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AN ECONOMETRIC MODEL OF THE SOUTH DAKOTA ECONOMY

BY

KIRK WRAY RUBIDA

A thesis submitted
in partial fulfillment of the requirements for the
degree Master of Arts, Major in
Economics, South Dakota
State University

1971
This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Arts, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser

Date

Head, Economics Department

Date
ACKNOWLEDGMENTS

Construction of a study of the magnitude and complexity represented by this first econometric model of the South Dakota economy incurs more debts of mind and spirit than any author could ever hope to acknowledge.

With the realization that any list will be grossly incomplete, special acknowledgments for assistance in the writing of this study must go to the following people.

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Senator George McGovern greatly assisted the initiation of empirical research by implementing an invaluable data search among national governmental departments and agencies for data relevant to the South Dakota economy.

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Mr. and Mrs. H. P. Rubida, parents of the author, gave generously of their financial assistance and personal encouragement, without which this study could never have been completed.

In addition to the assistance acknowledged above, special mention must be made of the fact that nearly every member of the Economics Department's faculty was contacted at some point during the construction of the South Dakota model. This contact was essential in insuring that the representatives of each economic specialization that exists within the Economics Department have an opportunity to provide guidance in the area of their specialization. Without this diversity of specializations, from which the researcher can draw both academic guidance and creative inspiration, the construction of this study would have been a virtually impossible task.

Finally, complete responsibility for all theoretical, mathematical, and statistical, misconceptions and omissions contained in this study must rest with the author who found the writing of this econometric analysis to be a challenging and rewarding endeavor.
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CHAPTER I

INTRODUCTION

...probably the greatest practical importance of econometrics lies in the contributions it can make to the formation of government economic policy. A sound economic policy depends more upon careful quantitative predictions than it does upon qualitative theoretical developments.¹

Problem

Barring the advent of apocalypse or perdition, continued economic growth promises to increase the growing complexities of state economic policy formation in the United States of the 1970's.

State governments throughout the country are preparing to meet the future's economic challenges by investing heavily in the financing of various kinds of positive economic analyses. The general purpose of these analyses is to delineate the macroeconomic constructs of a state economy in order to help define the feasible economic policy options open to the state administration.

Since World War II, the construction of input-output econometric models has been the most popular method of subnational macroeconomic analysis. An input-output econometric model may be defined as,

"...a general theory of production,"\(^2\) which delineates the economic interdependence of an economy's industries by showing, "...the final demand for goods and services and the interindustry transactions required to satisfy that demand."\(^3\) Operationally, the framework of an input-output econometric model, "...is essentially a system of double-entry bookkeeping."\(^4\)

Unfortunately, an input-output econometric model has one formidable and, in many cases, prohibitive disadvantage. Its implicit prerequisite of highly accurate and complete data of (1) the value of industry inputs and the sources of those inputs for each industry, and (2) the value of industry sales and the destination of those sales from each industry, virtually requires data collection by personal interview.\(^5\) This time-consuming and expensive process appears to be the key reason that many states have not undertaken the construction of statewide input-output econometric models or have settled for multi-county models.

With the construction of statewide input-output econometric models beyond the reach of many state governments, there remains a growing need for an alternative methodology by which a practical macroeconomic analysis can be made of an entire state economy.


\(^3\)Ibid., p. 30.

\(^4\)Ibid., p. 14.

\(^5\)Ibid., p. 75.
Purpose

The purpose of this study is to adapt and develop a methodology of practical statewide macroeconomic analysis that is an alternative to the construction of a statewide input-output econometric model.

Objectives

The three objectives of this study are:

1. to present an annual stochastic econometric model of the South Dakota economy which is an adaptation of Daniel B. Suits' 1962 annual stochastic econometric model of the United States economy; 6

2. to demonstrate the model's use as an effective instrument by which state economic performance can be forecast;

3. to begin exploration of the model's implications for the economic impact which selected exogenous economic policies may have on the state economy.

Review of Literature

Many stochastic econometric models exist for examining national economies, but research in the area of stochastic econometric models for examining a state economy is still in its infancy.

It is hypothesized that a constructive addition to this area of research may be made by adapting a stochastic national econometric model to the South Dakota economy.

6A stochastic econometric model is defined in this study as a system of statistically analyzed mathematical equations in which some or all of the equations contain a random disturbance term. See "Definitions" for a more detailed definition.
Because this study initiates research in a largely unexplored field of economic analysis, the purpose of this "Review of Literature" is to describe the source of the adapted national model and to list other sources which form the theoretical and procedural foundation of this study's analysis.

The most promising model available for adaptation, due to its relatively small size and simplicity, is a national model constructed by Daniel B. Suits, Professor of Economics at the University of Michigan. This study of the South Dakota economy relies heavily on Suits' stochastic econometric model of the United States economy, presented in the March, 1962, American Economic Review article, "Forecasting and Analysis with an Econometric Model."7

Three of Suits' other publications provide valuable theoretical and procedural instruction in this attempt to interpret his model and translate it for the South Dakota economy: Statistics: An Introduction to Quantitative Economic Research,8 The Theory and Application of Econometric Models,9 and An Econometric Model of the Greek Economy.10

Finally, two major college textbooks provide additional guidance for the student of econometrics: Preface to Econometrics by Michael


J. Brennan, Professor of Economics at Brown University,\textsuperscript{11} and Econometric Methods by J. Johnston, Professor of Econometrics at the University of Manchester, England.\textsuperscript{12}

Definitions

Economic Theory: "Economic theory consists of the study of various groups or sets of relations [among a system of observable and essentially measurable variables such as prices, costs, outputs, incomes, savings, and employment]\textsuperscript{13} which are supposed to describe the functioning of a part or the whole of an economic system."\textsuperscript{14}

Econometrics: "Econometrics is the application of modern statistical methods to economic theory that has been formulated in mathematical terms."\textsuperscript{15}

Econometric Model: "In econometrics a theory is called a model... [and]...one views economic life as explainable by a set of simultaneous mathematical equations. These equations express the relationships among economic magnitudes which guide economic behavior."\textsuperscript{16}

\textsuperscript{11}Brennan, op. cit.


\textsuperscript{14}Johnston, op. cit., p. 3.

\textsuperscript{15}Brennan, op. cit., p. iii.

\textsuperscript{16}Ibid., p. 9, 10.
econometric model, then, is a complete system of statistically analyzed mathematical equations in which each equation, "...involves at least one variable which also appears in at least one other relationship which is part of the model."\(^{17}\)

**Stochastic Econometric Model:** A stochastic econometric model completely avoids the strict requirements of double-entry bookkeeping and therefore differs from an input-output econometric model because different mathematical and statistical procedures are employed in constructing the model.

In a stochastic econometric model some or all of the equations are assumed to contain a random disturbance or stochastic term, \(u\), "...because it is unreasonable to presume that any relations in equation systems other than definitions should be satisfied exactly."\(^{18}\)

J. Johnston states in *Econometric Methods* that, "There are three possible, though not mutually exclusive, ways of rationalizing the insertion of the \(u\) term...":

(1) In explaining human behavior, no matter how meticulous may be the attempt to obtain an exhaustive list of all determining factors, the actual list of those factors extends ad infinitum.


This case then amounts to saying that \( Y = f(X_1, X_2, \ldots, X_n) \)
where \( n \) is an impractically large number, so that we choose instead to represent \( Y \) as an explicit function of just a small number of what are thought to be the more important \( X \)'s and let the net effect of the excluded variables be represented by \( u \). In the limiting case of a single, explicit variable we have \( Y = f(X_1, u) \).

(2) "A second justification for the presence of a disturbance term in economic relations is to assume that, over and above the total effect of all relevant factors, there is a basic and unpredictable element of randomness in human responses which can be adequately characterized only by the inclusion of a random variable term."

(3) "A third source of error lies in errors of observation or measurement."\(^{19}\)

In summarizing historically the difference between models in which all equations contain a disturbance term and models in which no equations contain a disturbance term, Richard Ruggles states that,

Prior to the introduction of the probability approach to model building, exact [or, technically defined, 'deterministic']\(^{20}\) equation systems without a random variable were used, and statisticians could legitimately claim that economists did not present their models in such a form that they represented well-specified statistical hypotheses. Models which have been made more realistic, in a statistical sense, by the introduction of a random disturbance into each of the structural equations (except the definitional equations) are termed 'stochastic models.'\(^{21}\)

\(^{19}\)Johnston, op. cit., p. 5, 6.


\(^{21}\)Ruggles, op. cit., p. 441-442.
Ruggles defines a stochastic model as a model in which, "...each of the structural equations (except the definitional equations)" contains a stochastic term. For simplicity of exposition, this study of the South Dakota economy defines a stochastic model as a model in which any, but not necessarily all, of the structural equations contains a stochastic term. In contrast, a deterministic econometric model, of which an input-output econometric model is the most widely known example, continues to be defined as a model in which none of the structural equations contains a stochastic term.

**Methodology of the Model**

This study's stochastic econometric model of the South Dakota economy consists of two types of theoretical equations: (1) definitional equations, and (2) structural equations that contain random disturbance terms and that are in the form of either simple or multiple linear regression equations.

Each equation in the model delineates one aspect of the relations of the state's various nonindustrial sectors of macroeconomic activity to each other and to other economic and noneconomic factors. Since the South Dakota model is stochastic, and not deterministic, the task of describing the state's economic activity consists of a process of selective aggregation. For example, all employees of private South Dakota businesses are combined under a single title of "wage and salary workers, private sector," and many similar products are combined under a single item of expenditure such as "durable goods." At a more advanced level of aggregation, infinitely complex
mathematical relations are simplified or reduced inductively, by the use of statistical inference, to the form of linear approximations of the relations between two or more data aggregates similar to those exemplified above.

Within the context of the above aggregation processes, a stochastic econometric model is constructed in three steps:

1. Empirical data are estimated, often by statistical inference, to represent the values of the data aggregates;
2. Statistical techniques, such as regression analysis, are employed to obtain numerical estimates of the coefficients of the individual linear equations;
3. The entire system of theoretical equations is solved by matrix inversion and transposition to provide estimates of policy multipliers which are used, first, to forecast state economic performance, and, second, to estimate the complex economic responses which may be called forth in the economy by specific simulated exogenous economic policies.

Schematic Example

Presented below is a simple illustrative example of the systematic structure of the stochastic econometric model of the South Dakota economy. Although the analyses made with this schematic example are completely unrelated to any known economy,
an understanding of the example's arbitrary structure greatly facilitates understanding of the structure of the actual South Dakota model.

Of the four equations in the exemplary model, equations (1), (2), and (3), are structural equations without a stochastic term, and equation (4) is a definitional equation. For simplicity, no structural equations with a stochastic term are included in the schematic example.

The contents of the schematic example, for the most part, consist of direct quotation of the schematic example presented by Suits as an aid in understanding his 1962 stochastic econometric model of the United States economy.

All alterations made in Suits' original text have been clearly marked with brackets and usually indicate a shift in reference from Suits' United States model to the South Dakota model which is presented in chapter three.

A. A Simple Illustrative Example

To illustrate the principles of application, let us suppose that the statistical procedure gave rise to the following, purely schematic, model of four equations.

\[
\begin{align*}
C &= 20 + .7(Y - T) \\
I &= 2 + .1Y_{-1} \\
T &= .2Y \\
Y &= C + I + G
\end{align*}
\]
According to equation (1), consumption (C) depends on current disposable income (Y - T). In equation (2), investment (I) depends on income lagged one period. The third equation relates taxes (T) to income, while the last defines income (Y) as the sum of consumption, investment and government expenditure G.

While this model is small, it illustrates most of the properties of the larger model [of the South Dakota economy]. The single consumption function in equation (1) corresponds to the set of four equations (01), (02), (03), and (04) that describe the behavior of the consumer sector in...[the South Dakota model]. The investment behavior represented in (2) corresponds to equations (05), through (10). The single tax equation (3) corresponds to a combination of the eleven tax and transfer equations, while the relationship of production to income embodied in equation (4) is indicated in much greater detail by equations (11) through (20).

This econometric model approximates the economy by a system of equations in which the unknowns are those variables—income, consumption, investment, and tax yield—whose behavior is to be analyzed. The "knowns" are government expenditure and lagged income. When projected values for the "knowns" are inserted in the equations, the system can be solved to forecast the values of the unknowns.

Quotation marks are used advisedly on the word "knowns." For, while some economic variables move so slowly along secular trends that their future values can be projected with considerable accuracy, others—for example new government expenditures—are unknown in advance of their occurrence, even in principle. Moreover, even the values of lagged variables are unknown at the time of the forecast, since a useful forecast must be made some months before the end of the preceding year.

At any rate, suppose we expect next year's government expenditure to be 20, and the preliminary estimate of this year's income is, say, 100. Substituting G = 20 and Y₁ = 100 into the equations above and solving gives C = 86.2, I = 12, T = 23.7, Y = 118.2.

B. Introducing Outside Information

It may appear from the foregoing that this kind of forecasting is a blind, automatic procedure; but while an
econometric model looks like a rigid analytical tool, it is actually a highly flexible device, readily modifiable to bring to bear additional information and judgment. For example, the investment equation in our little model is surely an unreliable predictor of capital formation. If no other information were available the equation would have to serve the purpose. But suppose we have available a survey of investment intentions reported by business. An estimate derived from such a survey is clearly superior to any that equation (2) could produce. To introduce the information into the forecast we simply remove equation (2) from the model and, in the remaining equations, set I equal to the survey value. Forecasts made from the [University of Michigan] Research Seminar [United States] model have frequently involved use of a figure for gross investment in plant and equipment derived from the McGraw-Hill Survey of Investment Intentions rather than from equation (05) of the model.

Information can also be used to modify individual relationships short of replacing them entirely. For example a prospective improvement in consumer credit terms—a variable that does not appear in our schematic model—would be expected to stimulate consumption expenditure. It is often possible to set an upper limit to this stimulating effect, and by increasing the constant term in the consumption function by this amount, to set an upper limit to the forecast economic outlook.

Using the flexibility to full advantage permits the forecaster to explore any desired number of alternative sets of projections and modifications, and to bring to bear all information and judgment he possesses. The econometric model is not, therefore, a substitute for judgment, but rather serves to focus attention on the factors about which judgment must be exercised, and to impose an objective discipline whereby judgment about these factors is translated into an economic outlook that is consistent both internally, and with the past observed behavior of the economic system.

