026	0.3578	0.3313	0.37	
1	Simulation	0.9698	0.3077	0.3393	0.366	3.3333
	Error by Simulation	-5.48%	-14%	+2.41%	-1.08%	
	Mathematical	0.9017	0.3363	0.4341	0.63	
2	Simulation	0.8340	0.2897	0.457	0.634	6.6667
	Error by Simulation	-7.5%	-13.85%	+5.01%	+0.63%	

TABLE 4-1d

COMPARISON OF SIMULATION STATISTICS WITH CALCULATED STATISTICS FOR M = 2

$\rho = 0.9$

Channel	Method	L	L_q	P_o^i	γ	μ
	Mathematical	4.817	3.8949	0.0789	0.34	
1	Simulation	4.8709	3.9535	0.0832	0.342	2.2222
	Error by Simulation	+1.12%	+1.5%	+5.45%	+0.59%	
	Mathematical	4.72	3.8299	0.1007	0.66	
2	Simulation	4.7628	3.8763	0.1142	0.658	4.4444
	Error by Simulation	+0.91%	+1.21%	+13.41%	-0.3%	

queue length and mean number[1] were smaller. Because of jockeying, the average queue length and the mean number of units at the channel at comparable channels tend to be the same, although the actual values of the statistics differ where there is a difference in the mean service rate.

For each model, the average queue length of each channel, the average number of units in each channel, the idle time of each channel and the portion of customers served by each channel are plotted against the utilization factor and are shown in Figures 4.1 through 4.4. Channels having the same service rate were found to have no significant difference in the values of their respective statistics and are represented by a single curve.

It can be seen in Figure 4.1 that as the utilization factor increases, the portion of customers served by the fastest server also increases, while that of the slowest decreases. This is due to the fact that as utilization factor value increases, more number of units jockey from the slower servers to faster servers.

Figure 4.2 indicates that the idle time of a server depends on both his service rate and the number of servers in the system. For a given number of channels and utilization factor, the fastest server will have more idle time than other servers. This is due to the ability of the fast channel to complete the service of units in queues of length one faster than the other channels. Further, at high utilization

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[1]Units at the channel is defined as the units in the queue plus units in service.

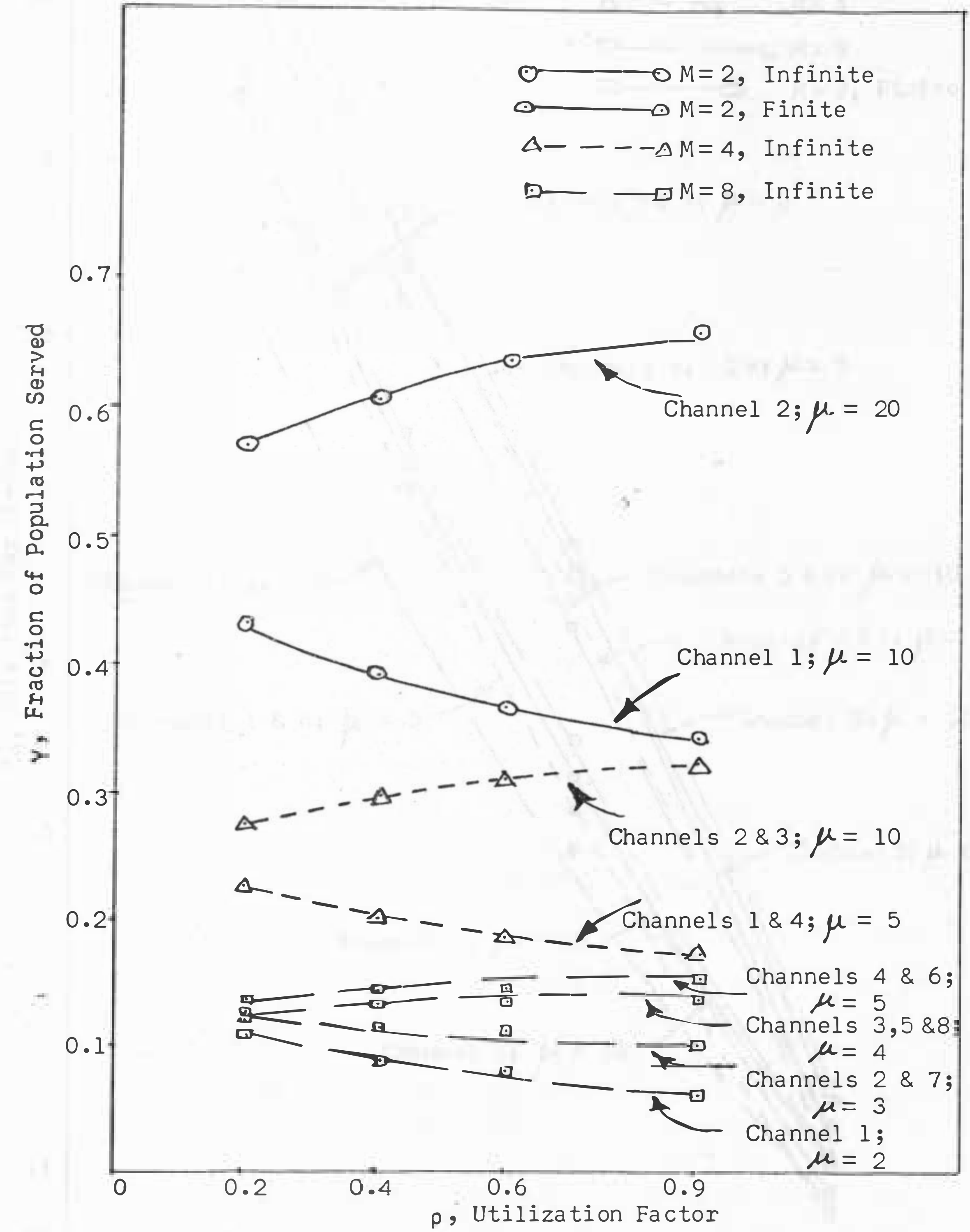

Figure 4.1. Fraction of Units Served for Models with an Infinite Queue as a Function of Utilization Factor.

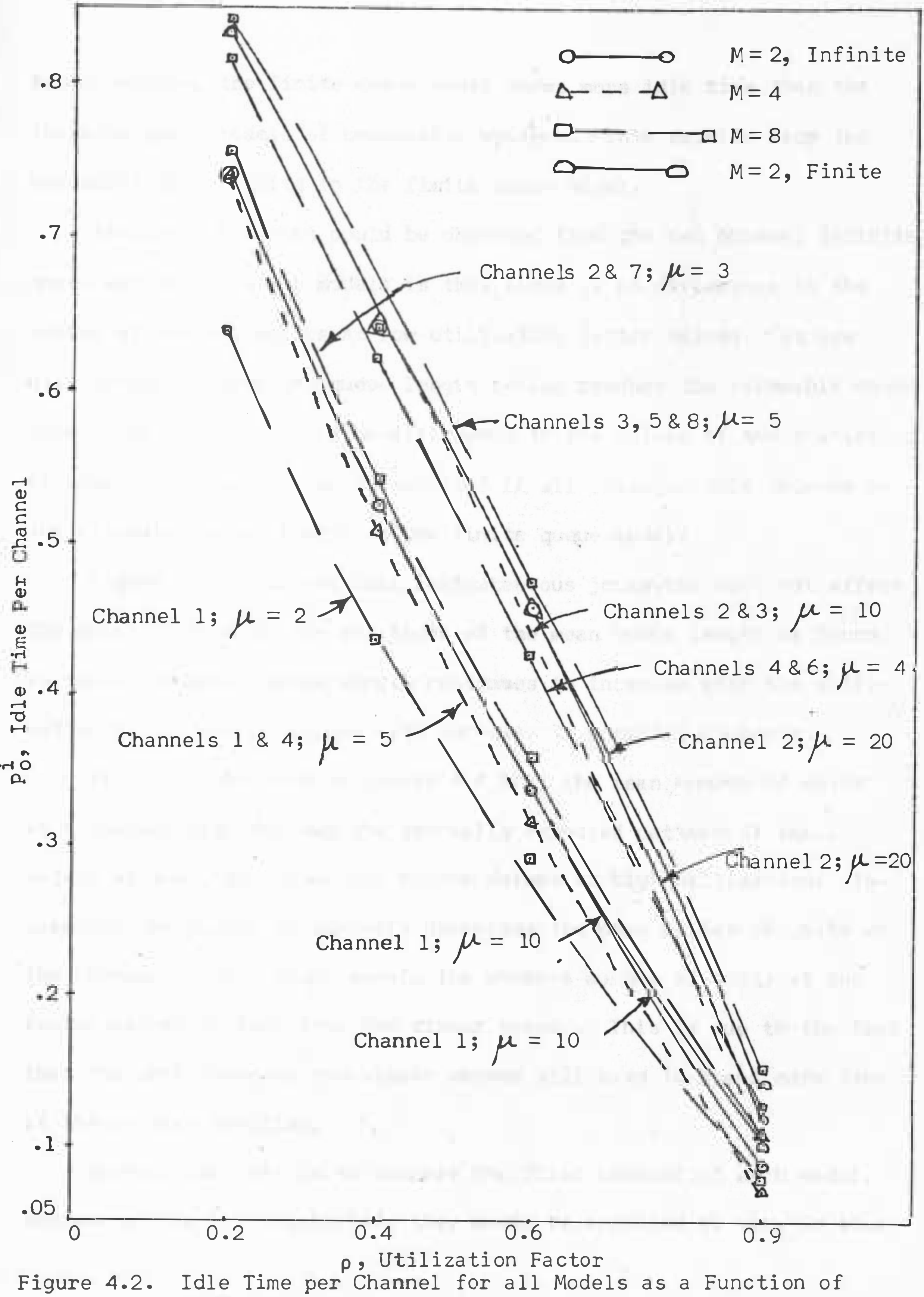

Figure 4.2. Idle Time per Channel for all Models as a Function of Utilization Factor.

factor values, the finite queue model shows more idle time than the infinite queue models of comparable systems. This results from the non-servicing of units in the finite queue model.

Another point that could be observed from the two channel infinite queue and finite queue models is that there is no difference in the values of the statistics at low utilization factor values. At low utilization values, the queue length seldom reaches the allowable queue length and hence there is no difference in the values of the statistics. Of course this can not be generalized in all cases as this depends on the allowable queue length in the finite queue model.

