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Integrated Numerical Simulation of Drip Irrigation With Subsurface Drainage

James Ray Hoover

INTEGRATED NUMERICAL SIMULATION OF DRIP

IRRIGATION WITH SUBSURFACE DRAINAGE

BY

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IRRIGATION WITH SUBSURFACE DRAINAGE

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Doctor of Philosophy, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser Date

Head, Agricultural Engineering Department Date

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VITA

James Ray Hoover was born April 7, 1941 in Decatur, Indiana to Mr. and Mrs. Leland L. Hoover of Lewistown, Ohio. He graduated from Washington Local High School in 1959, and enrolled at The Ohio State University where he graduated with the Bachelor of Agricultural Engineering degree in March 1965, and Master of Science degree majoring in Agricultural Engineering in June 1967.

He was employed by the Soil Conservation Service from 1964 to 1967, was employed by the Ohio Agricultural Research and Development Center during his M.S. graduate program from 1965 to 1967 and was employed by the Agricultural Research Service, United States Department of Agriculture, from August 1967 to the present. During the latter employment, he was stationed at Columbus, Ohio from 1967 through 1968, was located at Brookings, South Dakota from 1969 through June 1973, and was transferred to Orono, Maine, at that time. His research projects while with the Agricultural Research Service have been in the areas of drainage system design, evaporation suppression for moisture management, and modeling moisture movement in the soil.

In 1968 he married Martha Hellon. They have two daughters, Susan and Kathleen.

He is a member of the American Society of Agricultural Engineers, Soil Conservation Society of America, American Society of Agronomy, American Geophysical Union, Society of the Sigma Xi, Gamma Sigma Delta, Sigma Tau, Alpha Epsilon, and is a Registered Professional Engineer in South Dakota and in Maine.

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A	cross section of conductive media*
a	horizontal node spacing left of i,j node,* coefficient
ADI	alternating direction implicit method
b	vertical node spacing above the i,j node,* coefficient
c	horizontal node spacing right of i,j node*
C_i	coefficients in the $\theta(h)$ and $K(h)$ functions
C_d	drain resistivity
$C(\theta)$	specific water capacity
d	vertical node spacing below the i,j node,* depth from tile drain to the impervious layer
d_e	Hooghoudt's equivalent depth
$D(\theta)$	diffusivity coefficient
DELT	computer variable for time increment*
DH_j	computer variable for horizontal node distance*
DV_i	computer variable for vertical node distance*
Δ	change
E	electric potential
e	exponential function
f	soil porosity
H	computer variable for the water potential*
h	water potential
I	electric current flow
i	hydraulic gradient
i,j	Matrix subscript location indices

K	hydraulic conductivity*
K_B	hydraulic conductivity in the bottom layer
K_H	hydraulic conductivity in the horizontal direction
K_S	saturated hydraulic conductivity
K_T	hydraulic conductivity in the top layer
K_V	hydraulic conductivity in the vertical direction
K_X	hydraulic conductivity in the x direction
K_Y	hydraulic conductivity in the y direction
K_Z	hydraulic conductivity in the z direction
k	number of nodes in the horizontal direction
L	length of flow media*
l	number of nodes in the vertical direction
ln	natural logarithm function
M	height of the water table above the drain after t time
M_o	initial height of the water table above the drain
MC	soil moisture hysteresis code
n	finite difference time iteration index, coefficient
n+1	finite difference time iteration index
ϕ	hydraulic potential
PHI	computer variable for hydraulic potential*
π	pi = 3.14159
Q	flow through porous media*
R	media resistivity*
R'	specific resistance
R_i	path resistances obtained by addition of parallel block resistances

r	drain radius*
r_i	soil block resistivity
ρ	mesh size/drain radius
∂	partial derivative operator
s	drain spacing*
SOR	successive over-relaxation method
t	time*
θ	soil moisture content
THETA	computer variable for soil moisture content
τ	coefficient value from zero to one
ω	over-relaxation factor
x,y	point x,y
Y	elevation potential*
Z_0	conductor impedance

*Units for the length and time variables must agree with the units chosen for the hydraulic conductivity or soil resistivity.

INTEGRATED NUMERICAL SIMULATION OF DRIP IRRIGATION WITH SUBSURFACE DRAINAGE

Abstract

JAMES RAY HOOVER

Under the supervision of Professor John L. Wiersma

A numerical method and model were developed to simulate soil moisture during steady-state and transient flow toward a tile drain. Intermittent application of water by drip irrigation and rainfall were simulated by use of finite difference approximation equations. No known analytical solution for these flow situations exists because of the complex mathematics describing unsaturated flow in the soil.

The computer model is based on the electrical resistance network analog. The explicit finite difference equations were solved by the successive over-relaxation method for steady-state flow and by iteration for transient soil moisture flow. The computer program for the model consists of specialized subroutines which solve transient flow problems by a series of steady-state solutions. This series of solutions requires iterative application of the unsaturated flow equation to all unsaturated nodes and successive over-relaxation to all saturated nodes for each unsaturated iteration. Small time steps must be used during unsaturated flow to avoid an unstable solution.

The computer model was used to simulate soil moisture movement under three different regimes: (1) drip irrigation, (2) intermittent rainfall, and (3) rainfall on an inclined soil profile when hysteresis

was considered. An immediate response was obtained during irrigation or rainfall applications. The water table decline prior to water application and after the application was the typical exponential recession curve. The water table receded rapidly above the tile drain and drawdown progressed slowly toward the midplane boundary in the horizontal profile. This numerical technique and model could be a useful design tool to simulate soil moisture movement for evaluating proposed designs of drainage, irrigation, leach bed systems and other soil moisture flow problems.

INTRODUCTION

Our national population is growing by about two million people a year. At the present rate of growth, there may be 100 million more people in the United States by the year 2000. Stresses in the ability to consistently produce food and fiber for our population have been felt and will become more pronounced with increases in population. At the same time that more agricultural production is needed, more land is being taken out of production for homes, cities, factories, highways, waste disposal, etc. Unfortunately, only about 60 percent of the total land in the United States is suitable for crop production, and the above nonagricultural uses are removing many cropland acres permanently from agricultural production each year. Hence, researchers must find ways to produce maximum yields from limiting farmland.

One of the ways to maximize crop production is to improve the plant environment. Engineering wise, irrigation and drainage improvement practices have been very beneficial and economical. In the past, these systems have been designed from practical experience gained by trial and error and from costly field research.

Needed improvement in these design procedures fostered the development of theoretical studies to determine the physical laws of motion and conservation of mass governing soil moisture movement. These laws are expressed as a nonlinear, second order, partial differential equation. However, the complicated mathematics involved in solving the soil moisture flow equation has hindered the progress toward solutions of practical moisture flow problems. The only exact solutions apply to

steady-state flow problems. Therefore, other methods of solution have been developed including sand tank, electrical analog, and numerical analog models to analyze specific problems. These techniques bypass the mathematical complications inherent in theoretical analyses, but have their own handicaps.

The sand tank model did not become very popular, probably because of problems of obtaining a uniform conductivity and the necessary instrumentation to monitor the results. A modification of the sand tank was the glass bead-glycerol model which must be maintained at constant temperature and humidity.

Electrical analog models were an accurate method to simulate saturated soil moisture. However, a moving boundary, such as the water table in nonsteady flow was not easily solved. Therefore, most electrical analog solutions have been of steady-state flow problems or steady-state step solutions of nonsteady state problems.

Numerical solutions to soil moisture flow problems generally resulted from the solution of Darcy's and continuity equations or the diffusion equation. Due to the time required to solve most significant problems, numerical solutions were impractical until the advent of the electronic computer which could perform the needed calculations in a reasonable time. By using computers, researchers have been able to approximate solutions to steady and nonsteady-state flow problems which cannot be solved theoretically without making simplifying assumptions which destroy the usefulness of the solution.

The principal aim of this study is to develop the foundation skills and understanding of a simulation technique whereby a mathematical model of the equation of unsaturated flow of water through two-dimensional saturated and unsaturated porous media can be developed. The mathematical model will be used to simulate moisture flow characterized by saturated and unsaturated regions separated by a moving phreatic surface. The simulation will assume that at time zero the saturated soil profile will start draining to the tile. After an arbitrary time, the application of water to the profile surface will be simulated. The water applications represent drip irrigation from a line source and rainfall over the entire surface. The water applications will be repeated at a scheduled time interval and will represent typical application rates.

Drip irrigation will be simulated as line sources spaced at five-foot intervals, where the source is assumed to be one-foot wide. All emitters will operate at a constant rate during the application interval, whereas, rainfall application at any node will be proportional to the surface area represented by that node.

LITERATURE REVIEW

Theory

In 1856, Henry Darcy formulated the fundamental law of flow through porous media. The study showed that the quantity of water flowing through a sand column of length L and cross section A during the time t is given by

$$\frac{Q}{t} = KiA = K\frac{\Delta\phi}{L}A \qquad [1]$$

where the hydraulic conductivity K is the proportionality coefficient and the hydraulic gradient i is the ratio of the hydraulic potential difference $\Delta\phi$ at the ends of the sand column to the length of the column. The hydraulic potential at any point in the porous media is the sum of the gravitational, pressure, and osmotic potentials. Osmotic potential is usually ignored since solutes usually move more rapidly than the water in a soil. Potential is often expressed in terms of length of water column which produces the equivalent potential pressure. Therefore, the hydraulic potential ϕ is given as

$$\phi = h + y \qquad [2]$$

where h is the water potential and y is the elevation potential.

The hydraulic conductivity K is a measure of the permeability of the sand. Darcy's law is a dynamic equation of motion and when combined with an equation of state and an equation of continuity, the result is the general equation of flow through porous media

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(K_x(\theta)\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y(\theta)\frac{\partial\phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z(\theta)\frac{\partial\phi}{\partial z}\right) \qquad [3]$$

where θ is the volume of water per unit volume of porous media and K_x, K_y, and K_z are the hydraulic conductivities in the x, y, and z directions. This equation is nonlinear because θ, K, and ϕ are interdependent. To solve equation [3], the boundaries of the flow region, the conditions along these boundaries, and the conditions at some point in time which can be used as a starting point for the solution must be specified. Although equation [3] has no known analytical solution, its solution may be approximated through numerical analysis procedures using finite differencing techniques.

For saturated flow in an isotropic porous media, $\frac{\partial\theta}{\partial t} = 0$, $K_x = K_y = K_z = K$, and equation [3] can be simplified to the three-dimensional elliptical Laplace equation

$$0 = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} \qquad [4]$$

Additional reductions in equations [3] and [4] can be made when solving one- or two-dimensional problems by eliminating the terms containing the unnecessary direction x, y, or z.

The diffusivity concept of water flow was developed to simplify the mathematical and experimental solution of unsaturated flow problems by converting the general flow equation [3] to the diffusion and heat conduction equations for which solutions were available. This conversion assumes that the diffusion coefficient D is given as

$$D(\theta) = K(\theta)\, \partial h / \partial \theta \ . \qquad [5]$$

The ratio of hydraulic conductivity to diffusivity is sometimes called specific water capacity $c(\theta)$ where

$$c(\theta) = K(\theta)/D(\theta) = \frac{\partial \theta}{\partial h} \qquad [6]$$

which is the slope of the soil-moisture characteristic curve at moisture content θ.

Substitution of equations [2] and [5] into equation [3] gives

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial \theta}{\partial h}\left[\frac{\partial (h+y)}{\partial x}\right]\right) + \frac{\partial}{\partial y}\left(D(\theta)\frac{\partial \theta}{\partial h}\left[\frac{\partial (h+y)}{\partial y}\right]\right) + \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial \theta}{\partial h}\left[\frac{\partial (h+y)}{\partial z}\right]\right) \qquad [7]$$

which reduces to

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(D(\theta)\frac{\partial \theta}{\partial y} + D(\theta)\frac{\partial \theta}{\partial h}\right) + \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial \theta}{\partial z}\right) \qquad [8]$$

or

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(D(\theta)\frac{\partial \theta}{\partial y}\right) + \frac{\partial K(\theta)}{\partial y} + \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial \theta}{\partial z}\right) \qquad [9]$$

where y is in the vertical direction. In two dimensions, equation [9] becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(D(\theta)\frac{\partial \theta}{\partial y}\right) + \frac{\partial K(\theta)}{\partial y} \qquad [10]$$

which reduces to

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial y}\left(D(\theta)\frac{\partial \theta}{\partial y}\right) + \frac{\partial K(\theta)}{\partial y} \qquad [11]$$

for vertical flow or

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(D(\theta)\frac{\partial \theta}{\partial x}\right) \qquad [12]$$

for horizontal flow.

In the previous equations, the coefficients $K(\theta)$, $D(\theta)$, and $C(\theta)$ are given to show that these parameters are functions of the moisture content. The $K(\theta)$ and $\theta(h)$ functions are monotonic relations for moisture content θ increases <u>or</u> decreases. However, soils exhibit hysteresis if there is a reversal in the soil moisture trend. Topp (1971) has found no measurable hysteresis in the $K(\theta)$ function, but found considerable hysteresis in the $K(h)$ and $\theta(h)$ relationships. Specific water capacity $C(\theta)$ and diffusivity $D(\theta)$ also exhibit considerable hysteresis. Figure 1 from Klute, et al. (1964) shows the hysteresis behavior of a soil when it is dried and rewet producing the characteristic wetting and drying curves of the $\theta(h)$ relation. When the soil is so analyzed, the corresponding diffusivity to water content $D(\theta)$ or $D(h)$ relationship is composed of crossed curve segments which form a skewed or distorted "bowtie". As shown in Figure 1, the hysteresis behavior of the $D(\theta)$ and $C(\theta)$ does not form closed loops as in the case of the $\theta(h)$ function, but forms a "bowtie" type curve with jumps at the points corresponding to a reversal in moisture content or water potential.

Although equation [12] is of the same form as Fick's second law of diffusion and resembles the usual heat flow equations, there are several differences which do not permit the application of readily

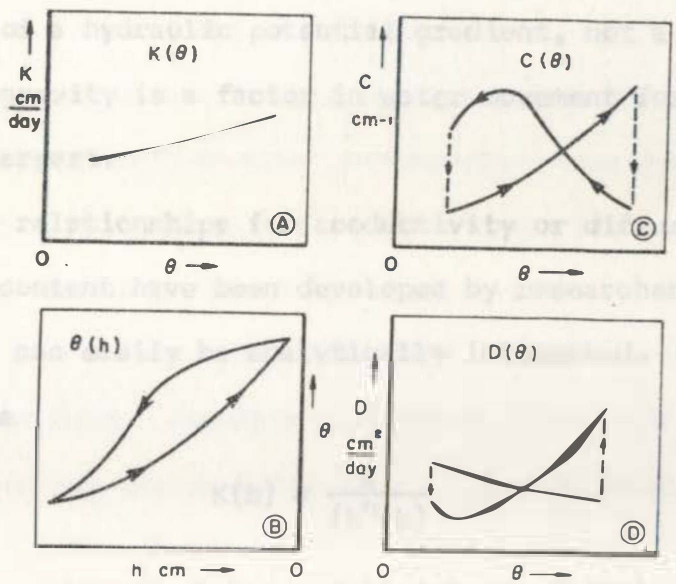

Figure 1. Hypothetical curves used in the discussion of the "bowtie" hysteresis effect in the diffusivity versus water content function.

available solutions of heat flow problems to soil moisture flow analysis. The major difference is that the specific heat is not a function of the temperature but that the moisture flow equivalent and the specific water capacity are a function of the moisture content and exhibit hysteresis in the $C(\theta)$ relation. The moisture flow is a result of a hydraulic potential gradient, not a moisture gradient. Also, gravity is a factor in water movement for which heat flow has no counterpart.

Special relationships for conductivity or diffusivity to water pressure or content have been developed by researchers which are empirical or can easily be analytically integrated. Examples of these functions are

$$K(h) = \frac{a}{(h^n+b)} \qquad [13]$$

used by Gardner (1958), Rubin and Steinhardt (1963), Taylor and Luthin (1969), Whisler, et al. (1968), and Whisler (1969);

$$K(h) = K_s e^{ah} \qquad [14]$$

used by Philip (1968) and (1972), Thomas, et al. (1974), Warrick (1974), and Zachmann and Thomas (1973);

$$D(\theta) = ae^{b\theta} \qquad [15]$$

utilized by Gardner and Mayhugh (1958) and Klute, et al. (1965), where K, K_s, D, θ, h, and e are the hydraulic conductivity, saturated hydraulic conductivity, diffusivity, moisture content, water potential, exponential function, and a, b, and n are constants, respectively.

Solution of the General Flow Equation

Analytical

Few analytical solutions of the general flow equations are available which consist of solutions of simple steady-state flow problems. Kirkham (1949) and (1951) solved two-dimensional tile drainage problems by assuming the tile drain to be a horizontal well and that flow is radial toward the drain. In the earlier publication, the soil was uniform with a constant hydraulic conductivity, whereas, the second solution was for a two-layered soil. Other analytical solutions of soil moisture flow have been found by Philip (1971) and (1972) for two-dimensional flow in a uniform soil from trickle or subsurface irrigation line sources. Zachmann and Thomas (1973) and Thomas, et al. (1974) also solved steady-state flow in a uniform soil with trickle or subsurface line sources.

Electric Analog

Investigators early recognized the similarity between Darcy's law and Ohm's law

$$I = E/R \qquad [16]$$

where the current flow I is equal to the ratio of the electric potential E to the resistance R which is proportional to the length L of the conductor per cross-sectional area A. Replacing the resistance $R = R'\frac{L}{A}$ where R' is the specific resistance of the conductor gives

$$I = \frac{EA}{R'L} \qquad [17]$$

Substituting the inverse of R' which is the specific conductivity K of the conductor, equation [17] becomes

$$I = \frac{KEA}{L} \qquad [18]$$

where I is the current flow, K is the conductivity, E is the potential difference over the length L of conductor with cross-sectional area A. Therefore, equation [18] is analogous to Darcy's law of water flow in porous media.

Early use of the analogy consisted of the use of conductivity paper made with graphite and a voltage analyzer by Childs (1943) to study steady-state tile drainage with a water table. In later publications, he analyzed the transient case of a falling or rising water table with tile drainage. The first to report the use of an electrical resistance network was Luthin (1953) making use of Kirchhoff's law

$$i_1 + i_2 + i_3 + i_4 = 0 \qquad [19]$$

for the currents through four resistors connected at point zero in the form of a five-point star as in Figure 2. Substituting equation [16] into equation [19] shows that

$$\frac{E_1-E_0}{R_1} + \frac{E_2-E_0}{R_2} + \frac{E_3-E_0}{R_3} + \frac{E_4-E_0}{R_4} = 0 \qquad [20]$$

defines the voltage anywhere within the resistance network formed by any combination of these five-point stars. Additional relationships

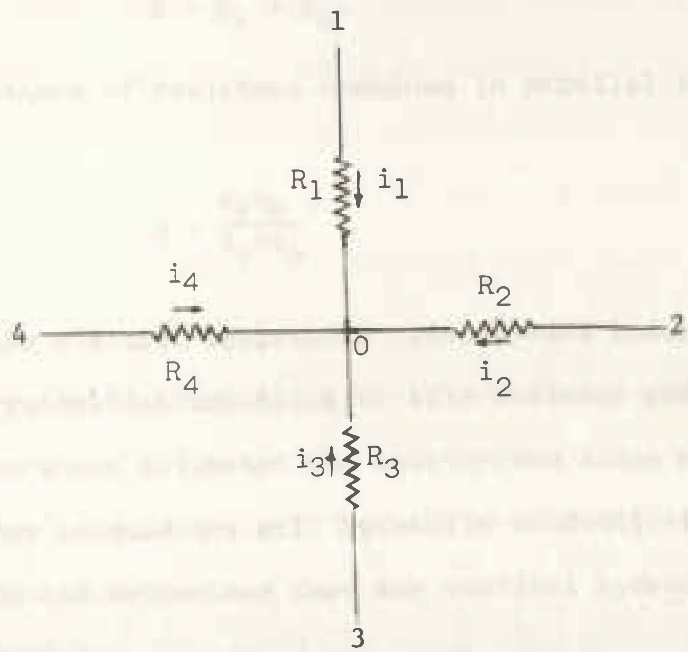

Figure 2. Five-point star used in resistance networks.

used are that the resistance of resistors combined in series is given by

$$R = R_a + R_b \tag{21}$$

and that the resistance of resistors combined in parallel is given by

$$R = \frac{R_a R_b}{R_a + R_b} \quad . \tag{22}$$

Next to use the electric resistance network were Bower and Little (1959), who found relaxation solutions to tile drainage and subsurface irrigation problems where saturated and unsaturated zones were united in one system. They assumed the soil hydraulic conductivity to vary linearly with depth and determined that the vertical hydraulic conductivity to be given by

$$K_V = \frac{K_T + K_B}{2} \tag{23}$$

and the horizontal conductivity to be given by

$$K_H = \frac{K_B - K_T}{\ln\left(\frac{K_B}{K_T}\right)} \tag{24}$$

where K_V, K_H, K_T, and K_B are the conductivities vertically, horizontally, at the top, and at the bottom of the layer, respectively. The ln is the natural logarithm function. They found the relaxation technique produced relatively rapid solutions by the resistance network system.

The next step in the development of the electric analog technique was the solving of transient flow problems by Brutsaert, et al. (1961) who compared their solution with sand tank model studies of falling water table with tile drainage by Luthin and Worstell (1957). They used a relation for drainable pore space as a function of capillary pressure as found in the Luthin-Worstell study. The electric analog solutions were in agreement with the sand tank solutions in the Luthin-Worstell experiment.

Vimoke, et al. (1962) developed greatly improved equations for the network resistance adjacent to the drain by using a logarithmic expression instead of a linear one. The solutions deviated less than two percent from the analytical solutions by Kirkham (1949) when this improved drain resistance expression was used. The network analog incorporated improved features in flexibility, accuracy, and ease of operation. Vimoke developed the "building block" approach to make it easier to understand and calculate the resistances of the network than the previously used node to node concept. The "building block" technique was published by Vimoke and Taylor (1962), wherein the mathematical relationships defining the solution of the electric network analog were developed. They described the use of mesh expansion, the solution of potential adjacent to the drain, and the procedure when solving a layered soil system. Thiel and Bornstein (1965) used the electric analog to study tile drainage of a layered soil inclined six percent.

Numerical Analysis

The nonlinearity of the general flow equation, because of the interaction of the hydraulic conductivity with water potential or moisture content and the hysteresis in the water potential to conductivity relation, has resulted in no known analytical solution of the equation for nonsteady flow cases. Solution of nonsteady flow problems can be accomplished with the electrical resistance network analog, but they are slow and laborous. Therefore, researchers turned to numerical methods to linearize the flow equation.

An early method was by relaxation of the potential variation by the five-point star equation such as [20]. Several versions of this method were tried by Kunz (1957) which included:

1. relaxing the residual at nodes of highest residual first (the Southwell method),
2. relaxing the residual of the nodes in succession without immediate replacement of the revised potentials until completion of a cycle (the Liebmann method),
3. sequential relaxation with immediate replacement (the improved Liebmann method), and
4. successively relaxing the potential by subtracting the product of a relaxation factor times the residual (called successive over-relaxation (SOR) if the relaxation factor is greater than one but less than two or is called successive under-relaxation if the relaxation factor is less than one).

Relaxation methods are explicit forms of solution since the unknown is calculated from known potentials.

Considering these four methods, the Southwell method is the fastest to converge to the solution but is not adaptable to use with modern computers. The Liebmann is slower to converge, but adaptable to computerization. The immediate replacement concept speeds convergence in the "improved" Liebmann method. A better method is to anticipate future changes needed to relax the residual by over-relaxation corrections. Over-relaxation factors ω greater than two result in divergence from the solution. Under-relaxation is often used when unstability of the solution is a problem. Another factor that affects the number of relaxations is the closeness of the first guess values for the nodes in the grid system. The better the initial values for the nodes, the quicker the solution will be obtained.

The improved Liebmann relaxation method was applied to saturated flow to tile drains in a uniform and a two layered soil by Luthin and Gaskell (1950). The steady-state solution for the uniform soil was within four percent of that found analytically by Kirkham (1949). No analytical solution exists for the two-layered drainage problem. Kirkham and Gaskell (1950) applied the same method to nonsteady flow to tile and ditch drains. Their results were in general agreement with those found by Childs in earlier work. An important observation was that the accuracy of the solution was improved with decreases in mesh size, but the labor involved in the hand calculations increased

rapidly with reductions in mesh size. They derived a relation for the rate that water declined and used the relaxation methods to solve the nonsteady flow problem by a series of steady-state solutions at constant water table levels. The same procedure, except utilizing a cylindrical coordinate system was used by Luthin and Scott (1952) to solve for flow through aquifers to gravity and artesian wells.

Klute (1952) developed the diffusion equation for horizontal infiltration and drainage cases. The nonsteady-state solutions of the diffusion equation were obtained by numerical iteration techniques and graphical determination of diffusivity from the moisture content versus water pressure relation.

Day and Luthin (1956) solved the general flow equation for one-dimension and utilized the relation between capillary conductivity and moisture content to similate vertical drainage. The results indicated the general trend of the drainage could be predicted when the capillary conductivity relation was used.

Gardner and Mayhugh (1958) determined from available data for some soils, that diffusivity was an exponential function of the moisture content as given in equation [15]. They used the diffusion equation [12] and the Boltzman transformation of variables

$$y = \frac{x}{\sqrt{D_o t}} \qquad [25]$$

which can only be used for semi-infinite, uniform porous media with a uniform initial moisture content, to convert the nonlinear general flow equation into an ordinary differential equation which they numerically

solved. They solved the nonsteady state, one-dimensional horizontal infiltration problem. Their results were in good agreement with data obtained from the pressure plate outflow method. Gardner (1959) again used the exponential relation for diffusivity to numerically solve the diffusion equation for horizontal flow occurring during evaporation from soil columns. The numerical solutions predicted accurately the rate of drying for laboratory soil columns.

Isherwood (1959) was the first to use a digital computer to solve the flow equation. He used the successive over-relaxation technique applied to the five-point star equation [20] with reflected images at the boundaries. He analyzed the falling water table in tile drained land as a series of steady-state solutions as conducted by Kirkham and Gaskell (1950). The assumptions for the solution were constant hydraulic conductivity and drainable porosity, hence neglecting the capillary fringe. He found that convergence was quite rapid if the over-relaxation factor was used. The water table fell exponentially with time after a slower initial rate.

Evans and Ashcroft (1961) analyzed the effect of a restricting layer on ponded flow to tile drains by a computerized relaxation method similar to that of Isherwood. Taylor and Luthin (1963) and Burke and Taylor (1965) analyzed other stratification problems with results within five percent of the analytical solutions by Kirkham.

In contrast to the explicit relaxation method, Ashcroft, et al. (1962) developed an implicit finite difference method of solution of

the diffusion equation. The implicit technique is the backward difference scheme instead of the explicit forward difference method. This means that the solution must consider all equations at once, such as a matrix solution of the coefficients of the linearized flow equation. Conveniently, the matrix involved is a tridiagonal matrix, wherein, only a tridiagonal band of coefficients are non-zero. This type of system can be solved rapidly by the Thomas algorithm described by von Rosenberg (1969). Ashcroft, et al. (1962) solved the one-dimensional, horizontal flow in a uniform semi-infinite medium and compared their results with those obtained analytically by use of the Boltzman transformation. Both methods gave very similar results.

Hanks and Bowers (1962) programmed the solution of infiltration into a layered soil by implicit finite difference methods. They solved the tridiagonal coefficient matrix by methods described by Richtmyer and Morton (1967). Their method required known relations between the moisture content and pressure head and between moisture content and diffusivity. The method did not require the soil to be homogeneous, semi-infinite, horizontal (gravity neglected), or the initial moisture content to be uniform. Their results compared well with work published by Philip in 1957.

A modified Gauss--Siedel iterative method with over-relaxation was used by Reisenauer (1963) to solve steady-state seepage flow from unlined canals through heterogeneous, partially saturated sands. His program could solve one-, two-, or three-dimensional flow problems

consisting of as many as 8,000 nodes. He used the optimum over-relaxation coefficient for stability as described by Young in 1954. Rubin and Steinhardt (1963) solved the problem of nonsteady, one-dimensional, vertical infiltration at rates less than and greater than the saturated hydraulic conductivity. They used the expressions

$$h = 11.3 + \frac{3.19}{\theta} - 0.05e^{15\theta} + e^{-575\theta+16.3} \qquad [26]$$

for the water pressure and

$$K = \frac{8400}{h^5 + 14.45^5} \qquad [27]$$

for the hydraulic conductivity of Rehovot sand. They assumed no hysteresis existed in these functions. Sewell and van Schilfgaarde (1968) solved two-dimensional, nonsteady, unsaturated flow to tile drains. They used an iteration technique and considered flow from the capillary fringe by a conductivity versus water potential relation.

Taylor and Luthin (1963) programmed the methods reported by Luthin and Gaskell (1950) to solve steady-state two-dimensional flow through a two-layered soil to tile drains. They used a constant value for the hydraulic conductivity. The publication gives a good description of their method to solve the Laplace equation.

Green, et al. (1964) used the method of Hanks and Bowers (1962) to study nonsteady vertical infiltration into a two-layered soil. Their infiltration rates were in good agreement with those measured in the field.

Wang, et al. (1964) analyzed nonsteady, unsaturated, vertical infiltration into soils, when the initial moisture content and the hydraulic and capillary characteristics of the soil are known. The step-by-step numerical procedure predicted the moisture content of the profile with depth as time of infiltration increased.

Burke and Taylor (1965) studied the effect of soil stratification in steady-state saturated flow to tile drains by numerical methods. They analyzed two- and three-layered soil arranged to form 144 different drainage situations. Deeper placement of drains resulted in higher flow rates only if the ratio of the conductivity of the lower layer to the upper layer was greater than one third.

Klute, et al. (1965) solved the nonlinear diffusion equation for horizontal infiltration and outflow through a soil column of finite length. They used the exponential relation between diffusivity and moisture content [15] and used an iteration method. For given boundary conditions and initial moisture content, they found that inflow occurs more readily than outflow.

