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COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS
PERTAINING TO R-CURVES IN LOW AND
HIGH DUCTILITY SOLIDS

By

RABINDRA NATH DASH

A thesis submitted
in partial fulfillment of the requirements for the
degree of Master of Science, Major in
Mechanical Engineering, South Dakota
State University
1978

COMPARISON OF THEORETICAL AND EXPERIMENTAL
RESULTS PERTAINING TO R-CURVES IN
LOW AND HIGH DUCTILITY SOLIDS

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Dr. M. P. Wnuk
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RND

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS
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HIGH DUCTILITY SOLIDS

Abstract

RABINDRA NATH DASH

Existing linear elastic fracture mechanics and J-integral analyses are well suited for safety assessments of high-strength, low-toughness materials. These analyses apply only to the onset of crack growth, which is usually tantamount to crack instability and structural failure in that class of materials. However, this is not the case for the low-strength, high-toughness materials in which crack instability may be preceded by extensive stable crack growth under rising loads. Here, a substantial margin of safety may exist even when the onset of crack growth is imminent.

This thesis describes research leading to a ductile fracture-mechanics methodology designed to treat two-dimensional, large-scale yielding and stable crack growth problems. The linear plasticity model of moving cracks (Wnuk, 1972-1978) is used to obtain predictions concerning the material toughness associated with the preliminary crack extension (R-curves), and to calculate the critical parameters, e.g., the load and crack size at which a transition to unstable brittle-like fractures will occur. One important finding of this work is that parameters truly reflecting the state of the crack-tip process zone are not functions of the extent of

NOTATION

$B, B_n, \phi_0, \phi'_0, m'$ = Shape factors

E = Young's Modulus

J_i = Initiation level of the J-integral

J_R = Toughness parameter (Rice's integral) in the sub-critical range

K = Toughness parameter taken as the stress intensity factor

K_f = Stress intensity factor attained at the point of terminal fracture

K_R = Toughness parameter (or stress intensity factor) in the subcritical crack growth range

P = Load

Q = Dimensionless load parameter

Q_f = Loading parameter at the instability point

R = Material resistance parameter taken as the extent of the yielded zone

R_f = Resistance at terminal fracture

R_R = Toughness parameter (length of the yielded zone) in the subcritical crack growth range

T_δ = Tearing modulus proportional to the crack tip opening angle, δ/Δ

X = Dimensionless crack length, $X = a/R_\infty$

X_0 = Non-dimensional initial crack size

Y_f = Non-dimensional resistance level at the critical point

Y_0 = Ductility parameter, a measure of the non-dimensional threshold toughness

Y_∞ = Non-dimensional, steady state limit of the material toughness

a = Crack length

a_f = crack length at the transition from stable to unstable propagation

b = Width of the plate

η = Ratio of the effective yield stress at the crack tip to the uniaxial yield stress, $\eta = \sigma_Y / \sigma_{Ys}$

λ_i = Initial stability index

σ_u = Ultimate stress

σ_Y = Yield stress encountered at the crack front

σ_{Ys} = Uniaxial yield stress

Δ = Process zone size

ϵ_f^p = Plastic Component of strain at fracture

ϵ_Y = Strain at yield

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CHAPTER 1

INTRODUCTION

Let us define the basic quantities pertinent in the fracture mechanics analyses developed in this thesis.

Crack growth-resistance can be measured by either one of the following parameters:

(a) Material toughness, K_R , is a quantity which encompasses the critical crack size, a_c , and the critical load, F_c . Dimensions of K_R are $\text{ksi} \sqrt{\text{in}}$ in the customary United States units, and $(\text{Pa} \sqrt{\text{m}})$, in the metric system.

(b) Stress intensity factor, K , defined by the relation

$$K = \lim_{r \rightarrow 0} \{ \sigma_y(r, \theta) \sqrt{2\pi r} \} \quad (\text{see Figure BB})$$

is a quantity which measures the increase in stress at the crack tip above that away from the crack. It encompasses such variables as the crack size, load and the dimensions of the specimen. The elastic-plastic stress field in the immediate vicinity of the crack tip can be described by two parameters K and J , as shown in Figure AA.

(c) J -integral, J or J_R , is a path-independent contour integral defined by Rice (1968) as follows

$$J = \int_{\Gamma} (U dy - T_i \frac{\partial u_i}{\partial x} ds)$$

Here U denotes the strain energy density, T_i are the tensile stress values at the points, i , chosen for evaluation along the contour Γ , and u_i are the displacements at the i points while ds is an increment of arc length measured along

the contour of integration (the contour Γ and coordinates x and y are shown in Figure BB).

(d) The extent of the plastic zone around the crack tip, R or R_R is a quantity shown in Figures AA and BB. a_0 is initial crack length before load is applied. a is the variable crack length after the load is applied, and

$$\Delta a = a - a_0$$

A plot of any of the parameters listed above against the extent of slow crack growth Δa , is referred to as an "R-curve", or "resistance curve". The quantities like load F and crack length " a " are customarily non-dimensionalized as follows:

- load is divided by the cross section area of the specimen A to give the nominal stress $\sigma = F/A$. This stress, in turn, is divided by the effective yield stress σ_Y encountered at the crack tip, and multiplied by the factor $\pi/2$, to give the dimensionless loading parameter, Q , (used in the plot shown in Figure A), i.e.,

$$Q = \pi \sigma / 2 \sigma_Y$$

- crack length " a " can be nondimensionalized in two different ways, namely

$$X = a/\Delta \quad \text{or} \quad x = a/R_{ss}$$

The linear dimension Δ is a microstructural constant dependent on the metallurgical processes involved in manufacturing a given alloy. (Δ is sometimes called "the process zone size"). The final act of decohesion (fracture is believed

to occur in the volume of the process zone. The quantity R_{ss} (or R_{∞}) corresponds to the steady-state extent of the plastic zone around the crack. R_{∞} is the upper limit of R beyond which the material fractures. It is rarely achieved in actual structures because they are designed to avoid this event. It is a useful quantity, though, in theoretical considerations aimed at predicting the occurrence of the stable/unstable transition of fracture mode in ductile metals.

Similarly, the extent of the plastic zone R is normalized as follows

$$Y = R/\Delta \quad \text{or} \quad y = R/R_{ss}$$

Quantities y and Y are nondimensional measures of material resistance to ductile fracture.

(e) Tearing modulus is defined as the ratio of the crack opening displacement δ (given as $\hat{\delta}$ in some papers), shown in Figure BB, and the crack tip process zone size Δ , i.e.,

$$\text{tearing modulus} = \hat{\delta} / \Delta$$

Equivalent definitions of the tearing modulus used on pages 38 and 47 are

$$\hat{R}/\Delta = (\pi E/8 \sigma_Y) (\hat{\delta}/\Delta)$$

$$n = \hat{R}/\Delta - \frac{1}{2} \log (2 e)$$

The physical meaning of the tearing modulus in the fracture theory is analogous to that of the strain hardening exponent used for the description of the deformation process.

The three modes of fracture (I, II, III) are shown in Figure CC.

The defect size is thus defined in this thesis as the initial crack length.

The fracture toughness of material varies from the plain-stress condition to the plain-strain condition. In the plain-strain condition, it is described by a single value, whereas this is not so in the plain-stress condition. In the plain-stress fracture, cracking occurs slowly so that fracture instability depends on the crack growth resistance, K_R , and the characteristics of the structure containing the crack. The relationship between K_R and crack length, a , has been studied by several authors, e.g. E. J. Ripling (1970), Cherepanov (1968), M. P. Wnuk (1971-1978).

The differential equations relating to the crack growth resistance and crack length have been established by Wnuk for plates in tension, and for thick-walled cylindrical pressure vessels weakened by a crack originating from the inner surface. It has been shown that the point of instability occurs in most cases when the stress intensity factor at the crack tip begins to increase more rapidly than the crack growth resistance, K_R .

In the work presented in this thesis, the differential equations describing material resistance in the subcritical crack growth range established by Cherepanov and Wnuk, have been integrated, and the values of fracture resistance measured by either R or K_R have been compared with experi-

mental data on crack propagation available in the literature. Experimental data were taken from Ripling (1970), Wilhem and FitzGerald (1975), Ratwani and Wilhem (1975, 1977), Fattorini, Mirabile, Podrini, and Spadaletti (1978), Ripling and Falkenstein (1970), the Armco Company, Griffis and Yoder (1976), and Creager (1973).

Existing linear elastic fracture mechanics and J-integral analyses are well suited for safety assessments of high-strength, low-toughness materials. These analyses apply only to the onset of crack growth, which is usually tantamount to crack instability and structural failure in that class of materials. However, this is not the case for the low-strength, high-toughness grades such as those used for nuclear reactor pressure vessel, see Table A, in which crack instability may be preceded by extensive stable crack growth under rising load. Here, a substantial margin of safety may exist even when the onset of crack growth is imminent.

In the cases considered in this thesis, all three ranges of material behavior, brittle, semibrittle, and ductile, are encountered successively, see Figure A. The crack initiates in the small-scale yielding range at $R/a \ll 1$, then continues to grow in a stable manner until the R/a ratio rises to values around (or near) one (intermediate range). Finally, the large-scale yielding range is entered. Since the final stage of the stable cracking process occurs at every increasing

R/a values, the terminal instability occurs in the near-yield or the post-yield range.

The upper toughness limit (R_{∞}) predicted for the small-scale yielding situation may never materialize in an experiment involving specimen sizes too small to ensure a containment of plasticity effects (indeed, very few experimentalists have ever seen the upper plateau for R_{ss} or J predicted from the small-scale yielding solution). The theory presented here allows one to extend the limits of applicability of the earlier solutions (of Wnuk (1972), Rice and Sorensen (1978)) valid within the contained plasticity range.

For large structures, however, such as nuclear reactor pressure vessels, there exists the interesting possibility of tapping the reserve strength that the materials possess in their inelastic region. This becomes apparent upon an examination of graphs shown in Figure B. Three fracture loci i.e., the initiation, terminal and the upper-plateau, steady-state limit of fracture toughness are shown. The domain contained between the lower and upper lines depicted in Figure A may serve as a measure of the unused portion of the theoretical strength of a ductile material. It is noteworthy that while the two extreme lines in Figure A represents material properties pertinent to fracture initiation and the maximum strength, respectively, the middle line can be shifted up and down depending on geometry and loading configuration

of a structural component. Therefore, it is in this region that design and materials science are expected to interact.

One important finding of this work is that parameters truly reflecting the state of the crack tip process zone are not functions of the extent of stable crack growth when the mode of fracture (full shear or flat) remains fixed. The possibility exists, therefore, that useful, stable growth parameters can be evaluated from the state of the crack tip at the onset of crack extension.

TABLE A

Crack growth parameters for specific pressure vessel steels

Material	δ_{Tc} (mm)	$\hat{\delta}/\Delta$	σ_0/L	J_{ss}/J_{IC}	T_{mi}
A533B	0.19	0.74	0.0024	8×10^{30}	555
A533B	0.18	0.20	0.0026	2×10^5	94.5
HY-80	0.125	0.39	0.0035	5×10^9	174
HY-80	0.045	0.12	0.0046	3.6	10.0
HY-130	0.06	0.13	0.0045	7.1	15.3

Fig. 1. Schematic of a thick-walled cylinder with a crack on the inner surface.

Fractured pressure vessel

Subcritical crack

Unstable fracture

Loading Parameter
 Q

Fig. A Phases of crack development in a thick
walled pressure vessel

Fig. B Three ranges of crack growth
 I No growth
 II Stable (or subcritical) growth
 III Unstable growth

Figure AA Schematic Representation of the Stress Fields of Cracked Bodies: (a) and (b) Linear Elastic Behavior Where LEFM is Applicable (c), Large-Scale Yielding Where J-Integral Analysis is Applicable, and (d) Crack Growth Accompanied by Large-Scale Yielding where no Current Analysis Techniques are Applicable.

Fig. BB Schematic representation of the elastic-plastic stress field present at the crack tip.

Fig. CC Modes of crack tip deformation field
(arrows indicate loads)

- I Opening Mode
- II In-plane shear Mode
- III Anti-plane shear Mode

CHAPTER 2

R-CURVE CONCEPT AND INSTABILITY CONDITIONS

The plastic-zone size for monotonic loading has been determined for edge and internal cracks in finite width plates in uniaxial tension at all stress levels up to general yielding. Also, for small-scale yielding, the plastic zone size may be symbolized in terms of the elastic stress intensity factor, or in terms of loading parameter, Q , and the current crack length, a , as

$$R = R(Q, a) \quad (2.1)$$

While the mathematical form of the strain singularities may be determined for extending (moving) crack, no complete solutions have been obtained. It is easy to show that the elastic/plastic boundary for monotonic loading cannot be the boundary for and extending crack under the same load, and at the same crack length. Thus, the strain-singularity equation serves only as an estimate of the extent of yielding in front of an extending crack. If a_0 is the initial crack length, then the crack extension is initiated at a value of the applied load satisfying the relation

$$R(Q, a_0) = [R_R(\Delta a)]_{a=0}$$

Here the symbol $R_R(\Delta a)$ denotes the material resistance to fracture which turns out to be a function of the amount of the true surface separation $\Delta a = a - a_0$.