Figure 4.3 indicates that instantaneous jockeying does not affect the general shape of the relations of the mean queue length as found in simple models. Queue length continues to increase with the utilization factor and decreases with increase in parallel channels.

It can be observed in Figure 4.4 that the mean number of units at a channel also follows the generally expected pattern of small values at low utilization and higher values at high utilization. Increasing the number of channels decreases the mean number of units at the channel. For a given model, the average number of units at the faster server is less than the slower server. This is due to the fact that the unit choosing the slower server will have to spend more time at the service facility.

Another analysis is to compare the first channel of each model. Because of the rules selected, they would be expected to vary in some

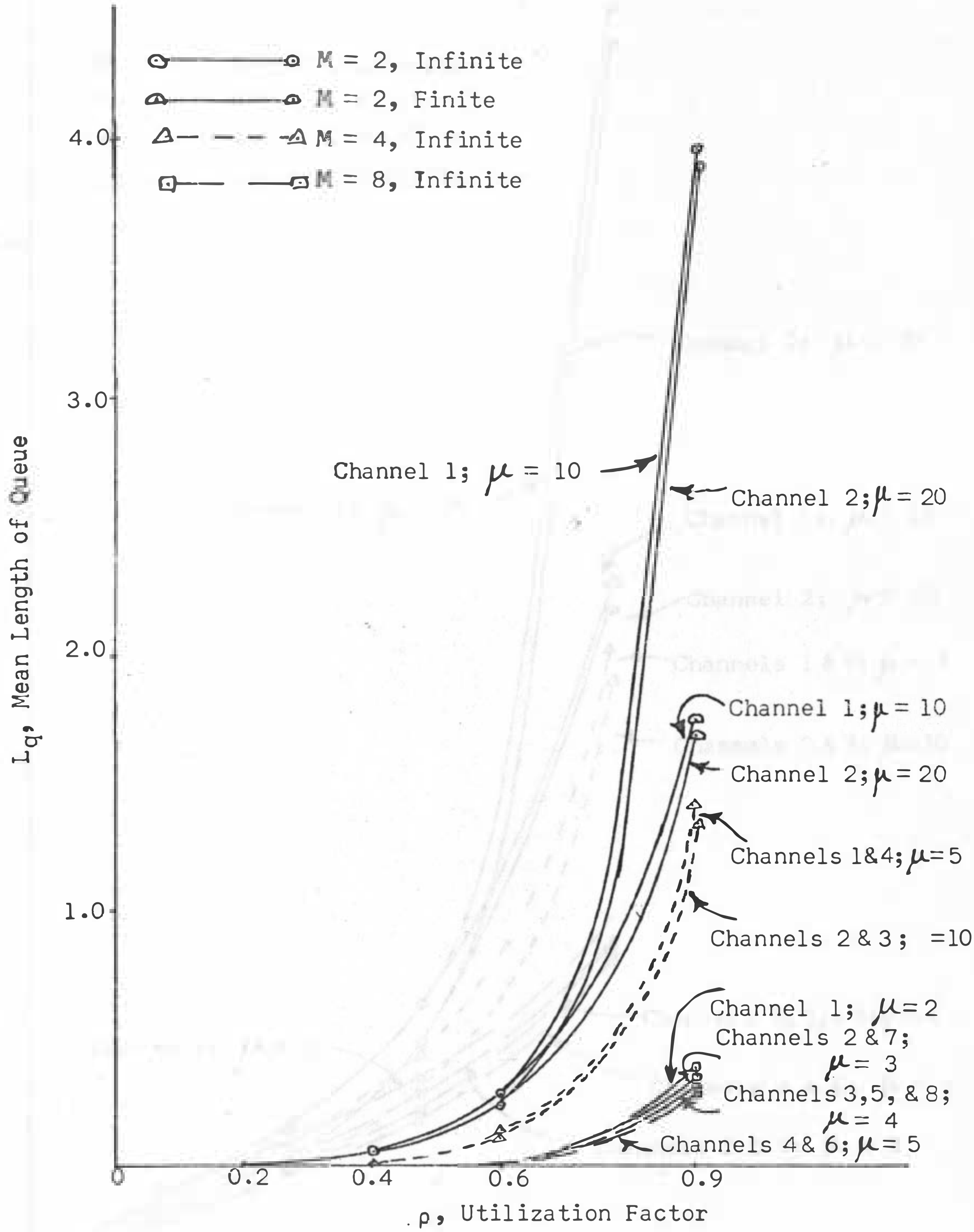

Figure 4.3. Mean Length of Queue as a Function of Utilization Factor.

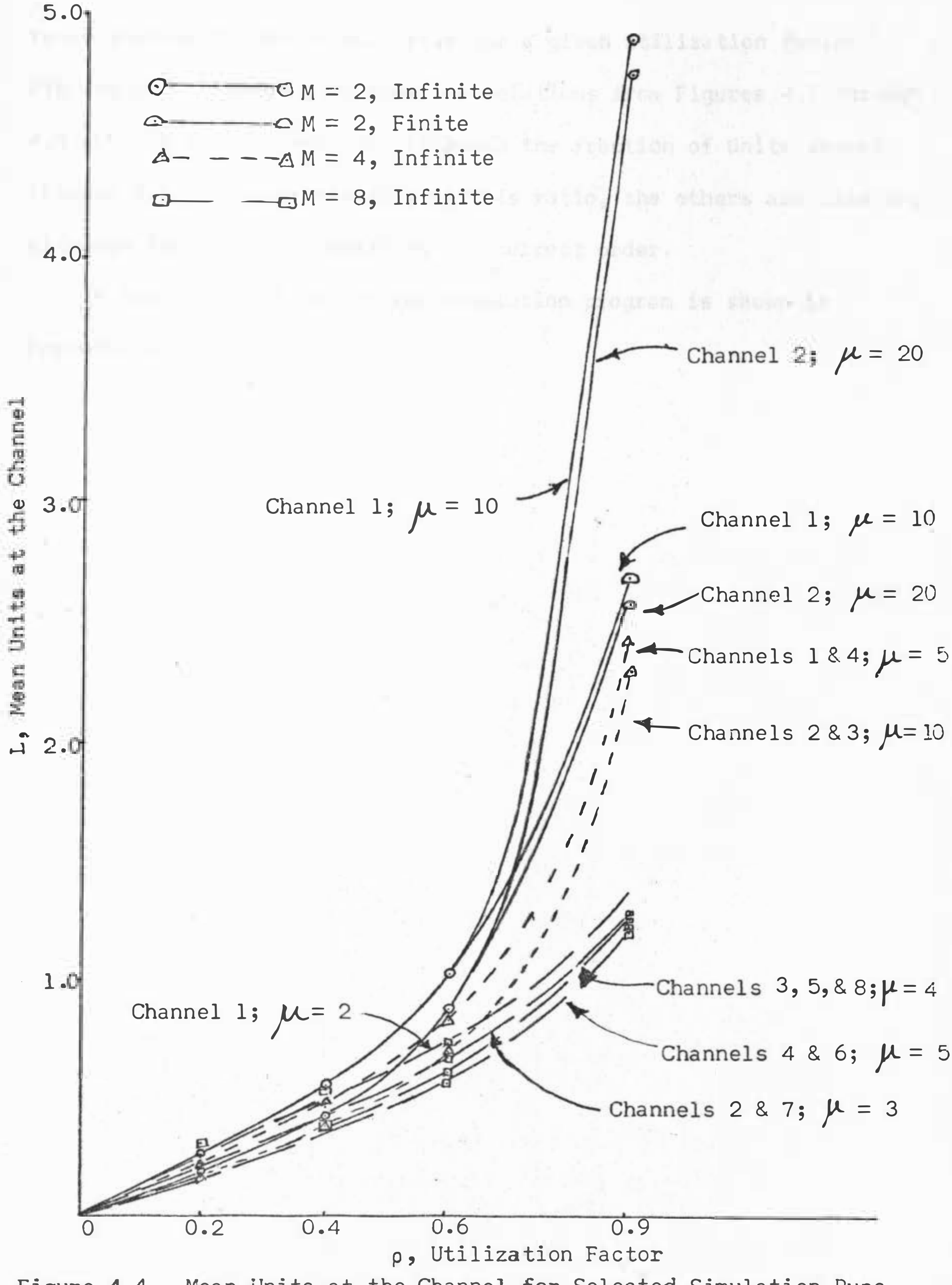

Figure 4.4. Mean Units at the Channel for Selected Simulation Runs as a Function of Utilization Factor.

ratio similar to the service rate for a given utilization factor. Figures 4.5a through 4.5d show the relations from Figures 4.1 through 4.4 for the first channels. Although the fraction of units served (Figure 4.5a) is approximately of this ratio, the others are less so, although they tend to remain in the correct order.

A typical print out of the simulation program is shown in Appendix D.

