10-1-1993

The Effect of Enforcement and Taxes in a Joint Export

Scott Fausti

South Dakota State University

Follow this and additional works at: http://openprairie.sdstate.edu/econ_staffpaper

Part of the Agricultural and Resource Economics Commons

Recommended Citation


This Article is brought to you for free and open access by the Economics at Open PRAIRIE: Open Public Research Access Institutional Repository and Information Exchange. It has been accepted for inclusion in Department of Economics Staff Paper Series by an authorized administrator of Open PRAIRIE: Open Public Research Access Institutional Repository and Information Exchange. For more information, please contact michael.biondo@sdstate.edu.
THE EFFECT OF ENFORCEMENT AND TAXES IN A JOINT EXPORT

by

Scott W. Fausti*

Economics Staff Paper 93-8**

October 1993

*Assistant Professor in the Economics Department at South Dakota State University.

**Papers in this series are reproduced and distributed to encourage discussion of research, extension, teaching, and economic policy issues. Although available to anyone on request, Economics Department Staff Papers are intended primarily for peers and policy makers. Papers are normally critiqued by some colleagues prior to publication in this series. However, they are not subject to the formal review requirements of South Dakota State University's Agricultural Experiment Station and Cooperative Extension Service publications.
The Effect of Enforcement and Taxes in a Joint Export Model of Smuggling

Scott Fausti

South Dakota State University

July 16, 1993

ABSTRACT

This paper proposes a model of joint product smuggling which explicitly examines the roles of uncertainty, enforcement, taxes, and the magnitude of a real resource cost in determining the firm's decision to smuggle and smuggling's impact on welfare and tax revenue collection. A framework is presented in which: 1) the tax rate, 2) the level of government enforcement, and 3) the real resource cost are analyzed to determine their impact on a firm's decision to smuggle or engage in strictly legal trade. The results derived in the paper indicate that the implied policy solution for the smuggling problem arrived at in the earlier literature of "the less smuggling the better" is at best misleading.

1 All correspondence should be directed to Scott Fausti, Assistant Professor, Department of Economics, South Dakota State University, Scobey Hall, Box 504A, Brookings, South Dakota 57007-0895.
I. Introduction.

The smuggling of imports and exports is a common phenomenon for lesser developed countries when high tariff or export tax rates are levied on traded goods. Smuggling therefore raises several serious economic issues concerning social welfare and trade tax policies for lesser developed countries. A literature review of these issues by Bhagwati (1981) reveals that the economic consequences for the interaction of government enforcement with smuggling have not been rigorously analyzed with respect to trade tax revenues and welfare.

In this essay the above issues are addressed. The starting point of the paper is the modification of Pitt's (1981) model of joint product export smuggling. The modification allows the incorporation of active government enforcement of the smuggling laws. Active enforcement is the assumption used to introduce uncertainty into the model.¹ The results of the model indicate that: 1) joint product smuggling does not have a strictly negative impact on welfare as compared to the non-smuggling case, 2) increased enforcement against smuggling does not have a strictly positive effect on welfare, 3) smuggling's impact on tax revenue collection is ambiguous, and 4) the presence of joint product smuggling reduces the revenue maximizing tariff rate.

¹ It is assumed that enforcement effort against smuggling incurs a zero real resource cost. This is a reasonable assumption if one assumes that increased enforcement effort against smuggling only requires a reallocation of resources within the legal system.
II. Assumptions.

The paper begins with the basic assumptions of Pitt's model of smuggling. Pitt assumes the small country case with the terms of trade fixed. The country produces two traded goods, an exportable (X) and an importable (M), employing primary factors purchased in competitive markets. Production and trade are carried out by identical firms. Domestic production is subject to diminishing returns. Legal and illegal trade in exports is carried out by the same firm. The law of one price holds in the domestic economy.