C. The Inverse Matrix

In principle, the exploration of a range of alternative projections and other modifications of the model consists of inserting each set of alternatives in turn as "knowns" in the equations and solving for the resulting forecast. The process is greatly expedited by further simplifying the model and by the use of the inverse matrix. Simplification of the model is made possible by the fact that one of the unknowns, I, depends only on knowns. I helps to determine the current values of C, T, and Y, but the latter do not, in turn, feed back into
the determination of the current value of I. As a result, once the knowns are given, I can be directly calculated from (2) without reference to any other part of the model, and hence, as far as the remaining equations are concerned, I can be treated as a known in the sense used above. (Indeed it is this fact that enables us to replace equation (2) with survey values for I.)

The process of solving the system of equations can then be divided into two parts. First: using the values of the knowns, calculate the value of I. Second: substitute the knowns (now including I) into the remaining equations, and solve for the other unknowns.

The inverse matrix facilitates the second step. For those unfamiliar with matrix manipulations the following will help clarify the nature and use of this table. Since I is now considered as known, the model is reduced to the system of three equations (1), (3) and (4) above. By transferring all unknowns to the left side, and representing the right sides by P₁, P₃, and P₄, these equations can be expressed as:

\[
\begin{align*}
(1) & \quad C - 0.7Y + 0.7T = 20 = P₁ \\
(3) & \quad -.2Y + 1.0T = 0 = P₃ \\
(4) & \quad -C + Y = I + G = P₄
\end{align*}
\]

Now using any convenient method to solve this system for C, Y, and T in terms of P₁, P₃, and P₄ will yield:

\[
\begin{align*}
C &= 2.273P₁ - 1.591P₃ + 1.273P₄ \\
T &= .455P₁ + .682P₃ + .455P₄ \\
Y &= 2.273P₁ - 1.591P₃ + 2.273P₄
\end{align*}
\]

That is, the value of each unknown is obtained as a specified weighted total of P₁, P₃, and P₄. Where a large number of equations is used, and a lot of calculating is to be done, it is convenient to display the weights used for each unknown as a column of numbers in a table, with the detail of the P's shown in a separate column at the right:

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>C</th>
<th>T</th>
<th>Y</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>2.273</td>
<td>.455</td>
<td>2.273</td>
<td>20</td>
</tr>
<tr>
<td>(3)</td>
<td>-1.591</td>
<td>.682</td>
<td>-1.591</td>
<td>0</td>
</tr>
<tr>
<td>(4)</td>
<td>1.273</td>
<td>.455</td>
<td>2.273</td>
<td>I + G</td>
</tr>
</tbody>
</table>
To make a forecast we first substitute $Y_{-1}$ into equation (2) and solve for $I$. Then $I$ and $G$ are substituted in the $P$ column of the table and the values of $P_1$, $P_3$, and $P_4$ calculated. These values, weighted by the numbers shown in the $C$ column of the inverse and summed, give the forecast value of consumption; use of the weights in column $Y$ gives the forecast for income, etc. For example if we set $Y_{-1} = 100$ and $G = 20$, we first find from (2) $I = 12$. Substituting these values in column $P$ of the table gives the forecast values: $C = 86.2$, $T = 23.7$, $Y = 118.2$.

D. **Short-Run Policy Multipliers**

It is an obvious step from economic forecasting to short-run policy analysis. To investigate any specified set of prospective government actions, we insert them in the proper place in column $P$ and solve for the forecast implied by these assumptions. The analysis is expedited if we first calculate short-run multipliers for the individual components of government action. These can then be applied in any desired policy mixture.

Short-run multipliers for any policy variable are readily calculated by inserting +1 for the variable everywhere it appears in column $P$, and then (ignoring all terms that do not contain the variable in question) extending a forecast using the columns of the inverse. For example, to calculate the government expenditure multiplier, set $G = 1$ in row (4) of column $P$. This makes $P_4 = 1$. To find the effect of this value of $G$ on, say, income, multiply this value of $P_4$ by the weight in row (4) of the $Y$ column to get $Y = 1 \times 2.273 = 2.273$. That is, the income multiplier on government expenditure is 2.273. Likewise, $T = 1 \times .455 = .455$. That is, the tax-yield multiplier on government expenditure is .455. In other words, for every dollar of additional government expenditure, tax receipts rise by nearly 46 cents. A corollary is that—according to our schematic model—an increase in government expenditure of 1 with no change in tax legislation will generate an increase in deficit of only:

$$G - T = 1 - .46 = .54$$

In addition to changing the value of exogenous variables like government expenditure, government policy can produce changes in the equations themselves. An extensive change—e.g. a substantial alteration in tax rates—can only be studied by replacing the old tax equation by a new one, but less extensive changes can be studied as shifts in the levels of existing equations, the coefficients being unaltered.
Multipliers for such shifts are easily determined by placing +1 in the row of column P that corresponds to the equation being shifted. The extensions are then made as before. For example, to calculate the multipliers on a +1 shift in the level of the tax equation, we put +1 in the row marked (3) of column P, since the tax equation is (3). The multiplier effect of this shift is then calculated by multiplying this 1 by the weight in the corresponding row of the appropriate column, as shown above.

For example for income:

\[ Y = 1 \times (-1.591) = -1.591 \]

For consumption:

\[ C = 1 \times (-1.591) = -1.591 \]

In other words, the multipliers associated with the shift of any equation are merely the weights in the row of the inverse corresponding to that equation.

Note that according to our simplified model, the tax-yield multiplier is .682. That is, an upward shift of $1 billion in the tax schedule actually increases yield by only $682 million. The difference is due to the decline in income arising from the shift in the tax schedule.

The small size of our illustrative model limits the policy variables to government expenditure and the level of taxes. In the more extensive model below, policy is given considerably more scope; a number of individual tax and transfer equations can be shifted, and a number of different kinds of expenditure altered. The number of possible combinations of action is correspondingly very large; but one important advantage to a linear system lies in the fact that once multipliers for the individual components have been calculated, the economic implications of a complete policy 'package' can be estimated by summing the effects of the individual components.

For example, an increase of $1 in government expenditure coupled with an upward shift of $1 in the tax schedule would generate a change in income given by the sum of the two individual multipliers:

\[ Y = 2.273 - 1.591 = .682 \]

This is what might be called an 'ex-ante-balanced' government expenditure multiplier. That is, the change in the law is such as to increase tax yield at the existing level of income by enough to balance the planned expenditure, but the budget will
not necessarily be balanced ex post. The tax and expenditure program will alter income, and hence will change tax yields. Analysis of the complete fiscal impact of the operation requires the examination of all revenue and outlay items combined. Adding together the two tax-yield multipliers we find that the additional expenditure of $1 is offset by a tax yield of:

\[.682 + .455 = 1.137\]

That is, the ex-ante-balanced expenditure of $1 billion would, in our example, be accompanied by an increase of $1.137 billion in tax yield and give rise to an ex-post surplus of $137 million.

Although the discussion has been focused on a highly simplified example, the principles developed apply equally to any linear econometric model. The presentation of the actual... [South Dakota]...model...will follow the same pattern as the...[above] illustration...\textsuperscript{22}

CHAPTER II

DATA COLLECTION

The quality of data used in constructing a stochastic econometric model is one of the most crucial indicators of the expected quality of the model's results.

The construction of this model of the South Dakota economy required that there be compiled or estimated, for the first time in one place, state macroeconomic data which previously have been available only from greatly scattered sources or not at all.

For the above reasons, this chapter presents, first, a short discussion of the general methodology of data collection pertinent to all economic variables in the model, and, second, a detailed explanation of data collection methodology and sources for each economic variable in the model.

General Methodology

Annual data from 1953 were assembled for fifty-four major macroeconomic variables relevant to the performance of the South Dakota economy.

All data used in the model are assumed to represent values for the calendar year. Although some data actually represent fiscal year analyses, it is assumed that unaltered fiscal year data compose the most accurate estimation available for calendar year data.

Data which were not available in constant dollars (1958 = 100.0) were deflated by means of the Commerce Department's implicit price
deflator for total gross national product (data taken from Economic Report of the President).23

First differences were calculated for all variables expressed in constant dollars. In keeping with Suits' terminology, first differences will be indicated throughout the study by prefixing the Greek letter delta, $\Delta$, to the symbol of the variable. Thus, the first difference for variable $Y$ for the year 1963 is defined symbolically as $\Delta Y_{1963} = Y_{1963} - Y_{1962}$.

First differences were calculated for all of the variables because five major advantages accrue from their use. The use of first differences, (1) reduces the downward bias in calculated standard errors that results from the autocorrelation of residuals from time series regression; (2) allows the use of lagged undifferenced values of certain variables to serve as proxy variables for the first differences in stocks, since data on many stocks are not available;24 (3) focuses emphasis on the changes in the status quo


24 "The change in the stock of a commodity is equal to additions, in the form of new purchases, less the withdrawals in the form [of] scrappage. Now, in any kind of durable good, the annual amount of scrappage is likely to be stable; there are no wide fluctuations in the rate at which, say, radios, kitchen stoves or automobiles wear out. The big variation in the stock arises from acquisitions. Thus last year's purchases can be used as a proxy variable for the increase in stock." Suits, The Theory and Application of Econometric Models, pp. 40-41.
that may result from changes in other factors; (4) minimizes trends caused by changes in exogenous variables such as population, tastes, technology, and habits, "...without explicitly introducing them into the analysis. The net effect of change in these factors is represented in the constant term of the equation."; (5) minimizes complications encountered in data revision since, "Revisions usually alter the level at which variables are measured, rather than their year-to-year variation."25

The year 1955 was selected as the initial year for operation of the model because it was desired that the model analyze the economic structure of the South Dakota economy from the post Korean War period to the present. Most of the 1955 variables employed in constructing the model are expressed as variables in first differences (ΔY1955 = Y1955 - Y1954) or as variables lagged one year (Y1955-1 = Y1954). These variables represent the first year of general economic recovery after (1) the Korean War, which ended on July 27, 1953, and (2) the 1953 - 1954 recession that followed the war.

The selection of 1953 as the initial year of collected data was technically required by the above selection of 1955 as the initial year for operation of the model. Specifically, the reason for the collection of data as far back as 1953 rests on the fact that eight

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of the fifty-four variables employed in constructing the model are expressed as lagged first differences. For example, the lagged first difference of variable Y for 1955 equals Y's 1954 value less Y's 1953 value. Symbolically, \( \Delta Y_{1955-1} = Y_{1954} - Y_{1953} \). Hence, 1953 data are prerequisite for making 1955 the initial year for operation of the model.

**Specific Methodology of Four Data Groups**

Each of the fifty-four major macroeconomic variables has been classified under one of four Data Groups by the nature of the source of its data.

**Data Group I:** This is the largest group and contains twenty-two of the fifty-four variables.

In the initial stages of data collection it was unknown exactly what data were in existence by states and what existing statewide data were collected by each national governmental department or agency.

In initiating the data search essential for filling the above information gap, Senator George McGovern most generously assisted the implementation of a comprehensive canvass of all relevant national governmental departments and agencies for whatever statewide data were currently available.

The extent to which the above data search was successful is summarized by the fact that all data for all twenty-two variables in Data Group I were received directly from national governmental departments and agencies by way of Senator McGovern's office.
Data Group I (continued)

The individual variables of Data Group I are defined below.

\[ E = \text{number of employees of private South Dakota businesses, including both farm and nonfarm employees.} \]

The values of this variable have been calculated from data received from the national departments of Defense and Labor. The equation used in defining \( E \) states that

\[ E = \text{LF} - \text{Eo} - \text{Eg} - \text{U}. \]

All four components of the definition of \( E \) are defined in Data Group I.

\[ \text{Eg} = \text{federal, state, and local government employees in South Dakota, including armed forces.} \]

The equation used in defining \( \text{Eg} \) states that \( \text{Eg} = \text{federal, state, and local government employment} \) (data received from Labor Department) + military personnel stationed in South Dakota (data received from Defense Department).

\[ \text{Eo} = \text{nonagricultural self-employed unpaid family workers in South Dakota, excluding child labor and housewives} \]

(data received from Labor Department).

\[ \text{LF} = \text{total civilian and armed forces labor force in South Dakota.} \]

The equation used in defining \( \text{LF} \) states that \( \text{LF} = \text{total employees of South Dakota civilian labor force} \) (data received from Labor Department) + military personnel stationed in South Dakota (data received from Defense Department).

\[ \text{M} = \text{federal military purchases of durable goods from private South Dakota businesses.} \]

According to the Defense Department, durable goods procurement for South Dakota consists mostly of missiles and space systems, non-combat vehicles, and electronics and communications
Data Group I (continued)
equipment, all of which are used or installed in South Dakota, but are purchased almost entirely out of state. For this reason the values of M are assumed to be negligible for the state of South Dakota.

\[ P^* \] = corporate profits of South Dakota corporations before tax (data received from Internal Revenue Service).

\[ Pf \] = farm income of South Dakota households (data received from Commerce Department).

\[ S1e \] = personal contributions for social insurance in South Dakota (data received from Commerce Department).

\[ Tcd \] = customs duties collected in South Dakota. According to the Internal Revenue Service, "Customs duties collected are shown by Bureau of Customs Regions and Districts which do not generally conform to individual State boundaries." Because no adequate methods could be developed at this time for estimating South Dakota's proportion of its customs region's total customs duties collected, the values of Tcd are assumed to be negligible for the state of South Dakota.

\[ Tfc \] = federal corporate income tax collected in South Dakota (data received from Internal Revenue Service).

\[ Tfe \] = federal excise tax collected in South Dakota (data received from Internal Revenue Service).

\[ Tfy \] = federal personal income tax collected in South Dakota (data received from Internal Revenue Service).

\[ T*fy \] = federal personal income tax liability in South Dakota (data received from Internal Revenue Service).
Data Group I (continued)

\[ U = \text{unemployed workers in South Dakota (data received from Labor Department).} \]

\[ U_c = \text{unemployed workers in South Dakota eligible for unemployment compensation (data received from Labor Department).} \]

\[ w = \text{average annual earnings of a full-time employee in private South Dakota businesses. The values of this variable have been calculated from data received from the national departments of Commerce, Defense, and Labor. The equation used in defining } w \text{ states that } w = \frac{W}{E}. \text{ Both components of the definition of } w \text{ are defined in Data Group I.} \]

\[ W = \text{wages paid by all South Dakota private businesses to households plus other labor income. The equation used in defining } W \text{ states that } W = \text{total wage disbursements in South Dakota} - W_g + \text{other labor income}. \text{ } W_g \text{ is defined in Data Group I, and the values of the other two components of } W \text{ were received from the Commerce Department.} \]

As part of the definition of \( W \), special reference must be made to the procedures used to calculate first differences for the fifty-four variables of the model.