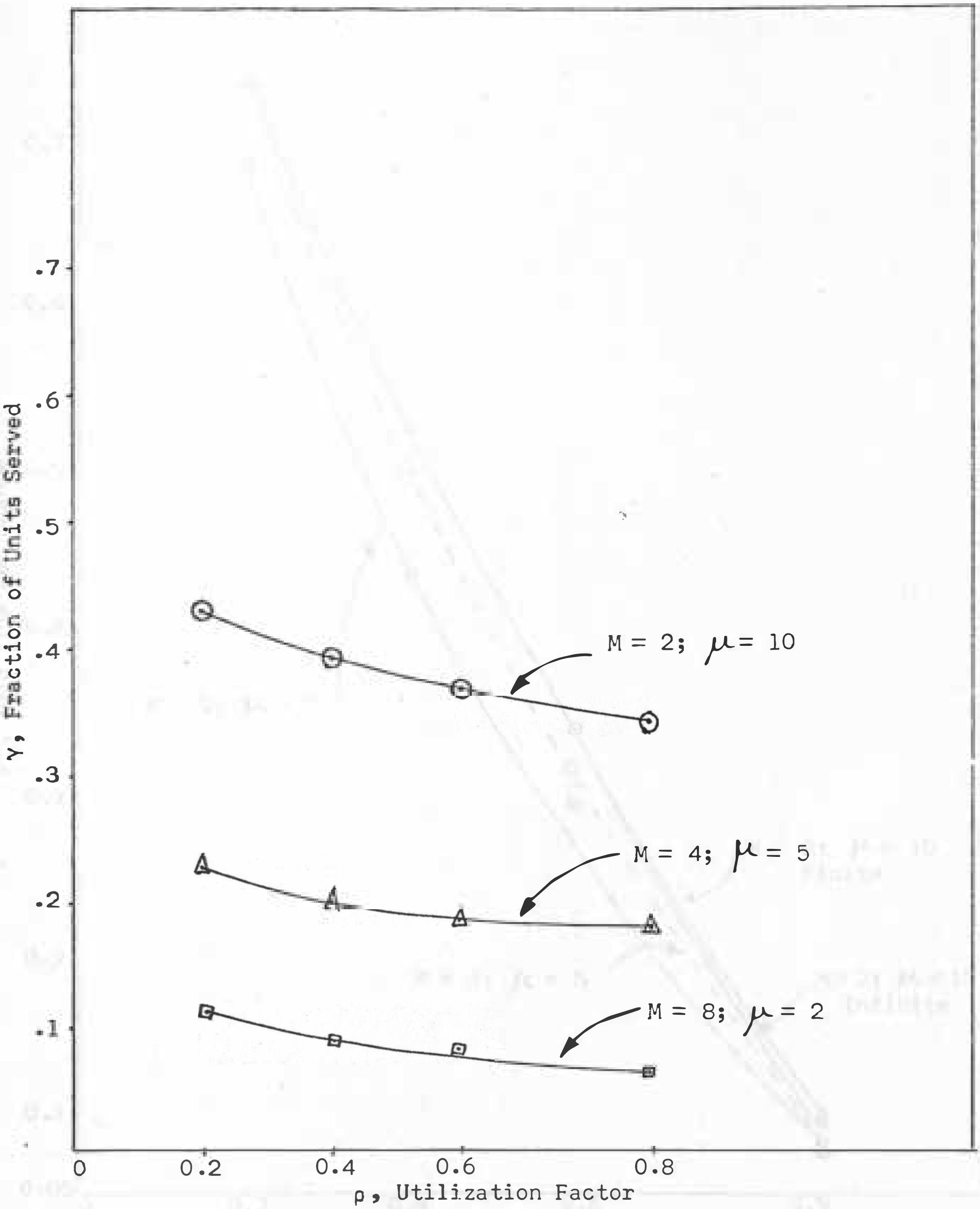

Figure 4.5a. Fraction of Units Served by First Channels of all Infinite Queue Models as a Function of Utilization Factor.

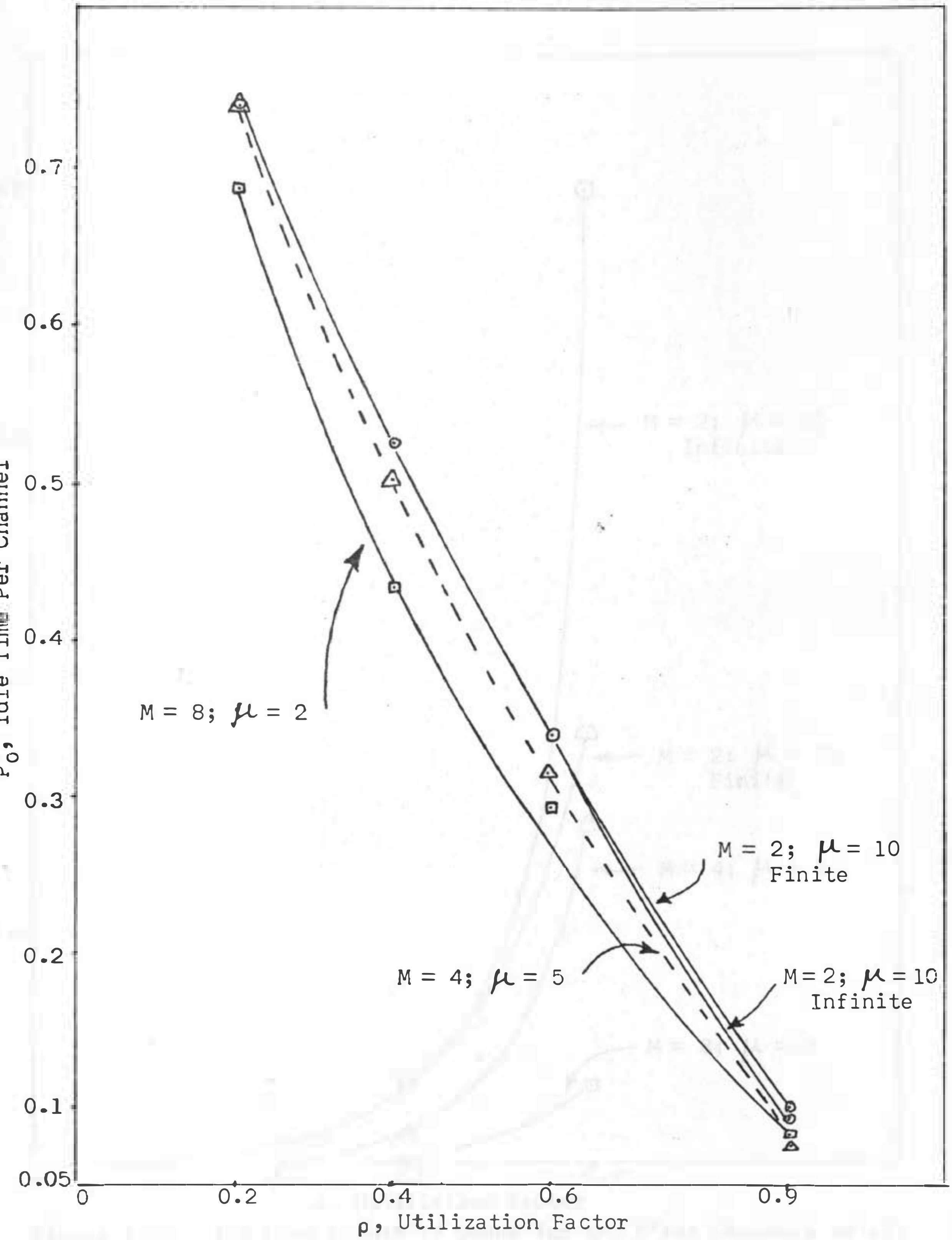

Figure 4.5b. Idle Time of the First Channels of all Models as a Function of the Utilization Factor.

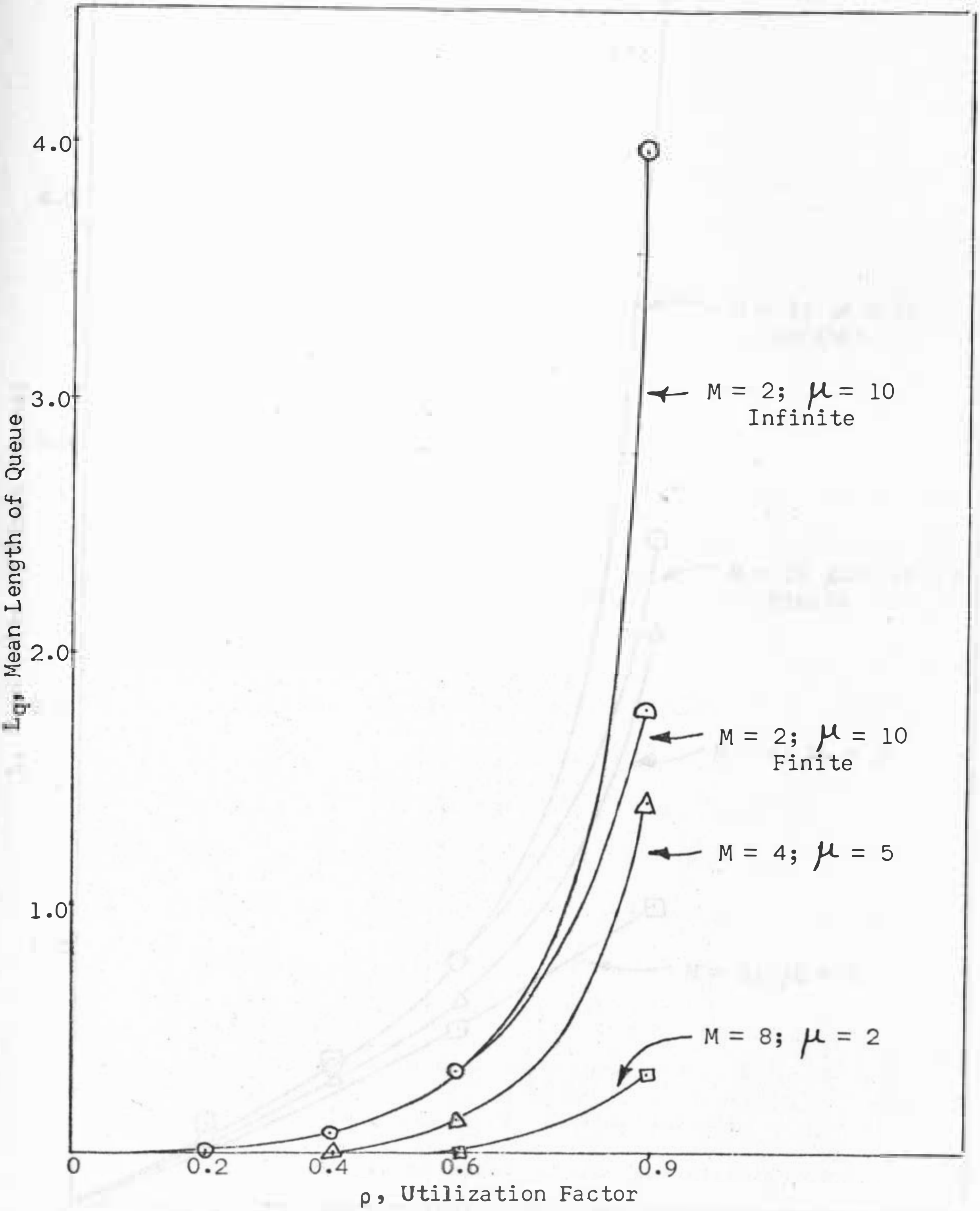

Figure 4.5c. The Mean Length of Queue for the First Channels of all Models as a Function of the Utilization Factor.

