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Jenny Blackburne
South Dakota State University

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Vibrations Analysis on Tennis Racket String

Author: Jenny Blackburne
Faculty Sponsor: Dr. Shawn Duan
Department: Mechanical Engineering

ABSTRACT

The purpose of this project was to investigate and analyse the vibration of a tennis racket string when a ball is hit. This was done using concepts of mechanical vibrations and modelled using Microsoft Excel. The displacement of the tennis string over time was graphed for two different positions on the racket and two different relative speeds of the tennis ball. It was found that the position of the tennis ball on the racket has very little effect on the displacement of the strings. If the ball or racket is moving faster when the ball is hit, then the displacement of the strings is proportionally greater. There were a few factors in the modelling that were unlike the actual vibration of a tennis string. This model did not take into account the fact that the tennis ball would deform when it is hit or the deflection of the tennis racket. The designed model will help compare different spring constants and ball speeds for different scenarios.

INTRODUCTION

Tennis is a sport enjoyed around the globe by participants of varying ages and abilities. It is important for a tennis racket to be well designed to maximize performance and minimize injury. There are many different aspects of a tennis racket that can affect how the tennis ball comes off the strings. One of the main factors to take into account is the vibration that occurs when the ball is hit. If there is too much vibration in the racket or the strings, this vibration can be carried down the racket handle to the player’s arm and cause wear or injury such as tennis elbow.

This study looks into the vibration of a tennis racket string during and after the ball is struck. Due to the large number of variables relating to the mechanisms of string vibration,
a simplified model was created in order to perform vibrational analysis. This model is shown in Figure 11. It assumes that rather than the racket frame and the tennis ball both moving with a certain velocity in opposite directions. The racket is fixed and the ball is moving at its speed plus the speed the racket is moving when it is hit. This is effectively the relative velocity of the ball in relation to the tennis strings. In the diagram, the spring and dampener are shown as fixed, this is to help visualize what is happening when the ball hits and the string starts vibrating. The mass of the tennis ball is involved in the diagram.

![Diagram of tennis string vibrations](image)

**Figure 11:** This is a simplified model of the tennis string vibrations.

**METHODS**

*Spring Constant Calculation*

To calculate the spring constant for the tennis racket, a re-stringer and a 4.5 kg weight were used. The re-stringer was strung with one string going into and out of the racket creating two lines of string. It was strung at tensions of 26 kg and 28 kg and the initial distance
from the top of the frame to the string was measured and tabulated. The 4.5 kg weight was hung from the string and the final displacement was measured and tabulated. The spring constant was found using Hooke’s Law as follows:

\[ F = kx \]  

(1)

By dividing the force of 4.5 kg by the displacement of the string, the spring constant can be calculated. This was done a total of six times, one for each of the three string set-ups and each of the two tensions. The three different string set-ups were as follows:

**Figure 12:** These are the three different string positions on a tennis racket.  

To find the value of the equivalent spring constant when the ball hits the racket, the assumption was made that the ball will hit four strings in the vertical direction and four strings in the horizontal direction. These strings will be the only strings that affected the spring constant. They were effectively connected in parallel; therefore, the spring constants were added together as follows:
\[ K_{eq} = 2K_a + 2K_b \] (2)

\( K_a \) and \( K_b \) are the average spring constant values at a given position.

For the analysis, two positions on the racket were investigated and compared. The two positions are shown in Figure 13.

**Figure 13:** Position 1 and Position 2 on the tennis racket were investigated and compared.

The spring constant used for position one was calculated using the values obtained for setup 1 and 3 spring constants. This calculation is given below.

\[ K_{eq} = 2K_1 + 2K_3 \] (3)

\[ K_{eq} = 2 \times 3,241 + 2 \times 4,684 \]
\[ K_{eq} = 15851 N/m \]

The spring constant for the second position was found in the same way, calculating to be 15,907 N/m.

A set up of the re-stringer is shown Figure 14. There is a weight tied to the strings and the deflection of the strings was measured. This picture shows the strings at position 1 as per Figure 12.

![Figure 14: The setup of a re-stringer in order to calculate the spring constant.](image)

**Assumptions and simplified system**

There are a vast number of factors involved in the vibration of a tennis string. In order to derive an equation of motion, a simplified system is needed to be made and assumptions are needed to be defined. Even though the simplified models created were not exactly the
same as the actual vibration of a fully strung tennis racket being used in a tennis match, they were similar enough to compare different scenarios. Some of the factors that could affect the vibration of a tennis racket string include:

i. Location where tennis ball is hit on the racket.
ii. Amount of mains and crosses on the tennis racket (the number of vertical and horizontal strings the racket has).
iii. The tension of the tennis strings.
iv. The speed of the ball and the speed of the racket when contact is made.
v. The time the ball is in contact with the tennis racket.
vi. The shape of the racket frame.
vii. The type of string used in the racket (some strings are more flexible or thinner than others).
viii. The type of tennis ball being hit (tennis balls are pressurized and will deform differently).