Whisler and Klute (1965) analyzed nonsteady vertical infiltration of soil columns which had been saturated and drained to give the initial conditions. They assumed hysteresis and iteratively removed the nonlinearity using a modified Crank-Nicolson finite difference method. They noted that each point in the sand column wetted along a different scanning curve of the moisture characteristic curves and that soils with steep moisture content versus water pressure curves exhibited a

sharp wetting front while in soils with more gently sloping characteristic curve, the wetting front was more diffuse.

Luthin and Taylor (1966) analyzed two-dimensional steady flow to ditches in a sloping profile where the outflow rate equaled the precipitation rate. They considered the capillary fringe by using equation [13] with a, b, and n equal to 3.6×10^6, 2.5×10^4, and 3, respectively. The ditches were assumed to reach the impermeable layer. Results consisted of water table profiles for different precipitation rates and different land slopes.

Staple (1966) used an implicit method to solve the diffusion equation for vertical infiltration followed by redistribution, considering hysteresis in the moisture content versus hydraulic conductivity relation. The results consisted of moisture content versus depth curves for time steps.

Remson, et al. (1967) solved the diffusion equation for vertical drainage without infiltration or evaporation. They used data for a diffusivity versus moisture content curve in the explicit forward difference solution to determine the computed moisture content versus depth curves for steps of flow time.

Rubin (1967) developed a numerical procedure to analyze post-infiltration redistribution of water in semi-infinite vertical soil columns. The method utilizes hysteresis relations making it possible to find the unique curve which characterises moisture transformations with depth. The method is applicable to cases of redistribution with or without a constant evaporation rate. The finite difference method

was an implicit type which was solved as in Rubin and Steinhardt (1963). He found that almost all the evaporated water came from soil layers with positive moisture gradients.

Rubin (1968) solved the flow equation for two-dimensional, transient, unsaturated or partially unsaturated soils by using alternating-directions implicit "ADI" finite difference methods. A solution by the ADI method consists of the solution of two sets of equations. The first set of equations is implicit in one direction only and forms a tri-diagonal matrix that can be solved easily by the Thomas algorithm. Then, the same type of equations, except implicit only in the direction perpendicular to that used in the first set of equations, are solved by the Thomas algorithm. The second solution is considered the solution to the problem. Hence, the ADI solution is like solving two one-dimensional, perpendicular flow problems sequentially, where the second solution is the answer to the problem. Rubin (1968) considered the problems of horizontal infiltration and ditch drainage. The results for the horizontal infiltration process involved upward flow-components, which are due, primarily, to a gravity induced variation in hydraulic conductivity along the inflow face. The drainage results demonstrate that transient water flow within the unsaturated zone and the outflow from the seepage zone may significantly affect the progress of the water table descent and the total outflow rates.

Wang and Lakshminarayana (1968) solved the nonsteady state diffusivity equation for vertical infiltration and vertical drainage through a layered soil using an explicit-implicit difference scheme.

Cumulative drainage and average rate of drying and wetting were in reasonable agreement with field measurements.

Amerman (1969) in his thesis used both explicit and implicit finite difference methods to determine the two-dimensional, partially unsaturated, nonsteady flow from surface irrigation into the soil profile. He had intended to use the ADI method entirely, but if a flow region contained fully saturated and partially saturated subregions separated by a moving phreatic surface, the ADI method failed due to the occurrence of singularities in the systems of simultaneous equations used in the ADI procedure. He did not consider hysteresis. He also solved the case of drainage from saturated, sloping land.

Freeze (1969) studied the problem of one-dimensional, vertical, nonsteady, unsaturated flow through homogeneous, isotropic soils in a recharging or discharging ground water flow system. He modeled infiltration and evaporation with hysteresis by implicit finite difference methods which were solved by recursion methods given in Richtmyer and Morton (1967). The solutions were given in the form of water potential, hydraulic potential, and moisture content profiles.

Hanks, et al. (1969) presented a numerical method to estimate one-dimensional infiltration, redistribution, evaporation, and drainage of water from the soil. They assumed hysteresis in the water content versus water potential relation, but not in the conductivity versus water content relation. Their estimation of infiltration, redistribution, and evaporation was in good agreement with measurements from soil columns.

Taylor and Luthin (1969) solved transient problems of drawdown around a pumped well in an unconfined aquifer. The solution, expressed in cylindrical coordinates, is a series of steady-state solutions of the saturated-unsaturated flow region. The analysis of the simultaneous solutions for flow in the saturated and unsaturated zones is similar to the procedure that will be described in this thesis.

Bolen (1970) analyzed transient water flow through layered soils to tile drains using the relaxation technique. The initial condition was a saturated profile. His numerical procedure was similar to that of Taylor and Luthin (1969). Results from the two-layered profile indicated that when the conductivity of the top layer to that of the bottom layer was less than or equal to 15, drain depth had more effect on drawdown than did the conductivity ratio. For ratios greater than 15, faster drawdowns were obtained by placing the drain in the more permeable top layer.

Brandt, et al. (1971) used the diffusion equation for nonsteady, unsaturated flow to analyze three-dimensional infiltration from a trickle source. They used a combination of the ADI noniterative procedure with Newton's iterative method. Their results compared well with those of Wooding for steady infiltration from a circular pond and with simple one-dimensional solutions.

Nwa, et al. (1971) used the ADI method to solve the problem of rainfall infiltration into a watershed surface. The slope of the watershed had no effect on the transient infiltration rate, but the steady-state rate increased as the land slope increased. They found

that the saturation starts from the top of the profile and moves down to the impermeable layer when the precipitation rate equals or exceeds the saturated hydraulic conductivity, but the situation is reversed if the rainfall rate is less than the saturated conductivity. They had no problem solving mixed flow problems by the ADI method.

Reichardt, et al. (1972) solved the soil moisture flow equation for nonsteady, two-dimensional horizontal infiltration into air-dry layered soils. The solution, based on a scaled soil water diffusivity function, was in good agreement with experimental and theoretical infiltration profiles for the layered soils.

Whisler, et al. (1972) analyzed ponded, nonsteady-state, infiltration into a heterogeneous porous medium in which the hydraulic conductivity varied linearly with the depth in the profile. The scaled hydraulic conductivity relation is a function of water pressure at the air entry value. Pressure head and water content profiles for two distributions of hydraulic conductivity were computed and compared with that for a homogeneous soil having an average conductivity.

Lomen and Warrick (1974) used the exponential relation between hydraulic conductivity and water potential to numerically, without finite differencing techniques, solve the linearized transient, two-dimensional flow equation. Their solution applies to single and parallel line sources on the ground surface. The solution applies to irrigation systems operated frequently, such as drip irrigation, for which the soil moisture at any point varies over a relatively small range. The results include lines of constant matric flux potential or

equal moisture content as a function of time. Warrick (1974) extended this method to time-dependent cases for point sources. His results for moisture front advance and cyclic water applications were similar to the finite difference results of Brandt, et al. (1971).

Summary

The basis of determining water flow in soil is by the general flow equation which is a combination of Darcy's law and an equation of continuity of mass. The flow equation is a nonlinear, partial differential equation which defies exact analytical solution, except for a few steady-state cases by Kirkham (1949, 1951) for tile drainage, Philip (1971, 1972), Zachmann and Thomas (1973) and Thomas, et al. (1974) for surface and subsurface line sources.

Hence, researchers developed methods to linearize the flow equation so that it could be solved for steady and nonsteady-state flow situations. The methods consist of the electric analog, numerical variable transformations, and finite difference techniques.

The first to utilize the analogy between Ohm's law and Darcy's law was Childs (1943) who used conducting paper as representing the porous media. The resistance network was developed and refined by Luthin (1953), Bower and Little (1959), Taylor, et al. (1960), Brutsaert (1961), Vimoke (1962), Vimoke and Taylor (1962), and Thiel and Bornstein (1965). Although the electric analog has been used mainly to solve steady-state flow problems, it can be used to solve nonsteady flow by a series of steady-state step solutions.

Numerical transformations have been limited to the Boltzman transformation and exponential relationships in conductivity and diffusivity functions. The Boltzman transformation has severe limitations including that it can only be used to study flow in a semi-infinite, uniform porous medium with a uniform initial moisture content. The Boltzman transformation was first used by Gardner and Mayhugh (1958). The conductivity exponential relation was used by Philip (1968, 1972), Thomas, et al. (1974), Warrick (1974), and Zachmann and Thomas (1973). Gardner and Mayhugh (1958) and Klute, et al. (1965) utilized the exponential diffusivity relation.

The first researchers to use finite difference methods to analyze soil moisture movement were Luthin and Gaskell (1950), who applied the relaxation technique to ponded water flow. Numerical finite difference methods were so successful that many researchers have turned to these methods. Others that solved steady-state ponded flow situations include: Luthin and Scott (1952), Evans and Ashcroft (1961), Reisenauer (1963), Taylor and Luthin (1963), and Burke and Taylor (1965) for various stratification conditions. Steady-state, unsaturated flow situations were studied by Reisenauer (1963), Sewell and van Schilfgaarde (1963), and Luthin and Taylor (1966).

Nonsteady (transient), unsaturated flow was first analyzed by Kirkham and Gaskell by successive steady-state steps by the relaxation method with calculations performed by hand. The method was very laborous. Isherwood solved the same problem with a digital computer. Their solutions were based on Kirchhoff's law for the electrical resistance network.

The first solution to the general flow equation was by Day and Luthin (1956). Klute (1952) modified the flow equation to replace the hydraulic conductivity by diffusivity times the rate of change in moisture content per change in water potential. Many researchers started using this or other diffusivity functions. Those utilizing the diffusivity concept include: Klute (1952), Gardner and Mayhugh (1958), Gardner (1959), Ashcroft, et al. (1962), Rubin and Steinhardt (1963), Green (1964), Klute, et al. (1965), Liakopoulos (1966), Staple (1966), Remson, et al. (1967), and Reichardt (1972).

The finite difference methods utilized by the above researchers have been categorized as explicit and implicit when the calculated value is computed from known values (forward difference scheme) or computed with unknown values (backward difference scheme), respectively. An interesting combination of both schemes forms the Crank-Nicolson equation derived from the central difference equation.

Researchers solving explicit methods using relaxation or iteration procedures include: Luthin and Gaskell (1950), Luthin and Scott (1952), Luthin and Day (1955), Day and Luthin (1956), Isherwood (1959), Evans and Ashcroft (1961), Reisenauer (1963), Sewell and van Schilfgaarde (1963), Taylor and Luthin (1963), Gupta and Staple (1964), Burke and Taylor (1965), Luthin and Taylor (1966), Remson, et al. (1967), Taylor and Luthin (1969), Bolin (1970), and Reichardt (1972). Explicit methods were solved by use of a tridiagonal matrix by Hanks and Bowers (1962), Green, et al. (1964), Hanks and Klute (1968), Hanks, et al. (1969).

The Crank-Nicolson modified method was used by Whisler and Klute (1965, 1967) and Whisler and Watson (1968). Freeze has developed the line successive over-relaxation (LSOR) method in Freeze (1971) and other recent publications which is said to be faster than the implicit alternating direction implicit (ADI) method.

Researchers solving implicit schemes are Ashcroft, et al. (1962), Rubin and Steinhardt (1963), Liakopoulos (1966), Wassmuth, et al. (1966), Rubin (1967), Freeze (1969), and Brutsaert (1971). A recent implicit scheme, the ADI method was developed by Rubin (1968) and used by Amerman (1969) and Nwa, et al. (1971).

Table 1 is a condensed summary of the techniques utilized in the research studies of the flow of moisture through porous media that have been described in detail. The summary indicates the applications of these analyses.

Table 1. Publication Summary of Some Solutions of the Soil Moisture Flow Equation.

Researcher	Conditions							
	Steady-State	Non-Steady-State	Method[1] of Solution	Diffusivity Concept	Coefficient[2] Function	Dimensions 1,2, or 3	Soil Layers	Application[3]
Childs (1943)	S		E		K(c)	2	1	Tile D.
Kirkham (1949)	S		A		K(c)	2	1	Tile D.
Kirkham & Gaskell (1950)		T	N		K(c)	2	1	Ditch D.
Luthin & Gaskell (1950	S		N		K(c)	2	2	Tile D.
Kirkham (1951)	S		A		K(c)	2	2	Tile D.
Klute (1952)		T	N	D	D(θ)	1	1	H.I. & D.
Luthin (1953)	S		E		K(c)	2	3	Tile D.
Day & Luthin (1956)		T	N		K(h)	1	1	V.D.
Gardner & Mayhugh (1958)		T	N	D	D(θ)	1,2, & 3	1	H.I.
Bower & Little (1959)	S		E		K(h)	2	1	Sub. I. & Tile D.
Gardner (1959)		T	N	D	D(θ)	1	1	H.E.
Isherwood (1959)		T	N		K(c)	2	1	Tile D.
Taylor et al. (1960)	S		E		K(c)	2	3	Tile D.

Table 1. Continued.

Researcher	Conditions							
	Steady-State	Non-Steady-State	Method[1] of Solution	Diffusivity Concept	Coefficient[2] Function	Dimensions 1,2, or 3	Soil Layers	Application[3]
Evans & Ashcroft (1961)	S		N		K(c)	2	3	Tile D.
Ashcroft et al. (1962)		T	N	D	D(θ)	1	1	H.I.
Hanks & Bowers (1962)		T	N	D	D(θ)	1	2	V.I.
Vimoke & Taylor (1962)	S		E		K(c)	2	2	Tile D.
Reisenauer (1963)	S		N		K(h)	2	3	Canal Seep.
Rubin & Steinhardt (1963-1964)		T	N	D	D($K\frac{\partial h}{\partial \theta}$)	1	1	V.I.
Sewell & van Schilfgaarde (1963)	S		N		K(h)	2	1	Tile D.
Taylor & Luthin (1963)	S		N		K(c)	2	2	Tile D.
Green (1964)		T	N	D	D(θ)	1	2	V.I.
Wang et al. (1964)		T	N		K(θ)	1	1	V.I. & E.

Table 1. Continued.

	Conditions							
Researcher	Steady-State	Non-Steady-State	Method[1] of Solution	Diffusivity Concept	Coefficient[2] Function	Dimensions 1,2, or 3	Soil Layers	Application[3]
Burke & Taylor (1965)	S		N		K(c)	2	3	Tile D.
Klute et al. (1965)		T	N	D	D(θ)	1	1	H.I. & H.E.
Thiel & Bornstein (1965)	S		E		K(c)	2	3	Tile D.
Whisler & Klute (1965)		T	N		K(h)*	1	1	V.I. & V.D.
Liakopoulos (1966)		T	N	D	D(h,θ)	1	1	E.
Luthin & Taylor (1966)	S		N		K(h)	2	1	Ditch D.
Staple (1966)		T	N	D	K(θ) & D(θ)*	1	1	V.I. & V.R.
Remson, et al. (1967)		T	N	D	D(θ)	1	1	V.D.
Rubin (1967)		T	N		K(θ)*	1	1	V.R.
Rubin (1968)		T	N		K(h)	2	1	H.I. & Ditch D.
Wang & Lakshminarayana (1968)		T	N		K(θ)	1	10	V.D. & V.I.

Table 1. Continued.

Researchers	Conditions							
	Steady-State	Non-Steady-State	Method[1] of Solution	Diffusivity Concept	Coefficient[2] Function	Dimensions 1,2, or 3	Soil Layers	Application[3]
Amerman (1969)		T	N		K(h)	2	1	Furrow I. & D.
Freeze (1969)		T	N		K(h)*	1	1	V.E.&R.
Hanks, et al. (1969)		T	N		K(θ)*	1	1	I,R,E, & D.
Taylor & Luthin (1969)		T	N		K(h)	2	1	Well
Bolin (1970)		T	N		K(h)	2	2	Tile D.
Brandt, et al. (1971)		T	N	D	D(θ)	2	1	Trickle I.
NWA, et al. (1971)		T	N		K(h)	2	1	I.
Philip (1971)	S		A	D	K(x,z)	2	1	Trickle & Sub I.
Philip (1972)	S		A		K(h)	2	1	Trickle I.
Reichardt, et al. (1972)		T	N	D	D(θ)	2	2& 3	H.I.

Table 1. Continued.

Researchers	Conditions							
	Steady-State	Non-Steady-State	Method[1] of Solution	Diffusivity Concept	Coefficient[2] Function	Dimensions 1,2, or 3	Soil Layers	Application[3]
Whisler, et al. (1972)		T	N		K(z)	1	2	V.I.
Zachmann & Thomas (1973)	S		A		K(h)	2	1	Sub I, S.I.&S.E.
Thomas, et al. (1974)	S		A	D	K(h) or K(θ)	2	1	Sub I.
Lomen & Warrick (1974)		T	N		K(h)	2	1	Trickle I.
Warrick (1974)		T	N	D	K(h)	2	1	Trickle I.

[1]Methods of Solution: A (Analytical, E (Electrical Analog), and N (Numerical).

[2]Coefficient Function: K (Hydraulic conductivity), D (Diffusivity), c (Constant), h (Water pressure), and θ (Moisture content), * Hysteresis considered.

[3]Application: V (Vertical), H (Horizontal), I (Infiltration), E (Evaporation), D (Drainage), R (Redistribution), and S (Surface).

METHODS AND PROCEDURES

Objectives

The objective of the study is to develop a numerical procedure for analyzing two-dimensional flow in a layered profile. The flow will be characterized by saturated and unsaturated regions separated by a moving phreatic surface. Both the saturated and unsaturated portions of the model will be verified by comparison with published results. The model will then be used to simulate the flow from a drip irrigation line source to a tile drain.

The simulation will assume that at time zero the profile will be saturated and start draining to the tile. After some arbitrary time, the dr p irrigation source will be started at a constant rate. One drip irrigation rate will be investigated which will consist of an application rate less than the drainage rate. In addition, soil moisture movement will be simulated during intermittent rainfall.

Methods

Several numerical methods were discussed in the review of literature and will be derived and analyzed in the process of selecting a method to use in this study. The basic concept of flow of water in porous media was presented as the general flow equation. For two-dimensional flow, the equation is:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K(\theta)\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K(\theta)\frac{\partial \phi}{\partial y}\right) \qquad [28]$$

The method of finite differences involves replacing the partial derivatives in this equation by an approximation finite difference. The finite difference approximations will be derived for an elemental volume where the height is $2\Delta y$, the horizontal distance is $2\Delta x$, and the depth is $2\Delta t$ as shown in Figure 3.

The finite difference approximations in the x direction are given by:

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)_{x,y} = \frac{\left(\frac{\partial \phi}{\partial x}\right)_{x+\Delta x/2,y} - \left(\frac{\partial \phi}{\partial x}\right)_{x-\Delta x/2,y}}{\Delta x} \qquad [29]$$

where

$$\left(\frac{\partial \phi}{\partial x}\right)_{x+\Delta x/2,y} = \frac{\phi_{x+\Delta x,y} - \phi_{x,y}}{\Delta x} \qquad [30]$$

and

$$\left(\frac{\partial \phi}{\partial x}\right)_{x-\Delta x/2,y} = \frac{\phi_{x,y} - \phi_{x-\Delta x,y}}{\Delta x} \qquad [31]$$

Therefore, equation [29] may be written as

$$\left(\frac{\partial^2 \phi}{\partial x^2}\right)_{x,y} = \frac{\phi_{x-\Delta x,y} - 2\phi_{x,y} + \phi_{x+\Delta x,y}}{(\Delta x)^2} \qquad [32]$$

Utilizing the common i,j subscript notation as shown in Figure 3, for the cartesian coordinate system, equation [32] becomes

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\phi^n_{i,j-1} - 2\phi^n_{i,j} \quad \phi^n_{i,j+1}}{\Delta x^2} \qquad [33]$$

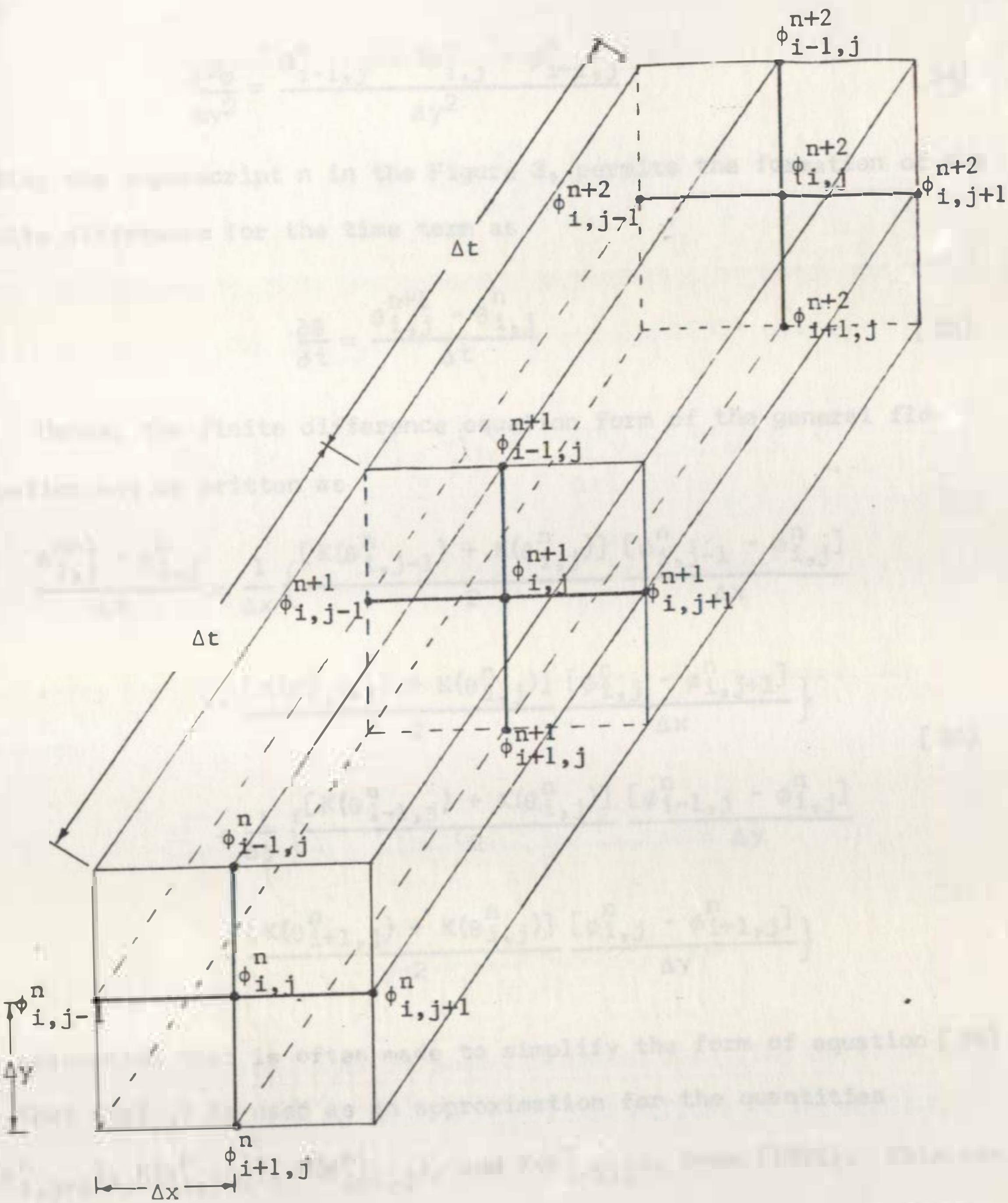

Figure 3. Elemental volume five-point stars at three time levels.

and

$$\frac{\partial^2\phi}{\partial y^2} = \frac{\phi^n_{i-1,j} - 2\phi^n_{i,j} + \phi^n_{i+1,j}}{\Delta y^2} \qquad [34]$$

Noting the superscript n in the Figure 3, permits the formation of the finite difference for the time term as

$$\frac{\partial\theta}{\partial t} = \frac{\theta^{n+1}_{i,j} - \theta^n_{i,j}}{\Delta t} \qquad [35]$$

Hence, the finite difference equation form of the general flow equation may be written as

$$\begin{aligned}\frac{\theta^{n+1}_{i,j} - \theta^n_{i,j}}{\Delta t} = {} & \frac{1}{\Delta x}\left\{\frac{[K(\theta^n_{i,j-1}) + K(\theta^n_{i,j})]}{2}\,\frac{[\phi^n_{i,j-1} - \phi^n_{i,j}]}{\Delta x}\right. \\ & \left. - \frac{[K(\theta^n_{i,j+1}) + K(\theta^n_{i,j})]}{2}\,\frac{[\phi^n_{i,j} - \phi^n_{i,j+1}]}{\Delta x}\right\} \\ & + \frac{1}{\Delta y}\left\{\frac{[K(\theta^n_{i-1,j}) + K(\theta^n_{i,j})]}{2}\,\frac{[\phi^n_{i-1,j} - \phi^n_{i,j}]}{\Delta y}\right. \\ & \left. - \frac{[K(\theta^n_{i+1,j}) + K(\theta^n_{i,j})]}{2}\,\frac{[\phi^n_{i,j} - \phi^n_{i+1,j}]}{\Delta y}\right\}\end{aligned} \qquad [36]$$

An assumption that is often made to simplify the form of equation [36] is that $K(\theta^n_{i,j})$ is used as an approximation for the quantities $K(\theta^n_{i,j+1})$, $K(\theta^n_{i,j-1})$, $K(\theta^n_{i+1,j})$, and $K(\theta^n_{i-1,j})$, Bear (1972). This assumption, although not theoretically accurate, permits equation [37] to be reduced to

$$\frac{\theta^{n+1}_{i,j} - \theta^{n}_{i,j}}{\Delta t \; K(\theta^{n}_{i,j})} = \frac{\phi^{n}_{i,j-1} - 2\phi^{n}_{i,j} + \phi^{n}_{i,j+1}}{\Delta x^2} + \frac{\phi^{n}_{i-1,j} - 2\phi^{n}_{i,j} + \phi^{n}_{i+1,j}}{\Delta y^2} \qquad [37]$$

Many researchers replace the hydraulic potential ϕ by water potential h, $\partial\theta$ by $C(\theta)\partial h$, and $K(\theta_{i,j})$ by $K(\theta)$ in equation [37] giving

$$\frac{C(\theta)}{K(\theta)}\left(\frac{h^{n+1}_{i,j} - h^{n}_{i,j}}{\Delta t}\right) = \frac{h^{n}_{i,j-1} - 2h^{n}_{i,j} + h^{n}_{i,j+1}}{\Delta x^2} + \frac{h^{n}_{i-1,j} - 2h^{n}_{i,j} + h^{n}_{i+1,j}}{\Delta y^2} \qquad [38]$$

Replacing $K(\theta)/C(\theta)$ by $D(\theta)$ given by equation [6], equation [38] becomes

$$\frac{h^{n+1}_{i,j} - h^{n}_{i,j}}{\Delta t \; D(\theta)} = \frac{h^{n}_{i,j-1} - 2h^{n}_{i,j} + h^{n}_{i,j+1}}{\Delta x^2} + \frac{h^{n}_{i-1,j} - 2h^{n}_{i,j} + h^{n}_{i+1,j}}{\Delta y^2} \qquad [39]$$

which is a form often used in recent publications. The equation [39] is said to be explicit or forward (in time) difference approximation since the one unknown $h^{n+1}_{i,j}$ is given in terms of known values which were determined during the <u>nth</u> time step. Equation [39] can be solved by iteration or matrix solution methods.

Another common practice is to assume that Δx and Δt can be chosen independently and the $\frac{\partial^2 h^n}{\partial x^2}$ can be replaced by $\frac{\partial^2 h^{n+1}}{\partial x^2}$, and similarily $\frac{\partial^2 h}{\partial y^2}$ can be replaced by $\frac{\partial^2 h^{n+1}}{\partial y^2}$. Making these modifications in the equation results in the implicit or backward (in time) difference approximation

$$\frac{h^{n+1}_{i,j} - h^{n}_{i,j}}{\Delta t\, D(\theta)} = \frac{h^{n+1}_{i,j-1} - 2h^{n+1}_{i,j} + h^{n+1}_{i,j+1}}{\Delta x^2} + \frac{h^{n+1}_{i-1,j} - 2h^{n+1}_{i,j} + h^{n+1}_{i+1,j}}{\Delta y^2} \qquad [40]$$

which can be solved only by matrix solution methods.

The solution of equations [39] and [40] can be shown to be of the accuracy, $O[\Delta t, \Delta x^2, \Delta y^2]$ by comparison with the exact Taylor series expansion about a given point 0 as

$$h_i = h_0 + (x_i - x_0)\frac{\partial h}{\partial x} + \frac{(x_i - x_0)^2}{2!}\left(\frac{\partial^2 h}{\partial x^2}\right)_0 + \frac{(x_i - x_0)^3}{3!}\left(\frac{\partial^3 h}{\partial x^3}\right)_0 + \ldots \qquad [41]$$

The degree of accuracy of the approximation is given in detail in pages 104-105 of Karplus (1958) and pages 65-67 of Remson, et al. (1971). If the accuracy could be improved to the order of Δt^2, the increased accuracy would result in faster convergence to the solution of the system of linear equations. One method would be to approximate $\partial h/\partial t$ over two time steps so that the left side of equation [40] is replaced by

$\frac{h_{i,j}^{n+2} - h_{i,j}^{n}}{2\Delta t \quad D(\theta)}$, however, the new equation always results in an unstable solution. Then Whisler and Klute (1965) decided to develop the finite difference approximation about the n+1/2 time level. The n+1/2 time level for the right hand portion of equation [40] was obtained by taking the average of equation [39] at the nth time level and equation [40] at the n+1 time level. The generalized equation is

$$\frac{h_{i,j}^{n+1} - h_{i,j}^{n}}{\Delta t \, D(\theta)} = \tau \left[\frac{h_{i,j-1}^{n} - 2h_{i,j}^{n} + h_{i,j+1}^{n}}{\Delta x^2} + \frac{h_{i-1,j}^{n} - 2h_{i,j}^{n} + h_{i+1,j}^{n}}{\Delta y^2} \right] + (1-\tau) \left[\frac{h_{i,j-1}^{n+1} - 2h_{i,j}^{n+1} + h_{i,j+1}^{n+1}}{\Delta x^2} + \frac{h_{i-1,j}^{n+1} - 2h_{i,j}^{n+1} + h_{i+1,j}^{n+1}}{\Delta y^2} \right] \quad [42]$$

When $\tau = 1$, equation [42] reduces to the fully explicit equation [39], when $\tau = 0$, [42] forms the implicit equation [40], and when $\tau = \frac{1}{2}$, the equation becomes the Crank-Nicolson equation. One-dimensional forms of these equations can be obtained by eliminating the Δx or Δy related terms.