The following additional assumptions are made so that a model of smuggling incorporating uncertainty can be developed: 1) firms that smuggle do incur a significant real smuggling cost; 2) smugglers (firms) are natives and therefore their utility functions are embodied in the country's social welfare function; 3) export taxes are assumed to be non-prohibitive; 4) firms must bear the risk of illegal activity and they cannot insure against criminal penalties; 5) exporting firms have a choice between strictly legal trade or smuggling, with the choice based on profit maximization; 6) firms are risk neutral; and 7) if the domestic exporting firm decides to smuggle, it will then produce a joint product, and legal trade will act as a cloak for the firm's illegal activity. The firm can use four methods to smuggle exports: a) under-invoicing of exports, b) falsely

---

2 Cooper (1974), and Deardorff and Stolper (1990) argue that smuggling may not impose any significant real cost on society over legal trade.
declared exports, c) under-assessment of exports, and d) clandestine smuggling of unreported production.\(^3\)

III. A Joint Product Model of Smuggling.

In addition to the assumptions made in the previous section, it is assumed each firm can trade illegally according to a modified Pitt smuggling function,

\[ S^* = G(L,S). \tag{1} \]

The variable \(S^*\) is the quantity of good \((X)\) made ready to be smuggled. The variable \(S^*\) in this model is defined as exports made ready for smuggling across the domestic border or, in other words, smuggling attempted. The variable \((L)\) is the quantity of good \((X)\) legally traded and \((S)\) is the quantity of good \((X)\) input into smuggling activity. The function \((G)\) is strictly concave and a twice differentiable linear homogeneous function. The function \((G)\) is assumed to have the following properties:

\[
\begin{align*}
G_L & \geq 0, \quad G_{LL} \leq 0, \tag{2} \\
1 & \geq G_S \geq 0, \quad G_{SS} \leq 0, \tag{3} \\
G(0,S) & = 0, \tag{4} \\
G(L,0) & = 0, \tag{5} \\
S - S^* & \geq 0, \quad \frac{\partial(S-S^*)}{\partial L} < 0, \quad \frac{\partial(S-S^*)}{\partial S} > 0. \tag{6}
\end{align*}
\]

\(^3\) Deardorff and Stolper (1990) discuss the widespread use of smuggling method (d) in a number of African countries.
Assumption (2) states that the marginal product of legal trade used in smuggling is non-negative and is declining in \((L)\). Assumption (3) states that a unit increase in the smuggling input \((S)\) results in a positive but less than unit increase in actual amount of the export made ready to be smuggled, and the marginal product of \((S)\) is declining. Assumption (4) states that legal trade is a necessary input into smuggling or the probability of detection is one. Assumption (5) states that firms can choose to engage in legal trade only. Assumption (6) prohibits the real resource cost of smuggling from being negative. The real resource cost of smuggling \((S-S^*)\) is the smuggler's selling cost in excess of legal trade selling cost. It is assumed that the actual magnitude of smuggling's real resource cost is exogenous to the model. However, a change in one of the endogenous variables, \((L)\) or \((S)\), affects the marginal resource cost of smuggling. A one-unit increase in \((L)\), ceteris paribus, reduces the marginal real resource cost of smuggling. A one-unit increase in \((S)\), ceteris paribus, increases the marginal real resource cost of smuggling.

In the literature, smuggling's ambiguous welfare effect is the direct result of how the real resource cost assumption is

---

*The excess selling cost is the consequence of firms engaging in cloaking activity to conceal illegal exports. Martin and Panagariya (1984), Thursby (1991), and Fausti (1992) also used this type of approach to generate the real resource cost associated with smuggling in their papers.*
modeled. A negative welfare effect results from an excessive real resource cost incurred by smugglers, while an insignificant real resource cost produces a positive welfare effect. As an example, Pitt assumes that the cost of smuggling is composed of either penalties and confiscation or a mixture of a real resource cost and penalties and confiscation. His welfare result is ambiguous because the composition of the cost mix is unknown.

Pitt's assumption is altered and it is assumed that the difference between \(S\) and \(S^*\) is a real resource cost incurred from the use of cloaking tactics employed to evade detection.