For all variables not otherwise defined, the values of the first differences of the variables may be calculated directly from the annual values of the variables as they are defined.

For certain variables, of which \( W \) is one, special definitions have been constructed in the process of constructing the model.
Data Group I (continued)

$W$ (continued)

$\Delta W$ may not be calculated directly from the annual values of $W$ because the theoretical definition of $\Delta W$ states that $\Delta W = \Delta (wE)$. This nonlinear equation is unacceptable because it is inconsistent with the processes of a model composed of otherwise linear equations.

Equation (15) of the model replaces the unacceptable equation, $\Delta W = \Delta (wE)$, with a linear approximation which states, for the purpose of the model, that, $\Delta W = w_{-1} \Delta E + E_{-1} \Delta W$.

$W_g = \text{wages received by South Dakota households from federal, state, and local, governments and armed forces (data received from Commerce Department)}$.

$X_f = \text{federal transfer payments received by South Dakota households. The equation used in defining } X_f \text{ states that } X_f = \text{total transfer payments received by South Dakota households} - X_{GI} - X_s - X_u$. $X_{GI}$, $X_s$, and $X_u$ are defined in Data Group I, and the values of total transfer payments received by South Dakota households were received from the Commerce Department.

$X_{GI} = \text{GI insurance dividends received by South Dakota households. More specifically, } X_{GI} \text{ represents a dividend, refunding overpayment of life insurance premiums, paid to ex-members of the United States armed forces (data received from Veterans Administration)}$.

$X_s = \text{state and local transfer payments received by South Dakota households (data received from Commerce Department)}$. 
Data Group I (continued)

Xu = unemployment compensation received by South Dakota households (data received from Commerce Department).

Data Group II: This is the second largest group and contains fourteen of the fifty-four variables.

One of the primary procedural challenges confronting successful construction of this stochastic econometric model of the South Dakota economy is the unavailability of data which, ideally, would be contained in a state income and product account similar to the Commerce Department's national income and product account. Because some essential data are available only on a national basis, procedures must be established for estimating a state's proportion of the values of national variables. This Data Group presents the procedures used in estimating the values of South Dakota data on the basis of known values of national data.

All data for all variables in Data Group II are estimated, at least in part, by means of an application of the fundamental assumption essential for the construction of an economic base study: that employment is proportional to income and can therefore serve as a proxy variable for production and income.26, 27


Data Group II (continued)

Both Tiebout and Nourse show that the reliance of past economic base studies on the above assumption, which will hereafter be called "Tiebout's fundamental assumption," has proven to be an effective stopgap tool in helping define the complex interrelationships of technologically advanced economies.28, 29

In a state economic base study, state employment data are used as proxy data for state production and income data. This econometric study of the South Dakota economy does not use employment data as proxy data, but rather uses employment data more indirectly in estimating a coefficient that determines the state's proportion of the values of national variables.

The values of all variables in Data Group II are estimated by using the assumption that the state's proportional representation in national aggregated macroeconomic variables is proportional to the state's percentage representation in the total civilian labor force of the United States.

For example, corporate dividends received by South Dakota households, \((\text{Div})\), may be estimated with the aid of the equation stating that, \(\frac{\text{(Div)}}{\text{(DivUS)}} = \frac{\text{CLF}}{\text{CLFUS}}\). After solving this equation for \((\text{Div})\) by multiplying both sides of the equation by \((\text{DivUS})\), the values of \((\text{Div})\) may be obtained using the equation stating that,

28Tiebout, op. cit., pp. 27 - 44.

Data Group II (continued)

\[(\text{Div}) = \frac{\text{CLF}}{\text{CLFUS}} \times (\text{DivUS})\]

given, that

\((\text{Div}) = \) corporate dividends received by South Dakota households,

\(\text{CLF} = \) total civilian labor force of South Dakota (data received from Labor Department),

\(\text{CLFUS} = \) total civilian labor force of the United States (data taken from Statistical Abstract of the United States),

\((\text{DivUS}) = \) corporate dividends received by United States households (national data taken from Survey of Current Business),

\(\text{CLF} = \) the coefficient whose application on the basis of Tiebout's fundamental assumption enables the determination of the state's proportion of the values of various national variables.

The theoretical nature of this application of Tiebout's fundamental assumption is that it results in estimates of values of variables, not for the real world economy of South Dakota, but for a state whose civilian labor force is the size of South Dakota's and whose economy is assumed to be in many respects a microcosm of the entire United States economy.

Data Group II, Subgroup I: Subgroup I contains variables whose values have been estimated solely with the aid of Tiebout's fundamental assumption whose application has just been demonstrated.

The individual variables of Data Group II, Subgroup I, are defined below.
Data Group II, Subgroup I (continued)

$A =$ expenditure for new automobiles and parts and net expenditure for used automobiles by South Dakota households. This definition is Suits' definition which is used in his national econometric model.

Unfortunately, data on a net basis were unavailable from any known source, and national values of gross auto product, available in *Survey of Current Business*, have been used as a substitute.

As a further demonstration of the application of Tiebout's fundamental assumption, values of $A$ are calculated by multiplying the coefficient, $\frac{CLF}{CLFUS}$, by the national values of gross auto product.

Algebraically, if $AUS =$ national values of gross auto product, then

$$A = \frac{CLF}{CLFUS} \times AUS.$$ 

$D =$ expenditure for all durable goods except automobiles and parts by South Dakota households. The equation used in defining the national value of the variable states that gross national $D =$ gross national household expenditure for all durables - gross national household expenditure for automotive durables. Again, the data available in *Survey of Current Business* provide a substitute in gross terms for the desired net figure.

$ND =$ expenditure for nondurable goods by South Dakota households. The same qualification holds for $ND$ that held for $A$ and $D$.

$S =$ expenditure for services by South Dakota households. The same qualification holds for $S$ that held for $A$, $D$, and $ND$. 
**Data Group II, Subgroup II**: Subgroup II contains variables whose values have been estimated with the aid of Tiebout's fundamental assumption and with the aid of an additional index of the level of industrial development in the state of South Dakota.

For the variables of aggregate demand in Subgroup I, (A, D, ND, and S), the use of Tiebout's fundamental assumption signified that the level of consumer expenditure in South Dakota is assumed to be at the level of an economy (1) whose civilian labor force is the size of South Dakota's, and (2) whose structure is assumed to be in many respects a microcosm of the entire United States economy. In simplest terms, it was assumed that all South Dakota consumers spend their incomes in a pattern identical to the expenditure pattern of the average consumer in the entire United States economy.

All variables in Subgroup II tend to be related to the level of industrial or corporate development in the state of South Dakota. Because of this relationship, application solely of Tiebout's fundamental assumption would imply that South Dakota's industrial or corporate development is representative of the industrial or corporate development of the entire country. Since South Dakota is, industrially, a relatively underdeveloped state, sole use of Tiebout's fundamental assumption would greatly overestimate the values of variables such as those involving the amount of South Dakota business depreciation and investment.
An index of the degree that South Dakota's industrial or corporate development varies from that of a microcosm of the entire United States economy was estimated using the following equation:

\[
\text{corporate proportionality coefficient} = \frac{P^*, \text{ defined in Data Group I as being actual corporate profits of South Dakota corporations before tax in constant dollars (data received from Internal Revenue Service)}}{\text{estimated corporate profits of South Dakota corporations before tax in constant dollars determined by application of Tiebout's fundamental assumption (national data taken from Survey of Current Business)}}
\]

Briefly described, the corporate proportionality coefficient for South Dakota is quite small. It increases gradually from 0.11010 in 1955 to 0.20699 in 1963, experiences a slight decline during the period 1964 through 1966, hits a high of 0.21435 in 1967, and then declines slightly to 0.20155 in 1968. Of course, the small size of the coefficient is not entirely surprising since most of South Dakota's businesses are small noncorporate enterprises involved in primary agricultural production.

The small size of the state's coefficient means that the level of South Dakota's corporate or industrial development is still well below the level of the corporate or industrial development of a microcosm of the national economy. For example, in 1968 South Dakota's level of corporate or industrial development is only about twenty percent (i.e., 0.20155) of the level of development of a microcosm of the entire nation.
Data Group II, Subgroup II (continued)

In reference to the rate of South Dakota's development in relation to the rate of development of a microcosm of the entire United States economy, it is encouraging to note that the state's corporate proportionality coefficient has nearly doubled since 1955.

The significance of the increase in the level of development from 0.11010 in 1955 to 0.20155 in 1968 is that the rate of South Dakota's corporate or industrial development is greater than the rate of the development of a microcosm of the entire United States economy. In short, the state has just recently begun to approach slowly the level of development of a microcosm of the entire nation. South Dakota would be at the level of development of a microcosm of the entire nation if its corporate proportionality coefficient equaled 1.00000.30

Practical application of the corporate proportionality coefficient can be shown with the use of the equation used as an example of the application solely of Tiebout's fundamental assumption. The only modification to be made of the value of (Div), derived by the above exemplary equation, is the multiplication of (Div) by the corporate proportionality coefficient. Thus:

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30One aspect of a partial explanation of the state's increase of level of development is suggested by the fact that South Dakota's level of corporate or industrial development in 1955, at a level of only ten percent, was so near nonexistence that, during the period 1955-1968, the addition of relatively few industries was required to increase the level of development to twenty percent.
Data Group II, Subgroup II (continued)

\[ \text{Div} = (\text{Div}) \times \text{CPC} \]

given, that

\[ \text{Div} = \text{corporate dividends received by South Dakota households} \]
after application of Tiebout's fundamental assumption and after
application of the corporate proportionality coefficient,

\[ (\text{Div}) = \text{corporate dividends received by South Dakota households} \]
after application of just Tiebout's fundamental assumption,

\[ \text{CPC} = \text{corporate proportionality coefficient}. \]

The individual variables of Data Group II, Subgroup II, are
defined below.

\[ \text{Dep} = \text{depreciation in South Dakota businesses (national data} \]
taken from Survey of Current Business). \]

\[ \text{Div} = \text{corporate dividends received by South Dakota households} \]
(national data taken from Survey of Current Business).

\[ g = \text{federal, state, and local, government and military purchases} \]
of goods and services from private South Dakota businesses. By
definition, \( g \) does not include \( Wg \) which is defined in Data Group I
as being wages received by South Dakota households from federal,
state, and local, governments and armed forces. The national variable
which is used to represent the values of \( g \), and whose data were taken
from the Statistical Abstract of the United States, is that of "capital
outlay." "Capital outlay" consists of expenditure by all levels of
government in South Dakota for construction, equipment, and land and
existing structures.
Data Group II, Subgroup II (continued)

ID = inventory stock of durable goods owned by South Dakota businesses. The national variable which is used to represent the values of ID, and whose data were taken from the Survey of Current Business, is that of "manufacturers' inventories, book value, seasonally adjusted, for durable goods industries."

\[ \text{ig} = \text{total net government interest payments received by South Dakota households.} \]

The equation used to define the national variable of ig states that \( \text{ig} = \text{total net federal government interest payments paid in the United States} + \text{total net state and local government interest payments paid in the United States} \) (data of both component variables taken from Survey of Current Business). Note that, "Government transfer and interest payments are treated as receipts of the household sector. Although in reality a large portion of the interest payments on public debt is received by banks, insurance companies, and other firms, these interest payments are considered in effect to pass through business firms and to become in their entirety receipts of the household sector." 31

IND = inventory stock of nondurable goods owned by South Dakota businesses. The national variable which is used to represent the values of IND, and whose data were taken from the Survey of Current Business, is that of "manufacturers' inventories, book value, seasonally adjusted, for nondurable goods industries."

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Data Group II, Subgroup II (continued)

\( L \) = South Dakota consumer liquid assets. The national variable which is used to represent the values of \( L \), and whose data were taken from the Economic Report of the President, is that of "selected liquid assets held by the public." This national variable is defined as consisting of demand deposits, currency, time deposits, postal savings system, savings and loan shares, United States government savings bonds, and United States government securities maturing within one year.

\( PD \) = expenditure for durable goods by South Dakota businesses. The national variable which is used to represent the values of \( PD \), and whose data were taken from the Survey of Current Business, is that of gross private domestic investment in "producers' durable equipment."

\( PE \) = expenditure for plant and equipment by South Dakota businesses. The equation used in defining the national value of the variable states that \( PE = \) all gross private domestic nonresidential investment in the United States + all gross private domestic farm residential investment in the United States (data of both component variables taken from Survey of Current Business).

Data Group II, Subgroup III: Subgroup III contains one variable whose values have been estimated with the aid of Tiebout's fundamental assumption and with the aid of the results of an economic base study of South Dakota.
Data Group II, Subgroup III (continued)

The single variable of Data Group II, Subgroup III, is defined below.

\[ F = \text{gross value of South Dakota exports (not just international exports).} \]

John Rapp's *An Economic Analysis of South Dakota* consists primarily of an economic base study of South Dakota constructed with the use of the methodology of Tiebout's *The Community Economic Base Study*.

With the aid of Tiebout's fundamental assumption that employment is proportional to income and can therefore serve as a proxy variable for production and income, Rapp estimates for 1965 the percentage of South Dakota's national income that is derived from exports by various industries. 32

For the purpose of constructing this study's stochastic econometric model of the South Dakota economy, the estimation of the dollar value for South Dakota's total gross exports is made in three steps with the assumption that Rapp's percentages hold constant for the entire period 1953 through 1968.

First, data were collected from the *Survey of Current Business* for the values of United States national income produced by all major industries.

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32 John Rapp, Table 10-2.—Final Allocation of Employment (Direct and Indirect [Sales]), *An Economic Analysis of South Dakota*, a report presented to the Governor of South Dakota by the South Dakota Industrial Development Expansion Agency, September 1, 1965, p. 118.
Data Group II, Subgroup III (continued)

industries in the nation. For convenience, the industries were combined into seven sets as follows: (1) total manufacturing, (2) agriculture, forestry, fishing, mining, and construction, (3) transportation, communication, and public utilities, (4) wholesale trade and retail trade, (5) finance, insurance, and real estate, (6) services including education, and (7) government excluding education.

Second, South Dakota's proportion of United States national income produced by each set of industries was estimated with the aid of the broad application of Tiebout's fundamental assumption as demonstrated in the introduction of Data Group II.