Stable crack extension begins under an increasing load

so that, when the crack has extended an amount Δa , the load is given by

$$R(Q, a_0 + \Delta a) = R_R(\Delta a) \quad (2.2)$$

Differentiating both sides with respect to Δa leads to

$$\frac{\partial R(Q, a_0 + \Delta a)}{\partial Q} \frac{dQ}{da} + \frac{\partial R(Q, a_0 + \Delta a)}{\partial a} = \frac{dR_R(\Delta a)}{da} \quad (2.3)$$

and the instability point is reached when no further load increase is required to maintain the crack extension, $\frac{dQ}{da} = 0$. At the instability point, equation (2.3) reduces to

$$\frac{\partial R(Q, a_0 + \Delta a)}{\partial a} = \frac{dR_R(\Delta a)}{da} \quad (2.4)$$

The load at the instability, and the amount of prior stable crack extension, are given by the simultaneous solution of equation (2.4) and equation (2.2).

The condition at instability could also be expressed in terms of variable fracture toughness, $K_R = K_R(\Delta a)$. For a small-scale yielding situation, there exists a simple, unique relationship between the resistance $R_R(\Delta a)$ and the fracture toughness $K_R(\Delta a)$, Rice (1965)

$$R_R(\Delta a) = \frac{\pi K_R^2(\Delta a)}{8 \sigma_{ys}^2} \quad (2.5)$$

in which Δa denotes the increment of crack length generated during the slow, stable-growth process, and σ_{ys} is the uni-axial yield stress. The stress intensity factor is a function of loading parameter, Q , and the crack half length " a "

$$K = K(a, Q(a)) \quad (2.6)$$

in which Q is an unknown function of a . The slope $\frac{dK}{da}$ should be computed as follows

$$\frac{dK}{da} = \frac{\partial K}{\partial a} + \frac{\partial K}{\partial Q} \frac{dQ}{da} \quad (2.7)$$

This slope which reflects the state of stress near the crack tip should be compared with the material resistance measured here by the slope of the K_R vs. a curve. At instability, both slopes become equal (note that $a = \Delta a + a_0$, while $\partial / \partial a = \partial / \partial (\Delta a)$):

$$\frac{dK(a_0 + \Delta a, Q(a_0 + \Delta a))}{d(\Delta a)} = \frac{dK_R(\Delta a)}{d(\Delta a)} \quad (2.8)$$

Recalling that dQ/da vanishes at the point of instability, we obtain the condition analogous to equation (2.4)

$$\frac{\partial K(Q, a_0 + \Delta a)}{\partial a} = \frac{dK_R(\Delta a)}{da} \quad (2.9)$$

This equation, and also the requirement of equality between the stress intensity factor and fracture toughness according to Irwin's criterion for failure:

$$K(Q, a_0 + \Delta a) = K_R(\Delta a) \quad (2.10)$$

supply all the information necessary to evaluate the parameters pertinent to the instability point. The usual parameters needed for such a description are given by one of the following pairs.

1) (K_f, a_f)

or

2) (Q_f, a_f)

or

3) (R_f, a_f)

The symbol a_f is used to denote the crack length at the transition from stable to unstable propagation, while K_f denotes the stress intensity factor attained at fracture; Q_f is the load at fracture, and R_f is the resistance (as defined by equation (2.5) at fracture).

Figure 1 describes the plastic-zone size, $R(a)$, Rice (1968b), for a quasistatic crack. It is a universal curve for a given material under the condition of small-scale yielding. The initiation and steady-state extension values are labeled as R_0 and R_∞ respectively. The family of lines originating at the origin ($\Delta a = -a_0$) represents the variation of the plastic-zone size with crack length at fixed values of the applied load Q . It is seen that the instability load satisfying the equations discussed above corresponds to the member of this family which tangentially touches the R curve (i.e., $R_R = R_R(a)$) for quasistatic extension. The amount of stable crack growth, and the final resistance level attained at the end of stable growth phase, are determined by the point of tangential contact, see Figure 1. It is seen that the resistance to failure attained in a fracture test which involves the slow stable phase of crack propagation is bounded by two material properties

$$R_0 \leq R_f \leq R \quad (2.11)$$

Therefore, it may be concluded that although both lower and upper bounds on the material toughness encountered at the instability point are material constants, the actual value of R at fracture (i.e. R_f) is not a material property. R has to be determined for a given geometrical and loading configuration from the equations (2.2) and (2.4) or, alternatively, from the set of equations (2.9) and (2.10).

Before these equations may be applied, though some additional information concerning the shape of the material R -curves (either R_R vs. a , or K_R vs. a curves) is required. Both these curves can be derived from the first principles of continuum mechanics combined with the theory of non-linear fracture mechanics. For this purpose certain non-linear differential equations of the general form have been derived by Wnuk (1972, 1973, 1974, 1977)

$$F(R_R, dR_R/da, \Delta a) = 0 \quad (2.12)$$

or

$$G(K_R, dK_R/da, \Delta a) = 0 \quad (2.13)$$

CHAPTER 3

DIFFERENTIAL EQUATIONS GOVERNING THE FRACTURE RESISTANCE DURING THE SLOW, STABLE CRACK-GROWTH STAGE

3.1 R vs. Δa Resistance Relation

The following differential equations governing crack-growth resistance within restrictions of the small-scale yielding condition are given by Cherepanov (1968) and Wnuk (1972) respectively

$$\frac{dR_R}{da} = \left\{ \begin{array}{l} \frac{3}{2} \frac{R_\infty - R_R}{R_R} \end{array} \right. \quad (3.1a)$$

$$\frac{1}{2} \log(R_\infty/R_R) \quad (3.1b)$$

These two equations can be transformed into the (K_R, a) plane. Assuming the small-scale yielding solution, which is valid for an arbitrary geometry, given by Wnuk (1972).

$$R_R = \frac{\pi K_R^2}{8 \sigma_{ys}^2} \quad (3.2)$$

we calculate the differential

$$dR_R = (\pi/8 \sigma_{ys}^2) 2K_R dK_R \quad (3.3)$$

Equations (3.2) and (3.3) combined with equations (3.1) yield

$$\frac{dK_R}{da} = \begin{cases} \frac{6\sigma_{ys}^2}{\pi} \frac{K_\infty^2 - K_R^2}{K_R^3} & (3.4a) \\ \frac{4\sigma_{ys}^2}{\pi K_R} \log(K_\infty/K_R) & (3.4b) \end{cases}$$

Before we proceed to solve (numerically) the non-linear differential equations given above, let us consider some closed-form solutions which can be obtained for an asymptotic case of near-critical crack as R_R approaches R . The physical meaning of such solutions is limited to the quasi-brittle solids which exhibits only a negligible amount of stable cracking. Nevertheless, it is of interest to study the solutions resulting for such a limiting case from equations (3.1) to (3.4).

Consider the difference

$$\mathcal{E} = 1 - R_R/R_\infty \quad (3.5)$$

to be a small quantity. Then the second of equations (3.1) can be linearized as follows

$$\log(R_\infty/R_R) = -\log(1-\mathcal{E}) = \mathcal{E} + \dots = 1 - R_R/R_\infty \quad (3.6)$$

The right hand side of equation (3.1b) becomes linear, if higher-order terms are neglected.

$$\frac{dR_R}{da} = \mathcal{E} = \frac{1}{2} \frac{R_\infty - R_R}{R_\infty} \quad (3.7)$$

This form can be integrated as follows

$$R_R(\Delta a) = R_\infty - (R_\infty - R_0) \exp - \frac{3(a - a_0)}{2 R_\infty} \quad (3.8)$$

which is identical to the solution of Cherepanov's equation (3.1a)

$$R_R(\Delta a) = R_\infty - (R_\infty - R_0) \exp \left\{ - \frac{3(a - a_0)}{2 R_\infty} \right\} \quad (3.9)$$

This result will be used as the R vs. (Δa) relation later.

3.2 K_R vs. Δa Resistance Relation

Equation (3.8) can be re-written in terms of the stress intensity factor K_R as

$$K_R(\Delta a) = \left\{ K_\infty^2 - (K_\infty^2 - K_0^2) \exp \left[- \frac{4\sigma_{ys}^2}{\pi K_\infty} (a - a_0) \right] \right\}^{\frac{1}{2}} \quad (3.10)$$

Figure 2 illustrates the accuracy of the linearized solution as compared with the numerical solutions of the non-linearized equation (the graphs are obtained for various initial conditions as indicated in Figure 2).

Let us now consider the transformation of equation (3.1a) into the domain of the variables loading parameter and crack length, (Q, a) . To this end we shall need to specify the geometrical configuration of the test piece in question.

Consider a central crack placed in an infinite plate. The small-scale yielding solution for such a geometry predicts the following relation between the length of the plastic zone, R , and crack size, a . (See Figure 1)

$$R = (1/2)Q^2 a, \quad Q = \pi \sigma_{ys} / 2 \quad (3.11)$$

Since in the subcritical range, the loading parameter, Q , is also a function of a , $Q = Q(a)$, we have

$$\frac{dR}{da} = \frac{1}{2} \left\{ 2Qa \frac{dQ}{da} + Q^2 \right\} \quad (3.12)$$

Substituted into equation (3.1b), this yields

$$\frac{dQ}{da} = \frac{\log (2R_{\infty}/aQ^2) - Q^2}{2 Qa} \quad (3.13)$$

We note, first of all, that the slow growth occurs only when the slope dQ/da is positive, while at $dQ = 0$ the transition to unstable fracture ensues. Therefore, the locus of final instability points is obtained by requiring

$$\log (2R_{\infty}/aQ^2) - Q^2 = 0 \text{ at instability.} \quad (3.14)$$

3.3 Terminal Q vs. (Δa) Relation

The resulting locus of critical loads and crack sizes (Q_f, a_f) is shown in Figure 3.

An example of the integral of equation (3.13) for

$$Q = Q(a) \quad (3.15)$$

was obtained from computer programs through integration and is given in Figure 4. It may be of interest to compare these results with the similar solutions obtained by other authors, such as Cherepanov (1970) and Morozov (1970). For the central crack in an infinite plane considered here, these authors obtained the following equations

$$\frac{dQ}{dx} = \frac{2 - xQ^2}{2 x^2 Q^3} \quad (\text{Cherepanov}) \quad (3.16a)$$

$$\frac{dQ}{dx} = \frac{2 - xQ^2}{x^2 Q} \quad (\text{Morozov}) \quad (3.16b)$$

These equations are both non-linear, but the second one has been integrated in a closed form by Wnuk (1972)

$$Q(x) = \frac{1}{x} \left\{ 4(x-x_0) + x_0^2 Q_0^2 \right\}^{\frac{1}{2}} \quad (3.17)$$

Here x denotes the dimensionless crack length, $x = a/R_\infty$

and $Q_0 = \pi \sigma_0 / 2 \sigma_{ys}$ is the load of fracture initiation.

Figure 5 graphs these three $Q(x)$ equations for the specified initial condition. The middle curve can be approximated by the closed-form solution which follows from the linearized theory of Wnuk, from equation (3.7). For

$R_R \rightarrow 1$ the following approximation may be used

$$Q = \sqrt{\frac{2}{x}} \left\{ 1 - \left(1 - \frac{Q_0^2 x_0}{2} \right) \exp \left[- \frac{x-x_0}{2} \right] \right\}^{\frac{1}{2}} \quad (3.18)$$

The curve $Q = Q(x)$ which results from this equation is plotted vs. non-dimensional crack length in Figure 6. It is seen that both the results converge when the quantity $Q_0^2 x_0 / 2$ approaches unity, i.e. ($R_0 \rightarrow 1$).

Let us consider now a finite-width panel containing a central crack. The pertinent equations were given by Dugdale (1960):

$$R = \frac{Q^2 a}{2} \sec \left[\frac{\pi a}{2b} \right] \\ \frac{R_\infty}{2} Q^2 x \sec \left[\pi x / x_0 \right] \quad (3.19)$$

The dimensionless quantities are defined as follows

$$x = a/R_\infty, \quad p = \pi a_0 / 2b \quad (3.20)$$

and $2b$ denotes the width of the plate.

Introducing equation (3.19) into (3.1b) gives

$$\frac{dQ}{dx} = \frac{\log \left[\frac{2}{Q^2} x \sec(px/x_0) \right] - Q^2 \left[1 + (px/x_0) \tan(px/x_0) \right] \sec(px/x_0)}{2 x Q \sec(px/x_0)} \quad (3.21)$$

Numerical integration of equation (3.21) yields the results which are plotted in Figure 7 for different values of p .

The terminal instability states are distinguished on the graphs of $Q = Q(x)$ shown in Figure 8 as maxima points, and they are denoted (Q_f, x_f) . Table 1 summarizes the results obtained from a series of computer runs, and describes the locations of the critical states obtained at various ratios of the initial crack size to the panel width $p = \pi a_0/2b$; $x_0 = a_0/R_\infty$.

Finally, Figure 9 shows the trend of the experimental (upper curve) and the theoretical data (lower curve) pertaining to the effect of the panel width, b , on the load at instability, Q_f . Although the curves shown differ in magnitude, they have similar trends. The experimental curve in Figure 9 was obtained from the data quoted by Creager (1973).