Smuggling is assumed to incur a risk of detection \(\rho\), \(1 \geq \rho \geq 0\) such that \(\rho = 1\) if \(L = 0\). The expected value of illegal goods intercepted as they are moved over the border is \(\rho \cdot P\cdot S^*\) or \(P\cdot S^*\) if \(L = 0\). The variable \(P\) is the world price of exports. The expected value of successful smuggling is \([(1-\rho)P\cdot S^*]\). The variable \(F\) is a multiple of the value of intercepted illegal goods which is imposed as a fine, \(F \geq 1\). The expected cost of interception to the smuggler is \(\rho \cdot F \cdot P\cdot S^*\) and

---


6 The result of the welfare effect being dependent on the magnitude of the real resource cost is what Pitt and Bhagwati and Hansen refer to as the ambiguous welfare result attributed to smuggling.

7 The real resource cost, for example, may take the form of: 1) special packing cost necessary to hide smuggled goods, and 2) the transport cost of shipping unreported production out of the country via clandestine ports.
is at least \((P^f \cdot S^*)\) if \((L=0)\). Expected smuggling revenue net of interception cost is equal to \([(1-\rho \cdot F) \cdot P^f \cdot S^*]\) and is non-positive if \((L=0)\).

The expected value of output price per unit of smuggled good at the border for the smuggling firm is \(E[P^s] = (1-\rho \cdot F) \cdot P^f\) and is non-positive if \((L=0)\). The expected value of revenue per input unit of the smuggled good at the border for the smuggling firm is \(E[P^s] \cdot (S^*/S) = (1-\rho \cdot F) \cdot P^f \cdot (S^*/S)\) and is non-positive if \((L=0)\). The expected value for the output price per unit of legally exported goods is \(E[P^l] = P^f \cdot (1-t) = P^f\) and represents the legal tax distorted price for exports. The variable \((t)\) denotes the ad valorem export tax. It is assumed the firm knows the values of these risk factors.

It is assumed each firm has a decision to make. The firm can engage in joint product smuggling or it can sell its output at the legal domestic export tax distorted price \((P^l)\), as implied by the assumption \(G(L,0)=0\).\(^8\) If the firm decides to smuggle, it receives the weighted average price for its total output.

If the firm decides to become involved in joint product illegal trade, then its situation can be thought of as a lottery. The expected value of the lottery is dependent on variables \((\rho, Y^*, F)\). The probability of apprehension \((\rho)\) is determined by the government. It is assumed the firm's probability of being caught is \((\rho)\) if it engages in cloaking activities. If it does

\(^8\) Strictly legal trade profits are derived from equation (7) when it is assumed \(S=0\).
not cloak its illegal activity, then the probability of apprehension is equal to one. The variable (F) determines the monetary equivalent of the punishment imposed on the firm by the government if the firm is caught in the illegal act of smuggling. Fines are considered a transfer to the government. As in Pitt's paper, profit maximization in production implies producing on the production possibility curve where the marginal rate of transformation equals domestic relative prices (P*). The variable (Y*) represents profits from joint product export trade.

Smugglers are assumed to be profit maximizers. Expected profit for the smuggling firm is given by equation (7),

\[ \text{E}(Y^*) = P^f \cdot G(L,S) - (\rho) \cdot F \cdot P^f \cdot G(L,S) + P^f \cdot (1-t) \cdot L - P^* \cdot (L+S). \]

The term \([P^f \cdot G(L,S) - (\rho) \cdot F \cdot P^f \cdot G(L,S)]\) denotes expected smuggling revenues; \(P^f \cdot (1-t) \cdot L\) represents revenues for legal trade. As in Pitt's article, firms earn zero economic profit in the long run. Setting equation (7) to zero and solving for \(P^*\) generates an expression for the long-run equilibrium domestic price ratio as a weighted average of prices received for goods legally exported in conjunction with goods illegally exported:

\[ P^* = \frac{(1-p \cdot F) \cdot P^f \cdot (S*)}{(L+S)} + \frac{P^f \cdot (1-t) \cdot (L)}{(L+S)}. \]

The exporting firm's decision of whether to engage in strictly legal trade (S=0) or engage in smuggling and produce a joint product (L+S*) will determine the long-run equilibrium

\[ \text{The first and second order conditions can be found in appendix (A).} \]
domestic price ratio (DPR). If firms smuggle, then Pitt's "price disparity" result is generated: \( p^* > p^L \).