Third, the gross value of South Dakota exports was estimated by multiplying the amount of national income produced by each set of industries in South Dakota (estimated in step two), by decimal equivalents of Rapp's percentages of South Dakota national income that is derived from exports by each of the seven sets of industries.

Critical analysis of the above three step procedure may induce the argument that use of Tiebout's fundamental assumption in step two probably leads to a significant underestimation of the value of agricultural exports in South Dakota. The reasoning behind the argument is that, in this case, Tiebout's fundamental assumption implies that South Dakota's exports, by each set of industries, should be representative of the value of national income produced by each of the seven sets of industries in the nation as a whole. This
argument is countered by the fact that while the values of South Dakota's agricultural exports are underestimated, there is a simultaneous offsetting overestimation of the values of South Dakota's manufacturing exports. This automatic offsetting effect helps produce a compromise level of total state exports.

The corporate proportionality coefficient was not applied to F because exports in South Dakota are not now very strongly related to industrial or corporate development. Use of the coefficient would fail to take into account the voluminous trade in South Dakota's agricultural exports which overshadows trade in all other state exports.

**Data Group III: This is the third largest group and contains twelve of the fifty-four variables. All data for all variables in Data Group III were compiled primarily from secondary sources available at South Dakota State University and, in a few cases, from direct communications with state governmental departments.**

The individual variables of Data Group III are defined below.

Aaa = national Aaa bond yield in percent (data taken from Economic Report of the President).

C = ratio of index of construction costs (Commerce Department's composite cost index taken from Construction Review)
Data Group III (continued)

\[ \text{FHA} = \text{national FHA ceiling interest rate in percent (data taken from Economic Report of the President)}. \]

\[ H = \text{expenditure for nonfarm public and private residential housing in South Dakota. The equation used to estimate the values of } H \text{ states that } H = \text{national average construction costs of new private nonfarm one family houses started (national data taken from Construction Review) } \times HS \text{ (defined in Data Group III). The above equation is only an approximation because it underestimates } H. \text{ The underestimation exists because the average construction cost used in the equation does not include houses and apartment houses built for more than one family. Further underestimation results from the use of the equation's other element, HS.} \]

\[ HS = \text{number of nonfarm public and private residential housing starts in South Dakota in units per year (data taken from Construction Review). HS contains an implicit underestimation of the actual values of HS because the Commerce Department collects data from only a limited number of permit-issuing places in the state.} \]

\[ Tbp = \text{business property tax collected in South Dakota. Local general property taxes consist of property taxes on lands, lots, and personal property. Because no business-nonbusiness property tax breakdown is available, it has been estimated, with the use of Commerce Department data, that in 1966 in South Dakota, 75.9 percent of local general property taxes was collected by means of taxes on} \]
Data Group III (continued)

Tbp (continued)

business property, and 24.1 percent of local general property taxes was collected by means of taxes on nonbusiness property.\textsuperscript{33}

It has been assumed further that the business-nonbusiness sources of South Dakota local general property taxes have existed in the 75.9/24.1 ratio for the entire period 1953 through 1968.

On the basis of these two assumptions, the relative contributions of business and nonbusiness property have been calculated by relating the above ratio to the data of total general property taxes which are available in the \textit{Annual Report of the South Dakota Department of Revenue}. Finally, total Tbp = local general property tax from business property + railroad property tax + telegraph tax + sleeping car tax + electric light, power, water, and gas tax + taxes on telephones within and outside corporate limits + grain tax + public shooting areas tax.

Although all data of total general property taxes in the \textit{Annual Report} consist of tax extensions or liabilities instead of actual tax collections, it is assumed that the difference between the values of South Dakota tax extensions and tax collections is small enough to be considered negligible for the purposes of the model.

Data Group III (continued)

Teg = state estate and gift taxes collected in South Dakota. Suits' usage of the symbol Teg will be maintained, although South Dakota has an inheritance tax instead of estate and gift taxes. All data available for Teg in the Annual Report of the South Dakota Department of Revenue have been multiplied by the coefficient (10/9). This has been done to compensate for the situation described in the Annual Report as follows: "Because the County Treasurer is permitted by law to retain 10% of all Inheritance Taxes collected, for county purposes, this amount [listed in the Annual Report] is but ninety percent of the fiscal year's total Inheritance Tax Collections..."34

Top = other state personal taxes collected in South Dakota. The equation used in defining Top states that Top = nonbusiness local general property tax (see Tbp in Data Group III for explanation of method of estimation) + school poll tax + dog tax + special assessments + road poll tax + money and credits + consumers use tax + auto registration tax + aircraft registration tax + motor fuel tax + butter substitute revenue stamp tax + cigarette revenue stamp tax + sales tax on liquor, beer, and cigarettes + liquor and wine revenue stamp tax (all data taken from Annual Report of the South Dakota Department of Revenue).

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Data Group III (continued)

\(T_{os} = \) other state \(T_{axes}\) on business collected in South Dakota.

The equation used in defining \(T_{os}\) states that \(T_{os} = sales\) tax licenses + ore tax + gross income tax + private car license tax + liquor licenses + beer licenses + cigarette licenses + peddlers and solicitors licenses + butter substitute licenses + trading stamp licenses + railway express tax + insurance company tax (data received from state Department of Insurance) + fire marshall tax (data received from state Department of Insurance) + tonnage tax (data taken from staff memorandum of state Legislative Research Council)\(^{35}\) + store licenses + retailers use tax + person weighing scale fee + vending machine licenses + racing track licenses (data received from state Department of Audits and Accounts).

South Dakota Departments of Insurance and of Audits and Accounts provided personal communications. Data source for all other components of the definition of \(T_{os}\), except tonnage tax, is the Annual Report of the South Dakota Department of Revenue.

\(T_{sc} = \) state corporate profits (or income) tax collected in South Dakota. \(T_{sc}\) consists solely of, "Bank Franchise Tax. An annual tax of 4 1/2% of the net income of banks and financial corporations (emphasis added) is levied for the privilege of doing business in this state."\(^{36}\) (Data taken from Annual Report of the South Dakota Department of Revenue).


\(^{36}\)Ibid., p. 3.
Data Group III (continued)

Tss = state sales tax collected in South Dakota (data taken from Annual Report of the South Dakota Department of Revenue).

Tsy = state and local income taxes collected in South Dakota (these taxes have never been used in the state as of the writing of this study).

Data Group IV: This is the smallest group and contains six of the fifty-four variables. All data for all variables in Data Group IV were compiled or estimated with the use of combinations of the sources of Data Groups I, II, and III.

The individual variables of Data Group IV are defined below.

G* = private gross state product of the South Dakota economy. The most important word in the above verbal definition of G* is the word "private." Suits' definition of private gross national product states that G* = total gross national product - Wg.

Equation (17) of the model provides the preliminary framework used in defining G*. Equation (17) states that,

\[ \Delta P = \Delta G^* - \Delta W - \Delta Dep - \Delta Tfe - \Delta Tcd - \Delta Tbp - \Delta Tss - \Delta Tos - \Delta SIr. \]

By solving equation (17) for \( \Delta G^* \), the equation states that,

\[ \Delta G^* = \Delta P + \Delta W + \Delta Dep + \Delta Tfe + \Delta Tcd + \Delta Tbp + \Delta Tss + \Delta Tos + \Delta SIr. \]

Going one step further and dropping the \( \Delta \)'s prefixing the variables, the equation states that,

\[ G^* = P + W + Dep + Tfe + Tcd + Tbp + Tss + Tos + SIr. \]

Since the values of all variables composing the equations of the model must be consistent throughout the model, the undifferenced
values of $G^*$ in equation (17), shown above, must be equal to the undifferenced values of $G^*$ in equation (11), shown below.

Values of $\Delta G^*$ are defined by definitional equation (11) of the model which states that,

$$\Delta G^* = \Delta(A + D + ND + S) + (\Delta F - \Delta R) + \Delta ID + \Delta IND + \Delta PE + \Delta H + \Delta g.$$

In undifferenced values equation (11) states that,

$$G^* = (A + D + ND + S) + (F - R) + ID + IND + PE + H + g.$$

At this point in the calculation of the undifferenced values of $G^*$, values of all variables in both equations (11) and (17) were known except the values of $(F - R)$.

Values of $G^*$ were calculated by solving equation (11) for $G^*$ in undifferenced values with the assumption that $(F - R) = 0$. This resulted in a disparity between the values of $G^*$ in equation (11) and the values of $G^*$ in equation (17).

Since the values of all component variables were known for equation (17), it was assumed that equation (17), solved for $G^*$ in undifferenced values, defined the correct values of $G^*$.

The disparity between the values of the two equations defining $G^*$ was then assumed to be due entirely to the previously excluded values of $(F - R)$.

With $(F - R)$ accounting for all disparity between the values of $G^*$ calculated by equations (11) and (17), the values of $G^*$ produced by both equations were forced to be numerically identical.
Data Group IV (continued)

G* (continued)

With the values of G* consistent with both major definitional equations (11) and (17), the next requirement was the calculation of the values of ΔG*.

There are four ways of calculating ΔG*:

1. Solve equation (17) for ΔG*.
2. Solve equation (17) for G* and take first differences of the values of G*.
3. Solve equation (11) for ΔG*.
4. Solve equation (11) for G* and take first differences of the values of G*.

The values of ΔG* produced by these four methods are identical. Of course, one major reason methods one and two produce values identical to those produced by methods three and four is that the original disparity has been assumed to be due entirely to (ΔF - ΔR).

First glances at equations (11) and (17) and at the above procedures for calculating the values of ΔG* may induce the erroneous conclusion that, although the values of G* have been forced to be identical, the values of ΔG* produced by the two equations cannot be identical.

This conclusion would be based on the observation that equation (17)'s inclusion of the linear approximation of the values of ΔW should lead to a disparity between the values of ΔG* produced by both equations.
The twofold reason that both equations produce identical values of $\Delta G^*$ is that, first, solving equation (17) for $\Delta G^*$ produces a definition for $\Delta G^*$ that includes a positive $\Delta P$, and, second, covert inclusion in variable $\Delta P$ of a negative $\Delta W$, by means of definitional equation (17), completely offsets the effect of the positive $\Delta W$ whose implications were questioned above.

$P = $ property income of South Dakota private businesses.

Suits' variables $(P - P^* + ig + Div)$, which are found in his equation (20) for disposable income, $Y$, represent the part of the Commerce Department's personal income accounts which consists of the sum of the Commerce Department's variables of property income and proprietors income, both farm and nonfarm.

Thus,

$$P - P^* + ig + Div = \text{sum of Commerce Department's variables of property income and proprietors income, both farm and nonfarm}$$

Since $P$ is the only variable whose values are unknown in the above equation, its values may be calculated as follows,

$P = \text{Commerce Department's variables} + P^* - ig - Div.$

Since $P^* = \text{undistributed corporate profits or UCP} + Div$,

$P = \text{Commerce Department's variables} + UCP + Div - ig - Div$,

or, $P = \text{Commerce Department's variables} + UCP - ig$.

The most important aspect of the above relationship is that the value of Suits' variable, $P$, which he calls property income, is much
Data Group IV (continued)

P (continued)
greater in value than the variable which the Commerce Department
defines as property income.

The Commerce Department's definition of property income is that
it, "...consists of dividends, personal interest income, and rental
income of persons." ³⁷

Suits' definition of property income is that it consists of
dividends + rental income of persons + proprietors income, both farm
and nonfarm + undistributed corporate profits.

Suits' definition of property income varies from the Commerce
Department's definition in that Suits' definition does not include
personal interest income, but it does contain the added variables of
proprietors income, both farm and nonfarm, and undistributed
corporate profits.

Values of ΔP are not calculated by taking first differences
of the values of P.

Values of P are defined, exactly, within the context of Commerce
Department data available for the definition of personal income, which
is a part of disposable income, Y, defined by equation (20). Equation
(20) involves an exact definition of W.

Values of ΔP are defined by definitional equation (17) of the
model. Equation (17) involves a linear approximation, rather than an
exact definition, of ΔW. This linear approximation of ΔW produces

³⁷Robert E. Graham, Jr., "Measuring Regional Market Growth,"
Data Group IV (continued)

P (continued)

the entire disparity that exists between \( \Delta P \) and the first differences of the values of \( P \).

In actuality, the procedures used in calculating the values of \( \Delta P \) for the purposes of this study were much more complicated than the above paragraphs suggest.

The procedures used in calculating values of \( \Delta P \), which would be consistent with the rest of the model, consisted largely of trial and error tempered with a little knowledge of macroeconomic theory.

In the early stages of investigation, values of \( \Delta P \) were originally calculated with the assistance of the equation that states that,

\[
\Delta P = \Delta G^* - \Delta W - \Delta W_g - \Delta Dep - \Delta Tfe - \Delta Tcd - \Delta Tbp - \Delta Tss - \Delta Tos - \Delta Sir.
\]

Note that the above equation varies from equation (17) by the amount of a negative \( \Delta W_g \). This variation resulted from a momentary lapse in memory which consisted of forgetting that \( G^* \) is defined in Suits' model as private gross national product, defined algebraically as \( G^* = \) total gross national product - \( W_g \).

Solving the above equation, defining \( \Delta P \), for \( \Delta G^* \), and then undifferencing all variables, the original value of \( G^* \) used in aiding the calculation of \( \Delta P \) stated that,

\[
G^* = P' + W + W_g + Dep + Tfe + Tcd + Tbp + Tss + Tos + Sir.
\]

Since the values of \( P \) were available as a result of data available for equation (20), and since data were available for all other variables on the right hand side of the equation, the values
Data Group IV (continued)

P (continued)

of $G^*$ were calculated easily. Then the values of $\Delta G^*$ were calculated, first, by relating equation (17) to equation (11), as described under $G^*$ in this Data Group IV, and, second, by taking first differences of the values of $G^*$ produced by either equation (11) or (17).

When it was realized that equation (17), solved for $G^*$, is correct as stated, and that no extra positive $W_g$ was needed, the value of $G^*$ was reduced by the value of $W_g$. Since $\Delta G^*$ may be calculated directly from the summation of undifferenced values of equation (17) or by means of equation (17) solved for $\Delta G^*$, reduction of the values of $G^*$ by the values of $W_g$ automatically reduced the values of $\Delta G^*$ by the values of $\Delta W_g$.

The reduction of the values of $\Delta G^*$ by the values of $\Delta W_g$ can be stated verbally as the addition of a negative $\Delta W_g$ to both sides of the equation which originally was used erroneously to define $\Delta G^*$ by means of including the extra positive value of $\Delta W_g$.