3.4 Transition to Unstable Fracture

The point of instability is obtained from the condition

$$\frac{dQ}{da} = 0 \quad (3.22)$$

It is of interest to derive an approximate, closed-form equation defining the location of the terminal instability points. This can be done by utilizing the equation (3.18) which resulted from the linearized theory of Wnuk. Computing

dQ/dx from equation (3.18), and setting it equal zero, we obtain

$$\{1 + 1.5(\Delta x + x_0)\} \exp[-\Delta x/2] - (1 - \mathcal{F}_0)^{-1} = 0 \quad (3.23)$$

in which $\Delta x = \Delta a/R_\infty$, $\mathcal{F}_0 = R_0/R_\infty$, or $\mathcal{F}_0 = Q_0^2 x_0/2$ and $x_0 = a_0/R_\infty$. This equation can be solved numerically for the critical crack length, x_f , or for the amount of slow, stable growth, Δx_f . For any given initial conditions (x_0, Q_0) , the transcendental equation (3.23) yields solutions $x = x_f$, given in Table 2.

Graphs shown in Figure 10 illustrate the dependance of the amount of slow crack growth on the initial crack size, a_0 , and the threshold load, Q_0 . The broken curves show the analogous predictions obtained from the closed-form solution of Morozov's equation (1969)

$$\frac{d}{dx} \left\{ \frac{1}{x} \left[4(x - x_0) + x_0^2 Q_0^2 \right]^{\frac{1}{2}} \right\} = 0, \quad (3.24)$$

hence

$$\Delta x_f = (1 - \mathcal{F}_0) x_0 \quad (3.25)$$

For a given set of the initial values describing the load and crack size at the initiation of growth, the factor $(1 - \mathcal{F}_0)$ can be evaluated. This factor appears as the slope of the straight lines shown in Figure 11.

To conclude this section, let us integrate the three differential equations describing load vs. the current crack length during the subcritical growth stage, $Q = Q(x)$. The equations are valid for the infinite plate/center crack geometry. They can be written in the following way

$$(a) \quad \frac{dQ}{dx} = \frac{2-xQ^2}{\frac{2}{3} x^2 Q^3} \quad (\text{Cherepanov})$$

$$(b) \quad \frac{dQ}{dx} = \frac{2-xQ^2}{x^2 Q} \quad (\text{Morozov})$$

$$(c) \quad \frac{dQ}{dx} = \frac{\log(2/xQ^2)}{2xQ} - Q^2 \quad (\text{Wnuk}) \quad (3.26)$$

Examples of the resulting integral curves are shown in Figure 5. The maxima points on these curves indicate the location of the terminal instability states. For a given set of the initiation values of crack size, x_0 , and load level, Q_0 , the computer program evaluates the critical point (x_f, Q_f) . The results are gathered in Table 3. To illustrate the effect of the initial parameters, x_0 and Q_0 , on the crack size and load at instability, the following graphs

x_f vs. x_0

x_f vs. Q_0

Q_f vs. x_0

Q_f vs. Q_0

are plotted as shown in Figures 11, 12, 13, and 14. It is seen that the critical parameters are only weakly dependent on the respective values of such parameters as the point of initiation of the crack. In other words, x_f vs. x_0 and Q_f vs. Q_0 are not rapidly changing functions. This is not so in the case of the relationships x_f vs. Q_0 and Q_f vs. x_0 . This can be seen from Figures 12 and 13 in which rapid

variations are observed.

The quantity

$$y_f = \frac{1}{2} x_f Q_f^2 \quad (3.27)$$

is a measure of the material toughness at the instability point. Table 4 shows the summary of results pertaining to the non-dimensional toughness $y_f = (R_f/R)$. Similar results obtained from Wnuk's equation

$$\frac{dy}{dx} = \frac{1}{2} \log \frac{1}{y} \quad (3.28)$$

are gathered in Table 5. The terminal toughness, y_f , is seen to decrease with an increase of the initiation load level if the initial defect size is kept constant. This is seen from the first two columns of Table 5: an increase in y_0 at the constant x_0 corresponds to an increasing threshold load, according to the equation

$$y_0 = \frac{1}{2} x_0 Q_0^2 \quad (3.29)$$

The last column of Table 5 representing a case of long cracks shows an almost constant toughness parameter, y_f (or a slightly reversed trend is observed). It is of interest to see that a long initial crack gives rise to a much more pronounced stable crack growth leading in effect to a higher terminal toughness parameter (both y_f and the amount of slow, stable growth, x_f , increase as the initial defect size is increased). These observations are drawn from examination of Figure 15 a, b, and c.

CHAPTER 4

MATCHING THE EXPERIMENTAL AND THEORETICAL RESULTS

4.1 R-Curve for the Small-Scale Yielding Case

Laboratory data pertaining to the measurements of fracture toughness associated with stable crack growth for various aluminum alloys and steels are now compared with the theoretical predictions derived on the basis of equations given in the preceeding sections.

The first alloy to be so compared is 7475-T61, which was thoroughly investigated by Heyer and McCabe (1971) at ARMCO, and later by Sullivan, Freed and Stoop (1973). Their results are replotted in a non-dimensional coordinate system

$$y = K_R/K_\infty, \Delta x = \Delta a/R_\infty \quad (4.1)$$

as shown in Figure 16. The normalizing constant R_∞ was obtained from the known material parameters

$$\sigma_{ys} = 73 \text{ ksi}, K_\infty = 125 \text{ ksi} \sqrt{\text{in}} \quad (4.2)$$

as follows

$$R_\infty = \pi K_\infty^2 / 8 \sigma_{ys}^2 = 1.142 \text{ in} \quad (4.3)$$

The curve (a) in Figure 16 shows the experimental results.

Curve (b) resulted from the numerical integration of the slow growth equation (3.4b) which is applicable to a small-scale yielding situation. In terms of the dimensionless variables, equation (3.4b) is

$$\frac{dy}{dx} = \frac{\eta^2}{2y} \log(1/y)$$

Here the coefficient η denotes the ratio of the effective yield stress $\bar{\sigma}_Y$ encountered at the crack tip to the uniaxial yield stress, $\eta = \bar{\sigma}_Y / \bar{\sigma}_{ys}$.

The discrepancy between experimental and theoretical results is too high to be blamed on numerical inaccuracy. It is likely that the theoretical model is deficient in that it applies only to a perfectly elastic-plastic behavior, while in fact the 7475-T61 exhibits a distinct work hardening. Such work hardening will tend to increase the slope dK_R/da , as plotted in Figure 16. If a two-fold increase of the initial slope of the theoretical result as given by equation (4.4) is allowed, the resulting curve approaches the experimental data, compare curves (a) and (c). Such a change in the slope of the R-curve corresponds to the effective yield stress/uniaxial yield stress ratio $\eta = \sqrt{2} = 1.41$ (or a 41% increase in the yield strength encountered at the element adjacent to the crack tip). If η is assumed $\sqrt{3} = 1.732$, then the theoretical curve (d) converges with the experimental curve for a substantial range of the stable growth.

It should be emphasized that the R-curve discussed here is a genuine material characteristic, independent of the initial crack length, specimen geometry, and boundary loading conditions. This is not the case for the large-scale yielding situation discussed in the following section.

A similar case is illustrated in Figure 17 in which the experimental R-curve for aluminum alloy 7075-T7, as given by Wilhem and FitzGerald (1975), is plotted in the non-dimensional coordinate system $\Delta x = (a - a_0)/R_\infty$ and $y = K_R/K_\infty$. The normalizing constants for the alloy 7075-T7 are

$$\sigma_{ys} = 67 \text{ ksi}, K_\infty = 80 \text{ ksi} \sqrt{\text{in}}, R_\infty = 0.56 \text{ in} \quad (4.5)$$

The theoretical curves (a) and (b) are obtained by numerical integration of equation (4.4) in which the hardening coefficient η was assumed 1 and $\sqrt{3}$, respectively.

As a third example, consider the R-curve, K_R vs. Δa , obtained experimentally by Ripling and Falkenstein (1970) for a low-strength cartridge brass. Its characteristics are (compare Metal Handbook, Vol. 1, p. 1019).

Ultimate strength, $\sigma_u = 94 \text{ ksi}$

Yield stress, $\sigma_{ys} = 65 \text{ ksi}$

Young modulus, $E = 16 \times 10^3 \text{ ksi}$

The data pertaining to the R-curve were obtained in a series of experiments carried out on contoured, double-cantilever specimens. The material resistance to cracking was proportional to load P , i.e.

$$K_R = 2 P \frac{m'}{B_n B} \quad (4.6)$$

in which the shape factors for the specimen used by Ripling and Falkenstein m' , B_n , and B were defined as follows

$$m' = 85.9 \text{ in}^{-1}$$

$$B_n = .016 \text{ in}$$

$$B = .516 \text{ in} \quad (4.7)$$

With these numbers equation (4.6) becomes

$$K_R = .207 \left[\frac{\text{ksi in}}{\text{lb}} \right] P \text{ [in lb]} \quad (4.8)$$

The amount of stable crack growth, Δa , is obtained from the reading of the COD* gage mounted on the CDCB (contoured, double cantilever beam) specimen. If this reading is denoted by $\Delta = \Delta(P)$, then the crack length corresponding to the current crack opening displacement can be computed from

$$\Delta a = \frac{EB}{8m'} \left\{ \frac{\Delta - \Delta_0}{P} \right\}_{P=P(\Delta)} \quad (4.9)$$

(if Δa is a small fraction of a_0 , and if Δ_0 denotes the COD at which the first deviation from the straight line, Δ vs. P , occurs). For a more pronounced stable growth, as is the case under consideration, one would need to integrate equation (4.9) as follows

$$\Delta a = \begin{cases} \frac{EB}{8m'} \int_{\Delta_0}^{\Delta} \frac{d\Delta}{P(\Delta)} \\ \text{or} \\ \frac{EB}{8m'} I(\Delta_0, \Delta) \end{cases} \quad (4.9a)$$

where the symbol $I(\Delta_0, \Delta)$ is introduced to denote the integral

$$\int_{\Delta_0}^{\Delta} \frac{d\Delta}{P(\Delta)} \quad (4.10)$$

This integral has dimensions of $[10^{-3} \text{ in/lb}]$, and it was obtained by numerical integration of the reciprocal of (*crack opening displacement)

force vs. crack opening relation, as indicated in Figure 18. Numerical values so generated are gathered in Table 6. This table also lists other values pertinent in the derivation of the normalized resistance curve, i.e.

$$\frac{K_R}{K_\infty} \text{ vs. } \frac{\Delta a}{R_\infty} \quad (4.11)$$

The normalization constants are as follows (taken from Ripling and Falkenstein's data) $K_\infty = 87.72 \text{ ksi } \sqrt{\text{in}}$, $R_\infty = (\pi/8) (K_\infty / \sigma_{ys})^2 = .715 \text{ in}$. For the case considered, the second of equations (4.9a) can be re-written

$$\Delta a \text{ in} = 12.014 \left[\frac{\text{in lb}}{\text{mil}} \right] I [\text{mil/lb}] \quad (4.12)$$

This latter form was used to generate the results listed in the 6th column of Table 6, and upon normalization by R , listed in column 10. These two columns are distinguished by the subscript "B" in order to distinguish them from columns 5 and 9 where analogous values are given (labeled "A"), but obtained from the linear equation (4.9). The final results are plotted in Figure 19.

4.2 R-Curve for the Large Scale Yielding Case

The J-curves recorded by Griffis and Yoder (1976) for the specimens of aluminum 2024 of three different thicknesses

$$B = \begin{cases} 0.64 & \text{cm} \\ 1.27 & \text{cm} \\ 2.36 & \text{cm} \end{cases}$$

are now compared with the theoretical predictions of

Wnuk (1978), pertaining to the large-scale yielding range.

The equations suggested by Wnuk are of the following form

$$\begin{aligned} R_R &= R_i (a/a_0)^{\lambda_i+1} \exp \left\{ -\frac{1}{4} \log^4(a/a_0) \right\} \\ J_R &= J_i (a/a_0) + \frac{8\sigma_Y^2}{\pi E} a \left\{ \lambda_i \log(a/a_0) - \frac{1}{4} \log^2(a/a_0) \right\} \end{aligned} \quad (4.13)$$

Let us rewrite the second equation in terms of the non-dimensional quantities

$y = J/J_i$, $x = a/a_0$, to get

$$y = x \left\{ 1 + \frac{8\sigma_Y^2}{\pi E} \frac{a_0}{J_i} \left[\lambda_i \log x - \frac{1}{4} \log^2(x) \right] \right\} \quad (4.14)$$

with λ_i denoting the initial stability index, defined as

$$\lambda_i = \frac{\pi E}{8\sigma_Y^2} \left[\left(\frac{dJ_R}{da} \right)_i - \left(\frac{\partial J_A}{\partial a} \right)_i \right] \quad (4.15)$$

The index "R" emphasizes the physical interpretation of the J-integral, which serves here as a measure of material toughness, while the index A denotes the intensity of the "applied" field. Note that while the initial stability index is a material property (as reflected by the constants E , σ_Y , $(dJ_R/da)_i$), it is also a geometry-dependent quantity. This geometric dependence enters through the second term in the square bracket $(\partial J_A / \partial a)_i$. For the geometry considered in the preceeding chapter (infinite center cracked plate subject to tension), the initial stability index was calculated by Wnuk as

$$\lambda_i = (\pi E / 8\sigma_Y^2) \left[\left(\hat{\delta} / \Delta \right) - 1 - \frac{1}{2} \log(2ea_0/\Delta) \right] \quad (4.16)$$

It is not known how relevant equations (4.13) and (4.15) are

for the three-point bend tests carried out by Griffis and Yoder (1976). We may, however, consider a limiting case of a linear J vs. (Δa) curve, for which Wnuk has computed the λ_i factor valid for an arbitrary geometry. Let us first linearize equation (4.4) by assuming x to be slightly greater than one, i.e. considering the amount of stable crack growth (Δa) to be a small fraction of the initial crack length.