IV. The Firm's Decision Mechanism

In the economic literature on uncertainty, it has been established that a risk neutral firm confronted with uncertainty will make its profit maximization decision as if it were operating in a certainty environment. Joint product illegal trade profit represents an uncertain prospect and legal trade represents a certain prospect. The firm's decision to engage in joint product smuggling \((L+S^*)\) or engage in strictly legal trade \((L)\) is based on a comparison of expected profits from joint product smuggling to profits earned from strictly legal trade.

The firm's decision will determine the domestic price ratio. If the firm to engages in joint product smuggling or strictly legal trade, it does so based on the following decision criteria condition statement:

\[
DPR = \max[P^L(1-t), P^L(S^*S)(1-\rho \cdot F)], \tag{9}
\]

where \((\rho \cdot F)\) is the expected value of punishment. Condition (9) leads to the first proposition of the paper:

\footnote{For a discussion of the behavior of a competitive firm operating under uncertainty see Hey (1979).}

\footnote{Condition (9) is derived from equation (8). Condition (9) makes a comparison of revenue coming from illegal trade \((P^L \cdot S^* \cdot (1-\rho \cdot F))\) to the revenue which would be earned by channeling illegal goods through legal channels, \((P^L \cdot S \cdot (1-t))\).}
PROPOSITION 1. If the firm receives a higher price via joint-product smuggling, then all firms will smuggle and Pitt's price disparity phenomena will result.

To establish proposition 1, the implications of condition (9) are examined. Condition (9) states that if the expected value of revenue per input unit of smuggled good is greater than the per unit revenue that could be earned by selling (S) through legal channels, then all firms smuggle and DPR=P*. If not, then DPR=P⁻. The following statements outline the firm's decision mechanism for engaging in joint product smuggling or the strictly legal trade alternative:

\[
P^f \cdot S^* \cdot (1-\rho \cdot F) - P^f \cdot S \cdot (1-t),
\]

if \((S^*+S) \cdot (1-\rho \cdot F) < (1-t)\), then \(S=0\), DPR is \(P^-\), \(11\)

if \((S^*+S) \cdot (1-\rho \cdot F) > (1-t)\), then \(S>0\), DPR is \(P^*\), \(12\)

if \((S^*+S) \cdot (1-\rho \cdot F) = (1-t)\),

then indeterminate, DPR is \(P^- = P^*\). \(13\)

Under the assumption of risk neutrality and a real cost associated with smuggling \((S-S^*)\), the domestic price ratio is determined by the higher of the legal and illegal trade prices. Equations (11) and (12) state that the firm's decision to smuggle or engage in strictly legal trade is dependent on that comparison. Equation (13) states that the coexistence of legal trade only firms exporting (L) with firms that smuggle (joint product exports) can only occur when the combined value of expected punishment and the real resource cost equals the export tax. Thus, condition (12) above establishes proposition 1. This model, unlike other models found in the previous literature,
requires the smuggling firm to account for the real resource cost it incurs in its output price structure and thus make it a contributing factor in the firm's decision to smuggle.

V. The Effect of Enforcement, Taxes, and the Terms of Trade on Smuggling Activity.

In this section a comparative static analysis is conducted. The analysis examines the effect of changes in enforcement activity, taxes, and the terms of trade on smuggling activity and the domestic price ratio. The comparative static results are then used in the analysis of how changes in these exogenous variables affect total exports, tax revenue collection, and social welfare.