Algebraically, the operation proceeds as shown below.

Original equation:

$$\Delta G^* + (-\Delta W_g) = \Delta P + \Delta W + \Delta W_g + \Delta Dep + \Delta Tfe + \Delta Tcd + \Delta Tbp + \Delta Tss + \Delta Tos + \Delta SIr + (-\Delta W_g)$$

= corrected equation:

$$\Delta G^* = \Delta P + \Delta W + \Delta Dep + \Delta Tfe + \Delta Tcd + \Delta Tbp + \Delta Tss + \Delta Tos + \Delta SIr.$$
Data Group IV (continued)

\( P \) (continued)

Note that although the same symbol has been used in both equations, the value of \( \Delta G^* \) in the corrected equation is less than the value of \( \Delta G^* \) in the original equation by the amount \((-\Delta W_g)\).

Solving the corrected equation for \( \Delta P \) produces an equation identical to equation (17). It is this equation, using the values of \( \Delta G^* \) estimated in the process, by which the values of \( \Delta P \) were calculated for the purposes of this study.

Hindsight shows that the above trial and mostly error procedure, of taking the long way around to calculate the values of \( \Delta P \), can be shortened considerably by defining \( \Delta G^* \) by means of solving equation (17) in undifferenced values for \( G^* \). Since values of \( P \) are available as a result of data available for equation (20), and since data are available for all other variables used to define \( G^* \), the values for \( G^* \) can be calculated easily, and then the values of \( \Delta G^* \) can be determined by taking first differences of the values of \( G^* \).

Then, with data available for \( \Delta G^* \) and for all other differenced variables on the right side of equation (17), values for \( \Delta P \) can be calculated as defined by equation (17). Please note that this procedure for calculating values of \( \Delta P \) still produces values which vary from the first differences of the values of \( P \) by the amount of disparity between actual \( \Delta W \) and the linear approximation of \( \Delta W \) which is used in the model.
Data Group IV (continued)

\[ R = \text{gross value of South Dakota imports (not just international imports).} \]

Values of \( R \) were estimated in four steps.

1. Values of \( G^* \) were calculated by solving equation (17) for \( G^* \) in undifferenced values. 
2. Values of \( G^* \) were calculated by solving equation (11) for \( G^* \) in undifferenced values with the assumption that \( (F - R) = 0 \).
3. The disparity between the results of step one and step two was assumed to be an estimate of the values of \( (F - R) \).
4. With data available for the values of \( F \) and \( (F - R) \), the following equation was used to estimate the values of \( R \): 
   \[ F - (F - R) = F - F + R = R. \]

Values of \( \Delta R \) were calculated by taking first differences of the values of \( R \).

\( S_{Ir} \) = contributions for social insurance made by South Dakota employers. Since no data could be found to represent the values of \( S_{Ir} \), a method of estimation was developed. Data are available for \( S_{Ie} \) (data received from Commerce Department), for national \( S_{Ie} \) (data taken from Survey of Current Business), and for national \( S_{Ir} \) (data taken from Survey of Current Business). By assuming that South Dakota's \( S_{Ir} \) is related to South Dakota's \( S_{Ie} \) in the same ratio that exists for the national values of these two variables, the values for South Dakota's \( S_{Ir} \) can be found with the following equations:
Assume that $S_{Ir} = S_{IrUS}$, with the suffix "US" denoting national variables.

Then, since $S_{Ir}$ is the only variable in the equation whose values are unknown, its values may be determined by the equation,

$$S_{Ir} = S_{Ie} \times S_{IrUS}/S_{IeUS}$$

Values of $\Delta S_{Ir}$ may be calculated by taking first differences of the values of $S_{Ir}$.

$T_{ref}$ = tax refunds received by South Dakota households and businesses. Suits' model contains separate structural equations for estimating the values of $\Delta T_{fy}$ and $\Delta T^{*}_{fy}$. Suits' method for defining $T_{ref}$ states that $T_{ref} = T_{fy} - T^{*}_{fy}$. Unfortunately, data inconsistencies between variables $T_{fy}$ and $T^{*}_{fy}$ make untenable for this study the use of Suits' equation for defining $T_{ref}$.

The equation used for defining $T_{ref}$ for the purposes of this study states that $T_{ref} = \text{federal tax refunds (data received from Internal Revenue Service)} + \text{South Dakota Department of Revenue tax refunds (data taken from Annual Report of the South Dakota Department of Revenue)} + \text{South Dakota Motor Fuel Tax Division tax refunds (data taken from Annual Report of the South Dakota Department of Revenue)}$.

The most important aspect of this study's definition of $T_{ref}$ is that $T_{ref}$ is defined differently in the South Dakota model than in the United States model. In the United States model $T_{ref}$ refers
Only to federal tax refunds. In the South Dakota model Tref refers to both federal and state tax refunds.

Values of ΔTref may be calculated by taking first differences of the values of Tref.

\( Y = \text{disposable personal income of South Dakota households.} \)

Values of ΔY are defined by definitional equation (20) of the model, and its undifferenced values may be ascertained by summing the undifferenced values of the component variables of equation (20).

There are two methods for calculating ΔY.

1. Take first differences of the values of \( Y \), with the values of \( Y \) having been calculated by solving equation (20) for \( Y \) in undifferenced values.

2. Solve for ΔY using equation (20) in its stated form with differenced values of the component variables.

The values of ΔY produced by these two methods are identical.

Equation (20) states that,

\[
\Delta Y = \Delta W + \Delta Wg + (\Delta P - \Delta P^*) + \Delta Div + \Delta ig + \Delta Xu + \Delta Xf + \Delta Xs \\
+ \Delta Xgi - \Delta TfY - \Delta TsY - \Delta Teg - \Delta Top - \Delta SIe + \Delta Tref.
\]

First glances at equation (20) may induce the erroneous conclusion that the above two methods cannot produce identical values. This conclusion would be based on the observation that method two involves the linearly approximated variable, \( \Delta W \), while method one involves the exact value of \( W \).
The reason both methods produce identical values is that the disparity expected to result from the second method's overt use of a positive $\Delta W$, is completely offset by the inclusion in equation (20) of variable $\Delta P$, which, by its definitional equation (17), covertly contains a negative $\Delta W$.

**Summary of Procedures for Making Consistent the Values of Definitional Equations (11), (17), and (20):** If the overall picture of the systematic procedures discussed in this Data Group, for making the model's definitional equations consistent, is still not clear, the following easy eight step recipe should provide added perspective.

1. Calculate the values of Suits' variable $P$ as described at the beginning of the discussion of variable $P$ in Data Group IV.

2. Use the values calculated in step one to calculate the values of $G^*$ by solving equation (17) for $G^*$ in undifferenced values.

3. Force equation (11)'s definition of $G^*$ to equal step two's definition of $G^*$ by the methods described under variable $G^*$ in Data Group IV.

4. Calculate the values of $\Delta G^*$ by taking first differences of the identical values of $G^*$ calculated in steps two and three.

5. Use the values of $\Delta G^*$ calculated in step four to calculate the values of $\Delta P$ by means of equation (17).
Data Group IV (continued)

6. Note that, after step five is completed, the values of $\Delta P$ vary from the values of the first differences of the values of $P$ because of the linear approximation of $\Delta W$.

7. Note that values of $\Delta G^*$, calculated by equations (11) and (17), will be identical because the effect of the positive linearly estimated $\Delta W$, in equation (17) solved for $\Delta G^*$, is completely offset by the covert inclusion of a negative $\Delta W$ in the definition of $\Delta P$.

8. Use known values of $F$ and the values of $(F - R)$ calculated in step three to calculate the values of $R$ as described in Data Group IV.

Chapter Summary

This chapter has presented the methodology of data collection used in constructing the first consistent set of macroeconomic data which defines the complex structure of the South Dakota economy.

The next chapter will present the results of using this set of data in constructing the South Dakota model's thirty-two equations.
CHAPTER III

MODEL SOLUTION

The South Dakota Model

This econometric model of the South Dakota economy consists of thirty-two equations: five definitional equations and twenty-seven least-squares regression equations fitted to annual first differences in the variables.

The equations of the 1969 model are presented and discussed below by sectors, and the symbol for each variable is identified the first time it appears. The discussions of the individual equations, unless otherwise indicated by brackets, consist of direct quotations of the discussions presented by Suits in his American Economic Review article as an aid in understanding the individual equations of his 1962 stochastic econometric model of the United States economy.

Note that lagged undifferenced values of certain variables appear at some points (e.g., in the automobile demand equation (01) below). These undifferenced values serve as proxy variables for first differences in stocks as explained in chapter two.

The figure in parentheses below each regression coefficient is the standard error of that regression coefficient. With the aid of its standard error, each regression coefficient has been analyzed by means of Student's t distribution to determine the degree of confidence that can be attributed to each regression coefficient. Those regression coefficients that are significant at the ten
percent level of significance are denoted by an underline placed below each significant coefficient's standard error. For those regression coefficients that are denoted as significant there is at least ninety percent probability that the regression coefficient's confidence limits enclose the true population parameter.


Although containing the 32 equations shown below, the basic structure of the [South Dakota] model parallels that of the schematic model of [chapter one], and it is useful to employ the latter as a kind of table of contents for the larger model. Thus before we begin an equation by equation analysis of the model, let us take a look at its four main components, corresponding to the four equations of the schematic model.

The first four equations in the [South Dakota] model correspond to the single consumption equation in the schematic model. In place of a single consumption aggregate, we now distinguish four components of consumption, each related to its own particular set of variables.

The second set of equations—the investment sector—corresponds to the single investment equation of the schematic model. In the six equations of the [South Dakota] model investment is disaggregated into five components—fixed plant and equipment, nonfarm residential housing, accumulation of inventories of durables and of nondurables, and net foreign investment. Moreover, each of these components is related to its own set of variables.

The third major component of the [South Dakota] model includes the equations that relate demand and production to employment and income. This part of the schematic model involved only equation (4), the definition of income as the sum of its parts.
Finally, the [South Dakota] model contains a fairly elaborate fiscal sector, representing many different kinds of taxes and transfers and their individual determinants. This section of the model corresponds to the tax equation (3) of the schematic model. 38

A. Aggregate Demand

1. Consumption

(01) Automobiles and Parts:

\[
\Delta A = +0.04143 \Delta(Y - Xu - Xf - Xs) - 0.00990 A_{-1} \\
\quad (0.03056) \\
+ 0.30227 \Delta L_{-1} - 1,750,455 \\
\quad (0.12488)
\]

Consumer expenditure for new and net used automobiles and parts (\(\Delta A\)) depends on disposable income (Y), net of transfers for unemployment compensation (Xu), and other federal (Xf) and state (Xs) transfers. These transfers are deducted on the ground that they are unlikely to find their way into the automobile market. Servicemen's insurance dividends (Xcr) are not deducted from disposable income. In addition, automobile demand depends on the stock of cars on the road (A_{-1}) and on the real value of consumer liquid assets at the end of the preceding year (\(\Delta L_{-1}\)).

(02) Demand for Other Durables:

\[
\Delta D = -0.00402 \Delta Y + 0.04456 D_{-1} + 0.07938 \Delta L_{-1} - 2,626,547 \\
\quad (0.01778) \\
\quad (0.10051) \\
\quad (0.06679)
\]

This equation relates \(\Delta D\), consumer expenditure for durables (other than automobiles and parts) to disposable income (\(\Delta Y\)), the accumulating stock of durables (D_{-1}) and liquid assets.

(03) Demand for Nondurable Goods:

\[
\Delta ND = + 0.04279 \Delta Y - 0.40627 \Delta ND_{-1} + 0.24978 \Delta L_{-1}
\]

\[
+ 12,141,837
\]

Nondurable expenditure depends on disposable income, liquid assets, and last year's nondurable expenditure (\( \Delta ND_{-1} \)). Notice the difference between this and the foregoing equations. In (01) and (02) the lagged values were undifferenced representing accumulation of stock. In this equation the difference itself is lagged, representing a dynamic adjustment in nondurable expenditure: an initial rise in level is followed by a subsequent secondary rise.

(04) Demand for Services:

\[
\Delta S = + 0.04555 \Delta Y - 0.24594 \Delta S_{-1} + 0.22696 \Delta L_{-1}
\]

\[
+ 14,557,674
\]

This equation is similar to (03) and relates expenditure for services (\( \Delta S \)) to disposable income, liquid assets, and lagged service expenditure. [In Suits' model \( \Delta S \) excludes imputed items; in the South Dakota model \( \Delta S \) includes them.]

These four equations constitute the demand sector. Note that the aggregate marginal propensity to consume can be estimated by summing the income coefficients in the four equations. The sum, [0.13], is an estimate of the marginal propensity to consume, at least as an initial impact. The lagged terms in the individual equations, however, generate a dynamic response of consumption to income.

2. Gross Capital Expenditure

(05) Plant and Equipment Expenditure:

\[
\Delta PE = + 0.08813 \Delta (P^*_{-1} - Tfc_{-1} - Tsc_{-1}) - 0.02997 \Delta PE_{-1}
\]

\[
+ 2,206,850
\]

\( \Delta PE \), expenditure for new plant and equipment, includes producers' durables, nonfarm nonresidential construction, and all farm construction. It is related to the preceding year's corporate profits (\( P^*_{-1} \)) after federal (Tfc) and state (Tsc)
corporate income taxes and to its own lagged, undifferenced value (PE_1). The latter represents growth in the stock of plant and equipment.

(06) Housing Starts:

\[ \Delta HS = -849.49683 \Delta(FHA - Aaa) - 0.55427 HS_1 + 684 \]

\[ (440.77734) \quad (0.22972) \]

This equation, which applies only to the postwar period, relates the number of nonfarm residential housing starts (\( \Delta HS \)), measured in...units per [year], to the gap between the simple average of the FHA and VA ceiling interest rates on the one hand, and the Aaa bond yield on the other (both expressed in percentage points). This interest rate differential reflects the substantial influence of credit availability on the volume of FHA and VA financed residential construction. It can function, however, only in the presence of a strong underlying housing demand. With the accumulation of a large stock as a consequence of construction in recent years, this interest rate differential may lose its role in the model. The term HS_1, the lagged undifferenced value of housing starts, only partially represents the effect of this accumulation, and equation (06) is probably due for revision.