The merits of each equation should be evaluated to determine which equation should be used in the model. Ideal conditions for the numerical model were given by Remson, et al. (1971) as :

1. the method should be convergent and unconditionally stable for Δt chosen independently of Δx or Δy,
2. the model should have relatively small truncation error without greatly increasing the complexity of the resulting equations, and
3. the solutions should be easy to compute.

The qualities of each equation can be determined for one- and two-dimensional forms of the flow equation.

In one-dimensional flow problems, the solution can be obtained rather easily. The explicit equation can be solved by iteration or matrix methods of solving simultaneous linear equations. Several iteration methods are the Jacobi, Liebmann, modified Liebmann or Gauss-Siedel, and the successive over-relaxation SOR, that are enumerated in order of increasing speed of convergence. The major asset of the explicit equation is its ease of solution, but the equation has a serious liability because of nonconvergence to a solution when Δt has been chosen too large. The explicit equation can be written as

$$h_i^{n+1} = \left(1 - \frac{2\Delta t\, D(\theta)}{\Delta x^2}\right) h_j^n + \frac{\Delta t\, D(\theta)}{\Delta x^2}\left(h_{j-1}^n + h_{j+1}^n\right). \quad [43]$$

Stability and convergence of the explicit equation will be maintained as long as the coefficient of h_j^n remains positive. Solving the coefficient equal to zero for Δt gives

$$\Delta t \le \frac{1}{2}\frac{\Delta x^2}{D(\theta)} \quad [44]$$

for stable, converging solutions. Hence, as one decreases Δx to improve the accuracy of the solution, the maximum time step is decreased, resulting in more time step iterations per solution.

Both the implicit and the faster converging Crank-Nicolson equations must be solved by matrix methods for solution of the system of linear equations. The coefficients form a tridiagonal matrix which may be readily solved by the Thomas algorithm or similar method with a medium amount of effort. The implicit and Crank-Nicolson equations should not be applied to nonlinear problems since the resulting equations are also nonlinear.

In summary of numerical methods of analyzing one-dimensional flow, the explicit method is the easiest to use, but it may require small time steps for stability of the solution. The implicit and Crank-Nicolson equations form a tridiagonal matrix which can be solved rapidly. For nonlinear problems, the Douglas-Jones predictor-corrector method described in detail in Remson, et al. (1971) should be used.

The explicit, implicit, and Crank-Nicolson forms of equation will be analyzed for two-dimensional flow situations. The explicit equation is solved in the same manner as described before, except that the iteration process is in both the x and y directions. The solution of the explicit equation is easy to calculate, but stability may be a problem. Equation [39] can be written as

$$h_{i,j}^{n+1} = \left(1 - \frac{2\Delta t D(\theta)(\Delta x^2+\Delta y^2)}{\Delta x^2\,\Delta y^2}\right) h_{i,j}^{n}$$

$$+ \frac{\Delta t D(\theta)}{\Delta x^2 \Delta y^2}\left(\Delta y^2\left[h_{i,j-1}^{n} + h_{i,j+1}^{n}\right]\right. \qquad [45]$$

$$\left. + \Delta x^2\left[h_{i-1,j}^{n} + h_{i+1,j}^{n}\right]\right).$$

Solving the coefficient of the $h_{i,j}^{n}$ term equal to zero for Δt gives

$$\Delta t \leq \frac{\Delta x^2\,\Delta y^2}{2D(\theta)(\Delta x^2+\Delta y^2)} \qquad [46]$$

for stable, converging solutions, Bear (1972). Hence, as one decreases either Δx or Δy to improve the accuracy of the solution, the maximum time step is decreased resulting in more time steps and more computer time per solution.

Both the implicit and Crank-Nicolson equations consist of five unknowns and form matrices with five non-zero diagonals. This type of matrix can be decomposed into kl tridiagonal matrices, where k and l are the number of nodes in the x and y directions, respectively. Hence, these methods are very difficult to solve.

Peaceman and Rachford (1955) developed a new method called the alternating direction implicit "ADI" which retained the best features of the implicit and Crank-Nicolson methods. The ADI procedure was convergent and unconditionally stable, resulted in a low truncation error,

and retained the desirable tridiagonal form of equations for two-dimensional flow problems. The procedure consists of two time steps. During the first step, one space variable is evaluated implicitly while the other space variable is evaluated explicitly. The second step consists of interchanging the roles of the space variables in step one and solving the new set of equations. Each step results in a tridiagonal matrix which can be readily solved by the Thomas algorithm. The solution of the second set of equations is considered the answer for the specified Δt. Although the equations utilized in each step result in an unstable solution, alternating between the two sets of equations, while Δt is held constant during each application of the ADI method, gives a solution that is stable for all ratios of $\Delta t/\Delta x^2$ and $\Delta t/\Delta y^2$.

For example, the equation for the first time step is

$$\frac{h^{n+1}_{i,j} - h^{n}_{i,j}}{\Delta t\ D(\theta)} = \frac{h^{n}_{i,j-1} - 2h^{n}_{i,j} + h^{n}_{i,j+1}}{\Delta x^2} + \frac{h^{n+1}_{i-1,j} - 2h^{n+1}_{i,j} + h^{n+1}_{i+1,j}}{\Delta y^2} \qquad [47]$$

and for the second time step is

$$\frac{h^{n+2}_{i,j} - h^{n+1}_{i,j}}{\Delta t\ D(\theta)} = \frac{h^{n+2}_{i,j-1} - 2h^{n+2}_{i,j} + h^{n+2}_{i,j+1}}{\Delta x^2} + \frac{h^{n+1}_{i-1,j} - 2h^{n+1}_{i,j} + h^{n+1}_{i+1,j}}{\Delta y^2} \qquad [48]$$

Remson, et al. (1971) states that the ADI method uses only four percent of the number of computation steps involved in the SOR method and only 14 percent of the number of steps used in the Crank-Nicolson method.

In summarizing numerical methods for two-dimensional flow, the explicit method is the easiest to solve, but may become unstable unless Δt is held small as indicated by equation [46]. The implicit and Crank-Nicolson methods do not result in a tridiagonal form of equations and, therefore, are very difficult to solve. The ADI method was developed to retain the assets of the implicit and Crank-Nicolson procedures, and still result in the tridiagonal matrix which can be readily solved. As noted in the literature review, Amerman (1969) had difficulty solving unsaturated-saturated flow separated by a phreatic surface by the ADI method. Current researchers are not apparently in agreement on the merits of the ADI method. For example, C. R. Amerman has gone back to the SOR method for future publications that the author has reviewed. Conversely, G. S. Taylor has recently changed from the SOR method to the ADI procedure. Still other researchers are developing a new finite element method. The discussion of the finite element procedure is too involved to present in this thesis.

Considering the ease of programming the SOR method, the uncertainty of the ADI procedure for mixed flow, and that modern digital computers are much faster than those used by previous researchers, the safest and easiest method to use in this report is the SOR method. If

stability or excessive computer operation time becomes a problem, the alternate method of solution would be by the ADI procedure.

The Model

The model for the solution of the general flow equation is based on an explicit finite difference procedure which is an adaptation of the building block approach developed by Vimoke and Taylor (1962) for the electrical resistance network analog. The equation defining current flow in the network at each grid intersection or node can be used to develop finite difference equations to model two-dimensional flow. The flow region is defined by a grid or mesh of points called nodes, separated from each other by finite differences, called the mesh increment. It is assumed that the mesh increments are small enough that each rectangle formed between the nodes can be considered to be of uniform moisture content and hydraulic conductivity and that they are referenced by the upper-left node of the rectangle. The soil is assumed to consist of a variable number of parallel horizon layers, in which the horizons are of arbitrary thickness. The horizontal mesh increments can also be selected arbitrarily except as specified later for the nodes adjacent to the drain.

The boundaries which identify the physical region to be modeled are zones of zero potential gradient, hence, no flow occurs across the boundaries. These zones of zero gradient result from impervious layers and lines of symmetry where the potential is a mirror image on each side of a line of symmetry. Boundaries in the model can be set in

any configuration to establish the limits of the grid to be analyzed, but are usually set along the lines of symmetry established by water sources, water sinks, and mid-planes between sources and sinks. The vertical boundaries are established by lines of symmetry and the bottom boundary results from an impervious layer. These boundaries are established by use of mirror image nodes to define the condition of zero potential loss across the boundary. The boundaries are considered to be in parallel pairs which define a rectangular flow area. The flow region consisting of zones of transient and steady flow separated by a moving water table is shown in Figure 4.

The finite difference equation for transient flow can be derived by utilizing Figure 5 and the resistance equations [22] and [23]. Figure 5(a) represents four blocks of soil with characteristic flow resistivity of r_1, r_2, r_3, and r_4. The dimensions of the blocks are a, b, c, and d as indicated in Figure 5(a). The resistance analog is shown in Figure 5(b), which results in 5(c) when the pairs of parallel resistances are combined according to equation [23]. The resistivities between nodes 1, 2, 3, 4, and node 0 are

$$R_1 = \frac{2br_1r_2}{cr_1 + ar_2} \qquad [49a]$$

$$R_2 = \frac{2cr_2r_3}{dr_2 + br_3} \qquad [49b]$$

$$R_3 = \frac{2dr_3r_4}{ar_3 + cr_4} \qquad [49c]$$

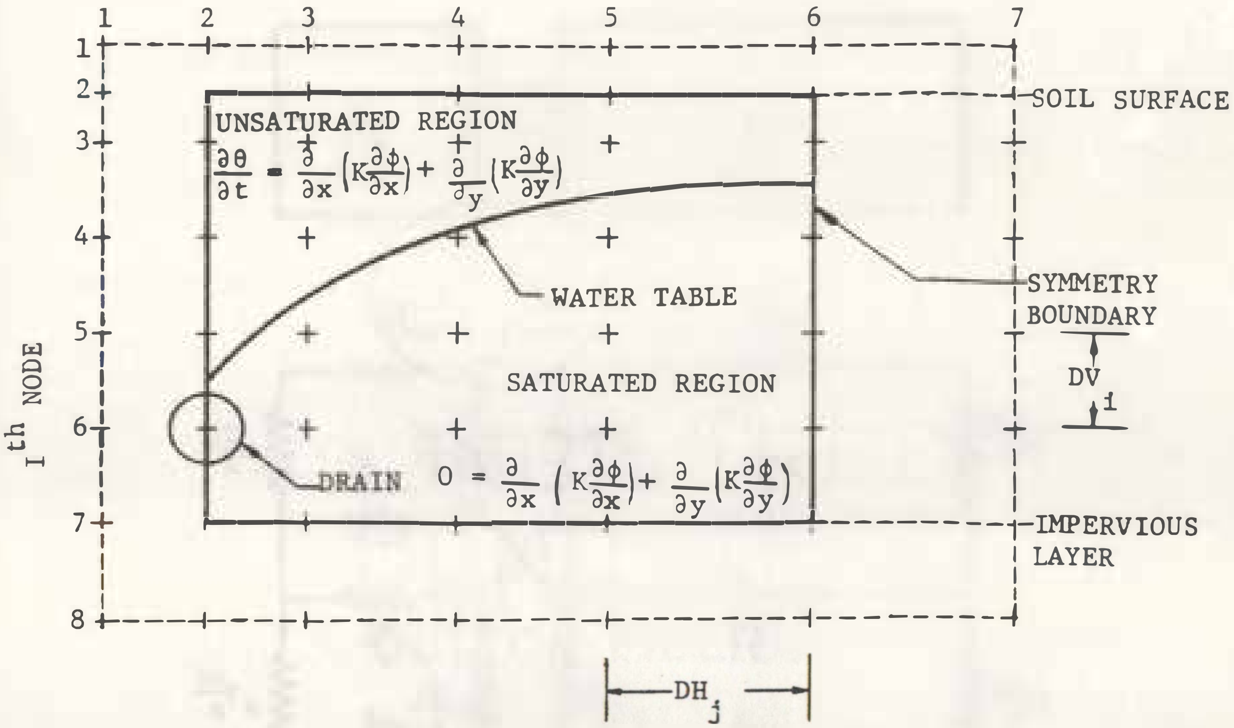

Figure 4. The flow region consisting of transient and steady flow separated by a moving water table.

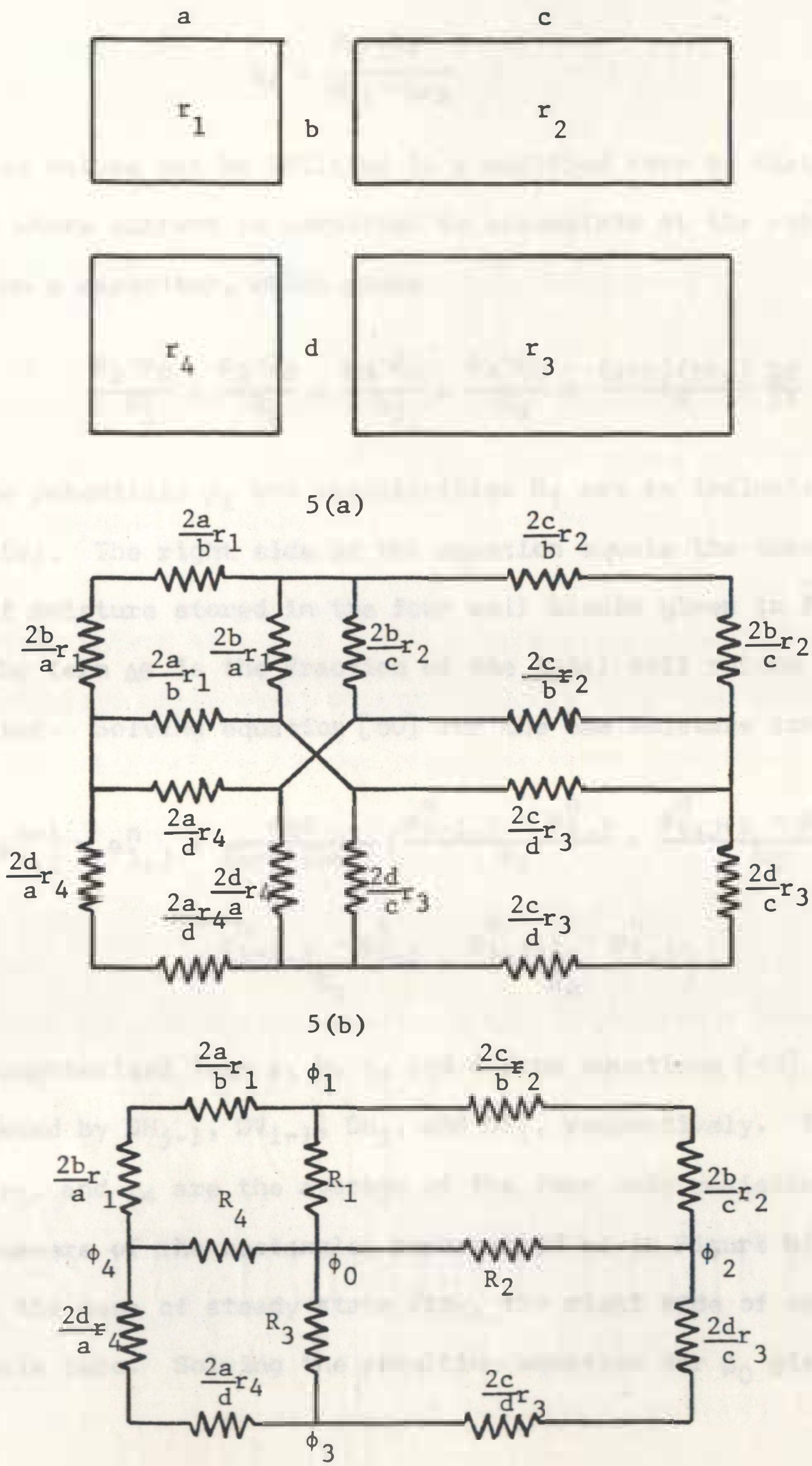

Figure 5. Building block analog of the electrical resistance network.

$$R_4 = \frac{2ar_1r_4}{dr_1 + br_4} \qquad [49d]$$

These values can be utilized in a modified form of Kirchhoff's law [20] where current is permitted to accumulate at the central node, such as on a capacitor, which gives

$$\frac{\phi_1-\phi_0}{R_1} + \frac{\phi_2-\phi_0}{R_2} + \frac{\phi_3-\phi_0}{R_3} + \frac{\phi_4-\phi_0}{R_4} = \frac{(a+c)(b+d)}{4}\frac{\Delta\theta}{\Delta t} \qquad [50]$$

where the potentials ϕ_i and resistivities R_i are as indicated in Figure 5(c). The right side of the equation equals the change in volume of moisture stored in the four soil blocks given in Figure 5(a). The term $\Delta\theta$ is the fraction of the total soil volume that contains water. Solving equation [50] for the new moisture content gives

$$\theta^{n+1}_{i,j} = \theta^{n}_{i,j} + \frac{4\Delta t}{(a+c)(b+d)}\left(\frac{\phi^{n}_{i-1,j} - \phi^{n}_{i,j}}{R_1} + \frac{\phi^{n}_{i,j+1} - \phi^{n}_{i,j}}{R_2}\right.$$

$$\left. + \frac{\phi^{n}_{i+1,j} - \phi^{n}_{i,j}}{R_3} + \frac{\phi^{n}_{i,j-1} - \phi^{n}_{i,j}}{R_4}\right) \qquad [51]$$

In the computerized form a, b, c, and d from equations [49] and [51] are replaced by DH_{j-1}, DV_{i-1}, DH_j, and DV_i, respectively. The values r_1, r_2, r_3, and r_4 are the average of the four node resistance values at the corners of the rectangles represented as in Figure 5(a).

For the case of steady-state flow, the right side of equation [50] equals zero. Solving the resulting equation for ϕ_0 gives

$$\phi_0 = \frac{R_2R_3R_4\phi_1 + R_1R_3R_4\phi_2 + R_1R_2R_4\phi_3 + R_1R_2R_3\phi_4}{R_2R_3R_4 + R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3} \qquad [52]$$

or in i,j notation is

$$\phi_{i,j}^{n+1} = \frac{R_2R_3R_4\phi_{i-1,j}^{n+1} + R_1R_3R_4\phi_{i,j+1}^{n} + R_1R_2R_4\phi_{i+1,j}^{n} + R_1R_2R_3\phi_{i,j-1}^{n+1}}{R_2R_3R_4 + R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3} \qquad [53]$$

where the n+1 superscript in the right side of equation [53] denotes using the most recent value. Utilizing new ϕ values as soon as they have been computed increases the rate of convergence to the correct solution.

Another method that will greatly improve the rate of convergence of the iterative application of equation [53] is to use successive over-relaxation, whereby the new potential value is given by

$$\phi_{i,j}^{(n+1)'} = \phi_{i,j}^{n} + \omega\left(\phi_{i,j}^{n+1} - \phi_{i,j}^{n}\right) = (1-\omega)\,\phi_{i,j}^{n} + \omega\phi_{i,j}^{n+1} \qquad [54]$$

where $0 \leq \omega \leq 2$. An optimum relaxation factor can be determined by Carre's method given on page 202 of Remson, et al. (1971) for a square mesh. Otherwise a table of values given in Taylor and Luthin (1963) may be used.

A cylindrical source or sink increases the resistance to flow by a factor $C_d/2$ as given by Vimoke and Taylor (1962). The drain resistance, C_d, is given by

$$C_d = \frac{8\,Z_0}{120\pi} = 0.021221 Z_0 \qquad [55]$$

and

$$Z_0 = 128 \log_{10}\rho + 6.48 - 2.34A - 0.48B - 0.12C, \qquad [56]$$

where $\rho = s/r$, s is the square mesh size, r is the drain radius and

$$A = \frac{1+0.405\rho^{-4}}{1-0.405\rho^{-4}}, \qquad [57]$$

$$B = \frac{1+0.163\rho^{-8}}{1-0.163\rho^{-8}}, \qquad [58]$$

$$C = \frac{1+0.067\rho^{-12}}{1-0.067\rho^{-12}}. \qquad [59]$$

When a cylindrical source or sink occurs at nodes 1, 2, 3, or 4 in Figure 5(c), the R value for that node determined with equation [49] should be replaced by the computed value times $C_d/2$.

Computational Procedures

The computer program establishes two-dimensional storage arrays corresponding to the grid nodes. Five arrays are used to store the current values of water potential (head) H, hydraulic potential PHI, resistivity R, moisture content THETA, and a hysteresis code MC. Fixed parameters are also stored in arrays to simplify the solution of several cases. These parameters are the horizontal and vertical mesh increments DH & DV, elevation Y, coefficients for the hysteresis relationships, and constants for the water content of the soil.

The initial values for the head, potential, resistivity, moisture content, and hysteresis code are assigned to each node. The potentials are defined such that the potential of the row corresponding to the soil surface is set equal to the elevation above the drain and the potential at the drain is equal to the drain radius. Initial resistivity values are determined from the moisture content versus hydraulic conductivity equations for the saturated moisture content. The equation for hydraulic conductivity is

$$K = \frac{C_1}{|H|^{C_3} + C_2} \qquad [60]$$

and the resistivity is 1/K or

$$R = C\,|H|^{C_3} + C_5 \qquad [61]$$

where K, R, H, C_1, C_2, C_3, $C_4 = 1/C_1$, and $C_5 = C_2/C_1$ are the hydraulic conductivity, resistivity, water head, and the C's are coefficients fit by least squares regression analysis, respectively. The relationship, moisture content versus water potential, is also fit by least squares regression analysis to the equation

$$\theta = \frac{C_1}{|H|^{C_3} + C_2} \qquad [62]$$

where θ, H, and the C's are the moisture content, head, and coefficients which have different numerical values than in equation [60], respectively.

Solution of flow in saturated soil is obtained by solving equation [53] for each node in a stepwise order beginning with the first node in the row immediately below the soil surface and progressing from left to right. The top boundary is not relaxed because the potential of this row must remain equal to the elevation. Also, the potential of the drain node is kept equal to the drain radius. This procedure is repeated row wise until the flow region is completely traversed. The sequence of relaxation of the nodes is indicated in Figure 6. At the completion of each traverse, each image boundary node potential is set equal to its counterpart within the flow region. The traverse is iterated repeatedly and the change in potential at every node during two consecutive iterations is computed by

$$\mathrm{ERROR} = \sum_{i=1}^{k} \sum_{j=1}^{l} \left| \phi_{i,j}^{n+1} - \phi_{i,j}^{n} \right| . \qquad [63]$$

When this value becomes less than some specified value, the SOR process is complete and the ϕ^{n+1} values are considered the steady-state solution. A limit is also placed on the maximum number of iterations that will be performed.

To initiate the unsaturated flow situation, the potentials of the row of nodes above the soil surface are set equal to the potential of the row below the surface. A specific time interval Δt is chosen and, beginning with the row of nodes coinciding with the soil surface, the finite difference equation [51] is solved for the change in moisture content at each node where the head H is less than or equal to zero.

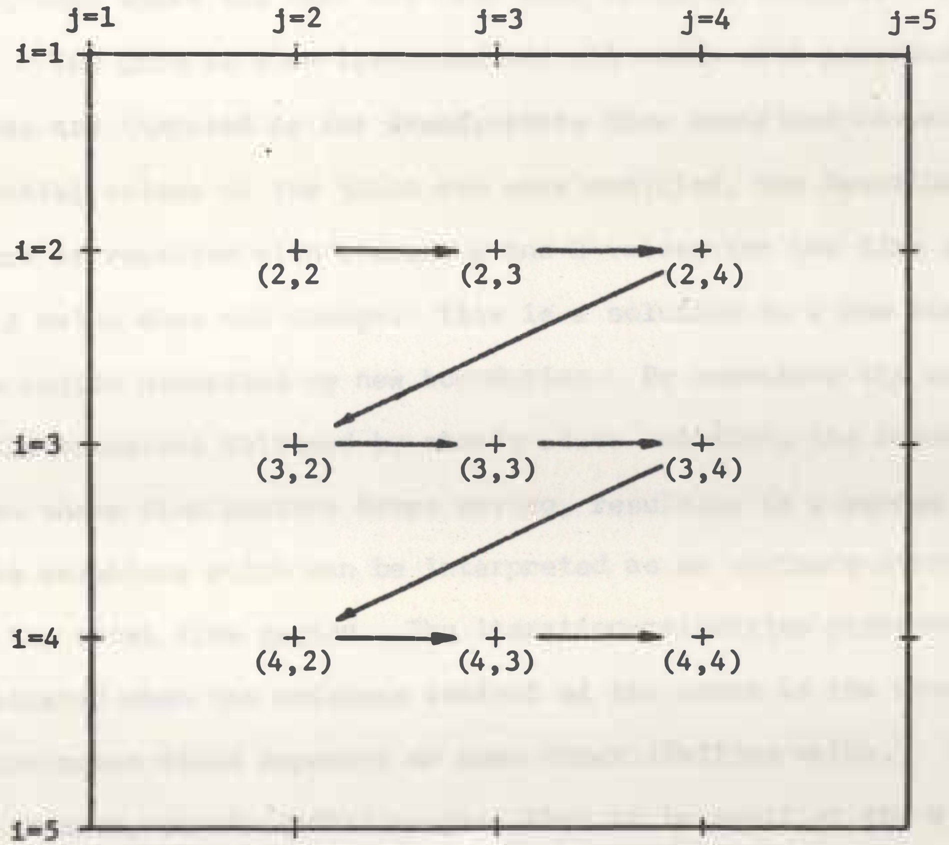

Figure 6. The sequence of progression during relaxation process.

When all nodes have been traversed, the head checked, and the unsteady flow equation has been applied resulting in new θ values, then new H, ϕ, and R values are calculated and placed in the storage arrays for every node where the head was less than or equal to zero.

The grid is then traversed and all nodes with positive head values are iterated as for steady-state flow described above. If the potential values of the third row were modified, the described procedure is repeated with average ϕ and R values for the time step until the ϕ value does not change. This is a solution to a new steady-state flow region described by new boundaries. By repeating the application of the transient followed by steady-state solution, the boundary between these flow regions keeps moving, resulting in a series of steady-state solutions which can be interpreted as an unsteady-state solution for the total flow region. The iteration-relaxation procedure can be terminated when the moisture content of the nodes in the unsaturated region reach field capacity or some other limiting value.

Remson, et al. (1971) suggest that Δt be small at the start and be increased at a linear rate, such as by:

$$\Delta t_n = (a + bt)(\Delta x)^2 \qquad [64]$$

where a and b are constants such that $a>0$ and $b>0$. Small time increments are used at first because too great a change in moisture content will cause the system to become unstable. As the soil drains and the flow rate decreases, longer time increments can be utilized up to the established maximum time interval.

Evaporation is quite similar to drainage except an incremental amount of water is subtracted from the nodes at the soil surface during each time period. Equation [50] is still valid for water moving in the flow region below the boundary in response to this imposed gradient. The amount of evaporation can be determined from the soil moisture and the corresponding climatological data or some other criteria.

Hydraulic conductivity is not constant when considering unsaturated flow since it is a function of the degree of wetness. Hysteresis occurs when the soil goes from a drying to a wetting cycle and vice versa. As noted in the literature review, the hysteresis is much greater in the moisture content to water potential relationship than in the moisture content to hydraulic conductivity relationship. Hysteresis is assumed for relating moisture content to water potential. Field or laboratory data can be used to generate these equations by a least squares regression analysis to fit equation [60] or [62] to the appropriate data. The moisture code MC value is used to determine whether drying or rewetting is taking place and for selecting the proper set of equations.

Simulation of infiltration is essentially the reverse of the evaporation procedure except that hysteresis occurs, requiring a different set of moisture content to water potential relationships. Also, the gradient at the interface is quite large, which implies that the time increment should be reduced compared with that for evaporation.

If infiltration occurs rapidly enough, the surface may become saturated, resulting in saturated flow near the soil surface.

The computer program model consists of several subroutines to perform specific operations in an iterated sequential order. Although many variables are in common among most of the subroutines, the use of subroutines has aided greatly in reducing the size of the program and in understanding the logic involved in the programmed solution. The program subroutines are called INPUT, OUTPUT, FLOW, SAT, UNSAT, NEWH, NEWR, NEWMC, LINRP, FTABLE, EQPLOT, and PABC. These subroutines are controlled by the main program and called either by the main program or by each other.

The INPUT subroutine reads the input data, calculates the drain resistivity, initializes all arrays for the solution of saturated flow, and calls LINRP to establish the coefficients of the moisture content to water potential and hydraulic conductivity to water potential relationships from tabular soils data. This subroutine prints a listing of all input data.

The subroutine OUTPUT writes all output from the computations, such as tables of THETA, MC, R, H, PHI, Q, and WTV, which are the moisture content, moisture hysteresis code, resistivity, water potential (head), hydraulic potential, flow or tile discharge, and water table elevations, respectively. The routine also calls EQPLOT, which results in a printer plot of equipotentials in the flow medium.

In subroutine FLOW, the water table elevations, the flow through the soil surface, and the discharge from the tile drain are calculated.

If the soil profile is inclined, the routine computes the discharge from each tile drain.

The subroutine SAT calculates new potentials for each node in the flow region by method of successive over-relaxation. The method is based on the building block method of combining network resistances as described by Vimoke and Taylor (1962). The boundaries are established using image nodes.

For unsaturated flow, UNSAT subroutine determines new moisture content, THETA, values for all nodes where the water potential H is less than or equal to zero. After each new THETA value is calculated, the moisture code is determined by calling NEWMC and a new H value is computed by calling NEWH. When an iteration has been completed, NEWR is called to determine new resistance R values and renew the image PHI and H values.

The subroutine NEWH utilizes the past moisture hysteresis code and the moisture content value to compute a new water potential H value from the correct wetting, drying, or scanning characteristic curve. The water potential is calculated by solving equation [62] for H.

In the NEWR subroutine, new hydraulic potentials are computed, the values of PHI and H are set for the image nodes, and the resistivity R values are calculated for all nodes including the image nodes. The value K(9), based on the resistivity change at each node is used to adjust the time increment, DELT, in the main program.

The NEWMC subroutine determines whether the flow is along the wetting or drying primary characteristic curves. If the moisture content trend changes, the flow is along a linear scanning curve until the other primary curve is reached. The moisture codes are 1, 2, 3, and 4 for drying, drying to wetting scanning, wetting, and wetting to drying scanning curves, respectively.