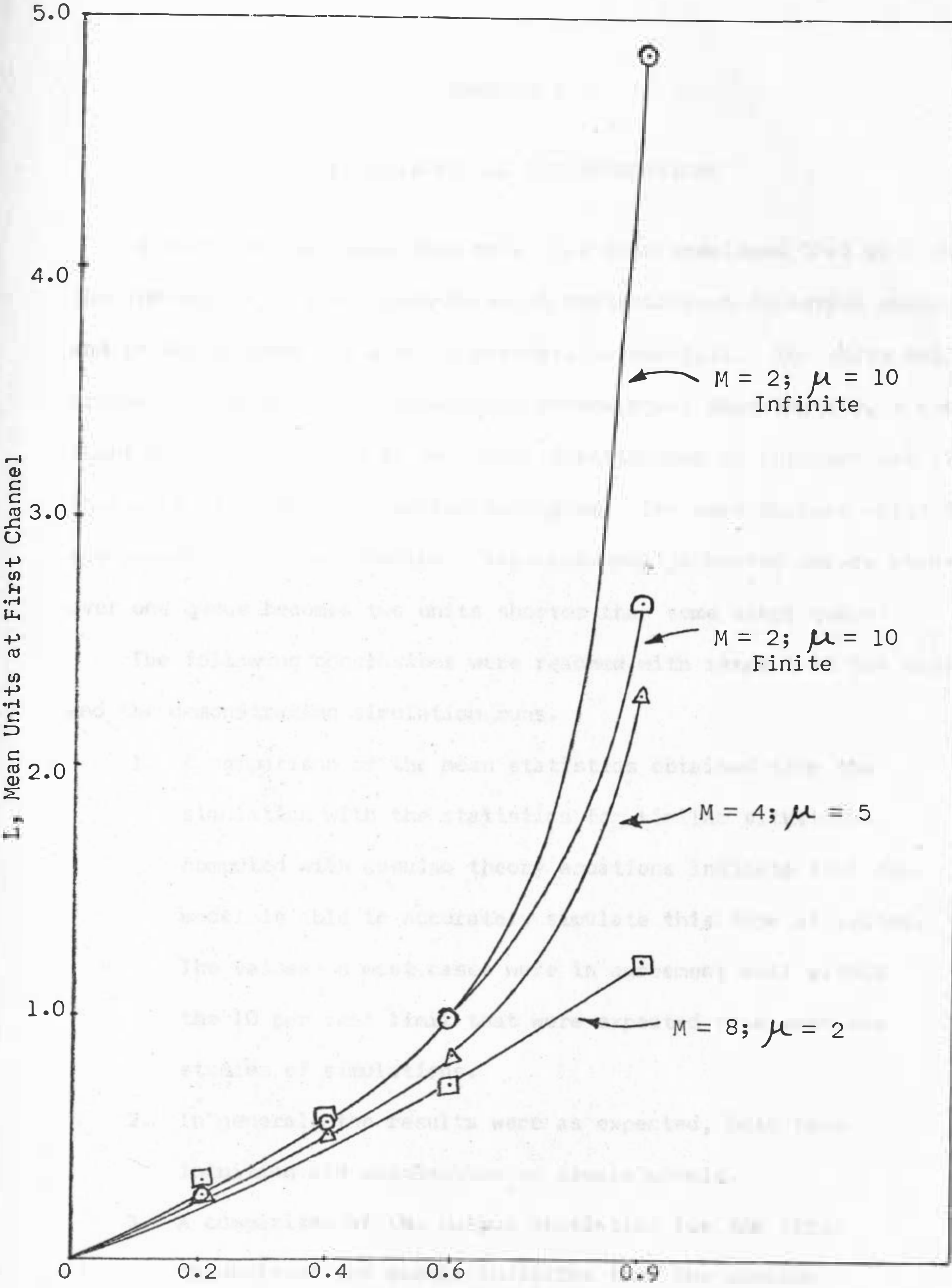

Figure 4.5d. Mean Units at First Channels for all Models as a Function of the Utilization Factor.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

A computerized simulation model has been developed that will allow the simulation of situations in which instantaneous jockeying occurs and in which there are 2 to 10 channels in parallel. The units may arrive according to the exponential interarrival distribution, a constant interarrival time or any other distribution of interarrival times that will fit into a ten-celled histogram. The same choices exist for the service time distribution. Instantaneous jockeying occurs whenever one queue becomes two units shorter than some other queue.

The following conclusions were reached with respect to the model and the demonstration simulation runs.

1. A comparison of the mean statistics obtained from the simulation with the statistics for similar situations computed with queuing theory equations indicate that the model is able to accurately simulate this type of system. The values in most cases were in agreement well within the 10 per cent limit that were expected from previous studies of simulations.
2. In general, the results were as expected, both from intuition and examination of simple models.
3. A comparison of the output statistics for the first channels of the models indicates that the various

statistics are not in proportion to the rates of service for the same utilization factors. It is not known whether this is a function of the jockeying feature or not.

The following suggestions are made for further investigation:

1. A system with a priority discipline and jockeying allowed should be studied.
2. A system with more than two channels and in which the customer has the option to jockey should be investigated.

LITERATURE CITED

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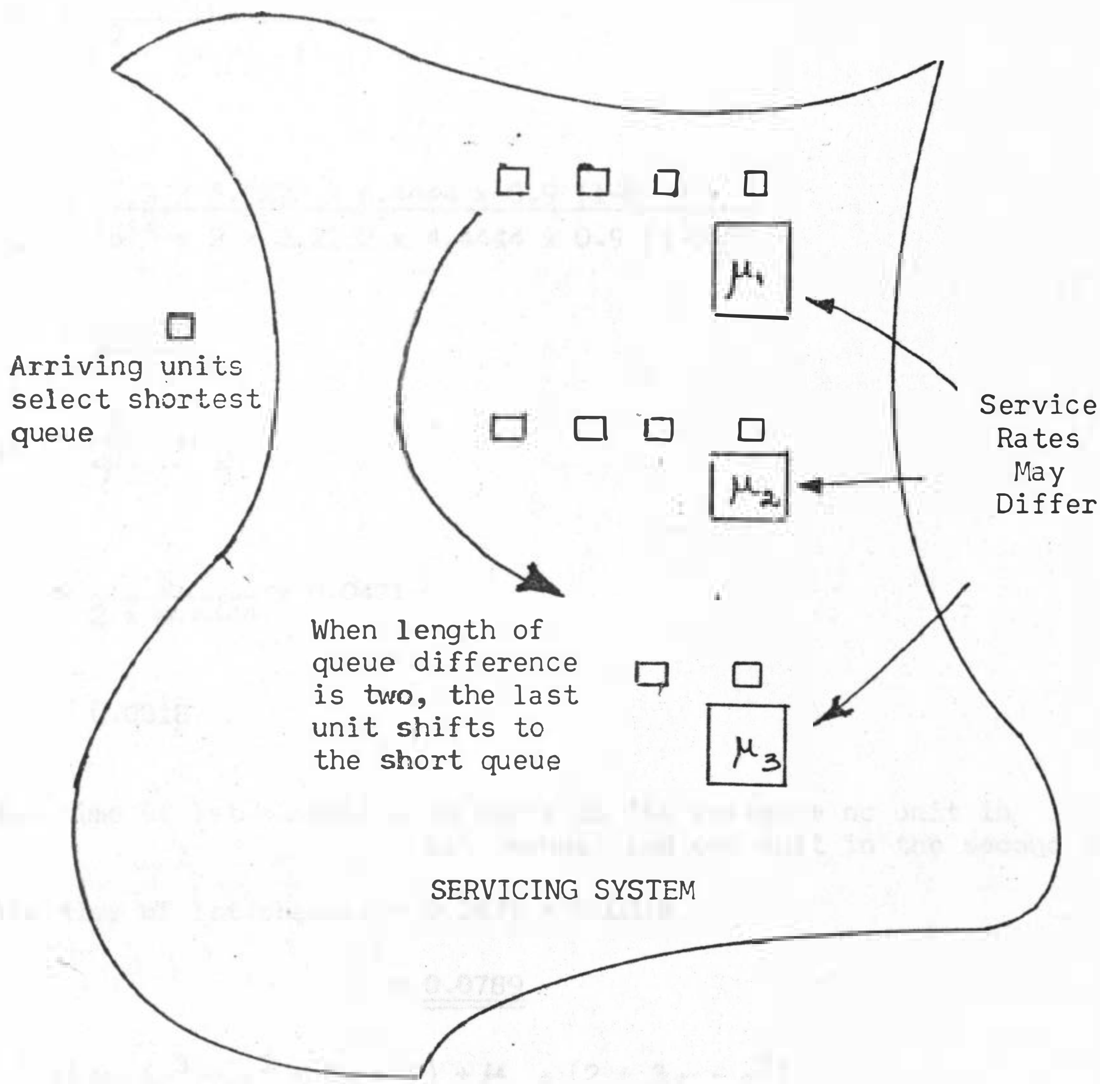
APPENDIX A

GENERALIZED SCHEMATIC DIAGRAM OF JOCKEYING SITUATION



APPENDIX B

CALCULATION OF THE STATISTICS

Two channel case with infinite queue.

λ = 6/time unit μ_1 = 2.2222/time unit

μ_2 = 4.4444/time unit $\rho = 0.9$

$$P_o = \frac{2\mu_1\mu_2\,\rho(1-\rho)}{\lambda^2 + 2\mu_1\mu_2\,\rho(1-\rho)}$$

$$= \frac{2 \times 2.2222 \times 4.4444 \times 0.9\,(1-0.9)}{(6)^2 + 2 \times 2.2222 \times 4.4444 \times 0.9\,(1-0.9)}$$

$$= \underline{\underline{0.0471}}$$

$$Q_{o1} = \frac{\lambda}{2\mu_2} \times P_o$$

$$= \frac{6}{2 \times 4.4444} \times 0.0471$$

$$= 0.0318$$

Idle time of 1st channel = no units in the system + no unit in 1st channel and one unit in the second channel.