The following linear form results

$$J_R = J_i \left\{ 1 + \left[1 + \frac{a_0}{J_i} \left(\frac{8\sigma_Y^2}{\pi E} \right) \lambda_i \right] (\Delta a/a_0) \right\} \quad (4.17)$$

or

$$y = 1 + \left(\frac{8\sigma_Y^2}{\pi E} \cdot \frac{a_0}{J_i} \lambda_i \right) \frac{\Delta a}{a_0} \quad (4.18)$$

This formula is used to plot graphs shown in Figures 20a and 20b illustrating the R-curves (both experimental and theoretical) for the 2024 AL specimens of thickness 0.64 cm and 1.27 cm. The third sample of thickness $B = 2.36$ units used in Griffis and Yoder experiments inhibited the stable growth through an increased effective yield strength σ_Y . This is reflected by the curvature of the R-curve which becomes slightly concave, as shown in Figure 21. To match this R-curve the complete form of equation (4.14) was used although it is doubtful that the same form should apply for the three-point bend test configuration*. The initial stability index can be "read" from the available data by using the relationship or variables in equation (4.15),

*(Figure BB)

$$\lambda_i = \frac{\pi E}{8G_Y^2} \left[\left(\frac{dJ_R}{da} \right)_i - \frac{J_i}{a_0} \right] \quad (4.19)$$

The values of

- 1) initial J occurring at the propagation threshold, J_i
- 2) initial crack size, a_0
- 3) initial slope of the J_R vs. (Δa) curve

can be read out directly from the graphs supplied by Griffis and Yoder (1976), namely

at $B = 0.64$ cm, $J_i = 13.7$ kPa-m, and

$$\begin{aligned} \text{(a) } a_0 &= 0.89 \text{ cm} \\ \text{(b) } a_0 &= 1.14 \text{ cm} \end{aligned} \quad (4.20)$$

at $B = 1.27$ cm, $J_i = 15.9$ kPa-m, and

$$\begin{aligned} \text{(a) } a_0 &= 1.78 \text{ cm} \\ \text{(b) } a_0 &= 2.26 \text{ cm} \end{aligned} \quad (4.21)$$

at $B = 2.36$ cm, $J_i = 14.8$ kPa-m, and

$$\begin{aligned} \text{(a) } a_0 &= 1.70 \text{ cm} \\ \text{(b) } a_0 &= 2.24 \text{ cm} \end{aligned} \quad (4.22)$$

The initial slopes $(dJ_R/da)_i$ are found to be as follows

$$\text{at } B = 0.64 \text{ cm } (dJ_R/da)_i = 14.2 \frac{\text{kPa-m}}{\text{mm}}$$

$$\text{at } B = 1.27 \text{ cm } (dJ_R/da)_i = 10.2 \frac{\text{kPa-m}}{\text{mm}}$$

$$\text{at } B = 2.36 \text{ cm } (dJ_R/da)_i = 12.0 \frac{\text{kPa-m}}{\text{mm}} \quad (4.23)$$

As can be seen the initial slopes of the J_R vs. (Δa) curve are independent of the initial crack size, but they

do vary with thickness. Therefore, the initial stability

index λ_i will reflect these variations, namely

at $B = 0.64$ cm and (a) $a_0 = 0.89$ cm, $\lambda_i = 0.332$

(b) $a_0 = 1.14$ cm, $\lambda_i = 0.598$

at $B = 1.27$ cm and (a) $a_0 = 1.78$ cm, $\lambda_i = 0.345$

(b) $a_0 = 2.26$ cm, $\lambda_i = 0.873$

at $B = 2.36$ cm and (a) $a_0 = 1.70$ cm, $\lambda_i = 0.030$

(b) $a_0 = 2.24$ cm, $\lambda_i = 0.031$

The effective yield stress encountered at the crack tip in the specimen of $B = 2.36$ cm was assumed to be about 10 times greater than the uniaxial yield stress, σ_y , which for 2024-T351 is 338 MPa. This leads to the initial stability index $\lambda_i = 0.03$. Note that the R-curve for the thinner specimens (at $B = 0.64$ cm and 1.27 cm) is linear in (Δa). Thus, the simplified equation (4.18) can be applied to match the experimental results. Combining equation (4.18) with the definition of the initial stability index, equation (4.19) may be written

$$J_R = J_i + \left(\frac{dJ_R}{da} \right)_i (\Delta a) \quad (4.24)$$

This gives a very good agreement with the data shown by Griffis and Yoder (1978) for the thin aluminum three-point bend specimens. Therefore, we conclude that, at least within the initial phase of growth, the R-curve does not exhibit any strong geometric dependence. The slope $(dJ_R/da)_i$, as can be seen from the Griffis and Yoder data, is not sensitive to

the initial crack length, but it does appear to vary with the thickness (decreasing thickness leads to a greater initial slope of the J_R vs. (Δa) curve).

The location of the terminal instability point, (a_f, P_f) , however, is affected by the geometry. In particular, the critical crack length, a_f , and the critical load, P_f , are influenced by the initial crack size as indicated by arrows in Figures 20 and 21. An estimate of the critical amount of slow crack growth, $\Delta a_f = a_f - a_0$, can be obtained from Wnuk's 1978 equation

$$a_f = a_0 \frac{2(\lambda_i + 1 - \phi_0)}{1 + \phi'_0 a_0} \quad (4.25)$$

in which the shape factors ϕ_0 and ϕ'_0 are defined by the formulas

$$\phi = \left\{ \frac{a}{J_A} \frac{\partial J_A}{\partial a} \right\} \text{ at } a = a_0 \quad \phi' = \frac{d\phi}{da}$$

$$\phi_0 = \phi[a_0] \quad \phi'_0 = \phi'[a_0]$$

At this time we are unable to verify equation (4.25) since the shape factors ϕ_0 and ϕ'_0 for the three-point bend specimen are not given analytically. (It appears that the numerical evaluation of ϕ_0 and ϕ'_0 based in the finite-element method should be possible). Equation (4.25) can be used to locate the critical point for all the geometrical configurations in which the factors ϕ_0 and ϕ'_0 are known functions of the initial crack size and the specimen dimension S .

CHAPTER 5

COMPARISON OF THE LARGE-SCALE AND SMALL-SCALE YIELDING THEORIES

The two limiting cases of considerable practical importance have been discussed in the preceeding chapters of this thesis. These are small-scale and large-scale yielding behaviors which may occur during the stable crack growth in a ductile metal, see Figure AA. Let us briefly summarize the essential features of both asymptotic solutions pertaining to the description of ductile fracture process. The governing equations of the problem relate the resistance, R_R or J_R , (measures of material toughness), developed during a quasi-static subcritical crack extension, to the geometrical and loading configurations (which affect the outer field parameters R or J) and to the material properties (such as the tearing modulus or opening constant, $\hat{R}/\Delta$ or $\hat{\delta}/\Delta$, and the size of the process zone Δ) in the following way. (The subscript "R" is dropped for convenience):

$$\begin{aligned} \text{(a)} \quad \frac{dR}{da} &= \frac{R}{\Delta} - \frac{1}{2} - \frac{1}{2} \log (4R/\Delta), \quad R_0/a_0 \ll 1 \\ \text{(b)} \quad \frac{dR}{da} &= \frac{a+R}{a} \left\{ \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left[\frac{4eaR}{(2R+a)\Delta} \right] \right\}, \quad R_0/a_0 \gg 1 \end{aligned} \quad (5.1)$$

Equation (5.1a) is valid for the contained plasticity ($R_0/a_0 \ll 1$) case, while equation (5.1b) describes the other limiting case of unconstrained yielding i.e. $R_0/a_0 \gg 1$.

There are two material constants which enter in both equations:

1) the $\hat{R}/\Delta$, which is directly related to the opening constants $\hat{\delta}/\Delta$ or to the 'tearing modulus' proposed by Shih (1978), $T_{\delta} = (E/\sigma_0)(\hat{\delta}/\Delta)$, or the dimensionless tearing modulus "n", proposed by Wnuk (1978). The relations between these constants are as follows

$$\frac{\hat{R}}{\Delta} = \begin{cases} \left(\frac{\pi E}{8\sigma_y} \right) \left(\frac{\hat{\delta}}{\Delta} \right) \\ \frac{\pi \sigma_0}{8\sigma_y} T_{\delta} \\ n + \frac{1}{2} \log (2e) \end{cases} \quad (5.2)$$

The symbol σ_0 is used to denote the uniaxial yield strength, while σ_y denotes the effective yield stress encountered at the crack front. If we assume that the McClintock (1958) relations, derived for Mode III ductile fracture, apply for the case considered here, we may then suggest that the tearing moduli, T or n, are the following functions of the initial ductility parameter, R_i

$$\frac{\hat{R}}{\Delta} = \frac{1}{2} \left\{ \left(2 \frac{R_i}{a_0} - 1 \right)^{\frac{1}{2}} + \log 4 \right\} \quad (5.3a)$$

or

$$T = \frac{4 \sigma_y}{\pi \sigma_0} \left\{ \left(2 \frac{R_i}{a_0} - 1 \right)^{\frac{1}{2}} + \log 4 \right\} \quad (5.3b)$$

or

$$n = \frac{1}{2} \left\{ 2 \left(\frac{R_i}{a_0} - 1 \right)^{\frac{1}{2}} + \log (2/e) \right\} \quad (5.3c)$$

It is noteworthy that, if McClintock's interpretation of the initial ductility R_i (1958) is to be extended to the Mode I fracture, then we would have a relation (See Figure CC)

$$\alpha = \epsilon_f^p / \epsilon_y = R_i / a_0 - 1 \quad (5.4)$$

The symbol ϵ_f^p is used to denote the plastic component of the total strain at fracture, while ϵ_y equals the strain at yield, $\epsilon_y = \sigma_y/E$ (McClintock (1958)) has derived the relationship between the initial toughness R_i and the shear strain encountered in Mode III fracture test. The validity of such a relation remains to be tested. Nevertheless, it has the plausible and interesting possibility of bridging the micro-structural fracture studies, which provide information concerning the plastic component of ultimate strain ϵ_f^p and the yield strain ϵ_y , to the continuum considerations concerned with the quantities such as the initial propagation threshold size of the plastic zone, R_i , and the fracture process zone size, Δ .

Let us observe that equation (5.1a) is valid for an arbitrary geometrical configuration, since it represents a universal R-curve applicable within the small-scale yielding range. (Solutions are easy to obtain numerically for specified initial conditions, as shown in Chapter 3). By contrast, equation (5.1b) has been derived for, and can only be applied to, one specific geometrical configuration. The center-cracked plate of infinite width is the configuration for which we shall illustrate the applications of the theory. Integration of equation (5.1b) leads to

$$R = R_i (a/a_0)^{\lambda_i + 1} \exp \left\{ - \frac{1}{4} \log^2 (a/a_0) \right\} \quad (5.5)$$

The nondimensional quantity λ_i , the stability index, named by Wnuk (1978) strongly depends on the choice of geometry

and boundary loading condition. For the case considered here the exponent λ_i is defined as follows:

$$\lambda_i = n - 1 - \frac{1}{2} \log (a_0/\Delta) \quad (5.6)$$

The direct proportionality of λ_i to the tearing modulus, n , indicates a type of dependence of the exponent λ_i on the material property which is descriptive of the slow stable cracking. Should another geometrical configuration be considered (for example a finite test piece), the index λ_i will also reflect the effect of specimen size and shape.

The effects of the initial crack length and the initial ductility parameter, R_i/Δ , are shown for the case of the small-scale yielding in Table 7 and in Figure 22.

It is of interest to note that the critical crack length, a_f , depends almost linearly on the initial crack size a_0 if plotted in a log log-scale (this observation agrees very well with the experimental data from the unpublished report of Epstein (1976)). The ductility level, as measured by the ratio R_i/Δ does not affect strongly the lines shown in Figure 22a, but it has a rather pronounced effect on the terminal toughness parameter R_f/a_f and/or R_f/R_i as demonstrated by graphs in Figures 22b, c, and d. The values used to make the plots shown in Figure 22 were obtained numerically from equation (3.1b) and (3.13) and they are listed in Table 7.

Analogous data pertaining to the large-scale yield have been used to plot the graphs shown in Figure 23.

Certain essential differences become evident upon a careful comparison between Figures 22 and 23. First, the crack size at instability for the large-scale yielding range does not depend perceptibly on the initial crack length, but only on the initial ductility parameter R_i/Δ . Second, the terminal toughness is a decreasing function of the initial crack size in Figures 23b and d. The terminal toughness is still strongly dependent on the ductility level R_i/Δ , just like it was in the case of small-scale yielding. see Figures 22c and 23c.