[Note that equation (06) has been altered in the South Dakota model. Suits' version of equation (06) in his 1962 national model states that, \( \Delta HS = +19.636 \Delta(FHA + VA - Aaa) \]

\[ (17.0) \]

\[ - 0.702 HS_1 + 66.147, \] with \( \Delta HS \) measured in thousands of units (0.312)

per month. Variable "VA" and the resulting need for a denominator of two have been excluded from the South Dakota model for the sake of simplicity. Justification for this alteration lies in the relatively dominant role the FHA plays in the government-underwritten residential loan market. The annual value of FHA residential loans is consistently greater than twice the annual value of VA residential loans.]

(07) Housing Expenditure:

\[ \Delta H = + 13,628.87109 \Delta HS + 104.78783 \Delta HS_1 \]

\[ (400.53638) \quad (388.48682) \]

\[ + 34,342,192.00000 \Delta C + 151,761 \]

\[ (16,506,437.00000) \]

Expenditure on housing, (\( \Delta H \)), depends on the rate at which residential construction is carried forward, and thus on current and lagged starts. In addition it depends on construction costs.
The term ΔC is the ratio of the index of construction costs to the GNP deflator.

(08) Durable Goods Inventory:

\[ ΔID = -0.04343 \Delta(A + D) + 0.85596 \Delta PD - 0.01966 ID_{-1} \]

\[ (0.01520) \quad (0.14067) \quad (0.03736) \]

+ 518,818 + [0.0 ΔM_{+1}]

Accumulation of durable inventories, ΔID, depends on sales of consumer durables, producers durables ΔPD, and the stock of inventory already accumulated ID_{-1}. In addition an important component of inventory is associated with government military orders. Production on such orders appears in the national accounts as goods in process, and exerts a strong impact on the economy long before delivery of the finished product materializes as government expenditure. A wide variety of arrangements and lead times are involved in this process. As a proxy for such orders in any given year, we use ΔM_{+1}, federal military purchases from private industry the following year. [Note that variable "ΔM_{+1}" is assumed to be zero for the purposes of this South Dakota model for reasons explained in chapter two.]

(09) Nondurable Goods Inventory:

\[ ΔIND = -0.02575 \Delta ND - 0.08410 IND_{-1} + 1,991,289 \]

\[ (0.03118) \quad (0.13926) \]

Accumulation of nondurable inventory, ΔIND, depends on consumer sales of nondurables and the stock already on hand, IND_{-1}.

(10) Imports:

\[ ΔR = -1.08316 ΔG* + 59,665,360 \]

\[ (0.16263) \]

This relates the aggregate level of imports to the private [gross state product] (G*).

3. Private Gross State Product

(11) ΔG* = \( Δ(A + D + ND + S) + (ΔF - ΔR) + ΔID + ΔIND \]

\[ + ΔPE + ΔH + Δg \]

Private GSP is defined as the sum of its parts including net exports (ΔF - ΔR) and government purchases from private firms (Δg).
B. Income and Employment

(12) Wage and Salary Workers, Private Sector:

\[ \Delta E = + 0.00001 \Delta G^* - 652 \]

(0.00001)

[This equation relates \( \Delta E \), the number of employees of private South Dakota businesses, including both farm and nonfarm employees, to the private GSP.]

(13) Unemployment:

\[ \Delta U = \Delta LF - \Delta Eo - \Delta Eg - \Delta E \]

Unemployment is the difference between labor force (\( \Delta LF \)) on the one hand, and the number of self-employed and unpaid family workers, (\( \Delta Eo \)), government workers, including armed services (\( \Delta Eg \)) and employees of private industry (\( \Delta E \)).

(14) Average Annual Earnings:

\[ \Delta w = - 0.01506 \Delta U - 0.00000 P_{-1} + 70 \]

(0.03234) (0.00000)

\( \Delta w \), average annual earnings (including wages and salaries plus "other labor income,") is related to unemployment and last year's profits. This relationship reflects two facts. First and probably more important, annual earnings are heavily influenced by overtime pay which varies inversely with the level of unemployment. Secondly, pressure of union demands varies directly with profits and inversely with the level of unemployment. The undifferenced level of profits is used since the existence of profits acts as a target for wage demands.

(15) Private Wage Bill:

\[ \Delta W = \Delta (\omega E) = \omega_{-1} \Delta E + E_{-1} \Delta w \]

By definition the wage bill is the product of average earnings and employment. To keep the model linear, this nonlinear relationship is replaced by the linear approximation shown.

(16) Depreciation:

\[ \Delta \text{Dep} = + 0.00834 \Delta G^* + 1,522,507 \]

(0.00756)
(17) Property Income:

\[ \Delta P = \Delta G^* - \Delta W - \Delta \text{Dep} - \Delta \text{Tfe} - \Delta \text{Tcd} - \Delta \text{Tbp} - \Delta \text{Tss} - \Delta \text{Tos} - \Delta \text{S1r} \]

Property income (\( \Delta P \)) is a residual from the GSP after deducting wage costs, depreciation (\( \Delta \text{Dep} \)), employer contributions for social insurance (\( \Delta \text{S1r} \)), and indirect business taxes: federal excises (\( \Delta \text{Tfe} \)), customs duties (\( \Delta \text{Tcd} \)), business property (\( \Delta \text{Tbp} \)), state sales (\( \Delta \text{Tss} \)), and other state taxes on business (\( \Delta \text{Tos} \)).

(18) Corporate Profits:

\[ \Delta P^* = + 0.08429 (\Delta P - \Delta P_f) + 1,187,438 \]

\[ (0.04502) \]

This relates profits (\( \Delta P^* \)) to total property income net of farm income (\( \Delta P_f \)).

(19) Dividends:

\[ \Delta \text{Div} = + 0.26542 (\Delta P^* - \Delta \text{Tfc} - \Delta \text{Tsc}) \]

\[ (0.12455) \]

\[ - 0.01184 (P^* - \Delta \text{Tfc} - \Delta \text{Tsc} - \Delta \text{Div})_{-1} + 286,799 \]

\[ (0.03548) \]

Current dividends (\( \Delta \text{Div} \)) depend on current profits after federal (\( \Delta \text{Tfc} \)) and state (\( \Delta \text{Tsc} \)) corporate profits taxes, and on last year's level of undistributed profits.

(20) Disposable Income:

\[ \Delta Y = \Delta W + \Delta \text{Wg} + (\Delta P - \Delta P^*) + \Delta \text{Div} + \Delta \text{ig} + \Delta \text{Xu} + \Delta \text{Xf} + \Delta \text{xs} + \Delta \text{XG1} - \Delta \text{Tfy} - \Delta \text{Tsy} - \Delta \text{Teg} - \Delta \text{Top} - \Delta \text{S1e} + \Delta \text{Tref} \]

Disposable income is the sum of wages, including government wages (\( \Delta \text{Wg} \)), noncorporate property income (\( \Delta P - \Delta P^* \)), dividends, government interest payments (\( \Delta \text{ig} \)), plus transfers, less personal taxes: federal (\( \Delta \text{Tfy} \)), and state (\( \Delta \text{Tsy} \)) income, estate and gift (\( \Delta \text{Teg} \)), other personal taxes (\( \Delta \text{Top} \)) and personal contributions for social insurance (\( \Delta \text{S1e} \)), all net of tax refunds (\( \Delta \text{Tref} \)).
C. Taxes and Government Transfers

1. Federal Taxes

(21) Federal Corporate Profits Tax:

\[ \Delta T_{fc} = + 0.31541 \Delta P^* - 308,665 \]
\[ (0.12183) \]

(22) Federal Personal Income Tax Receipts:

\[ \Delta T_{fy} = + 0.17176 (\Delta W + \Delta Wg) - 0.04225 (\Delta P - \Delta P^* + \Delta g) \]
\[ (0.11993) (0.02073) \]
\[ + 2.81521 \Delta D_{iv} + 831,561 \]
\[ (2.81658) \]

This equation relates income tax receipts in the form of withholding, quarterly payments on estimated tax, and final tax payment to the several income components.

(23) Federal Personal Income Tax Liability:

[In Suits' 1962 econometric model of the United States economy, equation (23) states:]

\[ \Delta T_{fy} = .100 (\Delta W + \Delta Wg) + .114 (\Delta P - \Delta P^* + \Delta g) + .154 \Delta D_{iv} \]

Tax receipts commonly exceed liability. The difference (\( \Delta T_{ref} \)) appears as a tax refund the following year.

[Suits' equation (23) has been disregarded entirely for the purpose of constructing an econometric model for the South Dakota economy. Four interrelated reasons prevail for disregarding the equation:

First, equation (23) exists within Suits' United States model solely for the purpose of helping to approximate (Tref) by means of the definition: \( T_{fy} - T_{fy} = T_{ref} \). See Suits' two statements which follow the equation presented above for verification of this fact.

Second, the summation of data of federal and state tax refunds provides a much better guide for approximating total \( \Delta T_{ref} \) than if \( T_{fy} \) were calculated solely for the purpose of fulfilling the less accurate definition \( T_{fy} - T_{fy} = T_{ref} \). Note that this means Tref is defined differently in the South Dakota model than in the United States model. In the United States model Tref refers only to federal tax refunds. In the South Dakota model Tref refers to total federal and state tax refunds.
Third, since $\Delta T^*_{fy}$ occurs no place else in the model, outside of equation (23), its exclusion can create no unexpected difficulties.

Fourth, that equation (23) is of only secondary importance to the model is confirmed by Suits' exclusion of equation (23) from the calculation of the model's inverse. In order to avoid confusion, no renumbering of Suits' equations (24) through (32) will take place. Equation (23) will be considered to continue in existence as an empty set. This allows numbers of all the equations of both the United States and South Dakota models to remain identical for easier reference.]

(24) Federal Excise Taxes:

$$\Delta T_{fe} = -0.00597 \Delta A - 0.03575 \Delta D + 0.00872 \Delta ND$$

\[\begin{align*}
\text{Coefficient} & \quad \text{Coefficient} \\
(0.03163) & \quad (0.07176) \\
+ 0.00507 & \Delta G^* - 0.00408 \Delta Y + 134,425 \\
(0.02662) & \quad (0.02586)
\end{align*}\]

(25) Customs Duties:

$$\Delta T_{cd} = 0.0 \Delta R + 0.0$$

[See chapter two for justification for assuming $\Delta T_{cd} = 0$.]

2. State and Local Taxes

(26) State Corporate Income Taxes:

$$\Delta T_{sc} = + 0.00762 \Delta P^* + 6,783$$

\[\begin{align*}
\text{Coefficient} \\
(0.00537)
\end{align*}\]

(27) State and Local Sales Taxes:

$$\Delta T_{ss} = -0.00290 (\Delta A + \Delta D + \Delta ND + \Delta S) + 1,102,038$$

\[\begin{align*}
\text{Coefficient} \\
(0.02262)
\end{align*}\]

(28) State and Local Personal Income Taxes:

$$\Delta T_{sy} = 0.0 (\Delta W + \Delta Wg + \Delta P - \Delta P^* + \Delta Div + \Delta ig) + 0.0$$

[See chapter two for justification for assuming $\Delta T_{sy} = 0$.]
3. Social Insurance Programs

(29) Private Employer Contributions for Social Insurance:

\[ \Delta \text{SIR} = + 144.08086 \Delta E + 1,431,856 \]
\[ (89.94777) \]

(30) Personal Contributions for Social Insurance:

\[ \Delta \text{SIE} = + 333.91260 (\Delta E + \Delta E_g) - 0.00850 (\Delta P - \Delta P*) \]
\[ (198.67662) \]
\[ (0.00665) \]
\[ + 2,559,600 \]

(31) Covered Unemployment:

\[ \Delta \text{Uc} = + 0.32484 \Delta U - 0.00171 (\Delta LF - \Delta LF - 1) - 33 \]
\[ (0.07638) \]
\[ (0.00996) \]

The relationship of unemployment covered by compensation programs (\(\Delta \text{Uc}\)) to total unemployment varies with the rate of increase in the labor force. When the labor force is growing rapidly, new entrants, not yet covered, make up a larger proportion of total unemployment.

(32) Unemployment Compensation:

\[ \Delta \text{Xu} = + 106.20935 \Delta \text{Uc} + 4,366,777 \]
\[ (2,065.46973) \]

Summary of the Statistical Significance of the Model's Regression Coefficients

In the South Dakota model, equations (5), (6), (7), and (23), are exogenous equations, equations (11), (13), (15), (17), and (20), are definitional equations, and equations (25) and (28) are empty sets due to lack of data. All remaining equations enter into the solution of the model by means of the forty-two regression coefficients they contain. Unfortunately, of the forty-two regression coefficients that enter into the solution of the model, only ten of them or
twenty-four percent were shown to be statistically significant at the ten percent level of significance.

At least two major reasons may account for the low confidence that must be placed in seventy-six percent of the model's regression coefficients. First, data for all variables contained in Data Group II and data for variables R and Slr are in great need of improvement. Second, even if all data used in solving the individual equations are accurate, there remains the possibility that significant linear relationships, which exist among the model's variables on a national level, may not be significant on a state level by virtue of the unique nature of the South Dakota economy.

Because of the low confidence that must be placed in most of the model's regression coefficients, a major decision had to be made at this point in the research. Two fundamental questions remained unanswered. First, would it be possible to improve both the data and the linear relationships of the regression equations in order to increase the confidence that could be placed in the model's regression coefficients. Second, given that a statistically unreliable set of equations was now available, could the correct methodology of forecast calculation and policy analysis be deciphered from Suits' writings.

Time limitations dictated that a choice be made concerning which of the two questions be answered. The decision was made to answer the second question, at the exclusion of the first, in order to assure completion of the outline of the model's methodology.
Guiding the decision was the awareness that the primary significance of this first South Dakota model lies not as much in its empirical results as in its establishing a workable methodology on which future research can be based.

**Chapter Summary**

This chapter has presented the thirty-two econometric equations that comprise the South Dakota model. Emphasis has been placed on the statistical unreliability of seventy-six percent of the regression coefficients that enter into the solution of the model. Because of this unreliability, time limitations dictated that a major decision be made to complete research on the methodology of the model at the expense of more accurate data estimates. Improvement of the data must be left to future research.

The next two chapters will present the methodology and results of forecast calculation and of policy analysis which have been established by using the equations contained in this chapter, chapter three.
CHAPTER IV

FORECAST CALCULATION

This chapter presents, first, an explanation of the methodology of forecast calculation, second, the forecast of the most recent year calculated, 1969, and, third, a review of past forecasts back to 1966.