Subroutine LINRP fits a least squares curve of the type given in equations [60] and [62] to the soil characteristic data. The curve is forced through the first data point assuming it to be the saturated value which is usually known. The subroutine calls FTABLE which determines the "F" tabular value for significance of fit of the curve. An option of plotting the data and fitted curve can be called by the PLOT subroutine from the IBM Scientific Subroutine Library.

The subroutine EQPLOT determines the equipotential curves for the flow region grid by linear interpolation and calls PABC which is a printer plot subroutine available at most computer centers or may be obtained from the company, Share. The subroutine EQPLOT will plot the equipotentials, drains, nodes, and the water table with the standard printer.

The main program's function is mainly the control of the subroutines. Its basic operations consist of calling: INPUT to set the initial conditions, SAT to calculate a saturated solution, FLOW to determine the flow, and OUTPUT to print the results. The next sequence is to set the top boundary and call UNSAT to compute new THETA, H, PHI, and R values for nodes where H is less than or equal to zero. After

UNSAT has been completed, SAT is called to relax the potentials of the nodes where H is greater than zero. This sequence of calling UNSAT and SAT is iterated until the top boundary value remains constant, then the FLOW and OUTPUT subroutines are called. The time increment is modified and the UNSAT-SAT sequence is repeated until a problem solution occurs.

The first part of the output consists of a complete listing of the input data, initialization values, and coefficients of the soil moisture and conductivity to water potential equations. Output from the model includes the tables H, PHI, THETA, MC, and R values which are the head, potential, moisture content, moisture hysteresis code, and the soil resistivity, respectively. Output also includes the flow across the upper boundary and through the drains or sources, the water table elevation at each vertical column of nodes, and a plot of equipotentials. Incremental time, accumulated time since the onset of unsaturated flow, accumulated number of iterations, and a measure of the completeness of the relaxation process are printed each time the moisture content table is printed. The frequency of these printed tables is controlled by input parameters.

Variations of the program can be used to simulate moisture movement in the profile under drainage, infiltration, evaporation, and irrigation processes. The procedure can be used to simulate both natural and man made moisture movement situations and can be used as a design tool to evaluate proposed designs for drainage, irrigation, and leach bed systems, etc. An example showing the contrast of time

and effort of an empirical design based on experimental plot results and that based on the numerical simulation model is given in Figure 7.

Figure 7. [illegible]

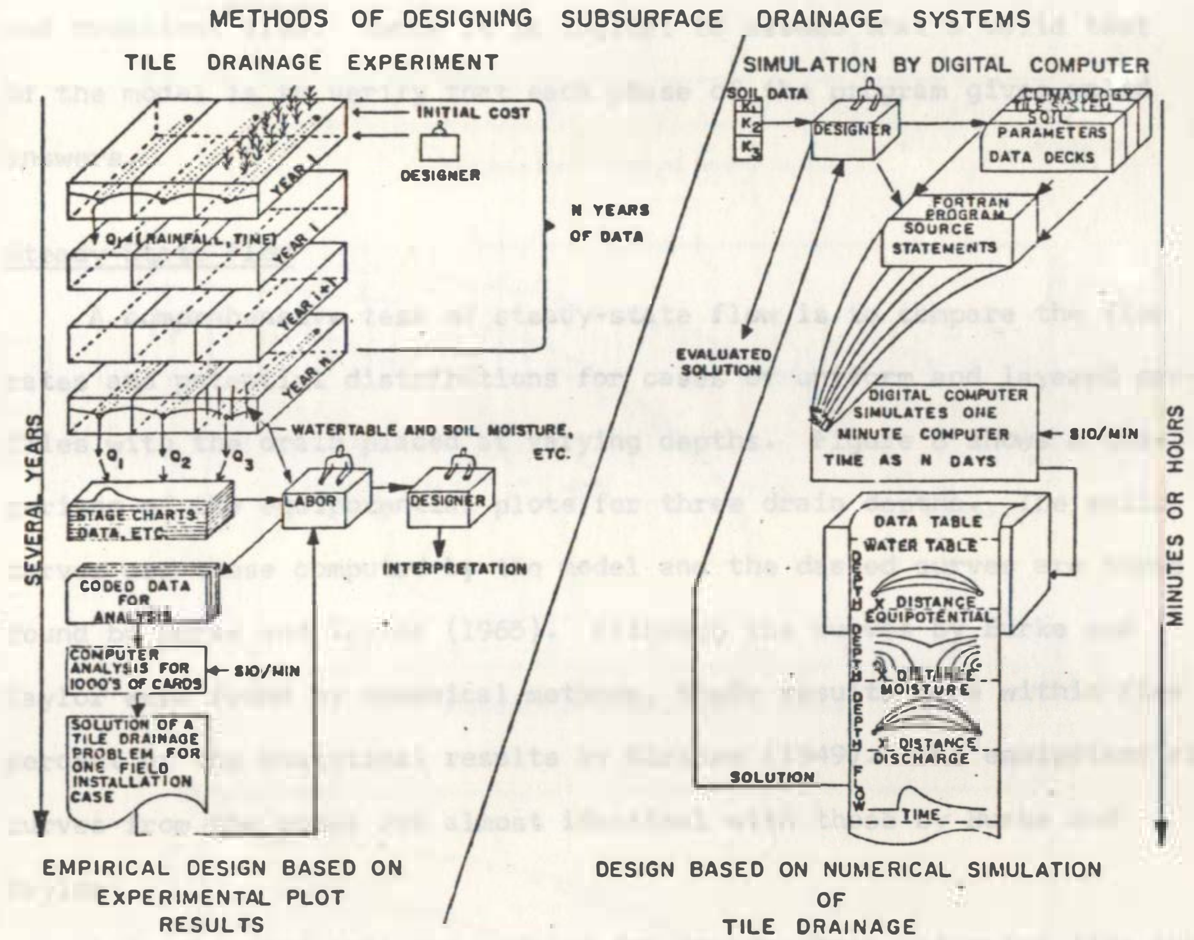

Figure 7. Contrast of experimental based design and numerical simulation design.

RESULTS AND DISCUSSION

After developing a model, an important step that should follow is the testing of the results of each part of the model or the model as a whole. There are two distinct phases of the model, steady-state and transient flow. Hence it is logical to assume that a valid test of the model is to verify that each phase of the program gives valid answers.

Steady-State Flow

A comprehensive test of steady-state flow is to compare the flow rates and potential distributions for cases of uniform and layered profiles with the drain placed at varying depths. Figure 8 shows a comparison of the equipotential plots for three drain depths. The solid curves are those computed by the model and the dashed curves are those found by Burke and Taylor (1965). Although the curves by Burke and Taylor were found by numerical methods, their results were within five percent of the analytical results by Kirkham (1949). The equipotential curves from the model are almost identical with those by Burke and Taylor.

A two-layered soil was modeled for steady-state saturated flow and the equipotentials are plotted along with those by Burke and Taylor in Figure 9. Again the drain depth was varied. The results of the model compare well with those of Burke and Taylor.

Table 1 is presented to show a comparison of the drain discharge per hydraulic conductivity of the top layer, Q/K_1. Again, the ratio is

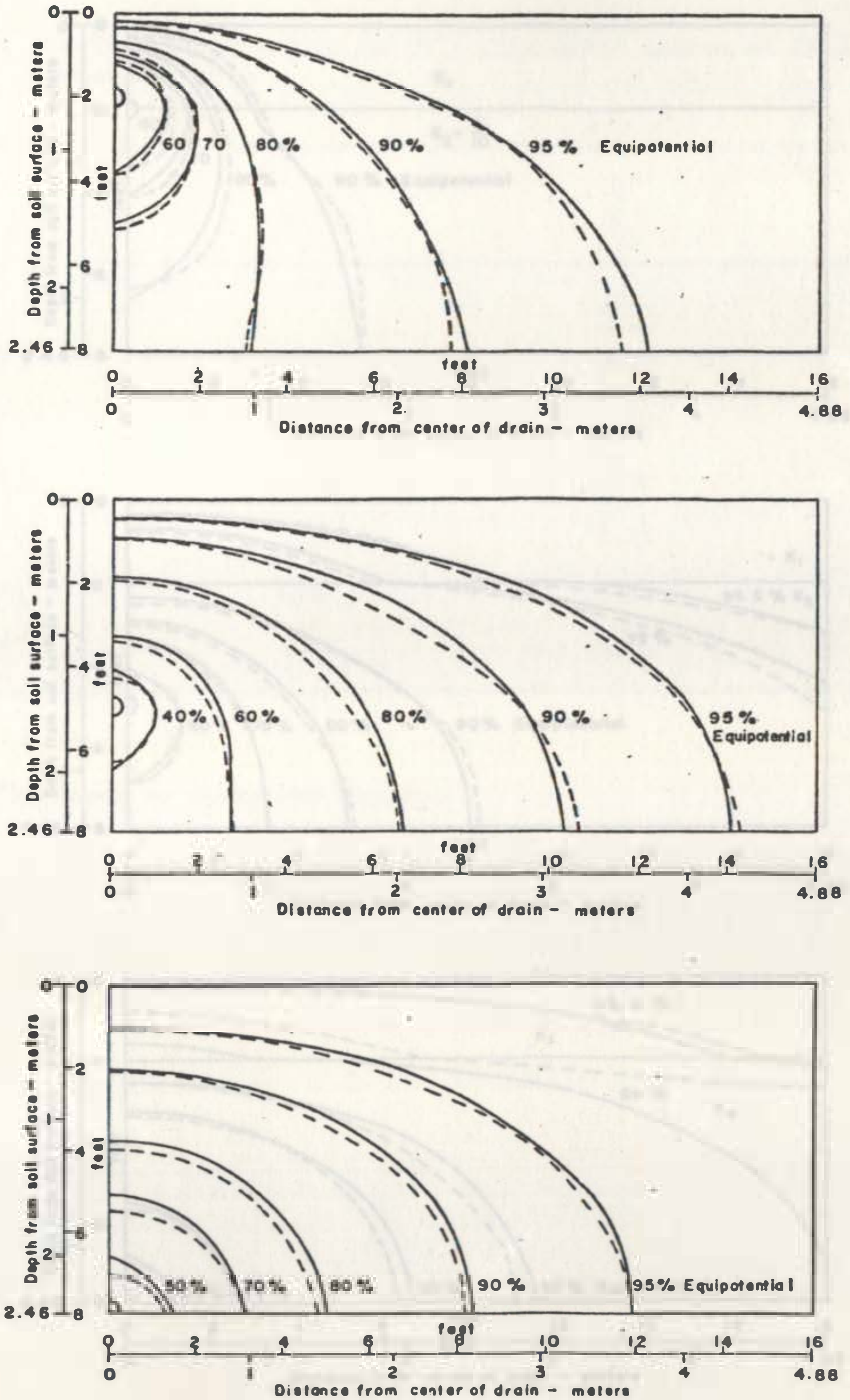

Figure 8. Equipotential in a uniform soil with three drain depths.

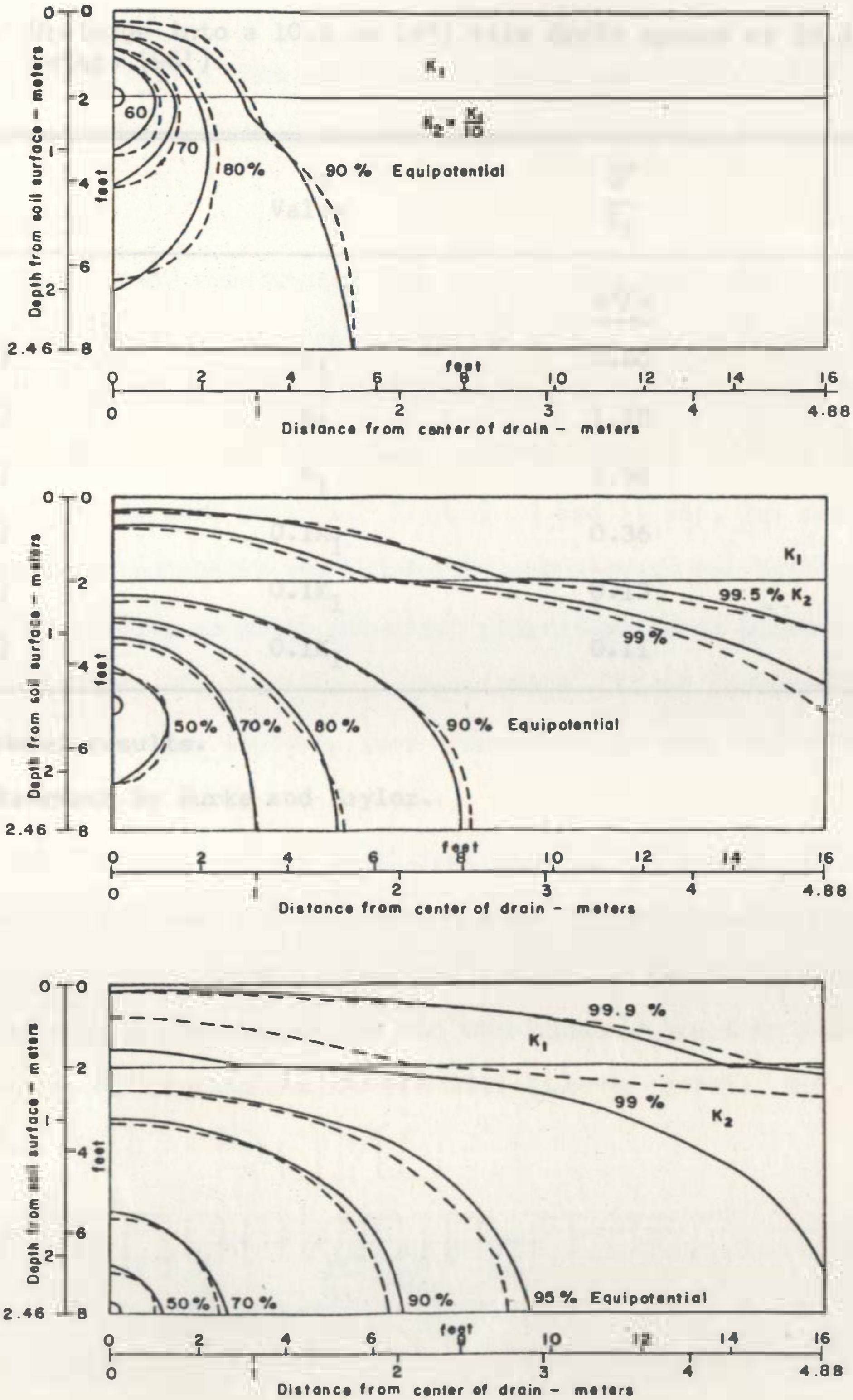

Figure 9. Equipotentials in a layered soil with three drain depths.

Table 2. Drainage into a 10.2 cm (4") tile drain spaced at 29.3 meters (96').

Drain Depth	K_2 Value	$\frac{Q^*}{K_1}$	$\frac{Q^{**}}{K_1}$
m		m^2/m	m^2/m
0.61 (2')	K_1	0.60	0.54
1.52 (5')	K_1	1.10	1.00
2.44 (8')	K_1	0.98	0.88
0.61 (2')	$0.1K_1$	0.36	0.34
1.52 (5')	$0.1K_1$	0.13	0.11
2.44 (8')	$0.1K_1$	0.11	0.10

*Model results.

**Research by Burke and Taylor.

in good agreement with that published by Burke and Taylor, thus verifying the steady-state portion of the model.

Transient Flow

The test of the unsaturated flow part of the model was conducted by using soil characteristic data supplied to the author by Dr. R. R. Bruce through personal communication. Graphs of this data have been published in Bruce (1972). Test data selected were for the B_1 horizon of the Cecil loamy sand profile. Figures 10 and 11 show the moisture content to water potential and hydraulic conductivity per saturated hydraulic conductivity to water potential relationships as plotted with a Wang 2200 plotting output writer, respectively. These characteristic curves could have been plotted, less accurately, by specifying NPLOT equal to one in the simulation program.

The profile selected was eight feet deep with the tile drain depth equal to six feet and the spacing equal to fifty feet. A plot of the computer model drawdown results is shown in Figure 12 along with a plot of the solution of the van Schilfgaarde equation solved for m iteratively as suggested by van Schilfgaarde (1974). His equation is

$$m = \frac{2d_e}{\dfrac{(2d_e+m_o)e^{\frac{9Kt}{d_e f}\left(\frac{d_e}{s}\right)^2}}{m_o} - 1} \qquad [65]$$

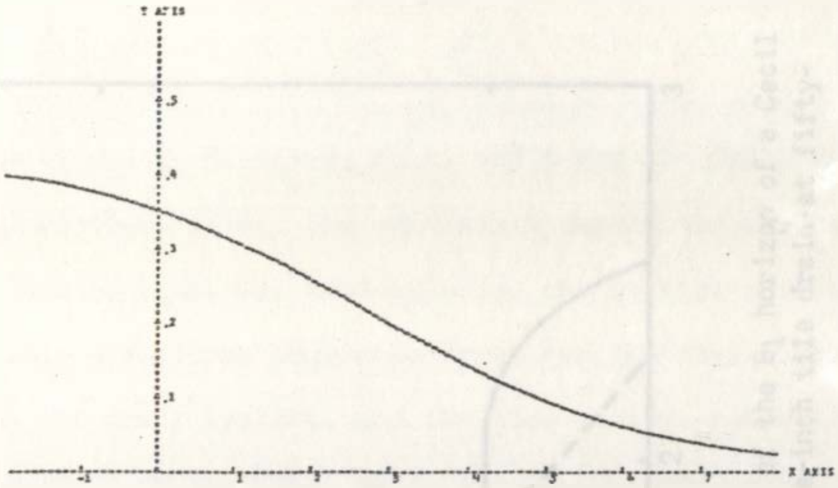

Figure 10. Desorption curve relating soil moisture content (y axis) to the $\log_{10}$ of the absolute value of the water potential, feet (x axis).

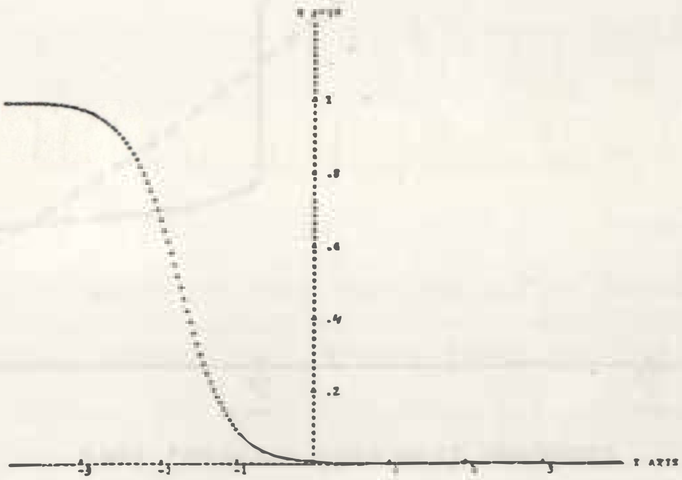

Figure 11. Ratio of the hydraulic conductivity to saturated hydraulic conductivity (y axis) to the $\log_{10}$ of the absolute value of the water potential, feet (x axis).

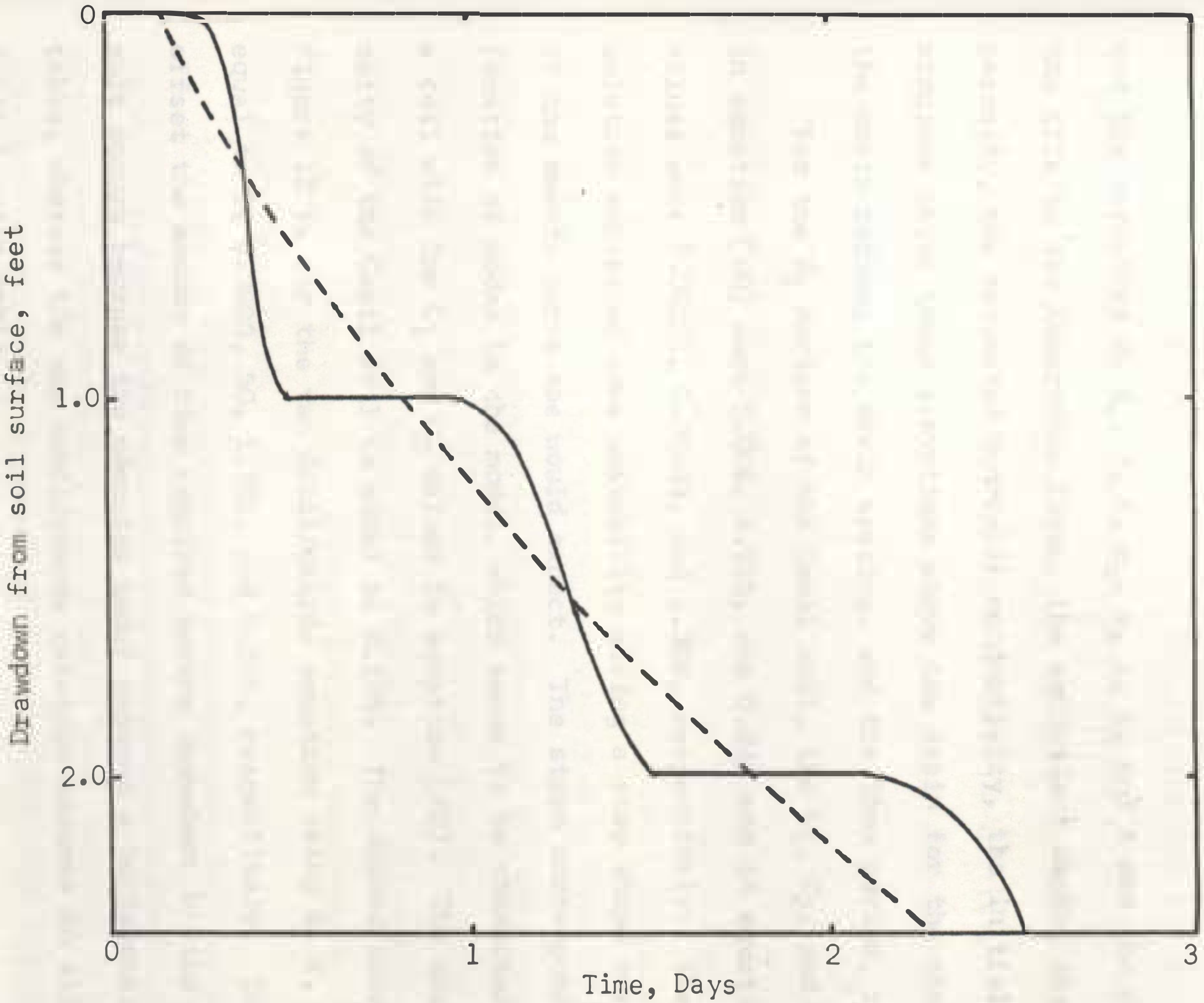

Figure 12. Drawdown for a soil profile composed of the B_1 horizon of a Cecil loamy sand with a six-foot deep, five-inch tile drain at fifty-foot spacing.

where

$$d_e = \frac{d}{1 + \frac{d}{s}(\frac{8}{\pi}\ln\frac{d}{r}) - 3.55 - 1.6\frac{d}{s} + 2\frac{d^2}{s^2}} \qquad [66]$$

and the variables d, d_e, f, K, m_o, m, r, s, and t are the depth from the tile to the impervious layer, the equivalent depth, the drainable porosity, the saturated hydraulic conductivity, the initial and final midplane water table elevations above the drain for the time period, the drain radius, the drain spacing, and the time period, respectively.

For the B_1 horizon of the Cecil soil, the C_1, C_2, and C_3 values in equation [60] were 2.006, 4.712, and 0.243 and in equation [62] the values were 0.00321, 0.00483, and 1.308, respectively. The computer solution exhibited some unstability giving a step shape curve instead of the smooth curve one would expect. The steps correspond to the location of nodes in the model, which seems to be characteristic of a soil with low C_1 and C_2 values in equation [62]. The drainable porosity of the Cecil soil is equal to 0.105. The dashed curve shown in Figure 12 is for the van Schilfgaarde equation using d, r, s, K, and f equal to 2, 0.2083, 50, 1.701, and 0.105, respectively. This curve is offset the amount of time required before drawdown in the computer result occurs because the computer model assumed a horizontal water table, whereas the van Schilfgaarde relation assumes an ellipse shaped initial water table.

The simulation for soils described by Taylor and Luthin (1969) had values of C_1, C_2, and C_3 in equations [60] and [62] of 0.1 to 10, 0.1 to 10, and 3, respectively. Plots of several characteristic curves in

these ranges where $A = 1/C_1 = 1/C_2$ are shown in Figure 13. A set of soil characteristics was arbitrarily chosen from the center curve; such that C_1, C_2, and C_3 were 2.0, 2.0, and 3, respectively. When this soil was used in the computer model, the smooth drawdown curve shown in Figure 14 was obtained. The time increment used in this analysis was equal to 0.001 days. Since this type of soil characteristic data are similar to data for several soils described in the literature and produced a smooth drawdown curve, these data were selected for simulation of drip irrigation.

Simulation of Drip Irrigation

Drip irrigation can be simulated by modifying the moisture content values at the soil surface to give the result of a given application of water at specific nodes. This is accomplished by determining the increase in moisture content of the surface row of nodes, evaluating equation [62] for a new H value associated with the new moisture content, computing a new value of R with equation [61] using the new H value, and solving the unsaturated-saturated sequence in the program as described previously.

Drip irrigation was simulated with the same soil as shown in Figure 13, $A = 0.5$ with a uniform profile of eight-foot depth. A tile drain was located six feet deep and at a forty-foot spacing. A typical emitter rate was assumed to be constant at 0.018 gallons per minute, therefore, the node spacing on each side of the drip irrigation line was also maintained constant at one foot. The irrigation lines, spaced at five feet with drip emitters every two feet were simulated as line sources. The

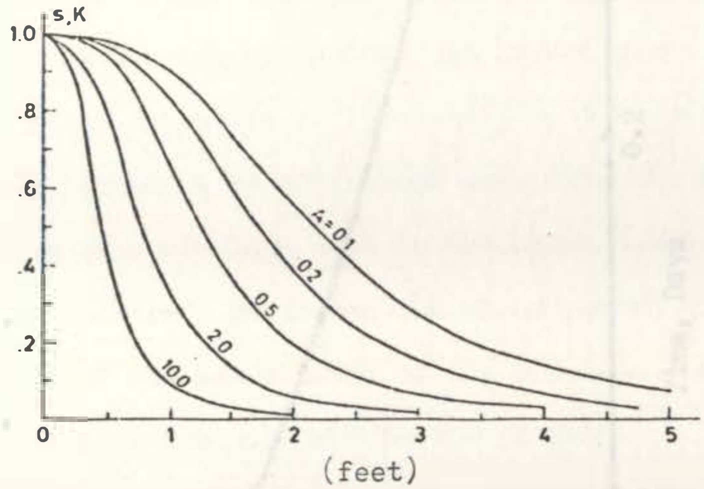

Figure 13. Functional relationships between water potential H, degree of saturation s, and hydraulic conductivity K for five hypothetical soils.

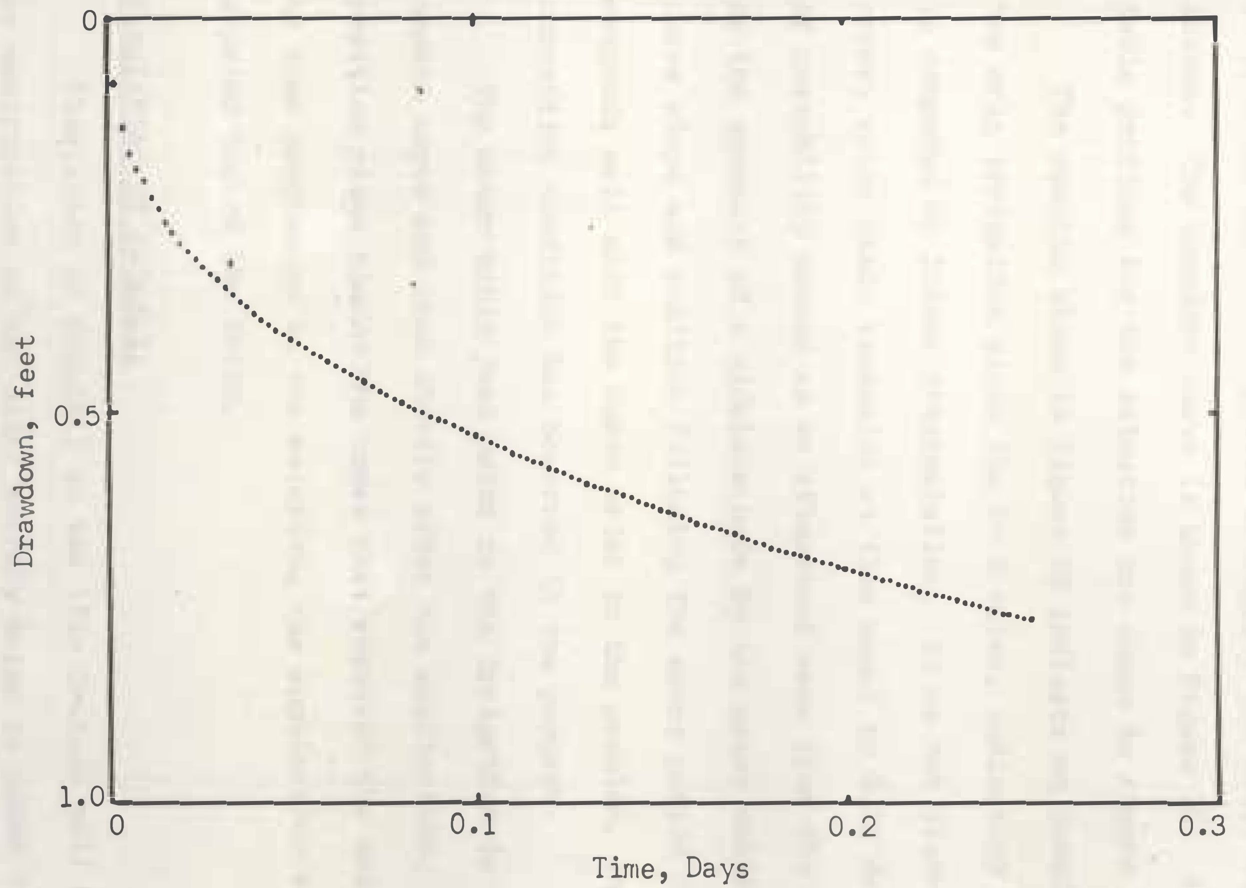

Figure 14. Drawdown for soil with A = 0.5 (see Figure 13) for an eight-foot deep profile with a tile drain at six feet deep and spacing of fifty feet.

drip irrigation commenced at one-tenth day after drawdown had started and continued for 0.02 of a day when four-tenths of an inch irrigation water would have been applied to replace the daily evapotranspiration losses. The drawdown curve is shown in Figure 15. A sequence of water table profiles for the situation are shown in Figure 16.