The following equations derived by Wnuk (1978) were employed in preparing the graphs shown in Figure 23: (a) the terminal toughness as a function of the initial crack size and the material ductility level R_i/Δ

$$R_f = R_i \exp (2\lambda_i + \lambda_i^2) \quad (5.7)$$

in which

$$\begin{aligned} \lambda_i &= n - 1 - \frac{1}{2} \log (a_0/\Delta) \\ n &= \frac{1}{2} \left[\sqrt{2(R_i/\Delta) - 1} + \log (2/e) \right] \end{aligned} \quad (5.8)$$

(a) the critical crack length

$$\begin{aligned} a_f &= a_0 \exp (2\lambda_i) \\ &\quad \Delta \exp (2(n-1)) \\ n &= \frac{1}{2} \left[\sqrt{2(R_i/\Delta) - 1} + \log (2/e) \right] \end{aligned} \quad (5.9)$$

(b) the toughness magnification ratio

$$\frac{R_f}{R_i} = \exp (2\lambda_i + \lambda_i^2) \quad (5.10)$$

To compare both the cases, the initiation and the instability loci were plotted on one diagram, see Figure 24 a and b. The broken vertical line indicates the range of applicability of the asymptotic solutions presented here; to the left of this line $R_i/a \gg 1$, so that large-scale yielding prevails. To the right of this line, $R_i/a_0 \ll 1$ and the small-scale yielding solutions should be used. One example of each case is given at a chosen initial ductility parameters of $R_i/a_0 = 40$ (Figure 24a) and $R_i/a_0 = 80$ (Figure 24b). The resulting load vs. crack extension curve (or Q-curve) is plotted in the same graph. We note that the Q-curve extends from the initiation (or threshold) point Q_i to the terminal (or critical) point Q_f . The initiation point for the Q-curve is obtained as follows

$$Q_i = \sqrt{2R_i/a_0}, \quad (5.11)$$

for small scale yielding, and

$$Q_i = \arccos \left(\frac{a_0}{a_0 + R_i} \right) \quad (5.12)$$

for large-scale yielding situation, of Dugdale (1960). Note that equation (5.12) reduces to equation (5.11) when the ratio R_i/a_0 is much less than one.

The terminal points of the Q-curves are obtained from the requirement of zero slope of the Q vs. (Δa) curve. The differential equations describing such a Q-curve for the small-scale yielding case (center-cracked plate of infinite width, w) is readily obtained from equation (5.1a) by

application of the well-known relations.

$$R(\text{center cracked plate, } W \rightarrow \infty) = \frac{1}{2} a Q^2(a), \quad Q = (\pi \sigma / 2 \sigma_y) \quad (5.13)$$

and

$$\frac{dR}{da} = \frac{\partial R}{\partial a} + \frac{\partial R}{\partial Q} \frac{dQ}{da} \quad (5.14)$$

The differential equation so obtained, see equation (3.13)

$$\frac{dQ}{dX} = \frac{\log(2A/XQ^2) - Q^2}{2XA} \quad (5.15)$$

in which

$$X = a/\Delta, \quad A = R_{ss}/\Delta = \exp\left\{\left[2(R_i/a_0) - 1\right]^{\frac{1}{2}} - 1\right\} \quad (5.16)$$

can be rewritten in the semi-logarithmic coordinates,

$z = \log X$ and Q , as follows

$$\frac{dQ}{dz} = \frac{\log(2A/Q^2) - Q^2 - z}{2Q} \quad (5.17)$$

This equation, when integrated numerically, yields the Q -curves of the type shown in Figures 24a and 24b in the small-scale yielding range. Obviously, for $dQ = 0$, we obtain the upper toughness locus, i.e.

$$\log X_f = \log (2A/Q_f^2) - Q_f^2 \quad (5.18)$$

in which the parameter A is defined by equation (5.16). For any given initial ductility parameter, the equation (5.18) yields a curve, shown as an upper toughness bound for the small-scale yield case in Figures 24a and 24b. Analogous lower and upper toughness bounds were obtained for the large-scale yielding situation by Wnuk (1978) as asymptotic solutions. The lower bound is described by equation (5.12) while for the geometry considered here, the upper bound

turns out to be independent of the crack length attained at the termination of the subcritical phase of crack growth, and equals

$$Q_f = \arccos \left(\frac{1}{1 + \frac{(R_i)^2}{a_0} \exp \lambda_i^2} \right) \quad (5.19)$$

regardless of the terminal crack size a_f/Δ , which can be computed as follows

$$\log (a_f/\Delta) = \log 2 - 3 + \sqrt{2(R_i/\Delta) - 1} \quad (5.20)$$

Here, the quantity λ_i is defined by equation (5.6). Therefore, for a given tearing modulus, n , and the initial ductility parameter, R_i/a_0 , both the terminal quantities $X_f = a_f/\Delta$ and the loading parameter Q_f can be computed. The points corresponding to the load at instability will thus fall on a vertical line, when plotted in a Q vs. $\log X$ diagram as was done in Figures 24a and 24b.

It should be noted that the formulas shown in Figure 24b did not qualify as asymptotic (either in small-scale yielding or large-scale yielding sense), and therefore, the data shown in this figure were obtained numerically by integration of the equations, derived by Wnuk (1978) valid for an arbitrary R/a ratio

$$\frac{dR}{dQ} = \frac{a + R}{a} \left\{ \frac{R}{\Delta} - \frac{1}{2} \log \left[\frac{4eRa}{\Delta(2R+a)} \right] \right\} \quad (5.21)$$

$$\frac{dQ}{da} = \frac{a(dR/da) - R}{(a + R) \sqrt{2aR + R^2}} \quad (5.22)$$

With non-dimensional variables $Y = R/\Delta$ and $X = a/\Delta$ these equations assume form

$$\frac{dY}{dX} = \frac{X + Y}{X} \left\{ \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left(\frac{4eYX}{2Y + X} \right) \right\} \quad (5.23)$$

$$\frac{dQ}{dX} = \frac{X(dY/dX) - Y}{(X + Y) \sqrt{2XY + Y^2}} \quad (5.24)$$

or, in a semi-logarithmic scale with, $X^1 = \log X$, they read

$$\frac{dY}{dX^1} = \left\{ (X + Y) \left[\frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left(\frac{4eYX}{2Y + X} \right) \right] \right\}_X = \exp X^1 \quad (5.25)$$

$$\frac{dQ}{dX^1} = \left\{ \frac{X}{X + Y} \frac{(dY/dX^1) - Y}{\sqrt{2XY + Y^2}} \right\}_X = \exp X^1 \quad (5.26)$$

Other data summarizing the mechanical tests performed for the large-scale yielding situation are gathered in Table 8.

CHAPTER 6

QUANTITIES PERTINENT TO THE STABILITY ANALYSIS OF
FRACTURE IN THE FULLY PLASTIC RANGE

While there is a general agreement concerning one of the two parameters involved, namely the threshold value of δ , R or J (or any other suitable measure of material toughness) occurring at the onset of crack extension, the choice of the second parameter is still not clear. Whether this should be a tearing modulus of Paris, T_J , or the crack opening angle, δ/Δ , or the initial stability index λ_i (which is a geometry-dependent quantity), as suggested in the present work, or yet another entity, remains to be seen.

Numerical examples show that the crack opening angle $d\delta/da$ (or $\hat{\delta}/\Delta$) remains constant over a significant range of subcritical growth; and thus the final stretch criterion which postulates a constant $\hat{\delta}/\Delta$ during the slow cracking process appears to gain an additional support. On the other hand, the dimensionless tearing modulus, $T_J = (E/\sigma_0^2) dJ/da$, proposed by Paris et al. (1977) for the description of slow growth, is not a constant, but a variable quantity which approaches a constant level in solids of high ductility. Therefore, the Paris modulus does not appear to be a suitable parameter for a fundamental measure of material toughness associated with stable crack growth.

Variation of the tearing modulus during the crack extension can be predicted from the closed-form solutions discussed in this thesis.

To achieve this objective, we have plotted the slopes of the R-curves predicted for the large-scale yielding situation by the asymptotic closed-form solution of Wnuk (1978) against the current crack length for 36 runs obtained under the initial conditions shown in Table 9. (See Figure 26)

The slopes of the R-curves for both the large- and small-scale yielding situations were computed from Wnuk's differential equations obtained for a quasi-static mode I crack described by a linear plasticity model, i.e.

$$\frac{dR}{da} = \frac{R}{a} \left\{ \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left(\frac{2ea}{\Delta} \right) \right\} \quad (6.1)$$

for the large-scale yielding situation and

$$\frac{dR}{da} = \frac{\hat{R}}{\Delta} - \frac{1}{2} \log (4eR/\Delta) \quad (6.2)$$

for the small-scale yielding case. At the incipient point of crack extension, $R = R_i$ and $a = a_0$, the equations (6.1) and (6.2) can be used to predict the initial slopes $(dR/da)_i$ for both the cases considered. We obtain, respectively,

$$\left(\frac{dR}{da} \right)_i = \begin{cases} \frac{Y_0}{2X_0} \left\{ \sqrt{2Y_0 - 1} + \ln(2/e) - \ln X_0 \right\} & (a) \\ \frac{1}{2} \left\{ \sqrt{2Y_0 - 1} - \ln Y_0 - 1 \right\} & (b) \end{cases} \quad (6.3)$$

The dimensionless quantities X_0 and Y_0 in equations (6.3) have been defined as follows

$$\begin{aligned} X_0 &= a_0/\Delta \\ Y_0 &= R_i/\Delta \end{aligned} \quad (6.4)$$

The symbol Δ denotes the size of the process zone, an intrinsic material constant characteristic of the micro-structure.

All the curves shown indicate that the slope of a resistance curve dR/da (or dJ_R/da) is not a constant over any significant range of crack extension, and thus could not be suggested as a suitable material property (supposedly independent of geometry and boundary loading conditions as required by Paris et al. (1977)). It is noteworthy, though, that for the tests performed on specimens of increasing initial ductility, R_i/a_0 a tendency is observed for the variations in slope dR/da to increase. (see graphs in Figure 25 corresponding to $R_i/a_0 = 17.5$ and 4.9). It is possible, therefore, that the tearing modulus, T_J , could retain its physical significance (of a material property) for tests performed on solids within a certain range of ductility.

The solutions obtained by Wnuk (1978) from his linear plasticity model of a stable crack under unconstrained yield condition do suggest two material parameters essential for characterization of fracture in the subcritical range: the initial crack opening δ_i (or initial toughness R_i), and the crack opening angle $\hat{S}/\Delta$ (proportional to the tearing modulus, n). The quantity $\hat{S}/\Delta$ may also be used, as

suggested by Shih, Delorenzi, and Andrews (1978) as a measure of the crack tip-opening angle, or as a new tearing modulus $T = (E/\sigma_0) (d\delta/da)$. Such a description of material toughness is somewhat analogous to the characterization of deformation behavior by the yield stress and strain hardening exponent.

The strain energy density, S , appearing in Shih's universal criterion of fracture is suggested as such "another entity". Indeed, a numerical example given by Shih (1978), and concerning a three-dimensional quasi-static crack, demonstrated subsequent shapes of the initially semi-elliptical surface crack extending into the unbroken material. The oval shapes shown resulted at the loci of points at which the minimum strain energy density, dW/dV , remains constant. It appears that the criterion of constant strain energy density would indeed be satisfactory for the description of a steady-state growth process. For the initial stages of crack extension, however, where the unsteady growth prevails, an incremental criterion of the type

$$\left[S(x_1) \right]_{x_1=0} - \left[S(x_1) \right]_{x_1=\Delta} = \hat{S}_c \quad (6.5)$$

would be more appropriate. It may be shown that such "incremental strain energy density" condition is equivalent to the criterion discussed at the beginning of this thesis, provided that the material constant S_c is chosen ^{as} $(\sigma_Y \hat{\delta}/2\Delta)$; where $\hat{\delta}$ denotes the final stretch, while the ratio $(\hat{\delta}/\Delta)$

measures the crack tip opening angle.