Methodology of Forecast Calculation

Simplification of the model is carried out as illustrated [in the schematic example presented in chapter one.] Inspection shows that in equation (05), plant and equipment expenditure (ΔPE) depends only on "known" values: last year's profits after taxes, and the stock of plant and equipment available at the beginning of the year. Similarly in equations (06) and (07), housing starts (ΔHS) and expenditure for nonfarm residential construction (ΔH) depend only on credit availability, construction costs, last year's starts, and the stock of houses at the beginning of the year. To make a forecast, therefore, we use the knowns to estimate ΔPE, ΔHS, and ΔH via equations (05), (06), and (07), and then use these values, together with the other knowns, to solve the [entire set of remaining equations by matrix inversion.] The inverse of the model is shown in Table [1]. This is merely an enlarged version of the little table shown earlier for the illustrative model of [chapter one], and is used in the same way.

For example, if the projected values of the variables presented in Table 2, which are the projections underlying the forecast of 1969, are (1) inserted in the projections column presented at the end of Table 1, (2) multiplied by the weights in the automobile column, ΔA, of Table 1, and (3) summed, the result is $-3,682,662. This is the
forecast decrease in automobile demand shown in Table 3 in the second section of this chapter. Table 3 presents the model's forecast of 1969.39

39Suits, "Forecasting and Analysis with an Econometric Model," American Economic Review, March, 1962, pp. 122, 126. In other words, as readers familiar with matrix algebra will recognize, the inverse matrix presented in Table 1 is tabulated in its transposed form, and it goes into the projections vector, presented at the end of Table 1, column by column.
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<td></td>
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<td></td>
</tr>
<tr>
<td>06</td>
<td>$+ 1,991,289 - 0.08410 \Delta N_{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>07</td>
<td>$+ 59,665,360$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>08</td>
<td>$0.0 + \Delta F + \Delta P_E + \Delta l + \Delta g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>09</td>
<td>$- 652$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>$0.0 + \Delta L_F - \Delta E_o - \Delta E_g$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>11</td>
<td>$+ 70 - 0.00000 P_{*-1}$</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>12</td>
<td>$0.0$</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>13</td>
<td>$+ 1,522,507$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>14</td>
<td>$0.0 - \Delta T_{bp} - \Delta T_{os}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>15</td>
<td>$+ 1,187,438 - 0.08429 \Delta P_f$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>$+ 286,799 - 0.01184 (P_{*} - T_{fc} - T_{sc} - \text{Div})_{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>17</td>
<td>$0.0 + \Delta W_g + \Delta i_g + \Delta x_f + \Delta x_s + \Delta X_{GI} - \Delta T_{op} - \Delta T_{eg} + \Delta T_{ref}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>$- 308,665$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>$+ 831,561 + 0.17176 \Delta W_g - 0.04225 \Delta i_g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>20</td>
<td>$+ 134,425$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>21</td>
<td>$0.0$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>22</td>
<td>$+ 6,783$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>$+ 1,102,038$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>$0.0 + 0.0 \Delta W_g + 0.0 \Delta i_g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>$+ 1,431,856$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>$+ 2,559,600 + 333.91260 \Delta E_g$</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>27</td>
<td>$- 33 - 0.00171 (\Delta L_F - \Delta L_{F-1})$</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>28</td>
<td>$+ 4,366,777$</td>
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<td></td>
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</table>
Table 2—PROJECTIONS UNDERLYING FORECAST OF 1969

<table>
<thead>
<tr>
<th>Equation no.</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>(A_{-1} = +101,320,000) (\Delta L_{-1} = -2,870,000) (\Delta x_f = +15,000,000) (\Delta x_s = +400,000)</td>
</tr>
<tr>
<td>02</td>
<td>(D_{-1} = +145,010,000) (\Delta L_{-1} = -2,870,000)</td>
</tr>
<tr>
<td>03</td>
<td>(\Delta N D_{-1} = +30,380,000) (\Delta L_{-1} = -2,870,000)</td>
</tr>
<tr>
<td>04</td>
<td>(\Delta S_{-1} = +31,600,000) (\Delta L_{-1} = -2,870,000)</td>
</tr>
<tr>
<td>05</td>
<td>(\Delta(P - T_{fc} - T_{sc} - D_i) = +36,251,486) (PE_{-1} = +44,383,000)</td>
</tr>
<tr>
<td>06</td>
<td>(\Delta(FMA - Aaa) = -0.10) (HS_{-1} = 1,086)</td>
</tr>
<tr>
<td>07</td>
<td>(\Delta HS = -300^a) (\Delta HS_{-1} = 161) (\Delta C = +0.010)</td>
</tr>
<tr>
<td>08</td>
<td>(\Delta PD = +2,000,000) (\Delta M_{-1} = 0.0) (ID_{-1} = +32,919,000)</td>
</tr>
<tr>
<td>09</td>
<td>(IND_{-1} = +17,863,000)</td>
</tr>
<tr>
<td>10</td>
<td>---</td>
</tr>
<tr>
<td>11</td>
<td>(\Delta P = +50,000,000) (\Delta PE = +4,071,534) (\Delta H = -3,576,607) (\Delta g = +500,000)</td>
</tr>
<tr>
<td>12</td>
<td>---</td>
</tr>
<tr>
<td>13</td>
<td>(\Delta LF = +3,400) (\Delta E_o = 0) (\Delta E_g = +100)</td>
</tr>
<tr>
<td>14</td>
<td>(P^*_1 = +2,000,000)</td>
</tr>
<tr>
<td>15</td>
<td>(W_{-1} = +2,782) (E_{-1} = +187,400)</td>
</tr>
<tr>
<td>16</td>
<td>---</td>
</tr>
<tr>
<td>17</td>
<td>(\Delta T_{bp} = +2,000,000) (\Delta T_{os} = +200,000)</td>
</tr>
<tr>
<td>18</td>
<td>(\Delta F = +2,000,000)</td>
</tr>
<tr>
<td>19</td>
<td>((P^* - T_{fc} - T_{sc} - D_i)_{-1} = +22,124,486)</td>
</tr>
<tr>
<td>20</td>
<td>(\Delta W_g = +10,000,000) (\Delta g = +500,000) (\Delta x_f = +15,000,000) (\Delta x_s = +400,000) (\Delta x_{GI} = -50,000) (\Delta T_{op} = +500,000) (\Delta T_{eg} = +100,000) (\Delta T_{ef} = +2,000,000)</td>
</tr>
<tr>
<td>21</td>
<td>---</td>
</tr>
<tr>
<td>22</td>
<td>(\Delta W_g = +10,000,000) (\Delta g = +500,000)</td>
</tr>
<tr>
<td>23</td>
<td>---</td>
</tr>
<tr>
<td>24</td>
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<td>25</td>
<td>---</td>
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<tr>
<td>26</td>
<td>---</td>
</tr>
<tr>
<td>27</td>
<td>---</td>
</tr>
<tr>
<td>28</td>
<td>(\Delta W_g = +10,000,000) (\Delta g = +500,000)</td>
</tr>
<tr>
<td>29</td>
<td>---</td>
</tr>
<tr>
<td>30</td>
<td>(\Delta E_g = +100)</td>
</tr>
<tr>
<td>31</td>
<td>((\Delta LF - \Delta LF_{-1}) = 0)</td>
</tr>
<tr>
<td>32</td>
<td>---</td>
</tr>
</tbody>
</table>

*Equation (06), employing the projections listed above, estimates \(\Delta HS = +167\). The figure, \(-300\), listed above and used in the model, is based on both equation (06)'s estimation and the anticipated effect rising interest rates will have on residential construction in 1969.

*This equation is excluded from the South Dakota model for reasons presented in chapter three.*
The Forecast of 1969

"The unknowns of the model are the 32 variables like automobile demand, disposable income, private G[S]P, etc. that stand on the left side of the equations. The knowns are variables like government purchases from private firms, labor force, household liquid assets, etc. that appear only on the right side of the equations, and whose values must be projected or assigned before the unknowns can be forecast." As explained in the previous section of this chapter, the forecast of 1969 employed the projected values shown in Table 2.

When the projections of Table 2 are inserted in the equations, (i.e., in the projections column of Table 1), the solution gives the outlook for 1969 shown in Table 3. The first two columns contain a detailed comparison of the forecast of 1968 with the actual values of 1968. The third column contains the solutions obtained from the equations. These are in first differences and are expressed as increases over 1968. When the forecast increase is added to the actual level for 1968 the result is the forecast level of 1969 shown in the fourth column. 40

---

Table 3--FORECASTS OF 1968 AND 1969

All data in constant dollars: 1958 = 100.0

<table>
<thead>
<tr>
<th></th>
<th>1968</th>
<th>1969</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Aggregate Demand</strong></td>
<td>Forecast</td>
<td>Actual</td>
</tr>
<tr>
<td><strong>1. Consumption Expenditure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A = Automobiles and Parts</td>
<td>79,375,350</td>
<td>101,320,000</td>
</tr>
<tr>
<td>D = Other Durables</td>
<td>147,432,117</td>
<td>145,010,000</td>
</tr>
<tr>
<td>ND = Nondurables</td>
<td>662,822,865</td>
<td>683,940,000</td>
</tr>
<tr>
<td>S = Services</td>
<td>576,880,347</td>
<td>602,740,000</td>
</tr>
<tr>
<td><strong>2. Gross Capital Expenditure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID = Durable Goods Inventory</td>
<td>33,498,252</td>
<td>32,919,000</td>
</tr>
<tr>
<td>IND = Nondurable Goods Inventory</td>
<td>18,531,294</td>
<td>17,863,000</td>
</tr>
<tr>
<td>R = Imports</td>
<td>1,553,862,227</td>
<td>1,222,930,318</td>
</tr>
<tr>
<td><strong>3. Private Gross State Product</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G* = Private GSP</td>
<td>1,002,515,863</td>
<td>1,394,270,988</td>
</tr>
</tbody>
</table>
Table 3—FORECASTS OF 1968 AND 1969—(continued)

<table>
<thead>
<tr>
<th>B. Income and Employment</th>
<th>1968</th>
<th>1969</th>
</tr>
</thead>
<tbody>
<tr>
<td>E = Private Wage and Salary Workers</td>
<td>178,608</td>
<td>187,400</td>
</tr>
<tr>
<td>U = Unemployment</td>
<td>16,892</td>
<td>8,100</td>
</tr>
<tr>
<td>w = Average Annual Earnings</td>
<td>2,739</td>
<td>2,782</td>
</tr>
<tr>
<td>W = Private Wage Bill</td>
<td>521,171,945</td>
<td>521,350,000</td>
</tr>
<tr>
<td>Dep = Depreciation</td>
<td>41,914,625</td>
<td>42,662,000</td>
</tr>
<tr>
<td>P = Property Income</td>
<td>340,139,663</td>
<td>688,675,000</td>
</tr>
<tr>
<td>P* = Corporate Profits</td>
<td>22,800,780</td>
<td>53,000,000</td>
</tr>
<tr>
<td>Div = Dividends</td>
<td>9,749,209</td>
<td>14,127,000</td>
</tr>
<tr>
<td>Y = Disposable Income</td>
<td>1,023,366,179</td>
<td>1,350,391,258</td>
</tr>
</tbody>
</table>

C. Taxes and Government Transfers

1. Federal Taxes

| Tfc = Federal Corporate Profits Tax | 6,707,680 | 16,267,700 | -183,974 | 16,083,726 |
| Tfy = Federal Personal Income Tax  | 107,783,968 | 137,149,400 | +3,366,714 | 140,516,114 |
| Tfe = Federal Excise Tax           | 5,113,169 | 5,603,400 | -52,558 | 5,550,842 |
| Tcd = Customs Duties              | 0 | 0 | 0 | 0 |
Table 3--FORECASTS OF 1968 AND 1969—(continued)

<table>
<thead>
<tr>
<th>C. Taxes and Government Transfers (continued)</th>
<th>1968</th>
<th>1969</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecast</td>
<td>Actual</td>
</tr>
<tr>
<td>2. State and Local Taxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tsc = State Corporate Income Tax</td>
<td>293,501</td>
<td>480,814</td>
</tr>
<tr>
<td>Tss = State and Local Sales Tax</td>
<td>27,198,303</td>
<td>26,571,626</td>
</tr>
<tr>
<td>Tsy = State and Local Personal Income Tax</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. Social Insurance Programs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sri = Private Employer Contributions for Social Insurance</td>
<td>22,228,712</td>
<td>24,660,000</td>
</tr>
<tr>
<td>SLe = Personal Contributions for Social Insurance</td>
<td>45,301,829</td>
<td>48,440,000</td>
</tr>
<tr>
<td>Uc = Covered Unemployment</td>
<td>4,097</td>
<td>1,421</td>
</tr>
<tr>
<td>Xu = Unemployment Compensation</td>
<td>77,654,741</td>
<td>74,291,500</td>
</tr>
</tbody>
</table>
Review of Past Forecasts

The forecast of 1969 was made before there was full knowledge of the true values of the projections in Table 2. Thus the values of the variables in Table 2 which were used in the model were, in the fullest sense of the word, projections or estimates. In like manner, because the forecast of 1969 was made before the end of 1969, the forecast of 1969 was, in the fullest sense of the word, a forecast or prediction.

In contrast, the "forecasts" of the years 1966, 1967, and 1968, were made after there was full knowledge of the true values of the variables, in tables similar to Table 2, which were used in the model for each of the above years. Thus the values of the variables in Table 2 for each of the above years were not projections, but accurate ex post data. In like manner, because the "forecasts" of the above years were made after the end of each year, the "forecasts" of these years, rather than being authentic forecasts or predictions, serve more as an ex post appraisal of the model's ability to represent the actual structure of the state's economy. This ex post appraisal is possible because the "forecasts" of the years 1966, 1967, and 1968, show how closely this econometric model's forecasts approach the true ex post values of each year if fully accurate projections or estimates of the usually inaccurately projected variables are possible. Thus, the quality of the results of the years 1966, 1967, and 1968, provides a standard for the amount of confidence which might be
placed in the forecast of 1969, given that the forecast of 1969 used estimates rather than ex post data for projections.