Idle time of 1st channel = 0.0471 + 0.0318

= $\underline{\underline{0.0789}}$

$$L_1 = \frac{\lambda[\mu_2(\rho^3 - \rho^2 + 2\rho + 2) + \mu_1\,\rho\,(2 + 3\rho - \rho^2)}{4\mu_1\mu_2\,(1-\rho^2)^2} \times P_o$$

$$L_1 = \frac{6[4.4444(0.729 - 0.81 + 1.8 + 2) + 2.2222 \times 0.9(2 + 2.7 - 0.81)]}{4 \times 2.2222 \times 4.4444 \times [1 - (0.9)^2]^2} \times 0.0471$$

$$= \underline{\underline{4.817}}$$

$$L_{q_i} = \frac{\lambda \rho[1+\rho) + 2\rho^2(\mu_2 + \mu_1 \rho)]}{4\mu_1\mu_2(1-\rho^2)^2} \times P_o$$

$$= \frac{6 \times 0.9[6(1+0.9) + 2(0.9)^2 (4.4444 + 2.2222 \times 0.9)]}{4 \times 2.2222 \times 4.4444 [1-(0.9)^2]^2} \times 0.0471$$

$$= \underline{\underline{3.8949}}$$

$$\gamma_1 = \frac{\mu_1}{\lambda}(1 - P_o^1)$$

$$= \frac{2.2222}{6}(1 - 0.0789)$$

$$= \underline{\underline{0.34}}$$

$$Q_{10} = \frac{\lambda}{2\mu_1} \times P_o$$

$$= \frac{6}{2 \times 2.2222} \times 0.0471$$

$$= \underline{\underline{0.0636}}$$

$$P_o^2 = P_o + Q_{10} = 0.0471 + 0.0636$$

$P_o^2 = \underline{\underline{0.1007}}$

$$L_2 = \frac{\lambda[\mu_1(\rho^3 - \rho^2 + \rho + 2) + \mu_2\rho(2 + 3\rho - \rho^2)]}{4\mu_1\mu_2(1-\rho^2)^2} \times P_o$$

$$= \frac{6[2.2222(0.729 - 0.81 + 1.8 + 2) + 4.4444 \times 0.9(2 + 2.7 - 0.81)]}{4 \times 2.2222 \times 4.4444 \times [1 - (0.9)^2]^2} \times 0.0471$$

$$= \underline{\underline{4.72}}$$

$$L_{q_2} = \frac{\lambda\rho[\lambda(1+\rho) + 2\rho^2(\mu_1 + \mu_2\rho)]}{4\mu_1\mu_2(1-\rho^2)^2} \times P_o$$

$$= \frac{6 \times 0.9[6(1 + 0.9) + 2(0.9)^2(2.2222 + 4.4444(0.9))]}{4 \times 2.2222 \times 4.4444 \; [1 - (0.9)^2]^2} \times 0.0471$$

$$= \underline{\underline{3.8299}}$$

$$\gamma_2 = 1 - \gamma_1 = 1 - 0.34$$

$$= \underline{\underline{0.66}}$$

APPENDIX C

TABLE C-1

Results of the Simulation Runs

M = 2; Infinite Queue Case

ρ	Channel	L	L_q	P_o^i	γ	μ
0.2	1	0.2743	0.0103	0.738	0.43	10
	2	0.1769	0.008	0.833	0.57	20
0.4	1	0.5519	0.0742	0.5527	0.394	5
	2	0.4289	0.0706	0.642	0.605	10
0.6	1	0.9698	0.3077	0.3393	0.366	3.3333
	2	0.8340	0.2897	0.457	0.634	6.6667
0.9	1	4.8709	3.8299	0.1007	0.66	2.2222
	2	4.762	3.8763	0.1142	0.658	4.4444

TABLE C-2

Results of Simulation Runs

M = 2; Finite Queue Case

ρ	Channel	L	L_q	P_o^i	γ	μ
0.2	1	0.2743	0.0103	0.738	0.43	10
	2	0.1769	0.008	0.833	0.57	20
0.4	1	0.5519	0.0742	0.5227	0.394	5
	2	0.4289	0.0706	0.6420	0.605	10
0.6	1	0.9698	0.3077	0.3393	0.366	3.3333
	2	0.8340	0.2897	0.4570	0.634	6.6667
0.9	1	2.6769	1.7737	0.0974	0.342	2.2222
	2	2.5747	1.7140	0.1399	0.657	4.4444

TABLE C-3

Results of Simulation Runs

$\rho = 0.2$; $M = 4$;

Channel	L	L_q	P_o^i	γ	μ
1	0.2669	0.0005	0.7348	0.226	5
2	0.1704	0.0013	0.8321	0.279	10
3	0.1676	0.0011	0.8348	0.267	10
4	0.2637	0.0003	0.7378	0.228	5

TABLE C-4

Results of Simulation Runs

$\rho = 0.4$; $M = 4$

Channel	L	L_q	P_o^i	γ	μ
1	0.5132	0.014	0.5022	0.199	2.5
2	0.3744	0.0189	0.6453	0.289	5
3	0.3789	0.0171	0.6390	0.302	5
4	0.5039	0.018	0.5155	0.210	2.5

TABLE C-5

RESULTS OF SIMULATION RUNS

$\rho = 0.6$; $M = 4$

Channel	L	L_q	P_o^i	γ	μ
1	0.8011	0.114	0.3131	0.187	1.6667
2	0.6836	0.1232	0.4397	0.313	3.3334
3	0.6723	0.1166	0.4455	0.313	3.3334
4	0.8255	0.1453	0.3200	0.188	1.6667

TABLE C-6

RESULTS OF SIMULATION RUNS

$\rho = 0.9$; $M = 4$

Channel	L	L_q	P_o^i	γ	μ
1	2.3246	1.3987	0.0749	0.180	1.1111
2	2.3022	1.4047	0.1034	0.330	2.2222
3	2.294	1.4003	0.1072	0.322	2.2222
4	2.4148	1.4879	0.0741	0.168	1.1111

TABLE C-7

RESULTS OF SIMULATION RUNS

$\rho = 0.2$; $M = 8$

Channel	L	L_q	P_o^i	γ	μ
1	0.3160	0.0	0.6848	0.111	2
2	0.2278	0.0	0.7728	0.118	3
3	0.1749	0.0	0.8260	0.118	4
4	0.1687	0.0	0.8319	0.140	5
5	0.1857	0.0	0.8149	0.124	4
6	0.1559	0.0	0.8449	0.134	5
7	0.2623	0.0	0.7385	0.125	3
8	0.1935	0.0	0.8070	0.130	4

TABLE C-8

RESULTS OF SIMULATION RUNS

$\rho = 0.4$; $M = 8$

Channel	L	L_q	P_o^i	γ	μ
1	0.5683	0.0005	0.4326	0.089	1
2	0.4689	0.0	0.5315	0.112	1.5
3	0.3692	0.0002	0.6313	0.128	2
4	0.3741	0.0	0.6263	0.141	2.5
5	0.3837	0.0001	0.6168	0.131	2
6	0.3458	0.0	0.6547	0.145	2.5
7	0.4522	0.00	0.5483	0.120	1.5
8	0.3831	0.0002	0.6176	0.133	2

TABLE C-9

RESULTS OF SIMULATION RUNS

$\rho = 0.6$; $M = 8$

Channel	L	L_q	P_o^i	γ	μ
1	0.7117	0.0062	0.2949	0.082	0.6667
2	0.6474	0.0064	0.3594	0.112	1
3	0.5911	0.0053	0.4146	0.130	1.3333
4	0.5354	0.0057	0.4707	0.141	1.6667
5	0.5771	0.0081	0.4315	0.133	1.3333
6	0.5274	0.0068	0.4799	0.152	1.6667
7	0.6432	0.0051	0.3623	0.109	1.0
8	0.5749	0.0090	0.4346	0.140	1.3333

TABLE C-10

RESULTS OF SIMULATION RUNS

$\rho = 0.9$; $M = 8$

Channel	L	L_q	P_o^i	γ	μ
1	1.2292	0.3075	0.0789	0.063	0.4444
2	1.2135	0.3226	0.1097	0.098	0.6668
3	1.2077	0.3419	0.1348	0.150	0.8888
4	1.1628	0.3150	0.1528	0.162	1.1111
5	1.2457	0.3635	0.1184	0.121	0.8888
6	1.2180	0.3558	0.1384	0.153	1.1111
7	1.2929	0.3846	0.0923	0.105	0.6668
8	1.2776	0.3914	0.1144	0.146	0.8888

APPENDIX D

PRINTOUT OF THE PROGRAM

PROBLEM NUMBER = 1

CHANNELS = 4 LAMDA = 6.0000

CHANNEL	MU	PREFERENCE RATE .
1	5.0000	0.2500
2	10.0000	0.2500
3	10.0000	0.2500
4	5.0000	0.2500

INFINITE QUEUE BEFORE EACH CHANNEL

FINAL STATISTICS

MEAN QUEUE	MEAN NUMBER	IDLE TIME	NO.OF UNITJOCKEYED TO	PORTION SERVED	CHANNEL NO.
0.0005	0.2669	0.7348	2	0.226	1
0.0013	0.1704	0.8321	2	0.279	2
0.0011	0.1676	0.8348	2	0.267	3
0.0003	0.2637	0.7378	1	0.228	4

MEAN WAITING TIME= 0.0005

MEAN TIME IN SYSTEM = 0.1445

P(0)	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8)	P(9)	P(10)
0.4205	0.3645	0.1587	0.0458	0.0091	0.0018	0.0006	0.0000	0.0	0.0	0.0
P(11)	P(12)	P(13)	P(14)	P(15)	P(16)	P(17)	P(18)	P(19)	P(20	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
P(21)	P(22)	P(23)	P(24)	P(25)	P(26)	P(27)	P(28)	P(29)	P(30)	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