The results shown in Figure 15 indicate an immediate response to the drip irrigation since the H = 0 value, indicating the water table, is computed by linear interpolation. It is not clear whether the temporary water table recession at time equal to 0.22 days is the result of unstability caused as an aftershock wave from the drip irrigation or the approach of a midplane node by the water table. The recession curve slope and position following the short period of unstability corresponds well with the curve prior to the problem, indicating a self-correcting condition has occurred in the program.

The water table just prior to the irrigation is shown to be a smooth curve and that shortly after the application, the water table position rises toward the nodes that received the water application. As time progressed in the solution, the application effects were dissipated toward the drain.

Simulation of rainfall

Simulation of rainfall on the tile drained soil may be conducted by modification of the data; whereby water is added to all surface nodes in proportion to the surface area represented by each node. Figure 17 shows the results of modeling rainfall on a complete Cecil loamy sand profile assumed to be eight-foot deep to the impervious

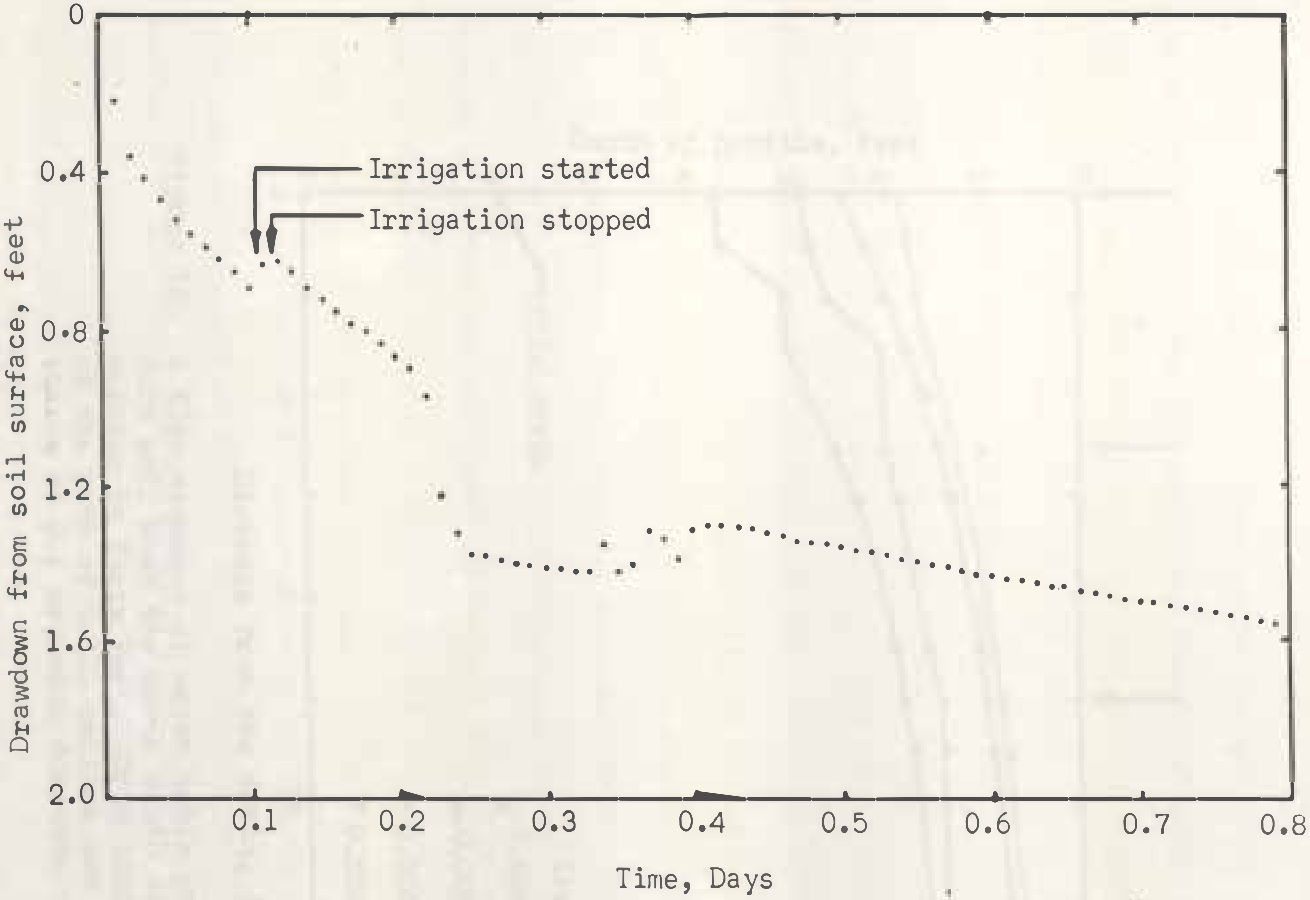

Figure 15. Drawdown in soil with A = 0.5 (see Figure 13). Drip irrigation every five feet on the eight-foot profile with a tile drain located six feet deep and at a forty-foot spacing.

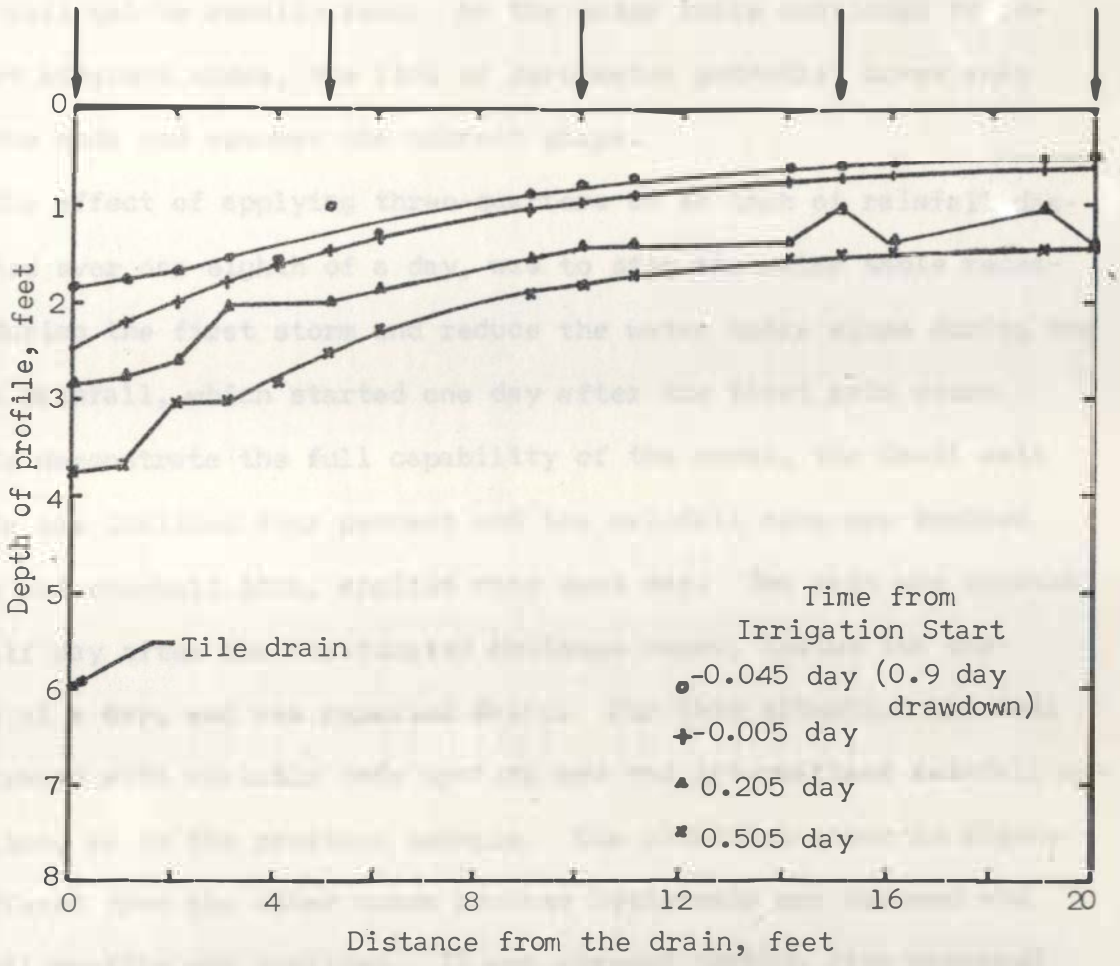

Figure 16. A time sequence of water table profiles for a uniform soil when the drain depth is six feet and spacing is forty feet. Drip irrigation (arrows) at the rate of 0.018 gallons per minute per emitter starts at 0.1 day with duration of 0.02 days.

layer. These soils data were used for this and the following analysis because they were readily available and represented an actual soil. The tile drain was located at the six-foot depth and at a fifty-foot spacing. Again the nodal effect on the water table position for the Cecil soil can be readily seen. As the water table continues to recede at adjacent nodes, the line of zero water potential moves away from the node and resumes the correct shape.

The effect of applying three-quarters of an inch of rainfall distributed over one-eighth of a day, was to stop the water table recession during the first storm and reduce the water table slope during the second rainfall, which started one day after the first rain storm.

To demonstrate the full capability of the model, the Cecil soil profile was inclined four percent and the rainfall rate was doubled to one and one-half inch, applied once each day. The rain was started one-half day after the unsaturated drainage began, lasted for one-eighth of a day, and was repeated daily. For this situation the soil was layered with variable node spacing and had intermittent rainfall application, as in the previous example. The situation shown in Figure 18 differed from the other cases because hysteresis was assumed and the soil profile was inclined. It was assumed that no flow occurred across the vertical lines of symmetry, the same as in the horizontal case. Although this assumption is not theoretically correct, the resulting error which occurs prior to drawdown to the tile was assumed to be small when the slope was small. The nodal effect is still evident in the curves. With this exception, the results are as expected. The results do demonstrate the capability of the program.

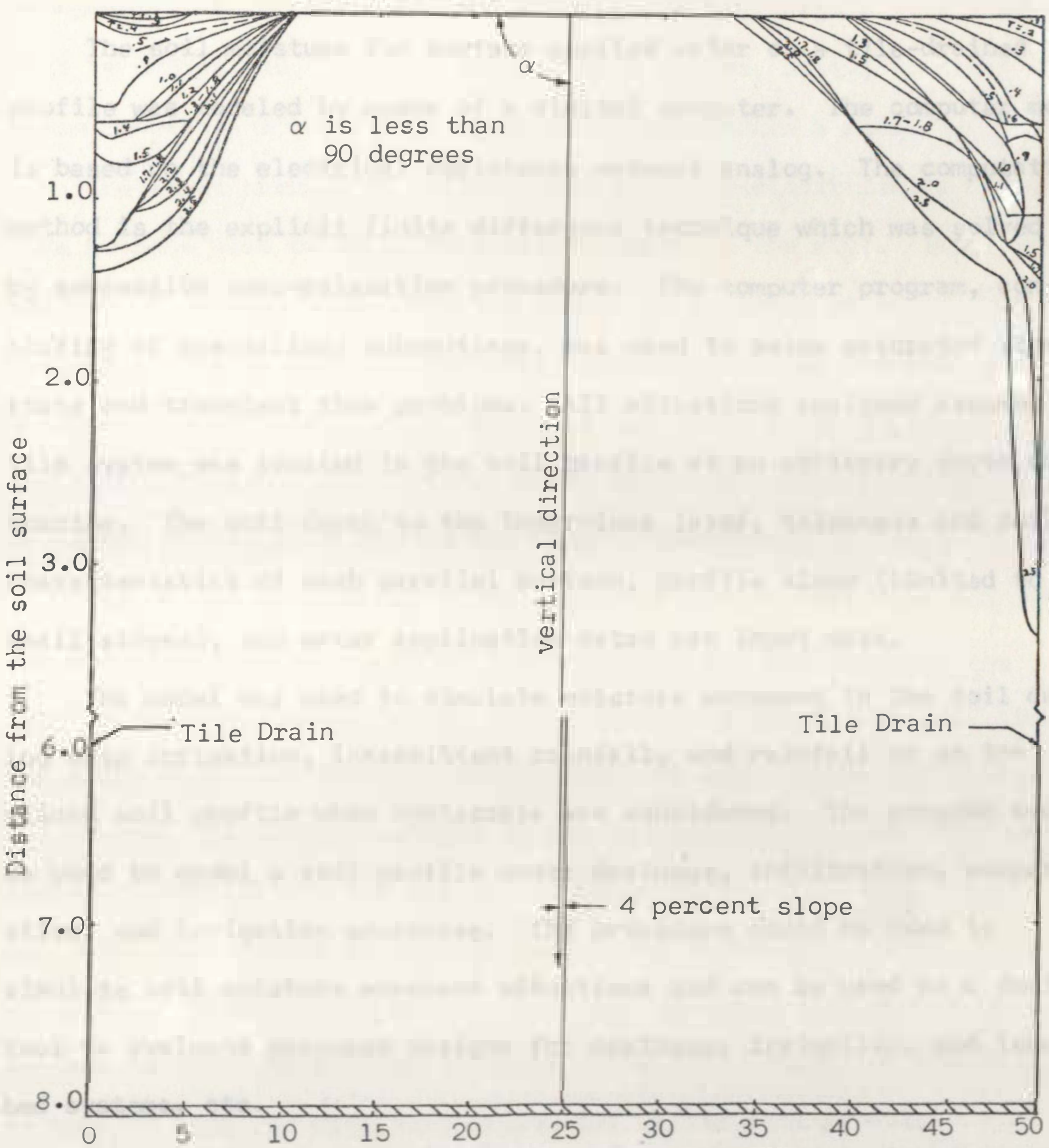

Figure 18. A time sequence of the water table profiles for a Cecil loamy sand profile of eight foot depth with tile drains located six feet deep and at a fifty-foot spacing. A 1.5 inch rainfall starts at 0.5 day for a duration of one-eighth of a day. Dashed curve represents the end of rainfall. The soil profile is inclined four percent.

SUMMARY AND CONCLUSIONS

The soil moisture for surface applied water on a tile-drained profile was modeled by means of a digital computer. The computer model is based on the electrical resistance network analog. The computation method is the explicit finite difference technique which was solved by successive over-relaxation procedure. The computer program, consisting of specialized subroutines, was used to solve saturated steady-state and transient flow problems. All situations analyzed assumed a tile system was located in the soil profile at an arbitrary depth and spacing. The soil depth to the impervious layer, thickness and soil characteristics of each parallel horizon, profile slope (limited to small slopes), and water application rates are input data.

The model was used to simulate moisture movement in the soil during drip irrigation, intermittent rainfall, and rainfall on an inclined soil profile when hysteresis was considered. The program could be used to model a soil profile under drainage, infiltration, evaporation, and irrigation processes. The procedure could be used to simulate soil moisture movement situations and can be used as a design tool to evaluate proposed designs for drainage, irrigation, and leach bed systems, etc.

The results showed an immediate response to drip irrigation and rainfall applications. The recession curve was like the exponential recession that was expected. The water table receded rapidly above the tile drain and drawdown progressed slowly toward the midplane boundary in the horizontal profile situations.

The model has two serious problems at its current stage of development. The solution becomes unstable if too large a time increment is chosen and, at least for one soil series, delays in water table recession occur as the water table approaches a node.

The first problem is a limitation of all explicit finite difference methods used to solve moisture movement in the soil. Implicit methods of solving these problems result in a stable solution, but must be solved by matrices methods. The matrices for two-dimensional problems are difficult to solve unless the alternating direction implicit method is used which produces two tridiagonal matrices which may be solved easily by the Thomas algorithm.

The second problem may be associated with the soil type and not the method, since smooth water table curves were obtained for the drip irrigation example before the irrigation started. At that time, the water table had receded past two rows of nodes above the drain.

Hence, the future research in the modeling of soil moisture will involve investigating the ADI method and probably replacing the SAT and UNSAT subroutines by the equivalent ADI procedure. Also, other situations involving the surface flow and geologic flow should be incorporated into the program to model rainfall in excess of the infiltration rate and deep percolation out of the soil profile.

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APPENDIX A

List of Symbols Used in Computer Program

LIST OF SYMBOLS USED IN COMPUTER PROGRAM

A(I,J)	Coefficients in the drying equation for soil layers, THETA(H)
ABSC	Horizontal heading - bottom of plot output
ACD	= 1 or drain resistivity
ADH	Temporary for DH(1)
ADV	Temporary for DV(1)
ALOG	Natural logarithm function
B(I,J)	Coefficients in the wetting equation for soil layers, THETA(H)
BCD	= 1 or drain resistivity
C(I,J)	Coefficient in the conductivity equation for soil layers, R(H)
CCD	= 1 or drain resistivity
C3L1	Lower limit of C_3, Level 1
C3L2	Lower limit of C_3, Level 2
COS	Trigonometric function
COSPIO	COS(PIO)
C3U1	Upper limit of C_3, Level 1
C3U2	Upper limit of C_3, Level 2
D(I)	Temporary storage for coefficient arrays
DCD	= 1 or drain resistivity
DELPHI	Change in PHI value at a node
DELT	Change in time or T increment
DEVMS	Deviation mean square
DEVSS	Deviation sum of square

DH(J)	Distance horizontally between nodes
DR(J)	Daily rate of water application at surface nodes
DV(I)	Thickness of each layer of the soil profile
E(I)	Scanning curve intersection with wetting curve
EQPLOT	Subroutine to locate and call plot of equipotentials
ERROR	Absolute sum of changes in potential during an iteration
EXP	Exponential library function
F(I)	Field capacity theta values for each soil layer
F1	Lower F value
F2	Upper F value
FDH	1 or 1/2
FL	F left side of interval
FLOW	Subroutine calculates drain outflow
FTHETA	Float theta value
F(I)	Saturated theta (moisture) for each soil layer
GA	Gamma of A
GB	Gamma of B
GC	Gamma of C
GE	Gamma of E
GMMMA	Library gamma function
H(I,J)	Finite difference estimate of pressure head, cm or feet of H_2O
H1	Left potential value of plotting rectangle
H2	Right potential value of plotting rectangle
HA	H average

HD	Temporary of HA, HD(I) plot page heading
HK	K - 1
HL	H - left side of interval
HN	Horizontal row number
HS	Sum horizontal space
HSUM	Sum of horizontal distance
I	Do loop counter for vertical direction
IA	Integer error code of gamma function
IB	Integer error code of gamma function
IC	Unsaturated iteration counter, error code of gamma function
IE	Integer error code of gamma function
IEQPLT	Integer to control output of equipotential plot, 1 = plot
IH	Integer output H control, 1 = print H table
IK	Saturated iteration counter
IMAGE( )	Character storage for points to be plotted
IMC	Integer to control output of moisture code, 1 = print MC table
INPUT	Subroutine to initialize arrays and determine soil coefficients
IPHI	Integer output phi control, 1 = print PHI table
IQ	Integer to control output of flow, 1 = print flow
IR	Integer to control output of resistances, 1 = print R table
ISAT	Control for saturated subroutine
ITHETA	Integer output theta control, 1 = print THETA table
IWT	Integer to control output of water table, 1 = print WTV table
IXMAX	Integerized X maximum

IYDIFF	Integerized maximum Y coordinate difference
IYMAX	Integerized Y maximum
J	Do loop counter for horizontal direction
K(I)	Constant control integer parameters for the drainage system
KA	Vertical node number at bottom of soil
KB	Image row number at bottom of grid
KC	Horizontal node number at the right boundary
KD	Horizontal column number for right image column
KE	Vertical node number above lower boundary
KF	Node to left of right boundary
KG	Vertical node number at left drain
KH	Vertical node number at right drain
KI	Maximum saturated iterations
L	Do loop vertical layer counter
LI	Control parameter
LINRP	Subroutine to fit given function to soils data
LS	Layers of soil data
MC(I,J)	Moisture hysteresis code storage array
MCIJ	Moisture code at I,J (1 = drying, 3 = wetting, 2 = and 4 = scan curves)
ML	Number of layers per rectangle plus one
N	Number of data points for each characteristic curve
NC	Card punch unit number
NDRAIN	Plot drain as D if NDRAIN = 1, NDRAIN = 2 plot right drain
NEWH	Subroutine to calculate new H values
NEWMC	Subroutine to determine moisture code

NEWR	Subroutine to calculate new R values
NH	Do loop counter in horizontal direction
NL	Number of segments per layer
NM1	N - 1
NM2	N - 2
NP	Printer unit number
NPL	Do loop potential line counter
NPLOT	= 1, plot by library routine
NR	Card reader unit number
NU	Number of points to be plotted with a given character
NV	Do loop counter in vertical direction
OR	Ordinate heading of plot
OUTPUT	Subroutine controls output
P(I)	Constant control floating point parameters
PO(I,J)	PHI(I,J) values at end of previous time step
P1	Mesh size at drain/drain radius
P2	Used in calculating drain resistivity
P3	Used in calculating drain resistivity
P4	Used in calculating drain resistivity
P5	Drain resistivity
PHI(I,J)	Hydraulic potential
PLOTA	Sets up plot, headings, scales, etc.
PLOTB	Stores characters to be plotted
PLOTC	Prints plot
PM	Maximum potential

P10	Radians of soil slope
PR	Sum of probability
PR1	Lower probability limit, level 1
PR2	Probability of level 2
P1R	Temporary of PR1
P2R	Temporary of PR2
Q(J)	Flow rate array
QBD	Flow in resistance below drain
QL	Flow out left`drain per unit length
QLD	Flow in resistance left of drain
QR	Flow out right drain per unit length
QRD	Flow in resistance right of drain
QTD	Flow in resistance above drain
R(I,J)	Hydraulic resistance values of the soil
RO(I,J)	R(I,J) values at end of previous time step
R1	Top arm of the resistance network star
R2	Right arm of the resistance network star
R3	Bottom arm of the resistance network star
R4	Left arm of the resistance network star
REGMS	Regression mean square
REGSS	Regression sum of squares
RESID	Residual
RESID2	Residual squared
SAT	Subroutine calculates saturated potentials
SC3	Standard deviation of C_3

SIN	Trigonometric function
SINPIO	Sine of land slope
SQRT	Library square root routine
SSU	Sum of squares of U
SSUV	Sum of squares of U * V
SSV	Sum of squares of V
SSY	Sum of squares of Y
SU	Sum of U
SUMT	Total sum of elapsed time of flow since saturation
SUU	Sum U squared
SUV	Sum U * V
SV	Sum of V
SVV	Sum of V squared
SY	Sum of Y
SYY	Sum of Y squared
TO(I,J)	THETA(I,J) values at end of previous time step
T1	T value (lower limit)
T2	T value (upper limit)
TEST(I)	Array to store NS, *, and **
THETA(I,J)	Moisture content array
THETAX	Temporary THETA value
TL	T left side of interval
U	Y transformed
UNSAT	Subroutine calculates unsaturated moisture contents
V	X transformed

V1	Upper potential value of plotting rectangle
V2	Lower potential value of plotting rectangle
VC	Vertical column number
VEXP	Vertical expansion factor in EQPLOT
VL	Value left side of rectangle.
VM	DV(I) * HK/HL
VS	Sum vertical space
VSUM	Sum of vertical distance
WTV(J)	Vertical height of water table above left drain centerline
WTX(J)	X coordinate for plotting the water table points
WTY(I)	Water table elevation in Y direction
X(J)	Horizontal distance from left drain
XMAX	Maximum horizontal scale value
XMIN	Minimum horizontal scale value
XX(I,J)	Node horizontal distance
Y(I,J)	Elevation head above center of left drain, also used Y(I)
YD	Elevation of drain
YDIFF	Y difference for plot
YH	Estimate of Y
YL	Y left side of interval
YM	Y at middle of integrated zone
YMAX	Maximum vertical scale value of plot
YMIN	Minimum vertical scale value of plot
YR	Right side of interval
YY(I,J)	Node elevation

APPENDIX B

Computer Model Soil Profile

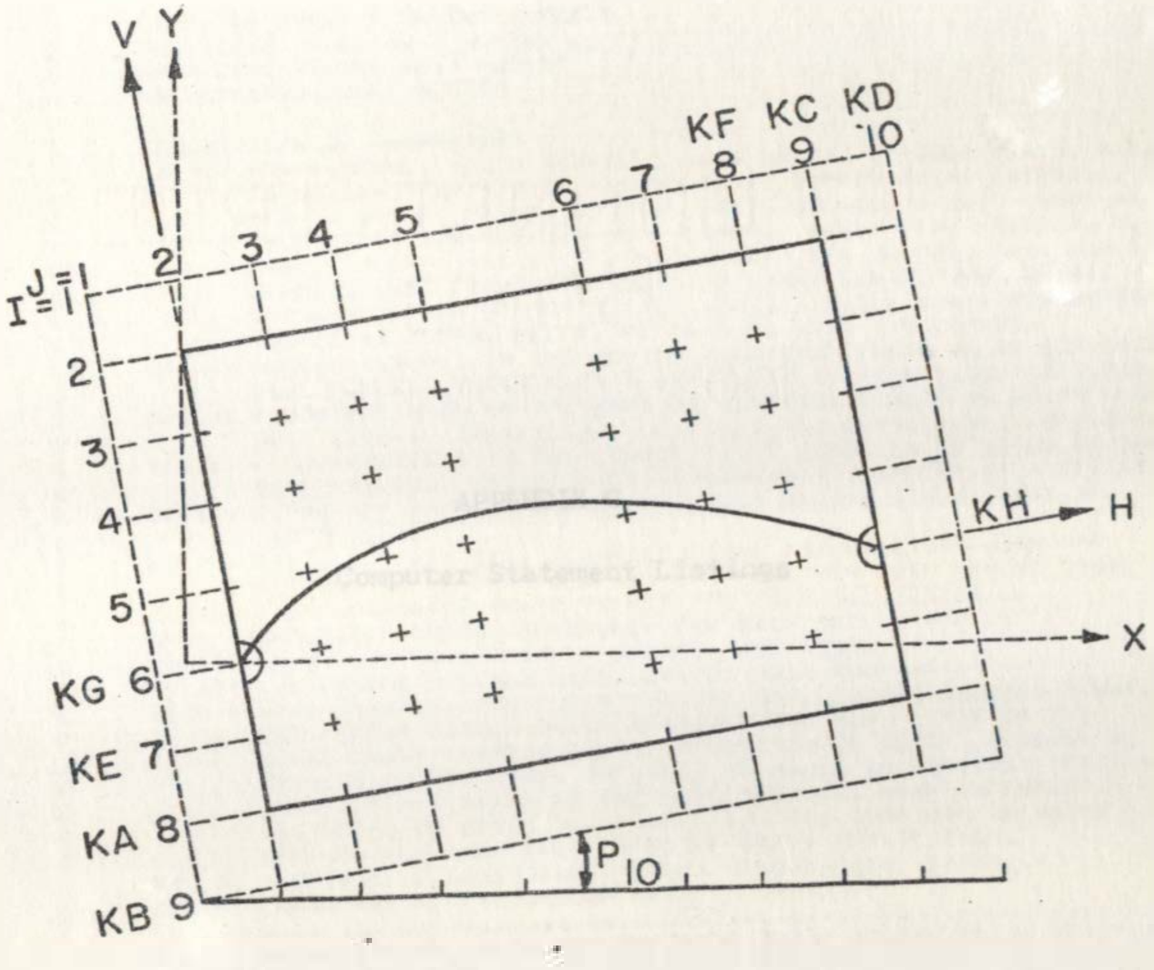

Figure 1. Typical computer model soil profile complete with parameters.