This is only a basic list of factors that affect the vibration of the tennis strings. It is noted that there may be many more not identified.

Furthermore, assumptions are made based on the level of analysis and the factors measured using basic equipment. The simplified system that we have created using a single string will have an amplified vibration rather than a fully strung racket with strings crossing over each other going horizontally and. This is because of the spring constant, k, of the single string being vastly smaller than an entire system. It would be difficult to measure the spring constant of the strings on a fully strung racket, because the force needed to be applied on the strings would be huge and the deflection would be small creating a large amount of error. Other assumptions that have been made in order to create an equation of motion are as follows:

1. Neglect mass of the strings and deformation of the tennis ball.

This can be done because the mass of the tennis strings is very small. This project looks at comparing different scenarios related to the length of tennis string and the tension which allows for the deformation of the tennis ball to
be neglected. Neglecting this deformation will mean that the modelled displacement of the tennis strings will be larger than the actual deformation.

2. The tennis ball (applied force) is hit in the exact middle (horizontally) of the racket.

   To simplify the equation of motion, it is necessary to make this assumption. Otherwise the equations created will be different for each point that the tennis ball is hitting. This assumption is valid because professional tennis players and college tennis players hit the middle of the strings a large percentage of the time in order to maximize power and accuracy.

3. The mass of the system is the mass of the tennis ball and it is constant the entire time the strings are moving.

   Since the tennis ball is in contact with the strings for a majority of the time the strings are vibrating and the vibration of the strings is tiny, this approximation can be used for the basic model.

4. The tennis ball is traveling perpendicular to the racket strings and parallel to the deformation of the tennis strings.

   Often when a tennis ball is hit, it does not come perpendicular to the strings, but this assumption is necessary to simplify the equation of motion and find the relative velocity of the tennis ball to the racket.

5. The spring constant of the strings is constant.

   Due to the fact that as the ball hits the strings and they deform (lengthen), the tension in the strings slightly increases. This means that the string deformation would be non-linear. It is necessary to assume that this is not the case, so that we can use the equations of motion we have learned in class.
RESULTS

*Free Body Diagram*

A free body diagram of the vibration system was created in order to form an equation of motion. The diagram is shown below in Figure 15.

![Free Body Diagram](image)

**Figure 15**: This figure is a free body diagram of vibrations system.

Where:

- $C\dot{x}$ = Damping Force
- $m\ddot{x}$ = Inertial Force
- $kx$ = Spring Force
- $F_0$ = Force of the tennis ball

*Equation of Motion*

The equation of motion for the system can be obtained by writing a force balance for the free body diagram in Figure 15. When the forces in the $x$-direction are balanced, we get the following equation:

$$m\ddot{x} + c\dot{x} + kx = F_0$$  \hspace{1cm} (4)
The convolution of integral was used to solve this equation where $F_0$ was applied for a small period of time and $F(\tau)$ is shown in Figure 16 (not to scale). $F(\tau)$ represents the force of the tennis ball over time.

![Graph](image)

**Figure 16:** This graph compares force vs time graph for convolution of integral.

The basic form for the convolution of integral equation was used to start with and is given below.

$$X(t) = \frac{1}{m\omega_d} \int_0^t F(\tau)e^{-\zeta\omega_d(t-\tau)}\sin\omega_d(t-\tau)d\tau \tag{5}$$

The force function shown in Figure 16 can be rewritten as follows:

$$F(t) = \begin{cases} F_0, & 0 < t < t_0 \\ 0, & t \geq t_0 \end{cases} \tag{6}$$

Where:  
$F_0 = \text{Maximum force of the tennis ball}$  
$t_0 = \text{Time the tennis ball is in contact with the strings}$

Substituting this into the convolution of integral equation and solving for the time interval $0 < t < t_o$ gives the following:
\[ x(t) = \frac{F_0}{m\omega_d} \int_0^t e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau) d\tau \]  

(7)

\[ x(t) = \frac{F_0}{m\omega_d (\zeta\omega_n)^2 + \omega_d^2} \left[ e^{-\zeta\omega_n(t-\tau)} \right] + \frac{1}{\omega_d} \omega_d \cos\omega_d(t-\tau) \]  

(8)

Evaluating the integral from 0 to \( t_0 \) for the interval of \( t > t_0 \) gives the following equation:

\[ x(t) = \frac{F_0}{m\omega_d (\zeta\omega_n)^2 + \omega_d^2} \left[ \omega_d e^{-\zeta\omega_n t} \right] - e^{-\zeta\omega_n t} \left[ \omega_n \sin\omega_d(t) + \omega_d \cos\omega_d(t) \right] \]  

(9)

Where:  
\( x(t) = \text{Displacement of the tennis racket strings over time} \)  
\( m = \text{mass of the system} \)  
\( \zeta = \text{Damping ratio} \)  
\( \omega_n = \text{Natural frequency of vibration} \)  
\( \omega_d = \text{Damping frequency of vibration} \)

**Graphical Analysis**

For the graphical analysis, values need to be calculated and input into the equations above.