APPENDIX E

COMPUTER PROGRAM

```
      QUEUE SIMULATION  THESIS  B.RABINDRANATH
C     THIS PROGRAM CAN TAKE A MINIMUM OF TWO CHANNELS AND A MAXIMUM OF
C     TEN CHANNELS
C     THIS PROGRAM CAN HANDLE EITHER INSTANTANEOUS JOCKEYING OR NO
C     NO JOCKEYING MULTIPLE QUEUES MODEL
C     VARIABLES DEFINED
C     AN(J) - AVERAGE NUMBER OF UNITS IN CHANNEL J, FINAL
C     AQ(J) - MEAN QUEUE LENGTH AT CHANNEL J, FINAL
C     TI(J) - IDLE TIME OF CHANNEL J, FINAL
C     NQ(J) - NUMBER OF UNITS JOCKEYED TO CHANNEL J, FINAL
C     PNP(J) - FRACTION SERVED BY CHANNEL J, FINAL
C     AWT - AVERAGE WAITING TIME OF A UNIT, FINAL
C     TIP - AVERAGE TIME A UNIT SPENDS IN THE SYSTEM, FINAL
C     PIPT - PROBABILITY THAT ZERO UNITS ARE IN THE SYSTEM,FINAL
C     P(IUM)-PROBABILITY THAT 'IUM' UNITS ARE IN THE SYSTEM,FINAL
C     CAL - FRACTION OF ARRIVALS TURNED AWAY,FINAL
C     AQL(J)-CUMU SUM OF MEAN QUEUE LENGTH IN CHANNEL J, ALL RUNS
C     ANL(J)-CUMU SUM OF MEAN NUMBER OF UNITS IN CHANNEL J,ALL RUNS
C     PNP(J)-CUMU SUM OF FRACTION OF UNITS SERVED BY CHANNEL J, ALL RUNS
C     TI(J)-CUMU SUM OF IDLE TIME OF CHANNEL J,ALL RUNS
C     AWAT -CUMU SUM OF MEAN WAITING TIME,ALL RUNS
C     TIS - CUMU SUM OF MEAN TIME A UNIT SPENDS IN THE SYSTEM,ALL RUNS
C     CAL - CUMU SUM OF FRACTION OF UNITS TURNED AWAY,ALL RUNS
C     PITD - CUMU SUM OF PROBABILITY THAT NO UNITS ARE IN THE SYSTEM,ALL
C               RUNS
C     PR(IUM)=CUMU SUM OF PROBABILITY THAT IUM UNITS ARE IN THE SYSTEM,
C               ALL RUNS
C     AVEQUE(J)-MEAN QUEUE LENGTH IN CHANNEL J,PER RUN
C     AVENUM(J)-MEAN NUMBER OF UNITS IN CHANNEL J,PER RUN
C     TID(J)-IDLE TIME OF CHANNEL J,PER RUN
C     AVEWAT- MEAN WAITING TIME ,PER RUN
C     TIMSYS-MEAN TIME IN THE SYSTEM,PER RUN
C     POR(J)=FRACTION SERVED BY CHANNEL J,PER RUN
C     FAR - FRACTION TURNED AWAY,PER RUN
C     PIDT - PROBABILITY THAT ZERO UNITS ARE IN THE SYSTEM,PER RUN
C     P(IUM)-PROBABILITY THAT IUM UNITS ARE IN THE SYSTEM,PER RUN
C     MARY(J)-CUMU SUM OF UNITS SERVED BY CHANNEL J,PER RUN
C     QLEN(J)- CUMU SUM OF NUMBER OF UNITS IN CHANNEL J,PER RUN
C     ZQUE(J)-CUMU SUM OF QUEUE LENGTH IN CHANNEL J,PER RUN
C     NJ(J)-NUMBER OF UNITS THAT JOCKEYED TO CHANNEL J,PER RUN
C     NAN - TOTAL NUMBER OF UNITS THAT ARE SERVICED,PER RUN
C     PIDTS - TOTAL TIME THAT ZERO UNITS ARE IN THE SYSTEM,PER RUN
C     PW(IUM)-TOTAL TIME THAT IUM UNITS ARE IN THE SYSTEM,PER RUN
C     I - TOTAL NUMBER OF ARRIVALS, PER RUN
C     CLOCK-PRESENT TIME
C     EVENT - NEXT EVENT OCCURRING TIME
C     LOTUS(J) - STATUS OF CHANNEL J, 0-IF NO UNIT IN THE STATION ,
C              &1-IF A UNIT IS IN THE SERVICE FACILITY.
C     IAT - NEXT UNIT ARRIVING TIME
C     IRUN - KEEPS COUNT OF THE NUMBER OF  RUNS
C     SECOMT(I,J)-SERVICE COMPLETION TIME OF ITH UNIT IN JTH CHANNEL
```

```
C      IWL(J)-NUMBER OF UNITS IN CHANNEL J AT ANY TIME
C      IWQ(J) - NUMBER OF UNITS IN QUEUE FOR CHANNEL J AT ANY TIME
C      DATA INPUT
C      CARD NUMBER 1
C      ALL ENTRIES ARE IN FIELDS OF I7
C      ENTRY 1 'L'NUMBER OF CHANNELS
C      ENTRY 2 'KB'  -1FOR TABULAR ARRIVAL ARTE, 0FOR CONSTANT ARRIVAL
C      RATE, &1 FOR POISSON ARRIVAL RATE
C      ENTRY 3 'KBRK' TOTAL NUMBER OF UNITS DESIRED TO BE SIMULATED
C      ENTRY 4 'IPRINO- &1 FOR RUN VALUES, -1 OR 0 FOR FINAL STATISTICS
C      ENTRY 5 'IPRI' THE NUMBER OF RUNS DESIRED
C      ENTRY 6 'MP'  &1 FOR INFINITE QUEUE, -1OR 0 FOR FINITE QUEUE
C      ENTRY 7 'MPL'  99 FOR INFINITE QUEUE , ALLOWABLE QUEUE FOR FINITE
C                     QUEUE
C      ENTRY 8 'JOCY' &1 FOR JOCKEYING, -1OR 0 FOR NO JOCKEYING
C      CARD NUMBER 2 (CONSTANT OR POISSON ARRIVAL RATE)
C      ENTRY 'AMQ' ARRIVAL RATE, FORMAT  F10.4
C      CARD NUMBER 2 (IN CASE OF TABULAR ARRIVAL RATE)
C      ALL ENTRIES ARE IN FIELDS OF F7.4
C      ENTRY  CUMULATIVE PROBABILITY VALUES
C      CARD NUMBER 3 (TABULAR ARRIVAL RATE)
C      ALL ENTRIES ARE IN FIELDS OF  F7.4
C      ENTRY - CORRESPONDING TIME VALUES
C      CARD NUMBER 3(POISSON OR CONSTANT ARRIVAL RATE) OR CARD NUMBER 4
C      ALL ENTRIES ARE IN FIELDS OF I3
C      FOR EVERY CHANNEL THERE SHOULD A VALUE OF KLY
C      ENTRY 'KLY'  -1 FOR TABULAR SERVICE RATE, 0 FOR CONSTANT SERVICE
C                   RATE, &1 FOR POISSON SERVICE RATE
C      THE FOLLOWING SET OF DATA CARDS DEPENDS ON THE NUMBER OF CHANNELS
C      FOR EACH CHANNEL THERE SHOULD BE AT LEAST ONE CARD
C      IN CASE OF POISSON OR CONSTANT SERVICE RATE
C      ALL ENTRIES ARE IN FIELDS OF  F10.4
C      ENTRY 1 'PQ' SERVICE RATE OF THAT CHANNEL
C      ENTRY 2 'PLAY' PREFERENCE FOR THAT CHANNEL
C      IN CASE OF TABULAR SERVICE RATE THERE SHOULD BE THREE CARDS FOR
C      THAT CHANNEL
C      CARD A
C      ALL ENTRIES ARE IN FIELDS OF  F7.4
C      ENTRY - CUMULATIVE PROBABILITY VALUES OF THE CELLS
C      CARD B
C      ALL ENTRIES ARE IN FIELDS OF  F7.4
C      ENTRY - CORRESPONDING TIME VALUES OF THE CELLS
C      CARD C
C      ENTRY 'PLAY' PREFERENCE FOR THAT CHANNEL, FORMAT F10.4
       DIMENSION AVENUM(10),AVEQUE(10),ABLE(2,10),TWT(10),LOTUS(10),W(10)
       DIMENSION SECOMT(500,10),NJ(10),TIDT(10),QLEN(10),ZQUE(10),IWQ(10)
       DIMENSION TM(50),AQL(10),ANL(10),AQ(10),AN(10),PW(100),P(100)
       DIMENSION TID(10),TI(10),NQ(10),PNP(10),POR(10),MARY(10),PR(100)
       COMMON IWL(10),L,KX,IX,KN,ST,TABLE(2,10),PQ(10),KLY(10),PLAY(10),P
      1L(30),NJQ,K,MPL
C      IX IS THE RANDOM NUMBER SEED
       IX=932684517
```

```
C      LEAP KEEPS COUNT OF THE NUMBER OF PROBLEMS
       LEAP=1
C      READ IN DATA
  730 READ(11,70,END=731)L,KB,KBRK,IPRINO,IPRI,MP,MPL,JOCY
       WRITE(12,735)LEAP
  735 FORMAT(1H1,' PROBLEM NUMBER = ',I3)
   70 FORMAT(8I6)
C      CHECK FOR ARRIVAL DISTRIBUTION OPTION
       IF(KB)91,92,92
   92 READ(11,181)AMQ
  181 FORMAT(F10.4)
       GO TO 93
C      READ IN  TABULAR ARRIVAL RATE , IF USED
   91 READ(11,82)((ABLE(KXX,LXX),LXX=1,10),KXX=1,2)
   82 FORMAT(10F7.3)
   93 READ(11,183)(KLY(J),J=1,L)
  183 FORMAT(10I3)
       DO 96 J=1,L
       IF(KLY(J))94,95,95
   95 READ(11,185)PQ(J),PLAY(J)
  185 FORMAT(2F10.4)
       GO TO 96
   94 READ(11,186)((TABLE(KRT,LRT),LRT=1,10),KRT=1,2)
  186 FORMAT(10F7.3)
       READ(11,99)PLAY(J)
   99 FORMAT(F10.4)
   96 CONTINUE
C      WRITING OUT DATA
       IF(KB)344,345,345
  344 WRITE(12,346)L
  346 FORMAT(1H0,'NO. OF CHANNELS =',I5)
       GO TO 4
  345 WRITE(12,343)L,AMQ
  343 FORMAT(1H0,'CHANNELS =',I3,5X,'LAMDA =',F10.4)
    4 WRITE(12,352)