An interesting example of matching the experimental data in both ranges, i.e., for small- and large-scale yielding is given by the following procedures. Consider the experimental data of Fattorini, Mirabile, and Podrini (1978) concerning ASA 52 steel tested at two temperatures: -80°C and $+20^{\circ}\text{C}$. (Figure 27 displays their data) The stable crack extension occurs within the small-scale yielding range at the low temperature, while the behavior changes over to large-scale yielding at the higher temperature. To match the -80°C data we apply Wnuk's equation of an R-curve:

$$\frac{dJ_R}{da} = \alpha \eta^2 \frac{4 \sigma_0^2}{\pi E} \log (J_{\infty}/J) \quad (6.6)$$

in where two dimensionless factors, α and η , are defined as follows

$$\eta = \sigma_Y / \sigma_0, \quad \alpha = J / (\sigma_Y \delta)$$

According to recent numerical studies of Rice and Sorenson (1978) the coefficient α is about 2. To obtain a satisfactory fit of the experimental data with equation (6.6) the coefficient η , which reflects the elevation of the yield strength occurring locally in the material adjacent to the crack front, was chosen to be $\eta = 3.9$. With these numbers, and with the material data provided by Fattorini, Mirabile, and Podrini

$$\begin{aligned} \sigma_0 &= 784 \text{ MPa at } -80^{\circ}\text{C} \\ E &= 207 \times 10^3 \text{ MPa} \\ J_i &= 40 \text{ kJ m}^{-2} \end{aligned} \quad (6.7)$$

the factor which multiplies $\log (J_{\infty}/J)$ in equation (6.6) becomes

$$(2)(3.9)^2 \left[\frac{4 (784)^2}{\pi^{207} \times 10^3} \right] = (2)(3.9)^2 (3.781) = 115 \frac{\text{kJm}^{-2}}{\text{mm}} \quad (6.8)$$

Combining this with

$$J_{\infty} \approx 1000 \text{ kJm}^{-2} \quad (6.9)$$

we obtain the following equation to integrate

$$\frac{dJ_R}{d(a-a_0)} = 115 \log \frac{1000}{J_R} \quad (6.10)$$

The resulting J_R vs. Δa curve is compared to their data in Figure 27 (note the metric units on both axes). It is seen that the continuous curve obtained is in a good agreement with the experimental points. With the data available, it is possible to give an estimate of the steady-state extent of the plastic zone which develops at the front of a moving crack, namely

$$R_{ss} = \frac{1}{\alpha \eta^2} \frac{1}{2} \frac{\pi E}{4 \sigma_0^2} J_{ss} = \frac{1}{2} \frac{1000}{115} = 4.35 \text{ mm} \quad (6.11)$$

The steel tested at room temperature (+20°C) exhibits a rather ductile behavior resulting in almost a linear J_R vs. Δa curve, as shown in Figure 27. The initial segment of the R-curve in the fully-plastic range should indeed be a linear function of (Δa) , according to the equation

$$J_R = J_i + (dJ_R/da)_i \Delta a \quad (6.12)$$

with

$$\begin{aligned} J_i &= 100 \text{ kJm}^{-2} \\ (dJ_R/da)_i &= 80 \text{ kJm}^{-2}/\text{mm} \end{aligned} \quad (6.13)$$

taken from the report of the Italian researchers, equation

$$(6.12) \text{ becomes } J_R = 100 + (80) (\Delta a) \quad (6.14)$$

The resulting straight line is shown in Figure 27.

CHAPTER 7

CONCLUSIONS

Existing linear elastic fracture mechanics and J-integral analyses are well suited for safety assessments of high-strength, -low toughness materials. These analyses apply only to the onset of crack growth, which is usually tantamount to crack instability and structural failure in that class of materials. However, this is not the case for the low strength, high-toughness grades in which crack instability may be preceded by extensive stable crack growth under rising load. Here, a substantial margin of safety may exist even when the onset of crack growth is imminent.

This thesis describes research leading to a ductile fracture mechanics methodology designed to treat two-dimensional, large-scale yielding and stable crack-growth problems. The line plasticity model of a moving crack by Wnuk (1972-1978) is used to obtain predictions concerning the material toughness associated with the preliminary crack extension (R-curves) and to calculate the critical parameters, i.e., load and the crack size, at which a transition to unstable brittle-like fracture will occur. One important finding of this work is that parameters truly reflecting the state of the crack-tip process zone are not functions of the extent of stable crack growth when the mode of fracture (full shear or flat) remains fixed. The

"final stretch" suggested by Wnuk in 1972 or the "crack tip-opening angle" could be used, for example, as such material characteristics, since they are constant over a considerable range of subcritical crack growth and obtainable from a series of relatively simple tests which involve direct measurements of the material toughness and the slope of the resistance curve at the onset of stable crack growth. The possibility exists, therefore, that useful, stable growth parameters can be evaluated from the state of the crack tip at the onset of crack extension.

An approach leading fairly accurate predictions of stable crack growth and crack instability directly from the input data obtained at the growth initiation was tested in this thesis and shown feasible. Resistance curves for three aluminum alloys (2024, 7075-T7 and 7475-T61), ASA 52 steel at temperatures of 20⁰C and -80⁰C, and cartridge brass were obtained from the theoretical model of Wnuk, which was applied in both small-scale and large-scale yielding ranges. The data so generated were compared with the published experimental results provided by Wilhem and FitzGerald (1975), Ratwani and Wilhem (1975, 1977), Fattorini, Mirabile, Podrini, and Spedaletti (1978), Ripling and Falkenstein (1970), the Armco Company, Griffis and Yoder (1976), and Creager (1973).

The agreement of the experimental and theoretical data varied from case to case depending on the ductility level,

geometry, and the boundary loading conditions as shown in the thesis. The over all convergence of the results discussed was good, and we may therefore conclude that the methodology discussed here for safety evaluation and fail-safe design of the pressure vessels, in nuclear reactors or other applications, offers a basis for a rational material and stress state evaluation in most of the cases where the linear elastic fracture mechanics is not applicable.

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Table 1. Critical parameters: crack size $x_f = a_f/a_0$ and the loading parameter $Q_f = \pi \sigma_f / 2 \sigma_y$ found for a center cracked panel of infinite width within small scale yielding range (Wnuk's non-linear equations of 1972 were employed).

$P \backslash x_0$	10	100	1000
0	$X_f = 13.098$ $Q_f = .36597$	$X_f = 106.91$ $Q_f = .13556$	$X_f = 1011.4$ $Q_f = .044429$
.2	$X_f = 12.898$ $Q_f = 0.35977$	$X_f = 106.81$ $Q_f = .13401$	$X_f = 1011.4$ $Q_f = .043974$
.4	$X_f = 12.699$ $Q_f = .34156$	$X_f = 106.51$ $Q_f = .12935$	$X_f = 1011$ $Q_f = .0426$
.6	$X_f = 12.199$ $Q_f = .31229$	$X_f = 106.21$ $Q_f = .12148$	$X_f = 1010.5$ $Q_f = .40274$
.8	$X_f = 11.799$ $Q_f = .27286$	$X_f = 105.51$ $Q_f = .11022$	$X_f = 1010.1$ $Q_f = .036924$

Table 2. Roots of the equation (3.23) representing the critical crack length attained at the end of slow stable crack growth phase (according to the linearized theory of Wnuk).

ρ_0	$c \backslash x_0$	10	100	1000
.4	.8	3.008	6.975	11.435
.6	.7	2.069	6.148	10.62
.8	.6	0.436	4.735	9.234
.9	.55	-1.000	3.322	7.845

$$c = 1 - \frac{\rho_0}{2}$$

Table 3. Critical states (X_f, Q_f) derived from the theory of (a) Cherepanov, (b) Wnuk, and (c) Morozov.

Q_0	X_0	5		10		100	
		Q_f	X_f	Q_f	X_f	Q_f	X_f
.05	a	.57053	6.1999	.419	11.199	.13906	104
	b	.46935	7.299	.36469	12.798	.13542	108
	c	.44795	9.897	.31715	19.594	.10315	187.95
.1	a	.57094	6.1993	.41797	11.399	.13938	102
	b	.46957	7.1988	.36540	12.998	.13604	106
	c	.45005	9.7974	.32020	19.403	.11538	149.97
.15	a	.57093	6.0994	.41803	11.399		
	b	.47022	7.1988	.36613	12.998		
	c	.45366	9.6974	.32544	18.802		
.2	a	.57103	6.0994	.418	11.2	OVER CRITICAL	
	b	.47135	7.1988	.36763	12.8		
	c	.45885	9.4975	.33347	17.998		
.25	a	.57130	6.0994	.41855	11.2		
	b	.47313	7.099	.36949	12.4		
	c	.46579	9.1977	.34437	16.799		

Table 4. Terminal toughness derived from the theory of

(a) Cherepanov, (b) Wnuk, and (c) Morozov.

 Y_i , $i=1,2,3$ denotes loading parameter, Q , T_i , $i=1,2,3$, denotes toughness, R/R_∞

	X_0	10	100	1000
Y_0	Q_0	.2	.063	.02
.2	a	$Y_1 = 0.41871$	$Y_1 = .13955$	$Y_1 = .04463$
		$T_1 = 0.9993$	$T_1 = .9991$	$T_1 = 1.0001$
	b	$Y_2 = .40383$	$Y_2 = .13567$	$Y_2 = .044436$
		$T_2 = 0.87201$	$T_2 = .98188$	$T_2 = .99806$
	c	$Y_3 = .33333$	$Y_3 = .10540$	$Y_3 = .03333$
		$T_3 = 1.0000$	$T_3 = 1.0000$	$T_3 = 1.0000$
	Q_0	.283	.089	.028
.4	a	$Y_1 = .41964$	$Y_1 = .13959$	$Y_1 = .044631$
		$T_1 = .9950$	$T_1 = .9996$	$T_1 = 1.000$
	b	$Y_2 = .37250$	$Y_2 = .13587$	$Y_2 = .044442$
		$T_2 = .86723$	$T_2 = .98119$	$T_2 = .99798$
	c	$Y_3 = .35355$	$Y_3 = .11180$	$Y_3 = .03535$
		$T_3 = 1.0000$	$T_3 = 1.0000$	$T_3 = 1.0000$
	Q_0	.346	.109	.0354
.6	a	$Y_1 = .42213$	$Y_1 = .13968$	$Y_1 = .044635$
		$T_1 = .9979$	$T_1 = .9999$	$T_1 = 1.0000$
	b	$Y_2 = .38142$	$Y_2 = .13624$	$Y_2 = .044456$
		$T_2 = .86559$	$T_2 = .98183$	$T_2 = 0.99803$
	c	$Y_3 = .37796$	$Y_3 = .11952$	$Y_3 = .037796$
		$T_3 = 1.0000$	$T_3 = 1.0000$	$T_3 = 1.0000$

Table 5. Terminal toughness ($Y_f = R_f/R_\infty$) and critical crack length ($X_f = a_f/R_\infty$) resulting from the interpretation of Wnuk's equation $dy/dx = 1/2 \log (1/Y)$ in the small scale yielding range.

$Y_0 \backslash X_0$	10	100	1000
.2	$X_f = 12.9$ $Y_f = .87173$	$X_f = 107$ $Y_f = .9861$	$X_f = 1010$ $Y_f = .9970$
.4	$X_f = 12.5$ $Y_f = .87315$	$X_f = 106$ $Y_f = .9801$	$X_f = 1010$ $Y_f = .9975$
.6	$X_f = 11.9$ $Y_f = .87016$	$X_f = 106$ $Y_f = .9851$	$X_f = 1010$ $Y_f = .9981$
.8	$X_f = 10.6157$ $Y_f = .85875$	$X_f = 105$ $Y_f = .9861$	$X_f = 1010$ $Y_f = .9989$
.9		$X_f = 103.3253$ $Y_f = .9820$	

Table 6. Experimental data (Δ and P) pertaining to cartridge brass of Ripling and Falkenstein (1972), and the reduced data needed for evaluation of the subcritical crack extension, Δa , and material toughness, K_R .

Δ mil	P [lb]	$\frac{I}{P}$ [1/lb]	I [mil/lb]	$(\Delta a)_A$ inch	$(\Delta a)_B$ inch	$K_R = .204P$ ksi $\sqrt{\text{in}}$	$\frac{K_R}{Y = K_\infty}$	$\frac{(\Delta a)_A}{R_\infty}$	$\frac{(\Delta a)_B}{R_\infty}$
0	0					0	0	0	0
5	37.64	.0265				7.67856	.0875		
10	75.28	.0132				15.35712	.175		
15	112.92	.0088				23.03568	.2626		
20	150.56	.0066				30.71424	.3501		
25	188.2	.0053	0	0	0	38.3928	.438	0	0
30	220	.00454	.0242	.27304	.2907	44.88	.5116	.382	.4065
35	250	.004	.0453	.48056	.5442	51.00	.5814	.672	.7611
40	280	.00357	.0639	.64360	.7677	57.12	.6511	.900	1.0737
45	310	.00322	.0806	.77509	.9683	63.24	.7209	1.084	1.354
50	330	.00303	.0961	.91015	1.1545	67.32	.7674	1.2729	1.6146
55	350	.00286	.1108	1.02977	1.3311	71.40	.8139	1.4402	1.8616
60	370	.00270	.1248	1.13645	1.4993	75.48	.8604	1.5894	2.097
65	390	.00256	.1374	1.23220	1.6507	79.56	.907	1.7233	2.3086
70	410	.00243	.1497	1.31861	1.7984	83.64	.953	1.8442	2.5152
75	420	.00238	.1617	1.43023	1.926	85.68	.9767	2.000	2.717
80	430	.00232	.1732	1.53667	2.0808	87.72	1.0	2.1491	2.910
85	430	.00232	.1847	1.67637	2.2189	87.72	1.0	2.344	3.103

Table 7. Summary of computer generated data concerning material resistance to fracturing developed in a small yield situation.