The "forecast" of 1968 was presented in Table 3 in the previous section of this chapter. The "forecasts" of 1966 and 1967, which are presented in Table 4, complete this chapter.
Table 4--FORECASTS OF 1966 AND 1967

Ex post data were used in calculating values of projections column vector for years 1966, 1967, and 1968. Similar values for 1969 were estimated.
All data in constant dollars: 1958 = 100.0

<table>
<thead>
<tr>
<th></th>
<th>1966</th>
<th>1967</th>
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<th></th>
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<td>Forecast</td>
<td>Actual</td>
<td>Forecast</td>
<td>Actual</td>
</tr>
<tr>
<td>A. Aggregate Demand</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Consumption Expenditure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A = Automobiles and Parts</td>
<td>84,231,156</td>
<td>88,650,000</td>
<td>90,881,172</td>
<td>84,820,000</td>
</tr>
<tr>
<td>D = Other Durables</td>
<td>136,728,053</td>
<td>140,750,000</td>
<td>148,342,852</td>
<td>138,780,000</td>
</tr>
<tr>
<td>ND = Nondurables</td>
<td>643,722,137</td>
<td>655,070,000</td>
<td>668,492,232</td>
<td>653,560,000</td>
</tr>
<tr>
<td>S = Services</td>
<td>555,806,101</td>
<td>555,260,000</td>
<td>575,331,217</td>
<td>571,140,000</td>
</tr>
<tr>
<td>2. Gross Capital Expenditure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID = Durable Goods Inventory</td>
<td>28,734,555</td>
<td>30,069,000</td>
<td>32,588,975</td>
<td>33,460,000</td>
</tr>
<tr>
<td>IND = Nondurable Goods Inventory</td>
<td>16,444,539</td>
<td>17,106,000</td>
<td>17,217,725</td>
<td>18,299,000</td>
</tr>
<tr>
<td>R = Imports</td>
<td>1,173,003,645</td>
<td>1,095,672,962</td>
<td>942,028,788</td>
<td>1,098,992,003</td>
</tr>
<tr>
<td>3. Private Gross State Product</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G* = Private GSP</td>
<td>1,275,778,427</td>
<td>1,372,976,576</td>
<td>1,567,281,252</td>
<td>1,376,549,647</td>
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</tbody>
</table>
### Table 4--FORECASTS OF 1966 AND 1967--(continued)

<table>
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<tr>
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<tbody>
<tr>
<td></td>
<td>Forecast</td>
<td>Actual</td>
</tr>
<tr>
<td><strong>B. Income and Employment</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E = Private Wage and Salary Workers</td>
<td>180,769</td>
<td>182,600</td>
</tr>
<tr>
<td>U = Unemployment</td>
<td>10,531</td>
<td>8,700</td>
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<tr>
<td>w = Average Annual Earnings</td>
<td>2,643</td>
<td>2,661</td>
</tr>
<tr>
<td>W = Private Wage Bill</td>
<td>477,791,783</td>
<td>485,940,000</td>
</tr>
<tr>
<td>Dep = Depreciation</td>
<td>39,321,403</td>
<td>38,342,000</td>
</tr>
<tr>
<td>P = Property Income</td>
<td>637,627,110</td>
<td>719,754,000</td>
</tr>
<tr>
<td>P* = Corporate Profits</td>
<td>44,296,626</td>
<td>50,602,000</td>
</tr>
<tr>
<td>Div = Dividends</td>
<td>11,447,989</td>
<td>12,981,000</td>
</tr>
<tr>
<td>Y = Disposable Income</td>
<td>1,257,939,708</td>
<td>1,330,945,125</td>
</tr>
<tr>
<td><strong>C. Taxes and Government Transfers</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Federal Taxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tfc = Federal Corporate Profits Tax</td>
<td>14,751,220</td>
<td>16,476,300</td>
</tr>
<tr>
<td>Tfy = Federal Personal Income Tax</td>
<td>99,918,300</td>
<td>106,275,000</td>
</tr>
<tr>
<td>Tfe = Federal Excise Tax</td>
<td>6,524,545</td>
<td>5,571,200</td>
</tr>
<tr>
<td>Tcd = Customs Duties</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

83
<table>
<thead>
<tr>
<th></th>
<th>1966</th>
<th>1967</th>
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</thead>
<tbody>
<tr>
<td></td>
<td><strong>Forecast</strong></td>
<td><strong>Actual</strong></td>
</tr>
<tr>
<td><strong>C. Taxes and Government Transfers (continued)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. State and Local Taxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_{sc} ) = State Corporate Income Tax</td>
<td>498,465</td>
<td>514,628</td>
</tr>
<tr>
<td>( T_{ss} ) = State and Local Sales Tax</td>
<td>16,581,647</td>
<td>23,450,455</td>
</tr>
<tr>
<td>( T_{sy} ) = State and Local Personal Income Tax</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3. Social Insurance Programs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( S_{lr} ) = Private Employer Contributions for Social Insurance</td>
<td>16,673,150</td>
<td>18,660,000</td>
</tr>
<tr>
<td>( S_{le} ) = Personal Contributions for Social Insurance</td>
<td>30,596,243</td>
<td>35,150,000</td>
</tr>
<tr>
<td>( U_c ) = Covered Unemployment</td>
<td>2,012</td>
<td>1,523</td>
</tr>
<tr>
<td>( X_u ) = Unemployment Compensation</td>
<td>70,943,267</td>
<td>69,355,900</td>
</tr>
</tbody>
</table>
Chapter Summary

This chapter has presented the methodology and results of forecast calculation which have been established by using the equations contained in chapter three.

At least two major sources of error may explain the inaccuracies of this chapter's forecasts. First, as explained in chapter three, the model's data and linear relationships are in need of more research. Second, there may be some unknown error in the methodology of forecast calculation.

The next chapter will use the results of this chapter on forecast calculation to begin an investigation of policy analysis.
CHAPTER V

POLICY ANALYSIS

By means of a brief analysis of a few short-run policy multipliers, this chapter begins exploration of the model's implications for the economic impact which selected exogenous economic policies may have on the state economy.

Unfortunately, data inadequacies and perhaps other unknown sources of error take their greatest toll in this final substantive chapter. While the methodology of this chapter is valid, all empirical results of this chapter appear irrational in the light of current economic theory.

The remainder of this chapter consists entirely of a paraphrase of Suits' analysis of his 1962 national model.

Short-run Policy Multipliers

Short-run multipliers for any policy variable are readily calculated as demonstrated in chapter one's schematic example. Specifically, they are calculated by inserting 1 for the variable in question everywhere it appears in the projections column of Table 1 and then (ignoring all terms that do not contain the variable in question) extending a forecast using the columns of Table 1.

For example, to find the multiplier on government purchases from private firms, set $\Delta g = +1$ everywhere it appears in the projections column of Table 1. The term $\Delta g$ is found in only one place:
in row (11) it is multiplied by 1. To find the effect of $\Delta g = 1$ on, say, private GSP we multiply the weight in row (11) of the GSP ($\Delta G^*$) column of Table 1 by 1: $\Delta G^* = 1 \times -5.10970 = -5.10970$.

That is to say, the short-run multiplier on government purchases is about -5.1 in 1969. Similarly the effect on, say, automobile demand is given by $\Delta A = 1 \times -0.19372 = -0.19372$, i.e., the short-run "automobile demand multiplier" on government purchases from the private sector is -0.19372 in 1969.

In working out a policy multiplier, care must be taken to include changes in all exogenous variables affected by the policy action. For example, an increase in government employment involves hiring additional people ($\Delta E_g$ in rows (13) and (30)) and paying them wages ($\Delta W_g$ in rows (20), (22), and (28)). At the average annual real wage for federal, state, and local government employees in South Dakota, including armed forces, of about $4,232 in 1968, $(1958 = 100.0)$, an addition of $50,000,000 to the government wage bill will hire about 11,815 additional employees. (In 1968 $E_g = 54,900$ employees and $W_g = 232,350,000$.)

To find the multipliers on government wages, therefore, set $\Delta E_g = +11,815$. This gives $(-1)(+11,815) = -11,815$ in row (13) and $(+333.91260)(+11,815) = +3,945,177$ in row (30) of the projections column of Table 1. Also set $\Delta W_g = 1$ to get 1 in row (20), $+0.17176$ in (22), and 0 in row (28) of the projections column of Table 1. The impact of additional government employment on private GSP is then found by extending these figures by the
weights in the corresponding rows of the GSP column: $\Delta G^* = (-11,815) (-204.37054) + (+1) (-0.63107) + (+0.17176) + 0.63108 + 0.63107 + (+3,945,177) (+0.63107) = \text{about} + 4,900,000.$

To find the effect of the action on total GSP, we must add the additional value added by government (i.e., government wages and salaries). Thus: Total GSP = + 4,900,000 + 1 = \text{about} + 4,900,001.

Finally, recall that government tax policy can be expressed by shifts in the equations themselves. As shown in the schematic example in chapter one, these shift multipliers are equal to the weights found in the row of the inverse matrix that corresponds to the equation being shifted. Thus, it is seen that from the + 0.63108 in row (22) of the GSP column that a $1,000,000 upward shift in the federal personal tax function will increase private GSP by $631,080. Note again [with a summation of row (22) of the three federal tax columns $\Delta T_f c$, $\Delta T_f y$, $\Delta T_f e$] that an upward shift of $1,000,000$ in the federal income tax schedule increases federal tax yield by only $1,028,750$ due to the decline in personal income and expenditure associated with the rise in taxes.$^{41}$

Chapter Summary

This chapter has presented the methodology and results of policy analysis which have been established using the results of chapter four's forecast calculation.

Because most of this chapter has consisted of a paraphrase of Suits' analysis of his 1962 national model, there is very little chance that an error has been made in the methodology of this chapter's policy analysis. It is therefore assumed that the seemingly irrational results of this chapter are due entirely to the sources of error that contributed to the inaccuracies of chapter four's forecast calculation.
CHAPTER VI

CONCLUSIONS, LIMITATIONS, AND NEED FOR FURTHER RESEARCH

The purpose of this study was to adapt and develop a methodology of practical statewide macroeconomic analysis that is an alternative to the construction of a statewide input-output econometric model. The study focused on three main areas: (1) presentation of an annual stochastic econometric model of the South Dakota economy which is an adaptation of Daniel B. Suits' 1962 annual stochastic econometric model of the United States economy, (2) demonstration of the model's use as an effective instrument by which state economic performance can be forecast, and (3) exploration of the model's implications for the economic impact which selected exogenous economic policies may have on the state economy.

Study Conclusions

The approximation of the actual macroeconomic behavior of South Dakota's complex economy by means of a system of thirty-two simple linear equations is a heroic simplification. Yet, while the system of equations is small in relation to the vast structure of a theoretically complete analysis, it was much more elaborate than other contemporary economic tools which can be used on a practical level. And, if nothing else, an econometric model proves to be a highly sophisticated method of observing past operation of the South Dakota economy and of giving order and meaning to the information obtained.
A stochastic econometric model, properly defined, is not a fully accurate replica of the real world economy. It is, instead, a system of econometric equations in which each equation is the representation of its own preliminary assumptions and theories. The totality of assumptions and theories, which makes up all the econometric equations in the model, composes a system of equations which may be viewed conceptually as one comprehensive macroeconomic theory (i.e., one econometric model = one complex macroeconomic theory).

The fundamental procedural problem in the construction of a new and improved economic theory is the selection of assumptions that collectively approximate the real world well enough to produce an economic model or theory that is simpler and more fruitful than previous theory. A theory is simpler if less knowledge is needed to make a prediction within a given field of phenomena, and it is more fruitful if its resulting prediction is more precise, if it yields predictions within a wider area, and if it suggests more additional lines for further research.

This study presents the first econometric model ever constructed of the South Dakota economy. By definition it therefore has replaced conjecture and guesswork with a macroeconomic theory. Because an economic theory is an economic hypothesis which has been tested and verified to some degree by empirical induction, it is of paramount significance that the construction of this or any econometric model is not a completed task. It is merely the beginning of a continuing process in which successively more and better information must be
incorporated in steadily improving the simple linear approximations of the real world economy. The model presented here should be looked upon not as a perfected tool, but as a first rough approximation whose real merit lies in its unlimited potential for improvement.

Limitations of the Study and Need for Further Research

Assuming that no errors of calculation were committed throughout the construction of the South Dakota model, the extent to which this model's forecasts varied inordinately from the actual values of the variables (for those years in which the actual values were known), was due to a number of limitations of the study.

The first major qualification of the model will be discussed in detail. It is that most of the data used in constructing the model consist of statistical estimations rather than the summation of accurate accounting figures. In addition, the values of some of the variables are estimated for the fiscal year while the values of other variables are estimated for the calendar year. Since this study emphasized that the quality of data used in constructing a stochastic econometric model is one of the most crucial indicators of the expected quality of the model's results, the primary area for further research lies in the improvement of the statistical estimations of the data on which the model was constructed.

For all variables in Data Group II and for variables R and S1r, original estimating procedures had to be created because no national or state source was able to provide official estimates. Special
attention should be devoted to improving the estimated values of these variables because they most likely contain a substantial amount of error.

Since the state of South Dakota, at the time of the writing of this study, has no form of macroeconomic data bank for use in economic and business research, the data estimates created in the writing of this study are the first attempt at constructing a set of consistent macroeconomic data. As the weak empirical results of the South Dakota model suggest, more research will be required to generate data of sufficient accuracy to allow the model to become a practical economic tool.

In order to make up the data deficiencies uncovered during the construction of the South Dakota model, it is recommended that the following actions be taken.

First, an economic data bank must be constructed for the purpose of compiling and estimating economic data relevant to the South Dakota economy if the methodology of econometrics is to make an effective contribution to the formation of state government economic policy.

Second, specifically, the Economics Department of South Dakota State University should take the initiative in developing such a data bank, (1) because this econometric model of the South Dakota economy provides the first prototype for further research, and (2) because the diversities of economic specialization inherent in
the Economics Department's faculty provides a most effective quasi interdisciplinary approach for analyzing the complex structure of the South Dakota economy.

A second major qualification of the model is that the model's many data inaccuracies make it impossible at this time to determine whether the significant linear relationships which were shown to exist in Suits' national model are significant on the state level.

A third major qualification of the model is that the statistical procedures used in constructing the model contain their own inherent technical limitations which have not yet been refined by further research on the frontiers of theoretical statistics.

A fourth major qualification of the model is that it is not a substitute for information and judgment. It is, instead, a theoretical structure that shows clearly where judgment has to be exercised and shows how to convert economic decisions into economic policies that are consistent internally and consistent with the environmental heritage of the economy.

And, finally, if errors of calculation are not assumed out of consideration, the next most important area for further research lies in a review and evaluation, in the light of more advanced information, of the methodology used in constructing the South Dakota model.
BIBLIOGRAPHY