APPENDIX C

Computer Statement Listings

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C     ..................................................................
C     SIMULATION OF TWO DIMENSIONAL SATURATED AND UNSATURATED DRAINAGE &
C            OPTIONAL WATER APPLICATION AT THE SOIL SURFACE
C
C     COMPUTER INPUT / OUTPUT NUMBERS
C     NC = CARD PUNCH UNIT NUMBER
C     NR = CARD READER UNIT NUMBER
C     NP = PRINTER UNIT NUMBER
C
C     DESCRIPTION OF PARAMETERS
C     K(I) = NO. I NODES, NO. J NODES, I LEFT DRAIN, I RIGHT DRAIN, MAX.
C            SATURATED ITERATIONS, BEGIN UNSAT. PRINT, PRINT INTERVAL,
C            END OF UNSAT. PRINT, CONTROL DELTT CHANGE, K(10) = NODE AT
C            THE TOP OF THE UNIFORM SOIL LAYER,  K(11) = 0 - READ A, B,
C            AND C COEFFICIENTSS IN E FIELD OR = 1 - COMPUTE A,B, AND C
C            COEFFICIENTS FROM LAYER DATA BY LINRP SUBROUTINE, UNSAT.
C            ITERATIONS FOR STEADY STATE, K(21) - K(28) USE 1 FOR OUTPUT
C            TABLES OF THETA, MC, R, H, PHI, Q, WTV, AND EQPLOT.
C     A(I,J) = COEFFICIENTS IN THE DRYING EQUATION (THETA VS H) FOR EACH
C            SOIL LAYER.  THETA(I,J) = A(I,1)/(H(I,J)**A(I,3) + A(I,2)).
C     B(I,J) = COEFFICIENTS IN THE WETTING EQUATION (THETA VS H) OF EACH
C            SOIL LAYER.  THETA(I,J) = B(1,1)/(H(I,J)**B(I,3) + B(I,2)).
C     C(I,J) = COEFFICIENTS IN THE CONDUCTIVITY ECUATION (R VS H) OF THE
C            SOIL LAYERS.  R(I,J) = C(I,4)*ABS(H(I,J))**C(I,3) + C(I,5).
C     D(I) = TEMPORARY STORAGE FOR COEFFICIENT ARRAYS A(I,J), B(I,J),
C            AND C(I,J).
C     E(I) = THETA / THETA SAT. SCANNING CURVE INTERSECTION WITH THE
C            WETTING CURVE FOR EACH 0.1 INTERSECTION WITH DRYING CURVE .
C     F(I) = FIELD CAPACITY THETA VALUES FOR EACH SOIL LAYER .
C     G(I) = SATURATED THETA (MOISTURE) FOR EACH SOIL LAYER .
C     DH(I) = DISTANCE HORIZONTALLY BETWEEN THE NODES .
C     DV(I) = THICKNESS OF EACH LAYER OF THE SOIL PROFILE .
C     P(I) = DRAIN RADIUS, PONDED THICKNESS, SAT. OVERRELAXATION CONST.,
C            SAT. ERROR LIMIT, DRAIN RESISTIVITY, MIN. LIMIT OF PHI
C            DIFFERENCE, INITIAL DELTT, REDUCTION OF DELTT, PERCENT R
C            CHANGES TO SET K(9), INCLINATION ANGLE IN RADIANS, P(11) =
C            LIMIT OF PHI ERROR AT THE SOIL SURFACE, VEXP IN EQPLOT,
C            BEGINNING OF DR(J) WATER APPLICATION, DURATION OF WATER
C            APPLICATION, AND CYCLE TIME OF WATER APPLICATION.
C     R(I,J) = HYDRAULIC RESISTANCE VALUES OF EACH SOIL LAYER .
C     DR(I) = NODE WATER APPLICATION RATE IN L**3/T.
C     ..................................................................
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      NR=5
      NP=6
      NC=7
      CALL INPUT
      CALL SAT (0)
      QL(1)=0.
      QR(1)=0.
      CALL FLOW (0)
      PHI(KB,1)=PHI(KB,3)
      PHI(KB,KD)=PHI(KB,KF)
      DO 1 J=1,KD
```

```
      PHI(1,J)=PHI(3,J)
      DO 1 I=1,KB
      MC(I,J)=1
      THETA(I,J)=G(I)
      H(I,J)=PHI(I,J)-Y(I,J)
      H(I,1)=H(I,3)
      H(I,KD)=H(I,KF)
      H(1,J)=H(3,J)
    1 H(KB,J)=H(KE,J)
      IC=0
      SUMT=0.
      DELT=P(7)
      CALL OUTPUT (1,1,1,1,1,1,1,1)
    9 DO 2 I=1,KB
      DO 2 J=1,KD
      TO(I,J)=THETA(I,J)
      PO(I,J)=PHI(I,J)
    2 RO(I,J)=R(I,J)
      SUMT=SUMT+DELT
      IC=IC+1
      KJ=K(12)
      DO 4 I=1,KJ
      PP=0.
      DO 3 J=1,KD
      PP=PP+ABS(PHI(1,J)-PHI(3,J))
    3 PHI(1,J)=PHI(3,J)
      CALL UNSAT (L1)
      CALL SAT (1)
      IF(PP.LE.P(11)*KD.AND.IC.NE.0) GO TO 5
    4 CONTINUE
    5 CALL FLOW (1)
      IF(IC/K(7).NE.1.*IC/K(7).OR.IC.LT.K(6)) GO TO 6
      CALL OUTPUT (K(21),K(22),K(23),K(24),K(25),K(26),K(27),K(28))
    6 IF(K(9).GT.0) DELT=DELT*P(8)
      IF(K(9).LE.0) DELT=DELT*(1/P(8))
      IF(IC.EQ.K(F)) GO TO 999
      IF(SUMT.GE.P(13)+P(14)) P(13)=P(13)+P(15)
      IF (SUMT+.0001.LT.P(13)) GO TO 7
      DO 8 J=2,KC
      THETA(2,J)=THETA(2,J)+DR(J)*DELT*4./((DH(J-1)+DH(J))*DV(2))
      IF(THETA(2,J).GT.G(2)) THETA(2,J)=G(2)
      CALL NEWMC (2,J,TO(2,J))
    8 CALL NEWH (2,J)
      CALL NEWR (L1)
    7 IF(L1.NE.0.OR.IC/K(7).NE.1.*IC/K(7)) GO TO 9
  999 CALL OUTPUT (1,1,1,1,1,1,1,1)
      STOP
      END
```

```
      SUBROUTINE INPUT
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      READ(NR,1) K
    1 FORMAT(10I7)
      KA=K(1)
      KB=K(1)+1
      KC=K(2)
      KD=K(2)+1
      KE=K(1)-1
      KF=K(2)-1
      KG=K(3)
      KH=K(4)
      KI=K(5)
      READ (NR,2) DH,DV,E,F,G,P,DR
    2 FORMAT(10F7.2)
      P1=DV(KG)/P(1)
      P2=(1.+.405*P1**(-4))/(1.-.405*P1**(-4))
      P3=(1.+.163*P2**(-8))/(1.-.163*P2**(-8))
      P4=(1.+.067*P3**(-12))/(1.-.067*P3**(-12))
      P5=60.*ALOG(P1)-2.34*P2-.48*P3-.12*P4+6.48
      P(5)=8.*P5/(120.*3.14159)
      SINP10=SIN(P(10))
      COSP10=COS(P(10))
      VS=-DV(KB)
      DO 010 I=1,KB
      VS=VS+DV(KB-I+1)
  010 Y(KB-I+1,2)=VS*COSP10
      YD=Y(KG,2)
      DO 020 I=1,KB
      Y(I,2)=Y(I,2)-YD
      HS=-DH(1)
      DO 020 J=1,KD
      IF(J.GT.1)HS=HS+DH(J-1)
      H(I,J)=0.0
      Y(I,J)=Y(I,2)+HS*SINP10
      PHI(I,J)=Y(I,J)+P(2)
  020 IF(I.GT.2) PHI(I,J)=PHI(2,J)/2.0
      PHI(KG,2)=P(1)
      IF(P(10).NE.0.)PHI(KH,KC)=P(1)+Y(KH,KC)
      LS=K(10)
      DO 030 I=1,30
      DO 030 J=1,5
      A(I,J)=0.
      B(I,J)=0.
  030 C(I,J)=0.
      IF(K(11).EQ.0) READ(NR,14) ((A(I,J),J=1,5),I=1,LS),((B(I,J),J=1,5)
     1,I=1,LS),((C(I,J),J=1,5),I=1,LS)
   14 FORMAT(5(1PE15.8))
      IF(K(11).EQ.0) GO TO 040
C     READ 6 CHARACTER NAME AND NUMBER OF OBSERVATIONS (A4,A2,I5)
C     READ N DATA CARDS FOR I TH LAYER, Y(I), X(I), (2F10.0)
      DO 050 I=1,LS
      CALL LINRP (D,I,.01,.05,NR,NP,NC)
      DO 050 J=1,5
```

```
 050 A(I,J)=D(J)
C       READ 6 CHARACTER NAME AND NUMBER OF OBSERVATIONS (A4,A2,I5)
C       READ N DATA CARDS FOR I TH LAYER, Y(I), X(I), (2F10.0)
        DO 060 I=1,LS
        CALL LINRP (D,I,.01,.05,NR,NP,NC)
        DO 060 J=1,5
 060 B(I,J)=D(J)
C       READ 6 CHARACTER NAME AND NUMBER OF OBSERVATIONS (A4,A2,I5)
C       READ N DATA CARDS FOR I TH LAYER, Y(I), X(I), (2F10.0)
        DO 070 I=1,LS
        CALL LINRP (D,I,.01,.05,NR,NP,NC)
        DO 070 J=1,5
 070 C(I,J)=D(J)
 040 DO 080 I=LS,KB
        DO 080 J=1,5
        A(I,J)=A(LS,J)
        B(I,J)=B(LS,J)
 080 C(I,J)=C(LS,J)
        DO 090 I=1,KB
        DO 090 J=1,KD
        RO(I,J)=C(I,5)
 090 R(I,J)=C(I,5)
        WRITE(NP,3) (I,X(I),DH(I),DV(I),E(I),F(I),G(I),P(I),R(I,1),DR(I),
       1I=1, 30)
     3 FORMAT('1',5X,1HI,4X,4HX(I),3X,5HDH(I),3X,5HDV(I),4X,4HE(I),4X,4HF
       1(I),4X,4HG(I),4X,4HP(I),2X,6HR(I,1),3X,5HDR(I),30(//2I8,8F8.4))
        WRITE(NP,4)(I,DV(I),(A(I,J),J=1,5),I=1,30)
     4 FORMAT('1H DRY I',2X,5HDV(I), 7X,6HA(I,1), 7X,6HA(I,2), 7X,6HA(I,3
       1), 7X,6HA(I,4), 7X,6HA(I,5),30(//I8,0PF7.3,5(1PE13.5)))
        WRITE(NP,5)(I,DV(I),(B(I,J),J=1,5),I=1,30)
     5 FORMAT('1H WET I',2X,5HDV(I), 7X,6HB(I,1), 7X,6HB(I,2), 7X,6HB(I,3
       1), 7X,6HB(I,4), 7X,6HB(I,5),30(//I8,0PF7.3,5(1PE13.5)))
        WRITE(NP,6)(I,DV(I),(C(I,J),J=1,5),I=1,30)
     6 FORMAT('1COND. I',2X,5HDV(I), 7X,6HC(I,1), 7X,6HC(I,2), 7X,6HC(I,3
       1), 7X,6HC(I,4), 7X,6HC(I,5),30(//I8,0PF7.3,5(1PE13.5)))
        IF((DH(1).NE.DH(2)).OR.(DH(KF).NE.DH(KC))) GO TO 100
        IF((DV(1).NE.DV(2)).OR.(DV(KE).NE.DV(KA))) GO TO 100
        IF((DV(KG-1).NE.DV(KG)).OR.(DV(KG).NE.DH(2))) GO TO 100
        IF(((DV(KH-1).NE.DV(KH)).OR.(DV(KH).NE.DH(KF))).AND.(P(10).NE.0.))
       1 GO TO 100
        IF((KG.GE.2).AND.(KG.LE.KA).AND.(KH.GE.2).AND.(KH.LE.KA))    RETURN
 100 WRITE (NP,7)
     7 FORMAT(7X,'MESH.NOT SQUARE AT THE DRAIN OR DRAIN NODE INCORRECT')
        STOP
        END
```

```
      SUBROUTINE OUTPUT (ITHETA,IMC,IR,IH,IPHI,IQ,IWT,IEQPLT)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      IF(ITHETA.EQ.0) GO TO 010
      WRITE(NP,1)IC,DELT,SUMT,(J,J=1,KD)
    1 FORMAT(1H1,' AFTER CHANGING WATER CONTENTS',I4,' TIMES, DELT IS'
     1,F7.4,' AND SUMT =',F8.4//' NODE MOISTURE CONTENT'//(I11,13I8))
      DO 100 I=1,KB
  100 WRITE(NP,2)I,(THETA(I,J),J=1,KD)
    2 FORMAT(/I3,2X,14F8.4/(5X,14F8.4))
  010 IF(IMC.EQ.0) GO TO 020
      WRITE(NP,3)(J,J=1,KD)
    3 FORMAT(1H1//19H NODE MOISTURE CODE//(I11,13I8))
      DO 200 I=1,KB
  200 WRITE(NP,4)I,(MC(I,J),J=1,KD)
    4 FORMAT(/I3,14I8/(3X,14I8))
  020 IF(IR.EQ.0) GO TO 030
      WRITE(NP,5)(J,J=1,KD)
    5 FORMAT(1H1//19H NODE RESISTIVITIES//(I11,13I8))
      DO 300 I=1,KB
  300 WRITE(NP,2)I,(R(I,J),J=1,KD)
  030 IF(IH.EQ.0) GO TO 040
      WRITE(NP,6)(J,J=1,KD)
    6 FORMAT(1H1//16H NODE WATER HEAD //(I11,13I8))
      DO 400 I=1,KB
  400 WRITE(NP,2)I,(H(I,J),J=1,KD)
  040 IF(IPHI.EQ.0) GO TO 050
      WRITE(NP,7)IK,ERROR,(J,J=1,KD)
    7 FORMAT(9H1    AFTER,I5,37H SATURATED ITERATIONS THE ERROR IS = ,
     1F8.3//15H NODE POTENTIAL//(I11,13I8))
      DO 500 I=1,KB
  500 WRITE(NP,2)I,(PHI(I,J),J=1,KD)
  050 IF(IQ.EQ.0) GO TO 060
      WRITE(NP,8) Q(1),(J,J=2,KC)
    8 FORMAT(/4X,'TOTAL AND FLOW THRU TOP',F8.3,' (L**3/T)'//(I11,13I8))
      WRITE(NP,2) (I,I=2,2),(Q(J),J=2,KC)
      WRITE(NP,9) QL(1),QL(2),SUMT,QL(3)
    9 FORMAT(/4X,'TILE FLOW =',F8.3,' (L**3/T)'/4X,'FLOW DURING DELT ='
     1,F8.3,' (L**3)'/4X,'DURING SUMT =',F8.3,' THE TOTAL FLOW =',F8.3)
      IF((P(10).NE.0.).AND.(IQ.NE.0)) WRITE(NP,9) QR(1),QR(2),SUMT,QR(3)
  060 IF(IWT.EQ.0) GO TO 070
      WRITE(NP,11)(J,J=2,KC)
   11 FORMAT(/4X,'WATER TABLE DEPTH IN THE V DIRECTION (L)'//(I11,13I8))
      WRITE(NP,2) IWT,(WTV(J),J=2,KC)
  070 IF(IEQPLT.EQ.0) GO TO 080
      CALL EQPLOT (KA,KC,KG,KH,DV,DH,P(10),PHI,P(2),P(12),IWT,WTY)
  080 RETURN
      END
```

```
      SUBROUTINE FLOW (ISAT)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      DO 1 J=2,KC
      DO 2 I=2,KA
      IF(H(I,J)) 2,3,4
    3 WTY(J)=Y(2,J)-Y(I,J)
      GO TO 1
    4 IF(H(I-1,J).GT.0.) GO TO 2
      WTY(J)=Y(2,J)-Y(I-1,J)+H(I-1,J)/(H(I-1,J)-H(I,J))*(Y(I-1,J)-Y(I,J)
     1)
      GO TO 1
    2 CONTINUE
    1 WTV(J)=WTY(J)/COS(P(10))
      Q(1)=0.
      DO 5 J=2,KC
      FDH=1.00
      IF(J.EQ.2.OR.J.EQ.KC) FDH=0.5
      RA3=(R(2,J)+RO(2,J)+R(2,J+1)+RO(2,J+1)+R(3,J+1)+RO(3,J+1)+R(3,J)+R
     1O(3,J))/8.
      RA4=(R(2,J-1)+RO(2,J-1)+R(2,J)+RO(2,J)+R(3,J)+RO(3,J)+R(3,J-1)+RO(
     13,J-1))/8.
      R3=2.*DV(2)*RA3*RA4/(DH(J-1)*RA3+DH(J)*RA4)
      Q(J)=FDH*(PHI(2,J)-PHI(3,J))/R3
    5 Q(1)=Q(1)+Q(J)
      QTD=(PHI(KG-1,2)-PHI(KG,2))*(R(KG-1,1)+R(KG-1,2))/(P(5)*R(KG-1,1)
     1*R(KG-1,2))
      QRD=(PHI(KG,3)-PHI(KG,2))*(R(KG-1,2)+R(KG,2))/(P(5)*R(KG-1,2)
     1*R(KG,2))
      QBD=(PHI(KG+1,2)-PHI(KG,2))*(R(KG,1)+R(KG,2))/(P(5)*R(KG,1)
     1*R(KG,2))
      QLD=(PHI(KG,1)-PHI(KG,2))*(R(KG-1,1)+R(KG,1))/(P(5)*R(KG-1,1)
     1*R(KG,1))
      QL(4)=QL(1)
      QL(1)=QTD+QRD+QBD+QLD
      QL(2)=ISAT*DELT*(QL(4)+QL(1))/2.
      QL(3)=ISAT*QL(3)+QL(2)
      IF(P(10).EQ.0) GO TO 6
      QTD=(PHI(KH-1,KC)-PHI(KH,KC))*(R(KH-1,KF)+R(KH-1,KC))/(P(5)
     1*R(KH-1,KF)*R(KH-1,KC))
      QRD=(PHI(KH,KD)-PHI(KH,KC))*(R(KH-1,KC)+R(KH,KC))/(P(5)*R(KH-1,KC)
     1*R(KH,KC))
      QBD=(PHI(KH+1,KC)-PHI(KH,KC))*(R(KH,KF)+R(KH,KC))/(P(5)*R(KH,KF)
     1*R(KH,KC))
      QLD=(PHI(KH,KF)-PHI(KH,KC))*(R(KH-1,KF)+R(KH,KF))/(P(5)*R(KH-1,KF)
     1*R(KH,KF))
      QR(4)=QR(1)
      QR(1)=QTD+QRD+QBD+QLD
      QR(2)=ISAT*DELT*(QR(4)+QR(1))/2.
      QR(3)=ISAT*QR(3)+QR(2)
    6 RETURN
      END
```

```
      SUBROUTINE SAT (ISAT)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      DO 010 IK=1,KI
      L=0
      ERROR=0.
      DO 020 I=3,KA
      DO 030 J=2,KC
      IF(H(I,J).LE.0.AND.ISAT.NE.0) GO TO 029
      L=L+1
      RA1=(R(I-1,J-1)+RO(I-1,J-1)+R(I-1,J)+RO(I-1,J)+R(I,J)+RO(I,J)+R(I,
     1J-1)+RO(I,J-1))/8.
      RA2=(R(I-1,J)+RO(I-1,J)+R(I-1,J+1)+RO(I-1,J+1)+R(I,J+1)+RO(I,J+1)+
     1R(I,J)+RO(I,J))/8.
      RA3=(R(I,J)+RO(I,J)+R(I,J+1)+RO(I,J+1)+R(I+1,J+1)+RO(I+1,J+1)+R(I+
     11,J)+RO(I+1,J))/8.
      RA4=(R(I,J-1)+RO(I,J-1)+R(I,J)+RO(I,J)+R(I+1,J)+RO(I+1,J)+R(I+1,J-
     11)+RO(I+1,J-1))/8.
      R1=2.*DV(I-1)*RA1*RA2/(DH(J)*RA1+DH(J-1)*RA2)
      R2=2.*DH(J)*RA2*RA3/(DV(I)*RA2+DV(I-1)*RA3)
      R3=2.*DV(I)*RA3*RA4/(DH(J-1)*RA3+DH(J)*RA4)
      R4=2.*DH(J-1)*RA4*RA1/(DV(I-1)*RA4+DV(I)*RA1)
      ACD=1.0
      BCD=1.0
      CCD=1.0
      DCD=1.0
      IF(J-3) 040,050,060
  040 IF(I.LT.K(3)-1.OR.I.GT.K(3)+1) GO TO 061
      IF(I-K(3)) 070,080,090
  070 CCD=2./P(5)
      GO TO 061
  080 DELPHI=0.
      GO TO 100
  090 ACD=2./P(5)
      GO TO 061
  050 IF(I.EQ.K(3)) CCD=2./P(5)
  060 IF(P(10).EQ.0.) GO TO 061
      IF(J-KF) 061,062,063
  063 IF(I.LT.K(4)-1.OR.I.GT.K(4)+1) GO TO 061
      IF(I-K(4)) 070,080,090
  062 IF(I.EQ.K(4)) BCD=2./P(5)
  061 DELPHI=(ACD/R1*PHI(I-1,J)+BCD/R2*PHI(I,J+1)+CCD/R3*PHI(I+1,J)+DCD/
     1R4*PHI(I,J-1))/(ACD/R1+BCD/R2+CCD/R3+DCD/R4)-PHI(I,J)
  100 ERROR=ERROR+ABS(P(3)*DELPHI)
      PHI(I,J)=PHI(I,J)+P(3)*DELPHI
      IF(PHI(I,J).GT.Y(2,KC)+P(2)) PHI(I,J)=Y(2,KC)+P(2)
      H(I,J)=PHI(I,J)-Y(I,J)
  029 IF(J.EQ.KF) PHI(I,KD)=PHI(I,KF)
      IF(I.EQ.KE) PHI(KB,J)=PHI(KE,J)
  030 IF(I.EQ.KE) H(KB,J)=H(KE,J)
      H(I,1)=H(I,3)
      H(I,KD)=H(I,KF)
  020 PHI(I,1)=PHI(I,3)
      IF(ERROR.LE.P(4)*L) GO TO 110
  010 CONTINUE
  110 RETURN
      END
```

```
      SUBROUTINE UNSAT (L1)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      PHI(2,1)=PHI(2,3)
      PHI(2,KD)=PHI(2,KF)
      DO 120 I=2,KA
      DO 120 J=2,KC
      IF(H(I,J).GT.0.) GO TO 120
      RA1=(R(I-1,J-1)+RO(I-1,J-1)+R(I-1,J)+RO(I-1,J)+R(I,J)+RO(I,J)+R(I,
     1J-1)+RO(I,J-1))/8.
      RA2=(R(I-1,J)+RO(I-1,J)+R(I-1,J+1)+RO(I-1,J+1)+R(I,J+1)+RO(I,J+1)+
     1R(I,J)+RO(I,J))/8.
      RA3=(R(I,J)+RO(I,J)+R(I,J+1)+RO(I,J+1)+R(I+1,J+1)+RO(I+1,J+1)+R(I+
     11,J)+RO(I+1,J))/8.
      RA4=(R(I,J-1)+RO(I,J-1)+R(I,J)+RO(I,J)+R(I+1,J)+RO(I+1,J)+R(I+1,J-
     11)+RO(I+1,J-1))/8.
      R1=2.*DV(I-1)*RA1*RA2/(DH(J)*RA1+DH(J-1)*RA2)
      R2=2.*DH(J)*RA2*RA3/(DV(I)*RA2+DV(I-1)*RA3)
      R3=2.*DV(I)*RA3*RA4/(DH(J-1)*RA3+DH(J)*RA4)
      R4=2.*DH(J-1)*RA4*RA1/(DV(I-1)*RA4+DV(I)*RA1)
      ACD=1.0
      BCD=1.0
      CCD=1.0
      DCD=1.0
      IF(J-3) 140,150,160
  140 IF(I.LT.K(3)-1.OR.I.GT.K(3)+1) GO TO 161
      IF(I-K(3)) 170,180,190
  170 CCD=2./P(5)
      GO TO 161
  180 THETA(I,J)=G(I)
      GO TO 200
  190 ACD=2./P(5)
      GO TO 161
  150 IF(I.EQ.K(3)) DCD=2./P(5)
  160 IF(P(10).EQ.0.) GO TO 161
      IF(J-KF) 161,162,163
  163 IF(I.LT.K(4)-1.OR.I.GT.K(4)+1) GO TO 161
      IF(I-K(4)) 170,180,190
  162 IF(I.EQ.K(4)) BCD=2./P(5)
  161 THETA(I,J)=TO(I,J)+DELT*4/((DH(J-1)+DH(J))*(DV(I-1)+DV(I)))*(ACD/R
     11*(PHI(I-1,J)-PHI(I,J)+PO(I-1,J)-PO(I,J))+BCD/R2*(PHI(I,J+1)-PHI(I
     2,J)+PO(I,J+1)-PO(I,J))+CCD/R3*(PHI(I+1,J)-PHI(I,J)+PO(I+1,J)-PO(I,
     3J))+DCD/R4*(PHI(I,J-1)-PHI(I,J)+PO(I,J-1)-PO(I,J)))/2.0
  200 IF(THETA(I,J).GT.G(I)) THETA(I,J)=G(I)
      IF(THETA(I,J).LT.F(I)) THETA(I,J)=F(I)
      CALL NEWMC (I,J,TO(I,J))
      CALL NEWH (I,J)
  120 CONTINUE
      CALL NEWR (L1)
      RETURN
      END
```

```
      SUBROUTINE NEWH (I,J)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      IF(H(I,J).GT.0.) GO TO 8
      MCIJ=MC(I,J)
      GO TO (1,2,3,2), MCIJ
    1 DO 10 L=1,5
   10 D(L)=A(I,L)
      TI=THETA(I,J)
      GO TO 5
    2 DO 20 L=1,5
   20 D(L)=A(I,L)
      ITHETA=THETA(I,J)/G(I)/0.1
      FTHETA=ITHETA/10.*G(I)
      TI=FTHETA
      GO TO 5
    3 DO 30 L=1,5
   30 D(L)=B(I,L)
      TI=THETA(I,J)
      GO TO 5
    4 DO 40 L=1,5
   40 D(L)=B(I,L)
      TI=E(ITHETA)
      MCIJ=0
      HD=HA
    5 IF(D(1)/TI-D(2).EQ.0) HA=0.
      IF(D(1)/TI-D(2).LE.0) GO TO 6
      HA=-(D(1)/TI-D(2))**(1./D(3))
    6 IF((MCIJ.EQ.1).OR.(MCIJ.EQ.3)) GO TO 7
      IF((MCIJ.EQ.2).OR.(MCIJ.EQ.4)) GO TO 4
      H(I,J)=((HA-HD)*(THETA(I,J)-FTHETA))/(E(ITHETA)*G(I)-FTHETA)+HD
      IF(HD-HA) 8,7,7
    7 H(I,J)=HA
    8 RETURN
      END
```

```
      SUBROUTINE NEWR (L1)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),Q(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,Q,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      L1=0
      K(9)=0
      DO 1 I=2,KA
      DO 2 J=2,KC
      PHI(I,J)=H(I,J)+Y(I,J)
      IF(PHI(I,J).LT.0.) PHI(I,J)=0.
      IF(PHI(I,J)-PHI(KG,2).GT.P(6)) L1=L1+1
    2 PHI(KB,J)=PHI(KE,J)
      PHI(I,1)=PHI(I,3)
    1 PHI(I,KD)=PHI(I,KF)
      PHI(KB,1)=PHI(KB,3)
      PHI(KB,KD)=PHI(KB,KF)
      PHI(1,1)=PHI(1,3)
      PHI(1,KD)=PHI(1,KF)
      DO 3 I=1,KB
      DO 3 J=1,KD
      H(I,J)=PHI(I,J)-Y(I,J)
      H(I,1)=H(I,3)
      H(I,KD)=H(I,KF)
      H(1,J)=H(3,J)
    3 H(KD,J)=H(KE,J)
      DO 4 I=1,KB
      DO 4 J=1,KD
      IF(H(I,J).GT.0) GO TO 4
      R(I,J)=C(I,4)*ABS(H(I,J))**C(I,3)+C(I,5)
      IF(ABS(RO(I,J)-R(I,J))/(RO(I,J)*100).GT.P(9)) K(9)=K(9)+1
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NEWMC (I,J,THETAX)
      DIMENSION PHI(30,30),H(30,30),THETA(30,30),R(30,30),MC(30,30),Y(30
     1,30),G(30),K(30),A(30,5),B(30,5),C(30,5),D(5),E(30),F(30),G(30),DH
     2(30),DV(30),P(30),WTY(30),WTV(30),QL(4),QR(4),PO(30,30),TO(30,30)
     3,RO(30,30),DR(30)
      COMMON KA,KB,KC,KD,KE,KF,KG,KH,KI,PHI,H,THETA,R,MC,Y,G,K,A,B,C,D,E
     1,F,G,DH,DV,P,NC,NR,NP,IK,ERROR,IC,DELT,SUMT,WTY,WTV,QL,QR,PO,TO,RO
     2,DR
      IF(H(I,J).GT.0.)  GO TO 10
      ITHETA=THETAX/G(I)/0.1
      FTHETA=ITHETA/10.*G(I)
      MCIJ=MC(I,J)
      GO TO (1,2,3,4), MCIJ
    1 IF(THETA(I,J).EQ.0.) MC(I,J)=3
      IF(THETA(I,J).LE.THETAX) GO TO 10
      MC(I,J)=2
      IF(ITHETA.EQ.0) GO TO 5
      IF(THETA(I,J).LT.E(ITHETA)*G(I)) GO TO 10
    5 MC(I,J)=3
      GO TO 10
    2 IF(THETA(I,J).GE.THETAX) GO TO 6
      MC(I,J)=4
      IF(THETA(I,J).LE.FTHETA) MC(I,J)=1
      GO TO 10
    6 IF(THETA(I,J).GE.E(ITHETA)*G(I)) MC(I,J)=3
      IF(THETA(I,J).EQ.G(I)) MC(I,J)=1
      GO TO 10
    3 IF(THETA(I,J).EQ.G(I)) MC(I,J)=1
      IF(THETA(I,J).GE.THETAX) GO TO 10
      MC(I,J)=4
      IF(ITHETA.EQ.0) GO TO 7
      IF((THETA(I,J).GT.FTHETA).AND.(THETA(I,J).LT.E(ITHETA)*G(I))) GO
     1 TO 10
    7 MC(I,J)=1
      GO TO 10
    4 IF(THETA(I,J).LE.THETAX) GO TO 8
      MC(I,J)=2
      IF(THETA(I,J).GT.E(ITHETA)*G(I)) MC(I,J)=3
      GO TO 10
    8 IF(THETA(I,J).LE.FTHETA) MC(I,J)=1
      IF(THETA(I,J).EQ.0.) MC(I,J)=3
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LINRP (C,IC,P1,P2,NR,NP,NC)
C     LINEAR REGRESSION WHICH FITS LN-LN CURVE THRU POINT (X(1),Y(1)).
C       1 READ PROBLEM PARAMETER CARD FOR LN-LN LINEAR REGRESSION.
C       2 READ X(I), Y(I) VALUES (FIRST CARD MUST CONTAIN FITTED POINT).
C       3 DATA FIT BY V=C(3)*U+ALOG(C(4)),WHERE V(I)=ALOG(1/Y(I)-1/Y(1))
C        ,U(I)=ALOG(X(I)-X(1)),AND C(1)=1/C(4),C(2)=C(1)*C(5),C(3) AND
C         C(4) BY REGRESSION,C(5)=1/Y(1). FITTING THRU 1 PT MEANS DF=N-1
      DIMENSION X(100),Y(100),U(100),V(100),P(300),C(5),TEST(3)
      DATA TEST /' NS ',' * ',' **'/
      READ (NR,1) PR,PR1,N,NPLOT
    1 FORMAT(A4,A2,I5,2X,I1)
C       PR,PR1..PROBLEM NUMBER (MAY BE ALPHAMERIC)
C        N..NUMBER OF DATA POINTS. NPLOT..OPTION PLOTTING CODE (1 = PLOT)
      READ (NR,2) X(1),Y(1)
    2 FORMAT(2F10.0)
      SU=0.
      SV=0.
      SUU=0.
      SVV=0.
      SUV=0.
      DO 10 I=2,N
      READ (NR,2) X(I),Y(I)
      V(I)=ALOG(ABS(1/Y(I)-1/Y(1)))
      U(I)=ALOG(ABS(X(I)-X(1)))
      SU=SU+U(I)
      SV=SV+V(I)
      SUU=SUU+U(I)*U(I)
      SVV=SVV+V(I)*V(I)
   10 SUV=SUV+U(I)*V(I)
      NM1=N-1
      NM2=N-2
      C(3)=(SUV-SU*SV/NM1)/(SUU-SU*SU/NM1)
      C(4)=EXP(SV/NM1-C(3)*SU/NM1)
      C(5)=1/Y(1)
      IF(X(1).NE.0) C(5)=1/Y(1)-C(4)*ABS(X(1))**C(3)
      C(1)=1/C(4)
      C(2)=C(1)*C(5)
      SSU=SUU-SU**2/NM1
      SSV=SVV-SV**2/NM1
      SSUV=SUV-SU*SV/NM1
      D=SSU/SSV*C(3)**2
      REGSS=SSUV**2/SSU
      DEVSS=SSV-REGSS
      R2=N-2
      REGMS=REGSS
      DEVMS=DEVSS/R2
      F=REGMS/DEVMS
      CALL FTABLE (1.,R2,P1,P2,F1,F2)
      IF (F.LT.F2) J=1
      IF (F.GE.F2) J=2
      IF (F.GE.F1) J=3
      SC3=SQRT(DEVMS/SSU)
      T1=SQRT(F1)
      T2=SQRT(F2)
      C3L1=C(3)-T1*SC3
      C3U1=C(3)+T1*SC3
      C3L2=C(3)-T2*SC3
      C3U2=C(3)+T2*SC3
      YH=Y(1)
      IF(X(1).NE.0) YH=C(1)/(ABS(X(1))**C(3)+C(2))
```