  352 FORMAT(1H0,T10,'CHANNEL',T23,'MU',T30,'PREFERENCE RATE')
  354 DO 350 J=1,L
       IF(KLY(J))35,34,34
   34 WRITE(12,348)J,PQ(J),PLAY(J)
  348 FORMAT(1H0,T10,I5,T18,F10.4,T30,F10.4)
       GO TO 350
   35 WRITE(12,360)J,PLAY(J)
  360 FORMAT(1H0,T10,I5,T30,F10.4)
  350 CONTINUE
       IF(MP)6,6,23
   23 WRITE(12,500)
  500 FORMAT(1H0,' INFINITE QUEUE BEFORE EACH CHANNEL ')
       GO TO 503
    6 WRITE(12,501)MPL
```

```
  501 FORMAT(1HO,' FINITE QUEUE OF ',I2,' BEFORE EACH CHANNEL')
C      INITIALIZE THE CUMU SUM OF MEAN VALUES,ALL RUNS
  503 DO 600 J=1,L
      TI(J)=0.0
      NQ(J)=0
      PNP(J)=0.0
      AQL(J)=0.0
      ANL(J)=0.0
  600 CONTINUE
      CAL=0.0
      DO 5 IUM=1,50
      PR(IUM)=0.0
    5 CONTINUE
      TIS=0.0
      AWAT=0.0
      PID=0.0
      IRUN=0
C      SET ALL INDIVIDUAL PARAMETERS = 0 INITIALLY
    1 I=0
      PAK=0.0
      NJQ=0
      TAT=0.0
      CLOCK=0.0
      EVENT=0.0
      PIDTS=0.0
      SPT=0.0
      NAN=0
      PAT=0.0
      DO 2 IUM=1,50
      PW(IUM)=0.0
      P(IUM)=0.0
    2 CONTINUE
      DO 13 J=1,L
      IWL(J)=0
      MARY(J)=0
      IWQ(J)=0
      POR(J)=0.0
      LOTUS(J)=0
      N(J)=0
      NJ(J)=0
      TWT(J)=0.0
      TIDT(J)=0.0
      QLEN(J)=0.0
      ZQUE(J)=0.0
      AVEQUE(J)=0.0
      AVENUM(J)=0.0
   13 CONTINUE
C      INCREASE THE TOTAL NUMBER OF ARRIVALS BY 1
   14 I=I&1
```

```
C     CHECK FOR ARRIVAL DISTRIBUTION
      IF(KB)813,814,815
C     CONSTANT ARRIVAL TIME IS GENERATED
  814 AT=1.0/AMQ
      GO TO 829
C     TABULAR ARRIVAL TIME IS GENERATED
  813 CALL ARAND
      KRT=1
      LRT=1
  816 IF(RN-ABLE(KRT,LRT))818,818,819
  819 LRT=LRT&1
      GO TO 816
  818 KRT=KRT&1
      AT=ABLE(KRT,LRT)
      GO TO 829
C     EXPONENTIAL ARRIVAL TIME IS GENERATED
  815 CALL ARAND
      AT=ALOG(RN)/(-AMQ)
C     NEXT ARRIVAL TIME IS OBTAINED
  829 TAT=CLOCK&AT
C     THE CHANNEL WITH THE SHORTEST QUEUE  IS SELECTED
      CALL SORT
C     CHECK WHETHER ANY UNIT IS REJECTED
      IF(K)24,24,25
   25 GO TO 75
   24 DO 15 J=1,L
C     CHECK FOR THE CHANNEL  WHICH THE UNIT HAS CHOSEN
      IF(J-KX)16,17,16
C     TEST THE STATUS OF THE SELECTED CHANNEL
   17 IF(LOTUS(KX)-1)18,19,19
C     IF FREE , GENERATE SERVICE TIME
C     QUEUE ,NUMBER OF UNITS,STATUS ARE ADJUSTED
   18 LOTUS(KX)=1
      IWL(KX)=IWQ(KX)&1
      IWQ(KX)=IWL(KX)-1
      CALL STR
      SPT=SPT&ST
      N(KX)=N(KX)&1
C     CALCULATE THE SERVICE COMPLETION TIME
      LPY=N(KX)
      IF(LPY-500)601,601,602
  602 LPY=LPY-500
  601 SECOMT(LPY,KX)=CLOCK&ST
      GO TO 15
C     IF SELECTED CHANNEL IS NOT FREE,ADJUST QUEUE AND NUMBER OF UNITS
   19 IWQ(KX)=IWQ(KX)&1
      IWL(KX)=IWQ(KX)&1
      GO TO 15
   16 IF(LOTUS(J)-1)21,15,15
```

```
C       UPDATE THE IDLE TIME
   21 TIDT(J)=TIDT(J)&AT
   15 CONTINUE
      GO TO 75
C       IF THE CLOCK TIME IS A SERVICE COMPLETION TIME,CHANGE THE STATUS
C       OF THE CHANNEL IN WHICH SERVICE IS COMPLETED
   50 LOTUS(KX)=0
C       UPDATE THE TOTAL NUMBER OF  UNITS SERVICED BY THAT CHANNEL
      MARY(KX)=MARY(KX)&1
      NAN=NAN&1
C       CHECK FOR QUEUE
      IF(IWQ(KX).EQ.0)GO TO 51
      GO TO 52
C       IF QUEUE EXISTS,PUT A UNIT IN THE FACILITY
C       ADJUST THE QUEUE,STATUS,NUMBER OF UNITS
C       GENERATE SERVICE TIME
C       CALCULATE THE SERVICE COMPLETION TIME
   52 IWQ(KX)=IWQ(KX)-1
      IWL(KX)=IWQ(KX)&1
      LOTUS(KX)=1
      CALL STR
      SPT=SPT&ST
      N(KX)=N(KX)&1
      LPY=N(KX)
      IF(LPY-500)603,603,604
  604 LPY=LPY-500
  603 SECOMT(LPY,KX)=CLOCK&ST
      GO TO 53
C       IF  THERE IS NO QUEUE
   51 IWL(KX)=0
C       TEST FOR JOCKEYING
   53 IF(JOCY)935,935,953
  953 DO 54 J=1,L
      JUE=IWL(J)-IWL(KX)
      LAMP=J
      IF(JUE-2)54,55,55
   54 CONTINUE
C       IF THERE IS NO JOCKEYING
  935 IF(IWL(KX).EQ.0)GO TO 58
      GO TO 75
C       A UNIT FROM THE LONGEST QUEUE JOCKEYS TO SHORTEST QUEUE CHANNEL
   55 NJ(KX)=NJ(KX)&1
C       ADJUST THE QUEUE,NUMBER OF UNITS IN THE CHANNEL FROM WHICH THE
C       UNIT JOCKEYED
      IWQ(LAMP)=IWQ(LAMP)-1
      IWL(LAMP)=IWQ(LAMP)&1
C       CHECK FOR THE STATUS OF THE CHANNEL
      IF(LOTUS(KX).EQ.0)GO TO 56
      GO TO 57
```

```
C     IF CHANNEL IS FREE,GENERATE SERVICE TIME
   56 LOTUS(KX)=1
      CALL STR
      SPT=SPT&ST
      N(KX)=N(KX)&1
C     CALCULATE THE SERVICE COMPLETION TIME
      LPY=N(KX)
      IF(LPY-500)605,605,606
  606 LPY=LPY-500
  605 SECOMT(LPY,KX)=CLOCK&ST
  886 IWL(KX)=1
      IWQ(KX)=IWL(KX)-1
      GO TO 75
C     IF CHANNEL IS NOT FREE,ADJUST QUEUE,NUMBER OF UNITS
   57 IWQ(KX)=IWQ(KX)&1
      IWL(KX)=IWQ(KX)&1
      GO TO 75
C     UPDATE IDLE TIME
   58 TIDT(KX)=TIDT(KX)&TAT-CLOCK
C     ARRANGE THE VARIOUS EVENTS IN AN ORDER
   75 LAB=0
      DO 76 J=1,L
      LPY=N(J)
      IF(LOTUS(J).EQ.0)GO TO 76
      GO TO 77
   77 LAB=LAB&1
      IF(LPY-500)925,925,926
  926 LPY=LPY-500
  925 TM(LAB)=SECOMT(LPY,J)
   76 CONTINUE
      LAB=LAB&1
      TM(LAB)=TAT
C     FIND THE NEXT EVENT TIME
      LART=1
      LARS=2
   80 IF(TM(LART)-TM(LARS))78,78,79
   79 LART=LARS
   78 LARS=LARS&1
      IF(LARS-LAB)80,80,81
   81 EVENT=TM(LART)
C     FIND WHETHER IT IS AN ARRIVAL OR SERVICE COMPLETION TIME
      IF(LART.EQ.LAB)GO TO 86
      GO TO 87
C     IF SERVICE COMPLETION TIME,FIND THE CHANNEL
   87 DO 88 J=1,L
      LPY=N(J)
      IF(LPY-500)924,924,923
  923 LPY=LPY-500
  924 IF(LOTUS(J).EQ.0)GO TO 88
```

```
      GO TO 89
   89 IF(EVENT-SECOMT(LPY,J))88,90,88
   88 CONTINUE
   90 KX=J
C     UPDATE THE STATISTICS
   86 DO 882 J=1,L
      QLEN(J)=QLEN(J)&IWL(J)*(EVENT-CLOCK)
      ZQUE(J)=ZQUE(J)&(IWQ(J)*(EVENT-CLOCK))
      TWT(J)=TWT(J)&(IWQ(J)*(EVENT-CLOCK))
  882 CONTINUE
      IUM=0
      DO 132 J=1,L
      IUM=IUM&IWL(J)
  132 CONTINUE
      IF(IUM)133,133,134
  134 IF(IUM-30)932,932,120
  932 PW(IUM)=PW(IUM)&EVENT-CLOCK
      GO TO 120
  133 PIDTS=PIDTS&EVENT-CLOCK
C     TEST FOR THE TOTAL NUMBER OF UNITS NEED TO BE SIMULATED
  120 IF(NAN-KBRK)83,84,84
C     IF LESS, UPDATE THE CLOCK TIME
   83 CLOCK=EVENT
C     CHECK WHETHER IT IS AN ARRIVAL OR SERVICE COMPLETION TIME
      IF(LAB.EQ.LART)GO TO 14
      GO TO 50
C     CALCULATE THE RUN STATISTICS
   84 DO 85 J=1,L
      AVEQUE(J)=ZQUE(J)/CLOCK
      AVENUM(J)=QLEN(J)/CLOCK
      TID(J)=TIDT(J)/CLOCK
      PAT=TWT(J)&PAT
C     TEST FOR FINITE QUEUE
      IF(MP)510,510,511
C     IF FINITE,FIND THE TOTAL NUMBER OF UNITS SERVICED
  510 PAK=PAK&MARY(J)
      GO TO 85
C     IF INFINITE,FIND THE FRACTION SERVED BY EACH CHANNEL