Run No.	X_0	Y_0	Q_f	$\log_{10} X_f$	$\log_{10} Y_f$	$\log_{10} (X_f - X_0)$
1	10^2	10	.5505	2.1467	1.3272	1.6042
2		20	.9129	2.2960	1.9158	1.9899
3		40	1.4478	2.4950	2.5133	2.3276
4		80	2.1347	2.7050	3.0627	2.6096
5		100	2.3841	2.7695	3.2231	2.6886
6	10^3	10	.2200	3.0540	1.4378	2.1218
7		20	.4628	3.1552	2.1849	2.6330
8		40	.9560	3.3687	3.0286	3.1262
9		80	1.6896	3.6480	3.8025	3.5374
10		100	1.9615	3.7365	4.0206	3.6485
11	10^4	10	0.0747	4.0109	1.4564	2.4062
12		20	.1820	4.0443	2.2635	3.0308
13		40	.5168	4.1840	3.3096	3.7223
14		80	1.2392	4.4898	4.3751	4.3200
15		100	1.5337	4.5997	4.6702	4.4740
16	10^5	10	.0239	5.0017	1.4585	2.5891
17		20	.0609	5.0082	2.2762	3.2777
18		40	.2117	5.0558	3.4063	4.1367
19		80	.7773	5.2993	4.7795	4.9966
20		100	1.0767	5.4260	5.1891	5.2219

Table 8. Data pertinent to stable extension of Mode I crack in fully yielded specimen

Fig. 1	ductility index n	initial crack length in a_0 -units	Ratio* $\frac{p}{\epsilon}$	Ratio R_{ss}/Δ	opening constant $\frac{R}{\Delta}$	initial value of stability index, λ_1	critical crack length in a_0 -units	amount of stable growth in a_0 -units	terminal toughness in R_1 -units	load at instability in ϵ_0 -units**
1	4	2	34.0	1.49×10^3	4.85	2.6534	201.71	200.71	230,389.0	0.355 1.000
2	4	10	34.0	1490.5	4.85	1.8487	40.34	39.34	1,230.4	0.353 0.954
3	2	2	8.77	27.30	2.85	0.6534	3.6945	2.6945	5.6622	0.3513 0.923
4	2	6	8.77	27.30	2.85	0.1041	1.2315	0.2315	1.2449	0.3515 0.733
5	2	10	8.77	27.30	2.85	-0.1513	1.000	-----	1.0000	0.3524

*Estimates of the ratios of the plastic component of fracture strain and the strain solution for Mode III.

**The upper value is obtained at the relative ductility index $R_1/a_0 = 25$, while the

at yield obtained from the analogy between Mode I and McClintock's

lower one corresponds to $R_1/a_0 = 5$.

Table 9. Initial slopes of the resistance curve, (dR/da) ; calculated under the following conditions: upper line: large scale yielding, asymptotic case, $R/a \gg 1$. middle line: intermediate range, arbitrary R/a ratio. lower line: small scale yielding, asymptotic case, $R/a \ll 1$.

$x_0 \backslash y_0$	10	20	40	80	100	140	160	180	200
	.7671	26.2250	78.8819	232.1904	327.6684	549.6152	674.42	807.63	948.75
2	8.283	25.304	75.807	224.040	316.800	533.113	655.033	785.32	923.48
	.5282	1.1246	2.0996	3.6137	4.2508	5.3808	5.8927	6.3772	6.8383
	.8747	3.6356	12.5575	40.000	57.7865	98.6570	122.01	147.04	173.66
10	1.462	4.748	14.258	42.154	59.691	100.768	124.004	148.885	175.32
	.5282	1.1246	2.0996	3.6137	4.2408	5.3808	5.8927	6.3772	6.8383
	-----	-----	-----	-----	4.5973	8.2539	10.3588	12.6317	15.063
100	.3000	1.136	2.866	6.741	8.907	13.684	16.286	19.022	21.889
	.5282	1.1246	2.0996	3.6137	4.2508	5.3808	5.8927	6.3772	6.8383
	-----	-----	-----	-----	-----	-----	-----	-----	-----
1000	.1934	0.8136	1.8632	3.6087	4.3949	5.8798	6.5945	7.2975	7.992
	.5282	1.1246	2.0996	3.6137	4.2508	5.3808	5.8927	6.3772	6.838

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25. Stability analysis for five cases of subcritical crack extension:

Run 1, $a_0 = 2$	$R_0 = 35.0019$	Tearing $n = 4$
Run 2, $a_0 = 10$	$R_0 = 35.0019$	modulous,
Run 3, $a_0 = 2$	$R_0 = 9.7745$	Tearing $n = 2$
Run 4, $a_0 = 6$	$R_0 = 9.7745$	
Run 5, $a_0 = 10$	$R_0 = 9.7745$	modulous

Note that the ordinate of the upper line (Slope dR/da) could be used as a measure of the energy required to propagate the crack while the function represented by the lower line (R/a) is proportional to the energy available for crack opening. Stability index λ is determined by the difference between the two functions shown.

The slope dR/da and the ratio R/a were obtained by integration of Wnuk's equation

$$\frac{dR}{da} = \frac{a + R}{a} \left\{ \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left[\frac{4eRa}{\Delta(2R+a)} \right] \right\} \quad \begin{matrix} 117 \\ 118 \end{matrix}$$

26. Initial slope of the resistance curve plotted vs. the non dimensional initiation toughness $Y_0 = R_0/\Delta$. Results shown were obtained under the following conditions.

(a) Large scale yielding range, asymptotic case, $R \gg a$. The governing differential equation is

$$\frac{dR}{da} = \frac{R}{a} \left\{ \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left(\frac{2ea}{\Delta} \right) \right\}$$

(b) Intermediate range involving an arbitrary R/a ratio. The governing equation for this range is

$$\frac{dR}{da} = \frac{a + R}{a} \left\{ \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left[\frac{4eRa}{\Delta(2R+a)} \right] \right\}$$

(c) Small scale yielding range, asymptotic case, $R \ll a$. The governing equation (b) is reduced to

$$\frac{dR}{da} = \frac{\hat{R}}{\Delta} - \frac{1}{2} \log \left(\frac{4eR}{\Delta} \right)$$

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27. Matching the experimental data of Fattorini, Mirabile, Podrini, and Spadaletti for ASA 52 steel by Wnuk's solution.

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Fig. 2. Matching the experimental data of Fattorini, Mirabile, Podrini, and Spadaletti for ASA 52 steel by Wnuk's solution.

Fig. 1. Resistance curve for a ductile material. Stable ($a < a_f$) and unstable ($a > a_f$) phases of crack extension are identified in the figure.

PAGE 9 12/13/77 LINEAR SOLUTION VS NON LINEAR , RO=9

Y3=1, Y33=2

LOAD Q →

Fig. 2. Normalized R-curves resulting from (a) the non linear Wnuk's equation and (b) from the linearized equation (3.18).

Fig. 3. Locus of terminal instability points on the (Load, Crack length) plane. Notation:

$$Q = \pi \sigma / 2 \sigma_Y, \quad x = a / R_{\infty}.$$

PROGRAM 2

Fig. 4. Loading parameter plotted vs. Current crack length for a centre cracked plate of infinite width. (Wnuk's solution for small scale yielding range.)

PROGRAM 3

Fig. 5 Load vs. crack length obtained for a centre cracked plate of infinite width according to the theory of (a) Cherepanov (b) Wnuk (c) Morozov

PAGE 6 12/13/77 LINEAR SOLUTION VS NON LINEAR, R0=.8

V3=1.0V33=2

LOADING PARAMETER

.402 12
12

.4015

.401

.4005

10.1

Fig. 6 • Load vs. crack length plots resulting from
(a) the non-linear Wnuk's theory and
(b) linearized equation, i.e., eq. (3.18)

PROGRAM 4

Fig. 7 (a) Effect of panel width on the critical crack length at constant initial crack length.

Fig. 7 (b) Effect of initial crack length on the critical load at constant panel width.

Fig. 7 (c) Effect of panel width on the critical load at constant initial crack length

Fig. 7 (d) Effect of initial crack length on the critical crack length at constant panel width.

Y1=1, Y2=2, Y3=3, Y4=4, Y5=5

LOADING PARAMETER

Fig. 9 (a) Critical load attained at the point of termination of the stable crack growth phase in a finite width panel

PROGRAM 10

Fig. 9 (b) The K_R vs. Δa curve obtained for a finite width panel from the crack theory of subcritical growth in small scale yielding range (eq. 3.1b), curve 1, and from experimental data of Creager (1972). (curve 2)

Fig. 10 Amount of slow stable crack growth as a function of the initial crack size and initiation threshold of toughness, P_0 . (linearized Wnuk's, Table 2, and Morozov's solutions for small scale yielding are shown)

Fig. 11 Amount of slow crack growth as a function of the initial crack length. Results shown were obtained from (a) Cherepanov, (b) Wnuk, (c) Morozov theory.

PROGRAM 3

Fig. 12 (a) Amount of slow crack growth as a function of the initiation threshold of loading parameter. Results shown were obtained from Cherepanov.

PROGRAM 3

Fig. 12 (b) Amount of slow crack growth as a function of the initiation threshold of loading parameter. Results shown were obtained from Wnuk.

FIG. 12 (c) Amount of slow crack growth as a function of the initiation threshold of loading parameter.
Results shown were obtained from Morozov.

Fig. 13 The critical load as a function of the initial crack length in the small scale yielding range. Results shown were obtained from (a) Cherepanov, (b) Wnuk, (c) Morozov theory.

Fig. 14 (a) The critical load as a function of the initiation threshold of loading parameter.
Results shown were obtained from Cherepanov.

PROGRAM 3

Fig. 14 (b) The critical load as a function of the initiation threshold of loading parameter. Results shown were obtained from knuk.

PROGRAM 3

Fig. 14 (c) The critical load as a function of the initiation threshold of loading parameter. Results shown were obtained from Morozov.

PROGRAM 3

Fig. 15 (a) Terminal toughness ($\gamma_f = R_f/R_\infty$) vs. the initiation toughness ($\gamma_0 = R_0/R_\infty$). All data shown were generated from the Wnuk's equation $dy/dx = .5 \log(1/y)$ valid within the small scale yielding range.

Fig. 15 (b) Terminal toughness ($y_f = R_f/R_\infty$) vs. the initial crack size ($x_0 = a_0/R_\infty$). All data shown were generated from the Wnuk equation $dy/dx = .5 \log(1/y)$ valid within the small scale yielding range.

Fig. 15 (c) The amount of slow crack growth ($\Delta x_f = \Delta a/R_\infty$) vs. the initial crack size ($x_0 = a_0/R_{ss}$). All data shown were generated from the Rankine equation $dy/dx = .5 \log (1/y)$ valid within the small scale yielding range.

Fig. 16. Matching the experimental data of ARMCO for aluminium alloy 7475-T61 (curve a) by Wnuk's solutions (curves b and c) obtained for small scale yielding range. The variations in slopes between curves a and b are explained in the text.

Fig. 17 • Matching the experimental data of ARMC0 for aluminium alloy 7075-T7 (curve a copied from Wilhem and Fitzgerald (1975)) by Wnuk's solutions (curves b & c) obtained for small scale yielding range. The variations in slopes between curves b and c are explained in text.

Fig. 18. The experimental data concerning Cartridge brass. Load vs. the load point opening displacement. copied from Ripling & Falkenstein(1970) work. The reduced data ($1/P$) needed for evaluation of the crack extension are also shown.

Fig. 19 Matching the experimental data of Ripling and Falkenstein for Cartridge brass by the small scale yielding solution of W_{mk} , see Table 6 .

Fig. 20 Matching the experimental data of Griffiths and Yoder (1976) for aluminium alloy 2024 by the large scale yielding solutions of Irwin (1976).

Fig. 21 Matching the experimental data of Griffis
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 the large scale yielding solution of Wnuk
 (1978). Equation (4.14), ratio $\bar{\sigma}_Y/\bar{\sigma}_0$ was
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Fig. 22 (a) Amount of crack growth X_f obtained for small scale yielding range, and plotted vs. the initial crack size.

Fig. 22 (b) Terminal toughness, Y_f , obtained for small scale yielding range, and plotted vs. the initial crack size.

Fig. 22 (c) Terminal toughness, Y_f , obtained for small scale yielding range, and plotted vs. the ductility level.

Fig. 22(d) The toughness magnification ratio Y_f/Y_0 obtained for small scale yielding range, and plotted vs. the initial crack size.

Fig. 23 (a) Amount of crack growth X_f obtained for large scale yielding range, and plotted vs. the initial crack size.

Fig. 23 (b) Terminal toughness, Y_f obtained for large scale yielding range, and plotted vs. the initial crack size.

FIG. 23 (c) Terminal toughness, Y_f obtained for large scale yielding range, and plotted vs. the ductility level.

Fig. 23 (d) The toughness magnification ratio Y_f/Y_0 obtained for large scale yielding range, and plotted vs. the initial crack size.

Fig. 24a Center cracked panel of infinite width subjected to Mode I loading. Q-curves for four runs are shown, runs i and ii are in the large scale yielding range, while runs iii and iv are in the small scale yielding range (all data are obtained from Whitt's closed form asymptotic solutions). The vertical broken line is drawn at $R/a = 1$, and it separates the small from large scale yielding range.