```
      RESID=Y(1)-YH
      WRITE (NP,3) PR,PR1,N,(I,C(I),I=1,5),D,X(1),Y(1),YH,RESID
    3 FORMAT('1',9X,'LINEAR LN-LN REGRESSION ANALYSIS FORCED THRU POINT
     1(X(1),Y(1))'/10X,'SOLUTION IS Y = C(1)/(X**C(3)+C(2)) OR 1/Y = C(4
     2)*X**C(3)+C(5)'/10X,'WHERE V(I)=ALOG(1/Y(I)-1/Y(1)) AND U(I)=ALOG(
     3X(I)-X(1)) IN'/10X,'V = C(3)*U + ALOG(C(4))'//10X,'PROBLEM NAME ..
     4.',A4,A2,10X,'NUMBER OF OBSERVATIONS =',I6/10X,'THE COEFFICIENTS A
     5RE'/5(10X,'C(',I1,')=',F17.6/),10X,'R2 (LN DATA)=',F9.6//30X,'TABL
     6E OF RESIDUALS'/10X,'I         U(I)          V(I)          X(I)          Y(I)
     7 YH(I)       RESIDUAL'//10X,'1',21X,4F10.5)
      SY=0.
      SYY=0.
      RESID2=0.
      DO 20 I=2,N
      YH=C(1)/(ABS(X(I))**C(3)+C(2))
      RESID=Y(I)-YH
      RESID2=RESID2+RESID*RESID
      SY=SY+Y(I)
      SYY=SYY+Y(I)*Y(I)
      WRITE (NP,4) I,U(I),V(I),X(I),Y(I),YH,RESID
    4 FORMAT(I11,F11.5,5F10.5)
      P(I)=X(I)
      P(N+I)=Y(I)
   20 P(N+N+I)=YH
      SSY=SYY-SY*SY/NM1
      D=1-RESID2/SSY
      R=SQRT(ABS(D))
      WRITE (NP,5) REGSS,REGMS,F,TEST(J),NM2,DEVSS,DEVMS,NM1,SSY,P2,F2,P
     11,F1,SC3,P2,T2,C3L2,C3U2,P1,T1,C3L1,C3U1,R,D
    5 FORMAT(/10X,42(1H*)//21X,'ANALYSIS OF VARIANCE'//10X,'SOURCE     DF
     1        SS          MS             F'//10X,'REGRESS     1',3F10.3,A4/10X,
     2'RESIDUAL',I4,2F10.3/10X,'TOTAL',I7,F10.3//10X,'TABLE VALUE',2('F(
     3',F3.2,') =',F6.2)//10X,42(1H*)//10X,'STANDARD DEVIATION OF C(3) =
     4',F14.3//10X,'CONFIDENCE LIMITS OF C(3)  LOWER       UPPER'/10X,'LEV
     5EL        T VALUES           LIMIT        LIMIT'//2(F14.2,F15.4,3X,2F10.3/
     6)/10X,'R (ORIGINAL DATA) =',F14.3/10X,'R2 (ORIGINAL DATA)=',F14.3)
      WRITE (NC,6) C(1),C(2),C(3),C(4),C(5)
    6 FORMAT(5(1PE15.6))
      IF (NPLOT.EQ.1) CALL PLOT (IC,P,N,3,0,1)
      RETURN
      END
```

```
      SUBROUTINE FTABLE (R1, R2, P1, P2, F1, F2)
      DATA DELT, DELF/0.005,0.001/,A, B, PR1, PR2/4*0.0/
      IF (A.EQ.R1/2.AND.B.EQ.R2/2.AND.P1.EQ.P1R.AND.P2.EQ.P2R) GO TO 1
      P1R=P1
      P2R=P2
      A=R1/2.
      B=R2/2.
      C=(R1+R2)/2.
      D=R1/R2
      E=(R2+1.0)/2.
      IF (C.GT.57) GO TO 2
      PR=0.0
      CALL GMMMA (A, GA, IA)
      CALL GMMMA (B, GB, IB)
      IF (R1.NE.1.) GO TO 3
      CALL GMMMA (E, GE, IE)
      PR1=0.5*(1.0-P1)*GB/GE*SQRT(3.14159*R2)
      PR2=0.5*(1.0-P2)*GB/GE*SQRT(3.14159*R2)
      TL=0.0
      YL=1.0
      DO 4 I=1,10000
      IF ((1.0+(TL+2*DELT)**2/R2)**E .GE. 1.0E75) GO TO 2
      YM=1.0/(1.0+(TL+DELT)**2/R2)**E
      YR=1.0/(1.0+(TL+2*DELT)**2/R2)**E
      PR=PR+DELT/3.0*(YL+4*YM+YR)
      IF (PR.LE.PR2) F2=(TL+3*DELT)**2
      F1=(TL+DELT)**2
      IF (PR.GE.PR1) GO TO 1
      YL=YR
    4 TL=TL+2*DELT
      GO TO 2
    3 CALL GMMMA (C, GC, IC)
      PR1=(1.0-P1)*GB/GC*GA/D**A
      PR2=(1.0-P2)*GB/GC*GA/D**A
      FL=0.0
      YL=0.0
      DO 5 I=1,16000
      IF ((1.0+D*(FL+2*DELF))**C .GE. 1.0E75) GO TO 2
      YM=(FL+DELF)**(A-1.0)/(1.0+D*(FL+DELF))**C
      YR=(FL+2*DELF)**(A-1.0)/(1.0+D*(FL+2*DELF))**C
      PR=PR+DELF/3.0*(YL+4*YM+YR)
      IF (PR.LE.PR2) F2=FL+3*DELF
      F1=FL+DELF
      F1=99.5-1.0/R1
      IF (PR.GE.PR1) GO TO 1
    5 FL=FL+2*DELF
      IF ((P1.EQ.0.01).AND.(R2.EQ.2.0)) GO TO 1
    2 F1=0.0
      F2=0.0
    1 RETURN
      END
```

```
      SUBROUTINE EQPLOT(KA,KC,KG,KH,DV,DH,P10,B,P2,VEXP,IWT,WTY)
C     EQUAL POTENTIAL PLOT
C     IMAGE (I) = ARRAY FOR STORING CHARACTERS TO BE PLOTTED
C     HD (I) = HORIZONTAL HEADING - TOP OF PLOT OUTPUT
C     OR (I) = VERTICAL HEADING - LEFT SIDE OF PLOT OUTPUT
C     ABSC (I) = HORIZONTAL HEADING - BOTTOM OF PLOT OUTPUT
C     DH(I) = SLOPE DISTANCE BETWEEN NODES
C     DV(I) = THICKNESS OF EACH LAYER
C     NOTE DIMENSION OF ARRAY B MUST BE SAME AS FROM THE CALLING PROGRAM
      DIMENSION IMAGE(1428),HD(25),OR(12),ABSC(25),X(900),Y(900),XX(30,3
     10),YY(30,30),B(30,30),DV(30),DH(30),WTX(30),WTY(30)
      DATA HD(1),HD(2),HD(3),HD(4),HD(5)/'EQUA','L PO','TENT','IAL ',
     1'PLOT'/, OR(1),OR(2),OR(3),OR(4),OR(5)/'VERT','ICAL',' DIS','TANC'
     2,'E   '/, ABSC(1),ABSC(2),ABSC(3),ABSC(4),ABSC(5)/'HORI','ZONT',
     3'AL D','ISTA','NCE '/
C     KC = NUMBER OF NODES HORIZONTAL + ONE
C     KA = NUMBER OF NODES VERTICAL + ONE
      KE = KA-1
      KF = KC - 1
      ADH = DH(1)
      ADV = DV(1)
      DH(1) = 0.0
      DV(1) = 0.0
C     FIND COORDINATES OF NODES
      VSUM = 0.0
      N = 0
      SINP10 = SIN(P10)
      COSP10 = COS(P10)
      DO 1 NV=2,KA
      VSUM=VSUM+DV(NV-1)
      HSUM=0.0
      DO 1 NH=2,KC
      N=N+1
      HSUM=HSUM+DH(NH-1)
      X(N) = HSUM*COSP10 + VSUM*SINP10
      Y(N) = HSUM*SINP10 - VSUM*COSP10
      XX(NV,NH) = X(N)
    1 YY(NV,NH) = Y(N)
C     XMAX AND XMIN = MAXIMUM AND MINIMUM HORIZONTAL SCALE VALUE
C     YMAX AND YMIN = MAXIMUM AND MINIMUM VERTICAL SCALE VALUE
      XMIN=0.
      IXMAX=XX(KA,KC)/5.0+1.0
      XMAX=5*IXMAX
      IYDIFF=(YY(2,KC)-YY(KA,2))/5.0+1.0
      YDIFF=5*IYDIFF
      IF(0.833*XMAX.GT.YDIFF) YDIFF=0.833*XMAX
      IF(YDIFF/0.833.GT.XMAX) XMAX=YDIFF/0.833
      IYMAX=YY(2,KC)+1.0
      YMAX=IYMAX
      IF(P10.EQ.0.) YMAX=0.
      YMIN=(YMAX-YDIFF)*VEXP
      IF(P10.EQ.0.) YMIN=YY(KA,2)*VEXP
      CALL PLOTA(IMAGE,XMIN,XMAX,YMIN,YMAX,0)
C     STORE COORDINATES OF NODES IN PLOT ARRAY
      CALL PLOTB(X,Y,'*',N)
C     SET NPL FOR PERCENT OF MAXIMUM POTENTIAL DESIRED
      PM=YY(2,KC)-YY(KG,2)+P2+B(KG,2)
      DO 2 NPL=1,9
      E = PM * 0.10 * NPL
C     SET I AND J FOR RECTANGLES TO BE CHECKED
```

```
      DO 2 I=2,KE
      DO 2 J=2,KF
      NU=0
C     ML = NUMBER OF LAYERS PER RECTANGLE PLUS ONE
      ML=DV(I)/COSP10*51./(YMAX-YMIN)
      IF(ML.LT.2) ML=2
C     NL = NUMBER OF SEGMENTS PER LAYER
      NL=DH(J)/COSP10*101./(XMAX-XMIN)
      IF(NL.LT.2) NL=2
      HL = ML - 1
      VC = NL - 1
C     CHECK EACH SIDE OF THE RECTANGLE TO SEE IF DESIRED POTENTIAL CRO
C     SSES IT
      IF((E .LE. B(I,J) .AND. E .GE. B(I+1,J)) .OR. (E .GE. B(I,J) .AND.
     1E .LE.B(I+1,J))) GO TO 3
      IF((E .LE. B(I,J) .AND. E .GE. B(I,J+1)) .OR. (E .GE. B(I,J) .AND.
     1E .LE. B(I,J+1))) GO TO 3
      IF((E .LE. B(I,J+1) .AND. E .GE. B(I+1,J+1)) .OR. (E.GE. B(I,J+1)
     1.AND. E .LE. B(I+1,J+1))) GO TO 3
      IF (.NOT. (E .LE. B(I+1,J) .AND. E .GE. B(I+1,J+1)) .OR. (E .GE. B
     1(I+1,J) .AND. E .LE. B(I+1,J+1)))  GO TO 2
C     SET K FOR DESIRED LAYER WITHIN RECTANGLE
    3 DO 4 K = 1, ML
      HK=K-1
      H1=B(I,J)-(B(I,J)-B(I+1,J))*HK/HL
      H2=B(I,J+1)-(B(I,J+1)-B(I+1,J+1))*HK/HL
C     DETERMINE IF EQUAL POTENTIAL LINE PASSES BETWEEN H1 AND H2.
      IF(.NOT.((E.LE.H1.AND.E.GE.H2).OR.(E.GE.H1.AND.E.LE.H2))) GO TO 4
      IF(H1.EQ.H2) GO TO 4
      NU = NU + 1
      HN=DH(J)*ABS((H1-E)/(H1-H2))
      VM=DV(I)*HK/HL
      X(NU)=HN*COSP10+VM*SINP10+XX(I,J)
      Y(NU)=HN*SINP10-VM*COSP10+YY(I,J)
    4 CONTINUE
      DO 5 L=1,NL
      VL=L-1
C     DETERMINE IF EQUAL POTENTIAL LINE PASSES BETWEEN V1 AND V2.
      V1=B(I,J)-(B(I,J)-B(I,J+1))*VL/VC
      V2=B(I+1,J)-(B(I+1,J)-B(I+1,J+1))*VL/VC
      IF(.NOT.((E.LE.V1.AND.E.GE.V2).OR.(E.GE.V1.AND.E.LE.V2))) GO TO 5
      IF(V1.EQ.V2) GO TO 5
      NU = NU + 1
      HN=DH(J)*VL/VC
      VM=DV(I)*ABS((V1-E)/(V1-V2))
      X(NU)=HN*COSP10+VM*SINP10+XX(I,J)
      Y(NU)=HN*SINP10-VM*COSP10+YY(I,J)
    5 CONTINUE
C     DETERMINE CORRECT CALL PLOTB FOR STORAGE IN PLOT ARRAY AND STORE
      GO TO (61,62,63,64,65,66,67,68,69),NPL
   61 CALL PLOTB(X,Y,'1',NU)
      GO TO 2
   62 CALL PLOTB(X,Y,'2',NU)
      GO TO 2
   63 CALL PLOTB(X,Y,'3',NU)
      GO TO 2
   64 CALL PLOTB(X,Y,'4',NU)
      GO TO 2
   65 CALL PLOTB(X,Y,'5',NU)
      GO TO 2
```

```
   66 CALL PLOTB(X,Y,'6',NU)
      GO TO 2
   67 CALL PLOTB(X,Y,'7',NU)
      GO TO 2
   68 CALL PLOTB(X,Y,'8',NU)
      GO TO 2
   69 CALL PLOTB(X,Y,'9',NU)
    2 CONTINUE
C     DETERMINE THE INTERSECTION OF THE WATER TABLE AND THE NODE COLUMNS
      IF(IWT.EQ.0) GO TO 7
      DO 8 NH=2,KC
      N=NH-1
      WTY(N)=YY(2,NH)-WTY(NH)
    8 WTX(N)=XX(2,NH)+WTY(NH)*SINP10/COSP10
      CALL PLOTB (WTX,WTY,'W',N)
    7 NDRAIN=2
      X(1)=XX(KG,2)
      Y(1)=YY(KG,2)
      X(2)=XX(KH,KC)
      Y(2)=YY(KH,KC)
      IF(P10.EQ.0) NDRAIN=1
      CALL PLOTB(X,Y,'D',NDRAIN)
C     PLOT OUTPUT GRAPH
      CALL PLOTC (HD,20,OR,17,ABSC,19)
      DH(1) = ADH
      DV(1) = ADV
      RETURN
      END
```

11	12	9	9	200	8	2	10	0	6
0	5	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0
1.00	1.00	1.00	3.00	10.00	10.00	10.00	10.00	3.00	1.00
1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5906	0.5906	0.4921	1.8373	0.4921	0.5879	1.00	1.00	1.00	1.00
1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.199	0.299	0.399	0.499	0.599	0.699	0.799	0.899	0.999	1.000
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.051	0.051	0.210	0.280	0.229	0.200	0.200	0.200	0.200	0.200
0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.399	0.399	0.425	0.436	0.430	0.411	0.411	0.411	0.411	0.411
0.411	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
.2083	0.00	1.50	0.001	0.00	0.01	0.010	1.00	0.05	0.00
0.0001	2.00	0.50	0.125	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	.1250	.1250	.2500	.8125	1.2500	1.2500	1.2500	.8125	.2500
.1250	.1250	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

4.777851E-01	1.197456E 00	3.654466E-01	2.092991E 00	2.506266E 00
4.777851E-01	1.197456E 00	3.654466E-01	2.092991E 00	2.506266E 00
2.006653E 00	4.721535E 00	2.432652E-01	4.983422E-01	2.352941E 00
2.349402E 00	5.388535E 00	2.094810E-01	4.256401E-01	2.293577E 00
2.572644E 00	6.432109E 00	2.730339E-01	3.880750E-01	2.500000E 00
2.232795E 00	5.432590E 00	3.196273E-01	4.478691E-01	2.433090E 00
1.814826E-01	4.548435E-01	1.830927E-01	5.510170E 00	2.506266E 00
1.814826E-01	4.548435E-01	1.830927E-01	5.510170E 00	2.506266E 00
1.143768E 00	2.691218E 00	1.396453E-01	8.743024E-01	2.352941E 00
2.441689E 00	5.600203E 00	2.405756E-01	4.095524E-01	2.293577E 00
1.752068E 00	4.360169E 00	2.202100E-01	5.707542E-01	2.500000E 00
2.052485E 00	4.973879E 00	3.326273E-01	4.872143E-01	2.433090E 00
1.504729E-02	2.341991E-03	1.962308E 00	6.645712E 01	1.556420E-01
1.504729E-02	2.341991E-03	1.962308E 00	6.645712E 01	1.556420E-01
8.211140E-03	4.827242E-03	1.308028E 00	1.217857E 02	5.878897E-01
5.226068E-03	3.159655E-02	1.359773E 00	1.913484E 02	6.045950E 00
1.816083E-02	1.921985E-01	1.799162E 00	5.506354E 01	1.058313E 01
1.893196E-02	1.336060E-01	1.369299E 00	5.282072E 01	7.057164E 00

APPENDIX D

Computer Model Output

I	K(I)	DH(I)	DV(I)	E(I)	F(I)	G(I)	P(I)	R(I,1)	DR(I)
1	11	1.0000	0.5906	0.1990	0.0510	0.3990	0.2083	0.1556	0.0
2	12	1.0000	0.5906	0.2990	0.0510	0.3990	0.0	0.1556	0.1250
3	9	1.0000	0.4921	0.3990	0.2100	0.4250	1.5000	0.5879	0.1250
4	9	3.0000	1.8373	0.4990	0.2800	0.4360	0.0010	6.0459	0.2500
5	200	10.0000	0.4921	0.5990	0.2290	0.4000	2.0686	10.5831	0.8125
6	8	10.0000	0.5879	0.6990	0.2000	0.4110	0.0100	7.0572	1.2500
7	2	10.0000	1.0000	0.7990	0.2000	0.4110	0.0100	7.0572	1.2500
8	10	10.0000	1.0000	0.8990	0.2000	0.4110	1.0000	7.0572	1.2500
9	0	3.0000	1.0000	0.9990	0.2000	0.4110	0.0500	7.0572	0.3125
10	6	1.0000	1.0000	1.0000	0.2000	0.4110	0.0	7.0572	0.2500
11	0	1.0000	1.0000	0.0	0.2000	0.4110	0.0001	7.0572	0.1250
12	5	1.0000	1.0000	0.0	0.0	0.0	2.0000	7.0572	0.1250
13	0	0.0	0.0	0.0	0.0	0.0	0.5000	0.0	0.0
14	0	0.0	0.0	0.0	0.0	0.0	0.1250	0.0	0.0
15	0	0.0	0.0	0.0	0.0	0.0	1.0000	0.0	0.0
16	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
28	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

H DRY I	DV(I)	A(I,1)	A(I,2)	A(I,3)	A(I,4)	A(I,5)
1	0.591	4.77785E-01	1.19746E 00	3.65447E-01	2.09299E 00	2.50627E 00
2	0.591	4.77785E-01	1.19746E 00	3.65447E-01	2.09299E 00	2.50627E 00
3	0.492	2.00665E 00	4.72153E 00	2.43265E-01	4.98342E-01	2.35294E 00
4	1.837	2.34940E 00	5.38853E 00	2.09481E-01	4.25640E-01	2.29358E 00
5	0.492	2.57284E 00	6.43211E 00	2.73034E-01	3.88675E-01	2.50000E 00
6	0.588	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
7	1.000	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
8	1.000	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
9	1.000	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
10	1.000	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
11	1.000	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
12	1.000	2.23279E 00	5.43259E 00	3.19627E-01	4.47869E-01	2.43309E 00
13	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0

H WET I	DV(I)	B(I,1)	B(I,2)	B(I,3)	B(I,4)	B(I,5)
1	0.591	1.81483E-01	4.54843E-01	1.83093E-01	5.51017E 00	2.50627E 00
2	0.591	1.81483E-01	4.54843E-01	1.83093E-01	5.51017E 00	2.50627E 00
3	0.492	1.14377E 00	2.69122E 00	1.39645E-01	8.74302E-01	2.35294E 00
4	1.837	2.44169E 00	5.60020E 00	2.40576E-01	4.09552E-01	2.29358E 00
5	0.492	1.75207E 00	4.38017E 00	2.20210E-01	5.70754E-01	2.50000E 00
6	0.588	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
7	1.000	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
8	1.000	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
9	1.000	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
10	1.000	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
11	1.000	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
12	1.000	2.05248E 00	4.99388E 00	3.32627E-01	4.87214E-01	2.43309E 00
13	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	-0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0

COND. I	CV(I)	C(I,1)	C(I,2)	C(I,3)	C(I,4)	C(I,5)
1	0.591	1.50473E-02	2.34199E-03	1.96231E 00	6.64571E 01	1.55642E-01
2	0.591	1.50473E-02	2.34199E-03	1.96231E 00	6.64571E 01	1.55642E-01
3	0.492	8.21114E-03	4.82724E-03	1.30803E 00	1.21786E 02	5.87890E-01
4	1.837	5.22606E-03	3.15965E-02	1.35977E 00	1.91348E 02	6.04595E 00
5	0.492	1.81608E-02	1.92198E-01	1.79916E 00	5.50635E 01	1.05831E 01
6	0.588	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
7	1.000	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
8	1.000	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
9	1.000	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
10	1.000	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
11	1.000	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
12	1.000	1.89320E-02	1.33606E-01	1.36930E 00	5.28207E 01	7.05716E 00
13	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0

```
LINEAR LN-LN REGRESSION ANALYSIS FORCED THRU POINT (X(1),Y(1))
SOLUTION IS Y = C(1)/(X**C(3)+C(2)) OR 1/Y = C(4)*X**C(3)+C(5)
WHERE V(I)=ALOG(1/Y(I)-1/Y(1)) AND U(I)=ALOG(X(I)-X(1)) IN
V = C(3)*U + ALOG(C(4))

PROBLEM NAME ...C-H B1           NUMBER OF OBSERVATIONS =    10
THE COEFFICIENTS ARE
C(1)=           2.006653
C(2)=           4.721535
C(3)=           0.243265
C(4)=           0.498342
C(5)=           2.352941
R2 (LN DATA)=  0.973835

                        TABLE OF RESIDUALS
 I       U(I)        V(I)        X(I)        Y(I)       YH(I)     RESIDUAL

 1                              0.0        0.42500    0.42500    0.0
 2    -0.17913   -0.85605   -0.83600    0.36000    0.35335    0.00665
 3     0.91988   -0.38955   -2.50900    0.33000    0.33599   -0.00599
 4     1.61303   -0.19643   -5.01800    0.31500    0.32355   -0.00855
 5     2.30628   -0.07715  -10.03700    0.30500    0.30995   -0.00495
 6     2.81708   -0.01980  -16.72800    0.30000    0.29924    0.00076
 7     3.51023    0.09106  -33.45599    0.29000    0.28381    0.00619
 8     4.20338    0.24953  -66.91199    0.27500    0.26749    0.00751
 9     4.89653    0.45020-133.82399    0.25500    0.25044    0.00456
10     6.21828    0.87920-501.83887    0.21000    0.21669   -0.00669

*******************************************

            ANALYSIS OF VARIANCE

SOURCE     DF        SS         MS          F

REGRESS     1      1.922      1.922     297.755 **
RESIDUAL    8      0.052      0.006
TOTAL       9      1.974

TABLE VALUEF(.05) =   5.31F(.01) = 11.25

*******************************************

STANDARD DEVIATION OF C(3) =            0.014

CONFIDENCE LIMITS OF C(3)   LOWER      UPPER
LEVEL         T VALUES      LIMIT      LIMIT

0.05          2.3049        0.211      0.276
0.01          3.3548        0.196      0.291

R (ORIGINAL DATA) =            0.989
R2 (ORIGINAL DATA)=            0.978
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LINEAR LN-LN REGRESSION ANALYSIS FORCED THRU POINT (X(1),Y(1))
SOLUTION IS Y = C(1)/(X**C(3)+C(2)) OR 1/Y = C(4)*X**C(3)+C(5)
WHERE V(I)=ALOG(1/Y(I)-1/Y(1)) AND U(I)=ALOG(X(I)-X(1)) IN
V = C(3)*U + ALOG(C(4))

PROBLEM NAME ...K-H B1            NUMBER OF OBSERVATIONS =      9
THE COEFFICIENTS ARE
C(1)=            0.008211
C(2)=            0.004827
C(3)=            1.308028
C(4)=          121.785736
C(5)=            0.587890
R2 (LN DATA)=  0.927139

                     TABLE OF RESIDUALS
I      U(I)        V(I)        X(I)        Y(I)       YH(I)     RESIDUAL

1                              0.0       1.70100    1.70100    0.0
2    3.75305    9.49014 -42.65099   0.00008    0.00006    0.00002
3    4.23263   10.63370 -68.89799   0.00002    0.00003   -0.00001
4    4.65384   10.43825-104.98700   0.00003    0.00002    0.00001
5    4.99476   11.52085-147.63799   0.00001    0.00001   -0.00000
6    5.33123   12.16743-206.69299   0.00001    0.00001   -0.00000
7    5.68791   12.32395-295.27588   0.00000    0.00000   -0.00000
8    5.97559   12.74085-393.70093   0.00000    0.00000   -0.00000
9    6.26327   12.59115-524.93384   0.00000    0.00000    0.00000

*******************************************

           ANALYSIS OF VARIANCE

SOURCE      DF        SS         MS          F

REGRESS      1      9.060      9.060     89.074 **
RESIDUAL     7      0.712      0.102
TOTAL        8      9.772

TABLE VALUEF(.05) =   5.59F(.01) = 12.28
*******************************************

STANDARD DEVIATION OF C(3) =            0.139

CONFIDENCE LIMITS OF C(3)   LOWER       UPPER
LEVEL       T VALUES        LIMIT       LIMIT

0.05         2.3649         0.980       1.636
0.01         3.5048         0.822       1.794