  511 PAK=MARY(J)
      POR(J)=PAK/KBRK
   85 CONTINUE
C     TEST FOR FINITE QUEUE
      IF(MP)513,513,514
C     IF FINITE,FIND THE FRACTION SERVED BY EACH CHANNEL
  513 DO 512 J=1,L
      PAC=MARY(J)
      POR(J)=PAC/PAK
  512 CONTINUE
C     CALCULATE THE MEAN WAITING TIME AND MEAN TIME IN THE SYSTEM
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  514 AVEWAT=PAT/NAN
      TIMSYS=(PAT&SPT)/NAN
C     TEST FOR FINITE QUEUE
      IF(MP)520,520,521
C     IF FINITE,CALCULATE THE FRACTION OF ARRIVAL TURNED AWAY
  520 CAT=NJQ
      FAR=CAT/I
C     CALCULATE THE SYSTEM PROBABILITY STATES
  521 DO 138 IUM=1,50
      P(IUM)=PW(IUM)/CLOCK
  138 CONTINUE
      PIDT=PIDTS/CLOCK
C     TEST FOR PRINTING OPTION
      IF(IPRINO)540,540,716
  716 LIFE=IRUN&1
C     PRINT THE RUN VALUES
      WRITE(12,717)LIFE
  717 FORMAT(1H0,10X,'RUN NUMBER =',I3)
  516 WRITE(12,610)
  610 FORMAT(1H0,'MEAN QUEUE     MEAN NUMBER     IDLE TIME     NO.OF UNITJO
     1CKEYED TO     PORTION SERVED      CHANNEL NO.')
      DO 609 J=1,L
      WRITE(12,101)AVEQUE(J),AVENUM(J),TID(J),NJ(J),POR(J),J
  101 FORMAT(1H0,F10.4,3X,F10.4,5X,F10.4,8X,I8,15X,F6.3,12X,I5)
  609 CONTINUE
      WRITE(12,612)AVEWAT,TIMSYS
  612 FORMAT(1H0,'AVERAGE WAITING TIME =',F8.4,5X,'MEAN TIME IN SYSTEM =
     1',F10.4)
      IF(MP)703,703,704
  703 WRITE(12,522)FAR
  522 FORMAT(1H0,15X,'FRACTION OF UNITS TURNED AWAY =',F8.4)
  704 WRITE(12,139)
  139 FORMAT(1H0,2X,'    P(0)       P(1)       P(2)       P(3)       P(4)
     1  P(5)       P(6)       P(7)       P(8)       P(9)       P(10)')
      WRITE(12,140)PIDT,(P(IUM),IUM=1,10)
  140 FORMAT(1H0,2X,F10.4,10F10.4)
      WRITE(12,141)
  141 FORMAT(1H0,2X,'    P(11)      P(12)      P(13)      P(14)      P(15)
     1  P(16)      P(17)      P(18)      P(19)      P(20')
      WRITE(12,142)(P(IUM),IUM=11,20)
  142 FORMAT(1H0,2X,10F10.4)
      WRITE(12,143)
  143 FORMAT(1H0,2X,'    P(21)      P(22)      P(23)      P(24)      P(25)
     1  P(26)      P(27)      P(28)      P(29)      P(30)')
      WRITE(12,144)(P(IUM),IUM=21,30)
  144 FORMAT(1H0,2X,10F10.4)
C     UPDATE THE RUN STATISTICS
  540 IRUN=IRUN&1
      DO 150 J=1,L
```

```
      TI(J)=TI(J)&TID(J)
      NQ(J)=NJ(J)&NQ(J)
      PNP(J)=PNP(J)&POR(J)
      AQL(J)=AQL(J)&AVEQUE(J)
      ANL(J)=ANL(J)&AVENUM(J)
  150 CONTINUE
C     TEST FOR FINITE QUEUE
      IF(MP)525,525,526
  525 CAL=FAR&CAL
  526 AWAT=AWAT&AVEWAT
      TIS=TIS&TIMSYS
      DO 168 IUM=1,30
      PR(IUM)=PR(IUM)&P(IUM)
  168 CONTINUE
      PID=PID&PIDT
      IF(IRUN-IPRI)1,3,3
C     SIMULATION STOPS
C        CALCULATE THE FINAL STATISTICS
    3 DO 151 J=1,L
      AQ(J)=AQL(J)/IPRI
      TI(J)=TI(J)/IPRI
      NQ(J)=NQ(J)/IPRI
      PNP(J)=PNP(J)/IPRI
      AN(J)=ANL(J)/IPRI
  151 CONTINUE
      IF(MP)530,530,531
  530 CAL=CAL/IPRI
  531 AWT=AWAT/IPRI
      TIP=TIS/IPRI
      PIPT=PID/IPRI
      DO 169 IUM=1,30
      P(IUM)=PR(IUM)/IPRI
  169 CONTINUE
C        PRINT THE FINAL STATISTICS
      IF(IPRINO)720,720,721
  720 WRITE(12,719)
  719 FORMAT(1H0,10X,' FINAL STATISTICS ')
      GO TO 515
  721 WRITE(12,723)
  723 FORMAT(1H1,10X,' FINAL STATISTICS ')
  515 WRITE(12,518)
  518 FORMAT(1H0,'MEAN QUEUE    MEAN NUMBER    IDLE TIME    NO.OF UNITJO
     1CKEYED TO    PORTION SERVED     CHANNEL NO.')
      DO 286 J=1,L
      WRITE(12,10)AQ(J),AN(J),TI(J),NQ(J),PNP(J),J
   10 FORMAT(1H0,F10.4,3X,F10.4,5X,F10.4,8X,I8,15X,F6.3,12X,I5)
  286 CONTINUE
      IF(MP)533,533,534
  533 WRITE(12,536)CAL
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536 FORMAT(1H0,15X,'FRACTION OF UNITS TURNED AWAY =',F8.4)
534 WRITE(12,311)AWT
311 FORMAT(1H0,T15,'MEAN WAITING TIME=',F8.4)
    WRITE(12,312)TIP
312 FORMAT(1H0,T15,'MEAN TIME IN SYSTEM =',F8.4)
    WRITE(12,439)
439 FORMAT(1H0,2X,'    P(0)       P(1)       P(2)       P(3)       P(4)
   1  P(5)       P(6)       P(7)       P(8)       P(9)       P(10)')
    WRITE(12,440)PIPT,(P(IUM),IUM=1,10)
440 FORMAT(1H0,2X,F10.4,10F10.4)
    WRITE(12,441)
441 FORMAT(1H0,2X,'    P(11)      P(12)      P(13)      P(14)      P(15)
   1  P(16)      P(17)      P(18)      P(19)      P(20')
    WRITE(12,442)(P(IUM),IUM=11,20)
442 FORMAT(1H0,2X,10F10.4)
    WRITE(12,443)
443 FORMAT(1H0,2X,'    P(21)      P(22)      P(23)      P(24)      P(25)
   1  P(26)      P(27)      P(28)      P(29)      P(30)')
    WRITE(12,444)(P(IUM),IUM=21,30)
444 FORMAT(1H0,2X,10F10.4)
    LEAP=LEAP&1
    GO TO 730
731 STOP
    END
C   THIS RANDOM NUMBER ROUTINE WILL GENERATE RANDOM NUMBERS GREATER
C   THAN 0 AND LESS THAN OR EQUAL TO 1
    SUBROUTINE ARAND
    COMMON IWL(10),L,KX,IX,RN,ST,TABLE(2,10),PQ(10)
    IX=IX*65539
    IF(IX)5,6,6
  5 IX=IX&2147483647&1
  6 RN=IX
    RN=RN*.4656613E-09
    IF(RN)7,7,8
  7 RN=1.
  8 RETURN
    END
C   THIS SUBROUTINE  IS USED TO SELECT A CHANNEL FOR A UNIT
    SUBROUTINE SORT
    COMMON IWL(10),L,KX,IX,RN,ST,TABLE(2,10),PQ(10),KLY(10),PLAY(10),K
   1IM(10),SAI(10),PAID(10),NJQ,K,MPL
    KA=1
    MJ=2
  6 IF(IWL(KA)-IWL(MJ))3,3,4
  4 KA=MJ
  3 MJ=MJ&1
    IF(MJ-L)6,6,7
  7 KX=KA
C   IF MORE THAN ONE CHANNEL HAVE THE SAME QUEUE,USE THE PREFERENCE
```

```
      PAI=0.0
      LIS=0
      DO 8 J=1,L
      IF(IWL(KX)-IWL(J))8,9,8
    9 LIS=LIS&1
      KIM(LIS)=J
      PAI=PLAY(J)&PAI
      SAI(LIS)=PAI
    8 CONTINUE
      IF(LIS-1)22,22,13
   13 CALL ARAND
      IF(RN-1)15,13,13
   15 DO 16 KIS=1,LIS
      IF(KIS-1)18,18,19
   19 PAID(KIS)=(SAI(KIS)-SAI(KIS-1))/PAI
      GO TO 20
   18 PAID(KIS)=SAI(KIS)/PAI
      GO TO 21
   20 PAID(KIS)=PAID(KIS)&PAID(KIS-1)
   21 IF(RN-PAID(KIS))17,17,16
   16 CONTINUE
   17 KX=KIM(KIS)
C     TEST FOR THE ALLOWABLE QUEUE
      IF(IWL(KX)-(MPL&1))22,23,23
C     UPDATE THE TOTAL NUMBER OF UNITS REJECTED
   23 NJQ=NJQ&1
C     K IS AN INDEX
      K=1
      GO TO 14
   22 K=0
   14 RETURN
      END
C     THIS SUBROUTINE IS USED FOR GENERATING SERVICE TIMES
      SUBROUTINE STR
      COMMON IWL(10),L,KX,IX,RN,ST,TABLE(2,10),PQ(10),KLY(10)
C     CHECK FOR THE SERVICE DISTRIBUTION OPTION
   16 IF(KLY(KX))20,21,22
C     GENERATE CONSTANT SERVICE TIME
   21 ST=1.0/PQ(KX)
      GO TO 30
C     GENERATE TABULAR SERVICE TIME
   20 CALL ARAND
      KKX=1
      LLX=1
   24 IF(RN-TABLE(KKX,LLX))25,25,26
   26 LLX=LLX&1
      GO TO 24
   25 KKX=KKX&1
      ST=TABLE(KKX,LLX)
```

```
      GO TO 30
C     GENERATE EXPONENTIAL SERVICE TIME
   22 CALL ARAND
      ST=ALOG(RN)/(-PQ(KX))
   30 RETURN
      END
```