PROGRAM 11

$$Y_0 = 80$$

Fig. 24 (b) Center cracked panel of infinite width subjected to Mode I loading. Q curves

for four runs are shown, runs a and b are in the large scale yielding range, while runs c and d are in the small scale yielding range (all data are obtained from Wnuk's Solution valid for an arbitrary R/a ratio)

FIG. 25(a) Stability analysis for five cases of subcritical crack extensions:

Note that the ordinates of the upper line (slope dR/da) could be used as a measure of the energy required to propagate the crack while the function represented by the lower line (R/a) is proportional to

the energy available for crack opening. Stability index is determined by the difference between the two functions shown. The slope dR/da and the ratio R/a were obtained by integration of Irwin's equation.

Fig. 25(b) Stability analysis for five cases of subcritical crack extension. Note that the ordinates of the upper line (slope dR/da) could be used as a measure of the energy required to propagate the crack while the function represented by the lower line (R/a) is proportional to the energy available for crack opening.

Fig. 26 Initial slope of the resistance curve plotted vs. the non-dimensional toughness, Y_0

Fig. 27 Matching the experimental data of Fattorini, Mirabile Podrini, and Spadaletti for ASA 52 steel with Wnuk's solution for (a) small scale yielding and (b) large scale yielding ranges.

MOORE, J. E.

COMPUTER PROGRAMS

PROGRAMME NO.1

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L   Y3.K=Y3.J+DT*FUN3.JK
R   FUN3.KL=(LGJN(2/(TIME.K*Y3.K*Y3.K))-Y3.K*Y3.K)/(2*TIME.K*Y3.K)
A   Y33.K=SQRT((2./TIME.K)*(1-A.K*EXP((TIN-TIME.K)*.5)))
A   A.K=1-.5*TIN*YIN*YIN
N   Y33=YIN
N   Y3=YIN
N   TIME=TIN
C   TIN=10/YIN=.1
PLUT Y3=1,Y33=2
PRINT Y3,Y33,FUN3
SPEC LENGTH=15/DT=.001/PLIPER=.1/PRIPER=.1
RUN  LINEAR SOLUTION VS NON LINEAR, RO=.95

```

PROGRAMME NC2

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```
L Y.K=Y.J+DT*FUN.JK
R FUN.KL=((LOGN(2*CCS(P*TIME.K/TIN)/(Y.K*Y.K*TIME.K)))*COS(P*TIME.K/TIN
X )-Y.K*Y.K*(1+((P*TIME.K/TIN)*SIN(P*TIME.K/TIN))/CCS(P*TIME.K/TIN)))
X /(2*TIME.K*Y.K)
A YIN.K=SQRT(.2*CCS(P)/TIN)
N Y=YIN
N TIME=TIM
C TIM=5
PLOT Y=1
PRINT Y
SPEC LENGTH=10/DT=.01/PLTPER=.1/PRTPER=.1
C P=0
RUN XC=5,P=0
```


PROGRAM NO.3

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```

E  Y1.K=Y1.J+DT*FUN1.JK
L  Y2.K=Y2.J+DT*FUN2.JK
L  Y3.K=Y3.J+DT*FUN3.JK
P  FUN1.KL=(1.5*(2-TIME.K*Y1.K*Y1.K)/(TIME.K*TIME.K*Y1.K*Y1.K))
P  FUN2.KL=(2-TIME.K*Y2.K*Y2.K)/(TIME.K*TIME.K*Y2.K)
P  FUN3.KL=(1.75*(2/(TIME.K*Y3.K*Y3.K))-Y3.K*Y3.K)/(2*TIME.K*Y3.K)
N  Y1=Y1N
N  Y2=Y2N
N  Y3=Y3N
N  TIME=TIM
PLNT  Y1=1,Y2=2,Y3=3
PPINT  Y1,Y2,Y3,FUN1,FUN2,FUN3
C  TIM=5/Y1N=.05
SPEC  LENGTH=7/DT=.00001/PLTPE=.04/PKTPCR=.04
RUN  YJ=5,NJ=.05

```

PROGRAMME NO. 4

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```

L  Y1.K=Y1.J+DT*FUN1.JK
L  Y2.K=Y2.J+DT*FUN2.JK
L  Y3.K=Y3.J+DT*FUN3.JK
L  Y4.K=Y4.J+DT*FUN4.JK
L  Y5.K=Y5.J+DT*FUN5.JK
R  FUN1.KL=((LOGN(2/(Y1.K*Y1.K*TIME.K)))-Y1.K*Y1.K)/(2*TIME.K*Y1.K)
R  FUN2.KL=((LOGN(2*COS(.2*TIME.K/TIN)/(Y2.K*Y2.K*TIME.K)))
X  *COS(.2*TIME.K/TIN)-Y2.K*Y2.K*(1+((.2*TIME.K/TIN)*SIN(.2
X  *TIME.K/TIN))/COS(.2*TIME.K/TIN)))/(2*TIME.K*Y2.K)
R  FUN3.KL=((LOGN(2*COS(.4*TIME.K/TIN)/(Y3.K*Y3.K*TIME.K)))
X  *COS(.4*TIME.K/TIN)-Y3.K*Y3.K*(1+((.4*TIME.K/TIN)*SIN(.4
X  *TIME.K/TIN))/COS(.4*TIME.K/TIN)))/(2*TIME.K*Y3.K)
R  FUN4.KL=((LOGN(2*COS(.6*TIME.K/TIN)/(Y4.K*Y4.K*TIME.K)))
X  *COS(.6*TIME.K/TIN)-Y4.K*Y4.K*(1+((.6*TIME.K/TIN)*SIN(.6
X  *TIME.K/TIN))/COS(.6*TIME.K/TIN)))/(2*TIME.K*Y4.K)
R  FUN5.KL=((LOGN(2*COS(.8*TIME.K/TIN)/(Y5.K*Y5.K*TIME.K)))
X  *COS(.8*TIME.K/TIN)-Y5.K*Y5.K*(1+((.8*TIME.K/TIN)*SIN(.8
X  *TIME.K/TIN))/COS(.8*TIME.K/TIN)))/(2*TIME.K*Y5.K)
A  YIN1.K=SQRT(.2/TIN)
A  YIN2.K=SQRT(.2*COS(.2)/TIN)
A  YIN3.K=SQRT(.2*COS(.4)/TIN)
A  YIN4.K=SQRT(.2*COS(.6)/TIN)
A  YIN5.K=SQRT(.2*COS(.8)/TIN)
N  Y1=YIN1
N  Y2=YIN2
N  Y3=YIN3
N  Y4=YIN4
N  Y5=YIN5
N  TIME=TIN
C  TIN=10
PLOT Y1=1,Y2=2,Y3=3,Y4=4,Y5=5
PRINT Y1,Y2,Y3,Y4,Y5
SPEC LENGTH=14/DT=.001/PLTPER=.1/PRTPER=.1
RUN XO=10

```

PROGRAMME NO. 5

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```
L Y1.K=71.3+DT*FUN1.JK
R FUN1.KL=((LOGN(1/Y1.K))/(Y1.K))*1.5
N Y1=YIN
N TIME=IIN
C IIN=1.75/YIN=.16
PLCT Y1=1,(0,1)
PRINT Y1,FUN1
SPEC LENGTH=5/DT=.001/PLTPER=.05/PRTPER=.1
RUN K CURVE,X0=1.75,YC=.16
```

PROGRAMME NC. 6

```

3      A  CQ.K=(1)/(1+(Y0/X0)*EXP((TIME.K-TIN.K)
4      X  *(L.K-0.25*(TIME.K-TIN.K))))
5      A  L.K=.5*(LOGN(2)-3.+SQRT(2*Y0-1)-LOGN(X0))
6      A  TIN.K=LOGN(X0)
7      A  XF.K=2*L.K+TIN.K
8      A  CQF.K=1/(1+(Y0/X0)*EXP(L.K*L.K))
9      N  TIME=TIN
10     PLOT CQ=*
11     PRINT CQ,L,XF,CQF
12     C  X0=4.5/Y0=40
13     SPEC LENGTH=7/DT=.01/PLTPER=.1/PRTPER=.1
14     RUN X0=4.5,Y0=40

```

PROGRAMME NO. 7

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L Y.K=Y.J+DT*FUN.JK
R FUN.KL=((LOGN(2*A.K/(Y.K*Y.K)))-TIME.K-Y.K*Y.K)
X /(2*Y.K)
A A.K=EXP(SQRT(2*Y0-1)-1)
A YIN.K=SQRT((2*Y0)/X0)
A TIN.K=LOGN(X0)
N Y=YIN
N TIME=TIN
PLOT Y=*
PRINT FUN,Y
C Y0=80/X0=400
SPEC LENGTH=12/DT=.01/PLTPER=.1/PRTPER=.1
RUN Y0=80,X0=400

PROGRAMME NO. 8

```

2
3   L   Y1.K=Y1.J+DT*FUN1.JK
4   L   Y2.K=Y2.J+DT*FUN2.JK
5   R   FUN1.KL=60.48*LOGN(1000./Y1.K)
6   R   FUN2.KL=115*LOGN(1150./Y2.K)
7   N   TIME=TIM
8   N   Y1=YIN
9   N   Y2=YIN
10  C   TIN=0
11  C   YIN=40
12  PRINT  Y1,Y2,FUN1,FUN2
13  PLOT  Y1,Y2
14  SPEC  LENGTH=5/DT=.01/PLTPER=.1/PRTPER=.1
15  RUN  MATCHING PROBLEM

```

PROGRAMME NO. 9

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```
I  Y1.K=Y1.J+DT*FUN1.JK
L  Y2.K=Y2.J+DT*FUN2.JK
R  FUN1.K1=1.5*(2-TIME.K*Y1.K*Y1.K)/(TIME.K*TIME.K
X  *Y1.K*Y1.K*Y1.K)
R  FUN2.K1=(LOGN(2/(TIME.K*Y2.K*Y2.K))-Y2.K*Y2.K)/
X  (2*TIME.K*Y2.K)
A  T1.K=.5*TIME.K*Y1.K*Y1.K
A  T2.K=.5*TIME.K*Y2.K*Y2.K
N  TIME=X0
N  Y1=Y0
N  Y2=Y0
PLOT Y1,Y2,T1,T2
PRINT Y1,Y2,FUN1,FUN2,T1,T2
C  X0=10/Y0=.2
SPEC LENGTH=15/DT=.01/PLTPER=.1/PRTPER=.1
RUN X0=10,Y0=.2
```

PROGRAMME NO. 10

```
2
3      L  Y1.K=Y1.J+DT*FUN1.JK
4      R  FUN1.KL=(15000/Y1.K)*LOGN(120/Y1.K)
5      A  EFF.K=TIME.K+.00007812*Y1.K*Y1.K
6      N  TIME=TIN
7      N  Y1=YIN
8      N  TIN=0
9      N  YIN=80
10     PRINT Y1,EFF
11     SPEC LENGTH=3/DT=.01/PLTPER=.1/PRTPER=.1
12     RUN TIN=0,YIN=80
```


PROGRAMME NO. 11

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```

L Y1.K=Y1.J+DT*FUN1.JK
L Y2.K=Y2.J+DT*FUN2.JK
R FUN1.K1=.5*(1.+(Y1.K/TIME.K))*(B.K-LOGN
X ((4*TIME.K*Y1.K)/(2*Y1.K+TIME.K)))
R FUN2.K1=(TIME.K*FUN1.JK-Y11.K)/((TIME.K+Y11.K)
X *SCR1(2*TIME.K*Y11.K+Y11.K*Y11.K))
A E.K=SCR1(2*YC-1)-1+LCN(4)
A R1.K=Y1.K/TIME.K
A LC1.K=((FUN1.JK*TIME.K)/Y1.K)-1
A Y11.K=Y1.K
A Y11.K=TIME.K*((1/COS(Y2.K))-1)

```

Y11 DEFINED PREVIOUSLY

N TIME=XC

N Y1=Y0

N Y2=Y1A

PLOT Y2,LC1

PRINT Y2,FUN2,LC1,Y1,R1

C XC=4.51/YC=80/Y1N=1.51740

SPEC LENGTH=50000/DT=1/PLTPER=500/PRTPER=500

RUN XO=4.51/YC=80

1 ERROR MESSAGES

PROGRAM 12

```

L  Y1.K=Y1.J+DT*FUN1.JK
L  Y2.K=Y2.J+DT*FUN2.JK
R  FUN1.KL=.5*(EXP(TIME.K)+Y1.K)*(B.K-LOGN((4*Y1.K)/
X  (2*Y1.K+EXP(TIME.K)))-TIME.K)
R  FUN2.KL=.5*Y2.K*(B.K-LOGN(2)-TIME.K)
A  TIN.K=LOGN(X0)
A  B.K=SQRT(2*Y0-1)-1+LOGN(4)
A  LL1.K=(FUN1.JK/Y1.K)
A  LL2.K=(FUN2.JK/Y2.K)
A  S1.K=(FUN1.JK)/(EXP(TIME.K))
A  S2.K=(FUN2.JK)/(EXP(TIME.K))
A  R1.K=Y1.K/EXP(TIME.K)
A  R2.K=Y2.K/EXP(TIME.K)
N  TIME=TIN
N  Y1=Y0
N  Y2=Y0
PLOT S1,R1
PRINT Y1,Y2,S1,S2,LL1,LL2
C  X0=2/Y0=35.0019
SPEC LENGTH=6/DT=.01/PLTPER=.1/PRTPER=.1
RUN X0=2,Y0=35.0019

```

NOT USED