To find \( F_0 \), first the velocity and mass of a tennis ball needed to be known. The official mass of a tennis ball is 56.7 g. For the velocity, the relative velocity needed to be used as the system is modelled as a stationary racket. To take an extreme case, the racket head speed of Roger Federer’s racket and the speed of a forehand hit by professional tennis players were used. The rotational speed of Federer’s racket when he is hitting a forehand is about 2400 rpm. A slower forehand would be around 700 rpm. If a lever arm value of 0.63 m is used, which includes the length to the center of the racket as well as 10 cm from the wrist up the forearm to the center of rotation, then the approximate speed of the racket at the point where the ball is hit can be obtained by the following equations:
\[ \omega = \frac{2400 \text{rev}}{\text{min}} \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) \left( \frac{1 \text{min}}{60 \text{sec}} \right) \]

\[ \omega = \frac{251 \text{ rad}}{\text{sec}} \]

\[ V_r = \omega r \]

\[ V_r = \frac{251}{s} (0.63m) \]

\[ V_r = 158 \frac{m}{s} \]

for a racket head speed of 2400 rpm, and

\[ V_r = 46.2 \frac{m}{s} \]

for a speed of 700 rpm.

The speed of the tennis ball used is 120.7 km/hr (75 mph) which would be the speed of a powerful forehand [2]. This can be converted into \( V_b = 33.5 \text{ m/s} \). Adding \( V_b \) and \( V_r \) together to obtain the relative velocity of the tennis ball on the strings gives:

\[ V_{rel} = 191.5 \frac{m}{s} \]

for a speed of 2400 rpm, and

\[ V_{rel} = 79.7 \frac{m}{s} \]

for a speed of 700 rpm.

From the R.Cross essay [1] the contact time the ball is on the racket is 4 m/s. If this value is used, then the average acceleration and force for a speed of 2400 rpm can be found as follows:

\[ a = \frac{v}{t} \]
\[ a = \frac{191.5}{0.004} \]

\[ a = 47875 \frac{m}{s^2} \]

\[ F = ma \]

\[ F = 0.0567 \times 47875 \]

\[ F = 2715N \]

For a speed of 700 rpm, the acceleration and force are:

\[ a = 19925 \frac{m}{s^2} \]

\[ F = 1130N \]

The natural frequency and the damped frequency can be calculated as shown:

\[ \omega_n = \sqrt{\frac{k}{m}} \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

For the values of \( \zeta \), a value of 0.03 was assumed as this made the vibration system look most realistic.

Figure 17 through Figure 19 show the displacement of the tennis strings given certain conditions. Figure 17 and Figure 18 show the displacement of the strings at the different positions of the racket given the different spring constants at the slower speed and Figure 19 shows the displacement at the larger speed of 2400 rpm.
**Figure 17**: The displacement of strings at position one.

**Figure 18**: The displacement of strings at position two is displayed in this figure.

**Figure 19**: The displacement of strings with a force of at 2400 rpm.
CONCLUSION

The purpose of this project was to measure the movement of tennis racket strings after they have been struck by a tennis ball. This was performed by doing a thorough analysis and finding an equation of motion for the strings. The spring constant value needed for the equation of motion was found using a racket re-stringer and applying a known amount of weight to different string tensions. The equation of motion was found by writing a force balance of the free body diagram of the vibration system. The string position was then solved for using the convolution of integral method. The equation of motion obtained was then graphed, and can be seen Figure 17 through Figure 19. Figure 17 and Figure 18 do not vary much as the variation of the spring constant was so small. This means that there will only be a slight decrease in string vibration when the position the ball is hit changes.

Figure 19 is an extreme example at a high speed; therefore, the results may be skewed. At such a high speed, the tennis ball deforms a greater amount, more than the strings. The deformation of the ball was not taken into account during this project, which would be the cause for some error in the results of the movement of the strings. It is unrealistic that the strings deform a whole 4 cm, so the deformation of the ball must be taken into account. This deformation would have meant that the force applied to the strings would have been less because the speed of the ball was effectively ‘cushioned’ by the ball deformation. This would have been the case in the other examples at a lower speed as well but not to such a large extent. The higher applied force on the tennis strings may even change the spring constant because the strings would lengthen as the force is applied.

The comparison of the two different speeds show, as expected, that the deformation of the strings is a lot higher with the higher relative speed of the tennis ball. For the slower speed, the maximum deformation read from the graph is about 1.85 cm. The faster speed had a maximum deformation of 4.4 cm. The difference in force between these two scenarios was 1130 N and 2715 N for the slower and faster speeds respectively. This was a 140% increase in force applied and it yielded a 138% greater maximum displacement. This shows that the increase in force is directly proportional to the increase in maximum displacement provided all other constants remain the same. This is expected from looking at the equation of motion.
REFERENCES