R (ORIGINAL DATA) =           0.950
R2 (ORIGINAL DATA)=           0.903
```

AFTER CHANGING WATER CONTENTS 0 TIMES, DELT IS 0.0100 AND SUMT = 0.0

NODE MOISTURE CONTENT

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990
2	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990
3	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250
4	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360
5	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
6	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
7	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
8	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
9	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
10	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
11	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

NODE WATER HEAD

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.5741	0.5734	0.5741	0.5757	0.5825	0.5892	0.5903	0.5904	0.5904	0.5904	0.5904	0.5904	0.5904
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.5741	0.5734	0.5741	0.5757	0.5825	0.5892	0.5903	0.5904	0.5904	0.5904	0.5904	0.5904	0.5904
4	0.9419	0.9359	0.9419	0.9562	1.0148	1.0709	1.0800	1.0813	1.0810	1.0808	1.0808	1.0809	1.0808
5	1.5938	1.5266	1.5938	1.7419	2.2990	2.8126	2.8963	2.9079	2.9051	2.9034	2.9036	2.9042	2.9036
6	1.7008	1.5842	1.7008	1.9240	2.6465	3.2806	3.3835	3.3978	3.3947	3.3927	3.3929	3.3936	3.3929
7	1.8899	1.6895	1.8899	2.2133	3.1044	3.8473	3.9674	3.9842	3.9811	3.9789	3.9791	3.9799	3.9791
8	2.1142	1.5509	2.1142	2.7157	3.9104	4.8163	4.9616	4.9322	4.9789	4.9765	4.9768	4.9776	4.9768
9	2.3003	0.2383	2.3003	3.3200	4.7639	5.7933	5.9575	5.9810	5.9778	5.9753	5.9756	5.9765	5.9756
10	3.4896	2.8585	3.4896	4.1937	5.6757	6.7793	6.9553	6.9805	6.9776	6.9751	6.9754	6.9763	6.9754
11	4.5960	4.2322	4.5960	5.1725	6.6468	7.7749	7.9549	7.9809	7.9783	7.9759	7.9761	7.9770	7.9761
12	3.4896	2.8585	3.4896	4.1937	5.6757	6.7793	6.9553	6.9805	6.9776	6.9751	6.9754	6.9763	6.9754

AFTER 48 SATURATED ITERATIONS THE ERROR IS = [illegible]

NODE POTENTIAL

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	5.9835	5.9828	5.9835	5.9851	5.9919	5.9986	5.9997	5.9998	5.9998	5.9998	5.9998	5.9998	5.9998
2	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
3	5.9835	5.9828	5.9835	5.9851	5.9919	5.9986	5.9997	5.9998	5.9998	5.9999	5.9998	5.9998	5.9998
4	5.8592	5.8532	5.8592	5.8735	5.9321	5.9882	5.9973	5.9986	5.9983	5.9981	5.9981	5.9982	5.9981
5	4.6738	4.6366	4.6738	4.8219	5.3790	5.8926	5.9763	5.9879	5.9851	5.9834	5.9836	5.9842	5.9836
6	4.2887	4.1721	4.2887	4.5119	5.2344	5.8685	5.9714	5.9857	5.9826	5.9806	5.9808	5.9815	5.9808
7	3.8899	3.6895	3.8899	4.2133	5.1044	5.8473	5.9674	5.9842	5.9811	5.9789	5.9791	5.9799	5.9791
8	3.1142	2.5509	3.1142	3.7157	4.9104	5.8163	5.9616	5.9822	5.9789	5.9765	5.9768	5.9776	5.9768
9	2.3003	0.2383	2.3003	3.3200	4.7639	5.7933	5.9575	5.9810	5.9778	5.9753	5.9756	5.9765	5.9756
10	2.4896	1.8635	2.4896	3.1937	4.6757	5.7793	5.9553	5.9805	5.9776	5.9751	5.9754	5.9763	5.9754
11	2.5960	2.2322	2.5960	3.1725	4.6468	5.7749	5.9549	5.9809	5.9783	5.9759	5.9761	5.9770	5.9761
12	2.4896	1.8685	2.4896	3.1937	4.6757	5.7793	5.9553	5.9805	5.9776	5.9751	5.9754	5.9763	5.9754

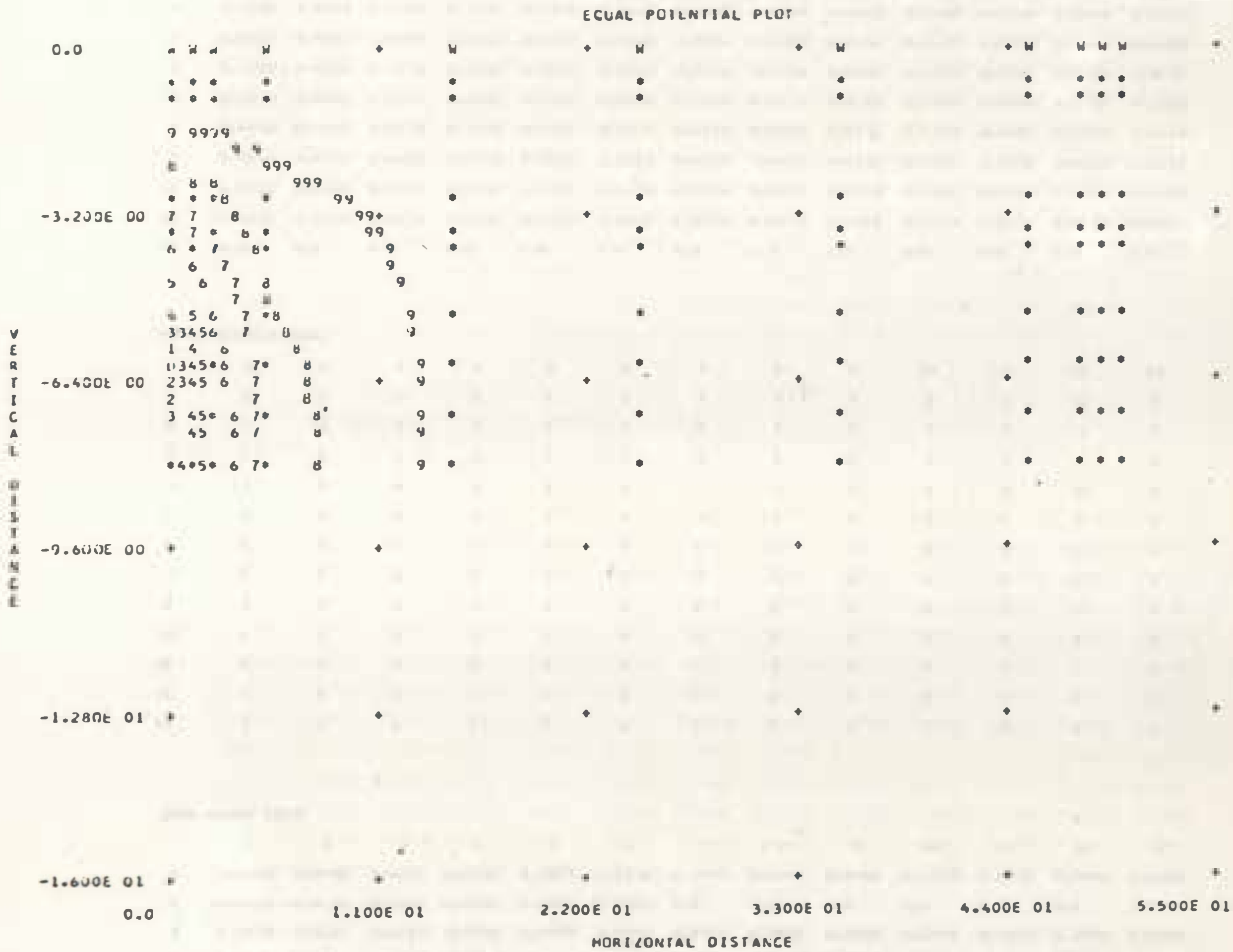
TOTAL AND FLOW THRU TOP 0.583 (L**3/T)

	2	3	4	5	6	7	8	9	10	11	12
2	0.0391	0.0753	0.1361	0.2394	0.0650	0.0151	0.0081	0.0064	0.0022	0.0011	0.0006

TILE FLOW = 1.122 (L**3/T)
FLOW DURING DELT = 0.0 (L**3)
DURING SUMT = 0.0 THE TOTAL FLOW = 0.0

WATER TABLE DEPTH IN THE Y DIRECTION (L)

	2	3	4	5	6	7	8	9	10	11	12
1	0.0	0.0	0.0	[illegible]	0.0	0.0	0.0	0.0	0.0	0.0	0.0


ECUAL POTENTIAL PLOT
0.0
-3.200E 00
-6.400E 00
-9.600E 00
-1.280E 01
-1.600E 01
VERTICAL DISTANCE
0.0
1.100E 01
2.200E 01
3.300E 01
4.400E 01
5.500E 01
HORIZONTAL DISTANCE

NPNP = 0

AFTER CHANGING WATER CONTENTS 10 TIMES, DELT IS 0.0100 AND SUMT = 0.1000

NODE MOISTURE CONTENT

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.3990	0.3735	0.3990	0.3730	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990	0.3990
2	0.3990	0.3546	0.3562	0.3561	0.3779	0.3954	0.3982	0.3987	0.3987	0.3986	0.3986	0.3986	0.3970
3	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250	0.4250
4	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360	0.4360
5	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
6	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
7	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
8	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
9	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
10	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
11	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110	0.4110
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

NODE MOISTURE CODE

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1

NODE WATER HEAD

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.5692	0.5580	0.5692	0.5720	0.5819	0.5892	0.5903	0.5905	0.5906	0.5905	0.5905	0.5905	0.5905
2	-0.0050	-0.0056	-0.0050	-0.0037	-0.0006	-0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.5693	0.5581	0.5693	0.5720	0.5819	0.5892	0.5904	0.5905	0.5906	0.5905	0.5905	0.5905	0.5905
4	0.9378	0.9315	0.9378	0.9527	1.0144	1.0714	1.0807	1.0822	1.0823	1.0823	1.0823	1.0823	1.0823
5	1.5932	1.5259	1.5932	1.7415	2.3005	2.8168	2.9021	2.9155	2.9167	2.9165	2.9166	2.9167	2.9166
6	1.7007	1.5541	1.7007	1.9242	2.6435	3.2855	3.3962	3.4067	3.4083	3.4080	3.4081	3.4082	3.4081
7	1.8903	1.6498	1.8903	2.2138	3.1065	3.8525	3.9746	3.9939	3.9958	3.9955	3.9956	3.9957	3.9956
8	2.1149	1.5513	2.1149	2.7167	3.9128	4.8221	4.9595	4.9928	4.9952	4.9950	4.9950	4.9952	4.9950
9	2.3012	0.2583	2.3012	3.3213	4.7664	5.7493	5.9559	5.9921	5.9949	5.9947	5.9947	5.9949	5.9947
10	3.4907	2.8695	3.4907	4.1951	5.6782	6.7853	6.9635	6.9917	6.9948	6.9946	6.9947	6.9949	6.9947
11	4.5971	4.2933	4.5971	5.1738	6.6491	7.7806	7.9528	7.9916	7.9949	7.9947	7.9949	7.9950	7.9949
12	3.4907	2.8695	3.4907	4.1951	5.6782	6.7853	6.9635	6.9917	6.9948	6.9946	6.9947	6.9944	6.9942

NODE RESISTIVITIES

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556
2	0.1576	0.1581	0.1576	0.1568	0.1557	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556	0.1556
3	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879	0.5879
4	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459	6.0459
5	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831	10.5831
6	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572
7	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572
8	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572
9	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572
10	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572
11	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572
12	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572	7.0572

AFTER 1 SATURATED ITERATIONS THE ERROR IS = 0.018

NODE POTENTIAL

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	5.9786	5.9774	5.9786	5.9414	5.9913	5.9986	5.9997	5.9999	6.0000	5.9999	5.9999	5.9999	5.9999
2	5.9450	5.9744	5.9450	5.9963	5.9934	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
3	5.9787	5.9775	5.9787	5.9914	5.9913	5.9986	5.9993	5.9999	6.0000	5.9999	5.9999	5.9999	5.9999
4	5.8551	5.8488	5.8551	5.8700	5.9317	5.9887	5.9980	5.9995	5.9976	5.9996	5.9976	5.9996	5.9996
5	4.6732	4.6659	4.6732	4.8215	5.3605	5.8968	5.9821	5.9955	5.9967	5.9965	5.9966	5.9967	5.9966
6	4.2886	4.1720	4.2886	4.5121	5.2364	5.8734	5.9781	5.9946	5.9962	5.9959	5.9960	5.9961	5.9960
7	3.8903	3.6698	3.8963	4.2135	5.1065	5.8525	5.9746	5.9939	5.9958	5.9955	5.9956	5.9957	5.9956
8	3.1149	2.5513	3.1149	3.7167	4.9128	5.8221	5.9595	5.9928	5.9952	5.9950	5.9950	5.9952	5.9950
9	2.3012	0.2363	2.3012	3.3213	4.7664	5.7993	5.9558	5.9921	5.9949	5.9947	5.9947	5.9949	5.9947
10	2.4907	1.8595	2.4907	3.1951	4.6782	5.7853	5.9635	5.9917	5.9943	5.9946	5.9947	5.9948	5.9947
11	2.5971	2.2333	2.5971	3.1733	4.6491	5.7806	5.9628	5.9416	5.9949	5.9947	5.9948	5.9950	5.9948
12	2.4907	1.8595	2.4907	3.1951	4.6782	5.7853	5.9635	5.9917	5.9949	5.9946	5.9947	5.9948	5.9942

TOTAL AND FLOW THRU TOP 0.566 (L**3/T)

	2	3	4	5	6	7	8	9	10	11	12
2	0.0384	0.0743	0.1355	0.2392	0.0621	0.0112	0.0030	0.0013	0.0005	0.0002	0.0001

TILE FLOW = 1.122 (L**3/T)
FLOW DURING DELT = 0.011 (L**3)
CURING SUMT = 0.1CC THE TOTAL FLOW = 0.112

WATER TABLE DEPTH IN THE V DIRECTION (L)

	2	3	4	5	6	7	8	9	10	11	12
1	0.0057	0.0351	0.0038	0.0005	0.0000	0.0	0.0	0.0	0.0	0.0	0.0

APPENDIX E

Characteristic Curves for a Cecil Loamy Sand

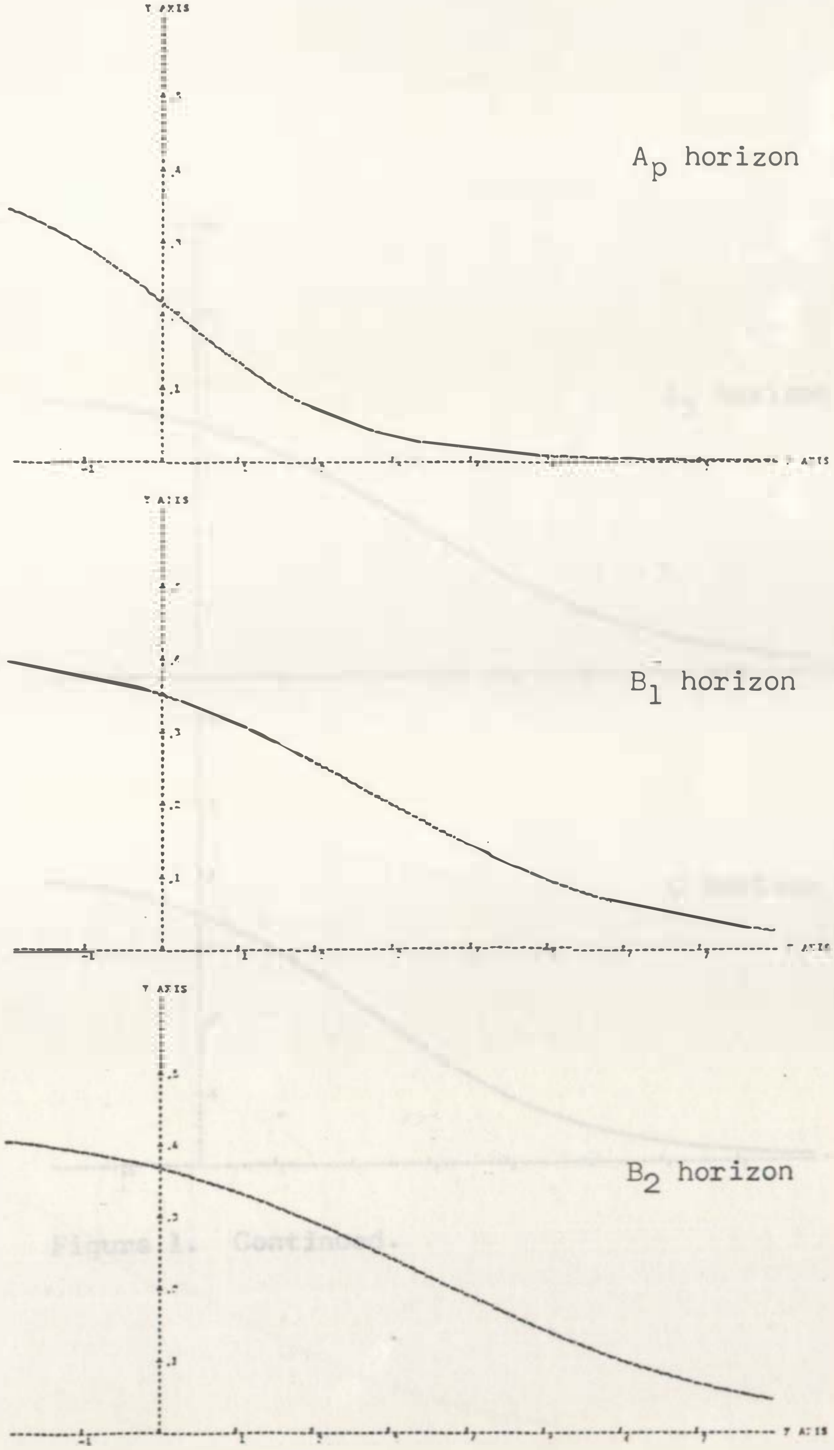


Figure 1. Desorption curves relating soil moisture content (y axis) to the log of the $\log_{10}$ of the absolute value water potential, feet (x axis).

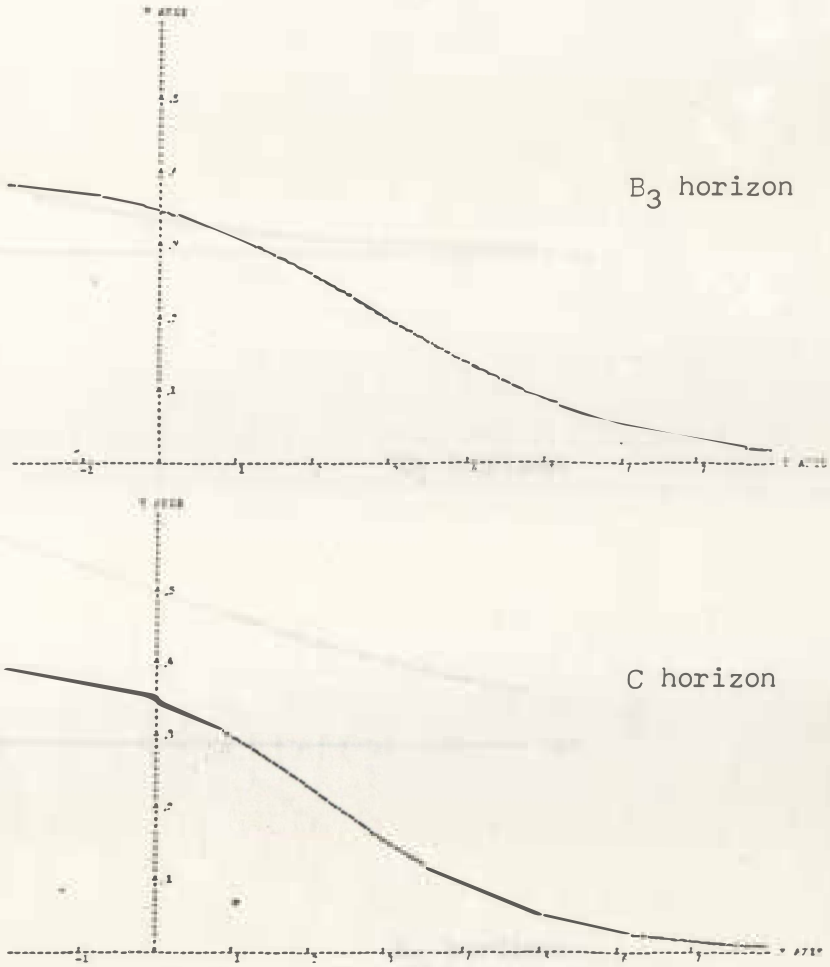


Figure 1. Continued.

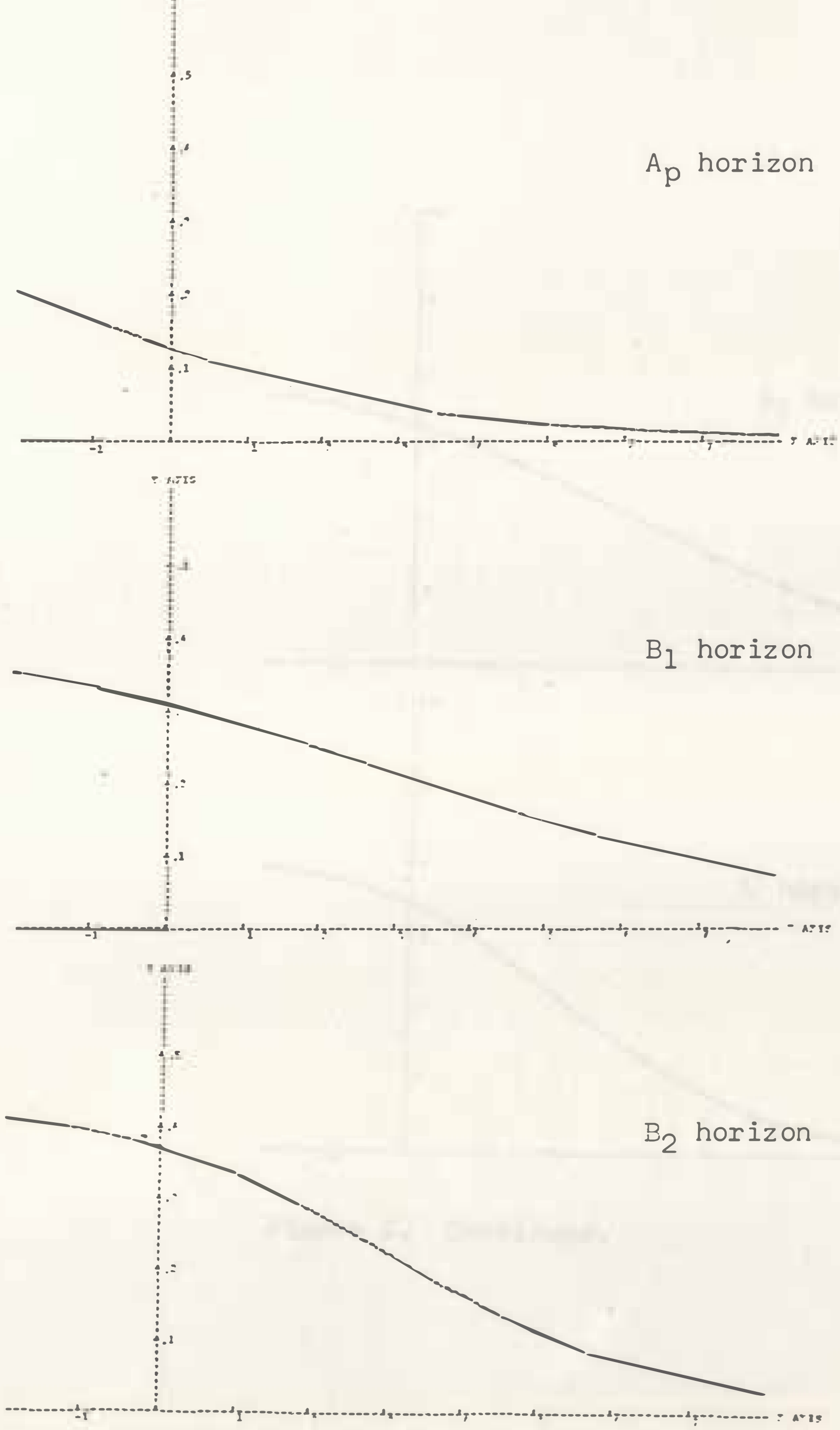


Figure 2. Adsorption curves relating soil moisture content (y axis) to the $\log_{10}$ of the absolute value of the water potential, feet (x axis).

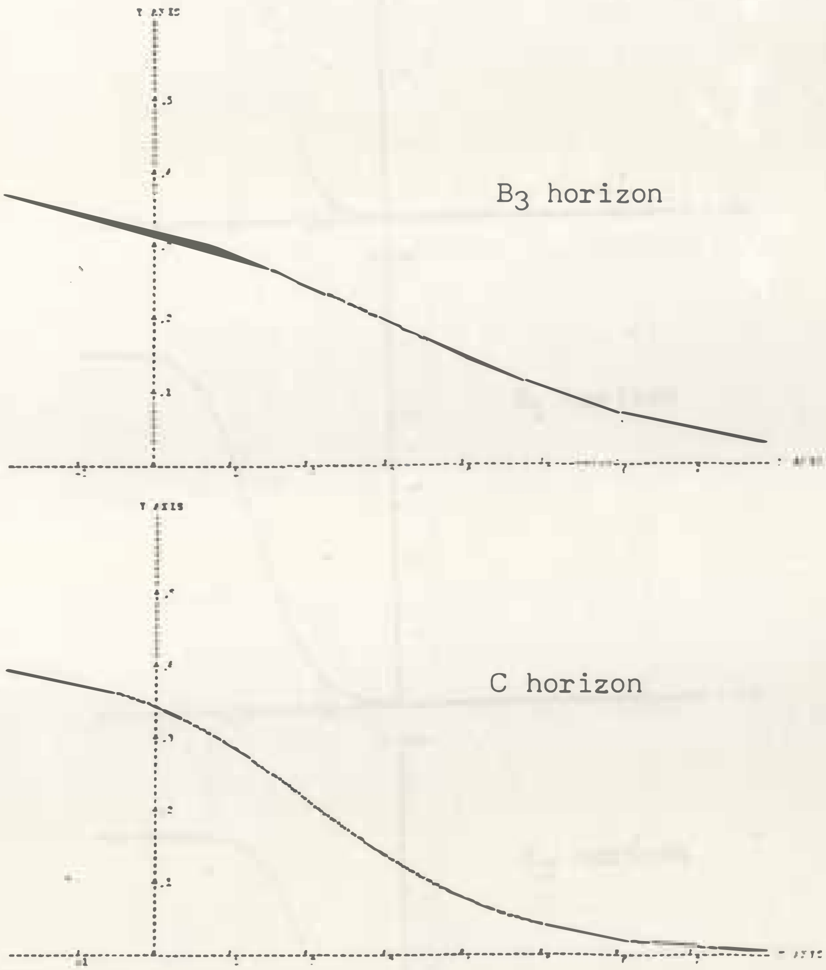


Figure 2. Continued.

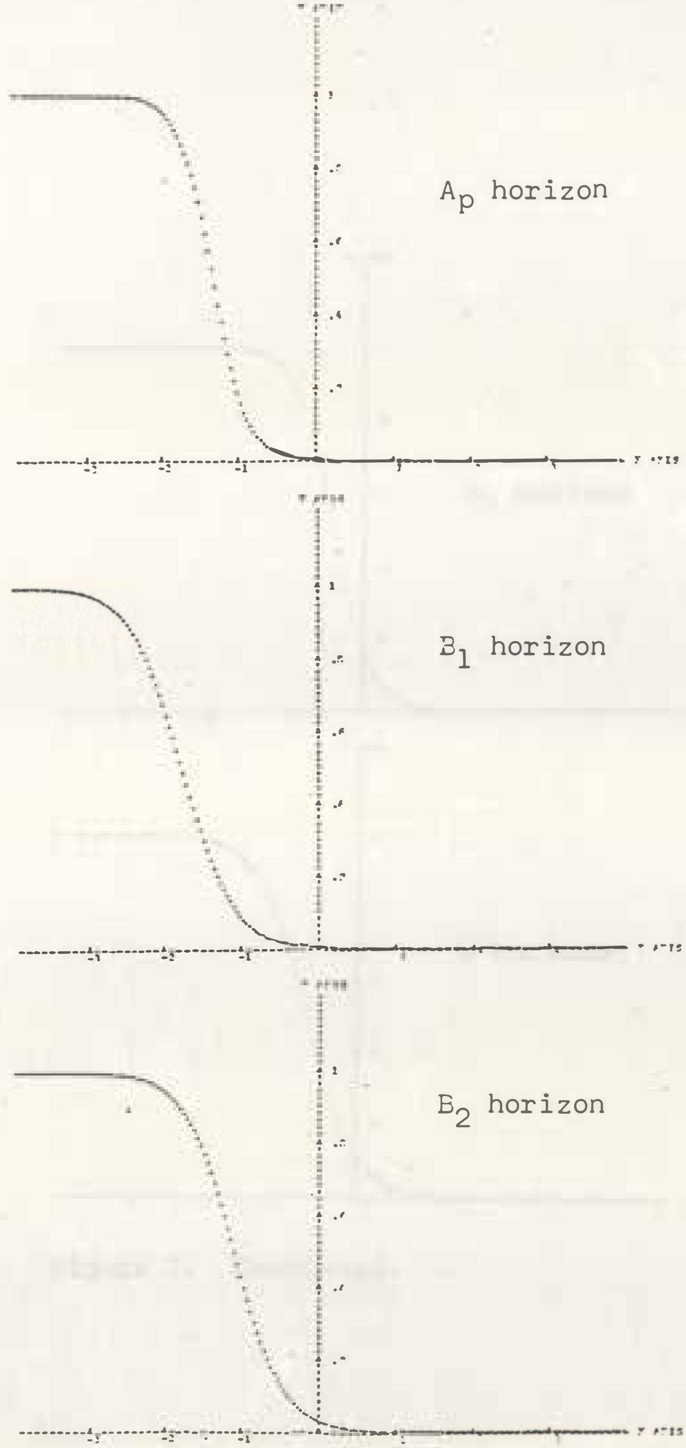


Figure 3. Ratio of the hydraulic conductivity to saturated hydraulic conductivity (y axis) to the $\log_{10}$ of the absolute value of the water potential, feet (x axis).

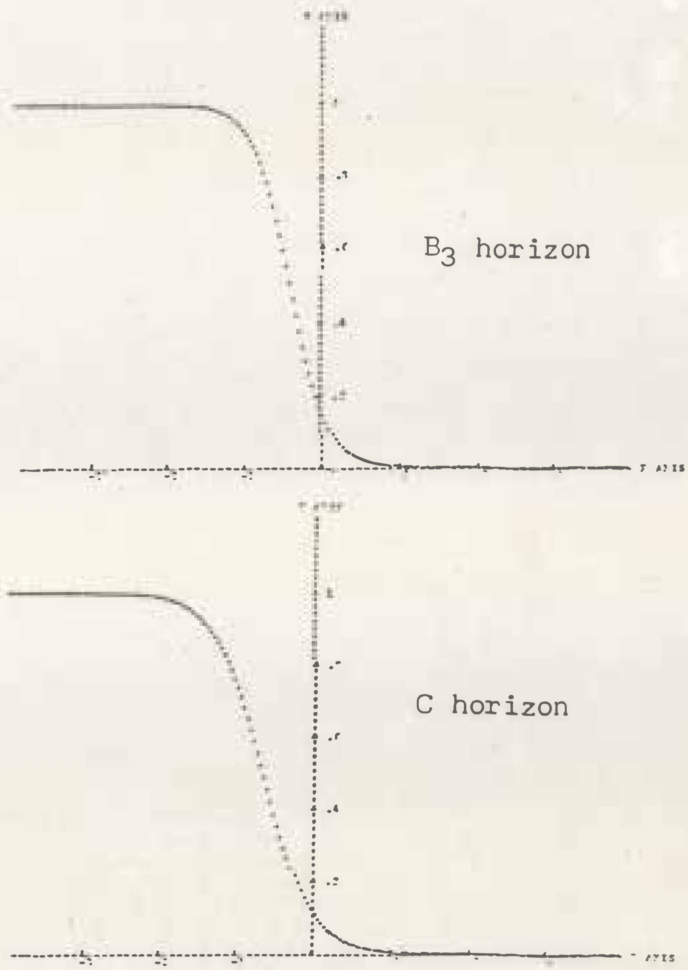


Figure 3. Continued.