Beginning with the affect of a change in enforcement:

PROPOSITION 2. In the presence of smuggling, increasing the monetary penalty or the probability of detection will: 1) reduce legal and illegal exports; 2) have a strictly negative effect on the DPR; and 3) have an ambiguous effect on the real resource cost of smuggling.

To establish proposition 2, the comparative static results derived in appendices (B&C) are examined: \( \partial L/\partial \rho < 0, \partial S/\partial \rho < 0, \partial L/\partial F < 0, \partial S/\partial F < 0, \partial P^*/\partial \rho < 0, \partial P^*/\partial F < 0. \) What is interesting about the comparative static results is that they are counter intuitive. One would normally expect increased enforcement to reduce illegal exports and increase legal exports. However, this result is attributed to the nature of joint product smuggling in
conjunction with declining average export prices as enforcement activity increases.\textsuperscript{12}

With respect to the real resource cost of smuggling, an increase in either enforcement instrument ($\rho$ or $F$) has an ambiguous effect on $(S-S^*)$, via the negative effect enforcement has on $(L)$ and $(S)$, as equation (6) indicates. This result is in contrast to the positive effect (increased cost) derived in the paper by Martin and Panagariya. The ambiguous effect derived here results from increased enforcement reducing smuggling, which reduces the total real resource cost. But at the same time, increased enforcement reduces legal trade, which reduces the protection from detection and requires smugglers to increase cloaking activities. It is the cloaking activities that generate the real resource cost in this paper. The above discussion establishes proposition 2.

The next issue is the affect of an increase in the export tax:

PROPOSITION 3. In the presence of smuggling, increasing the export tax will: 1) reduce legal and illegal exports; 2) have a strictly negative effect on the DPR; and 3) have an ambiguous effect on the real resource cost of smuggling.

\textsuperscript{12} The results derived for the effect of changes in enforcement activity on smuggling and the domestic price ratio are consistent with the literature. Martin and Panagariya (1984), Sheikh (1989), Thursby (1991), and Fausti (1992) all show that increased enforcement has a negative effect on smuggling volume and the domestic price ratio. However, the effect of increased enforcement on legal trade volume has been addressed in the papers by Martin and Panagariya, and Thursby. The former derived an ambiguous result and the latter a positive result.
To establish proposition 3, the comparative static results derived in appendices (B&C) are examined: $\frac{\partial L}{\partial t} < 0$, $\frac{\partial S}{\partial t} < 0$, $\frac{\partial P^*}{\partial t} < 0$. These results are counter intuitive with respect to smuggling activity. One would expect smuggling effort to increase as the export tax rises, however, since legal trade is a necessary input into smuggling, as legal exports decline, export smuggling declines.\(^{13}\) The tax increase has an ambiguous effect on smuggling's real resource cost and this result can be explained in a manner similar to the enforcement discussion. The above discussion establishes proposition 3.

The next issue is the affect of an increase in the world price of exports:

**PROPOSITION 4.** In the presence of smuggling, an increase in $p^f$ will: 1) increase legal and illegal exports; 2) have a strictly positive effect on the DPR; and 3) have an ambiguous effect on the real resource cost of smuggling.

To establish proposition 4, the comparative static results derived in appendices (B&C) are examined: $\frac{\partial L}{\partial p^f} > 0$, $\frac{\partial S}{\partial p^f} > 0$, $\frac{\partial P^*}{\partial p^f} > 0$. In the presence of smuggling, the comparative static results demonstrate that if the terms of trade ($p^f$) improve, which implies the world price of exports increase, the amount of

\(^{13}\) The results derived above for the effect of an increase in the tax rate on smuggling, legal trade, and the domestic price ratio are in contrast to the results found in the earlier literature. Sheikh (1989), and Fausti (1992) demonstrate that an increase in the tax rate does increase the volume of smuggling and has a negative effect on the domestic price ratio. However, these authors do not address the issue of the effect on legal trade volume. The results found in the paper by Martin and Panagariya show an ambiguous effect on smuggling volume and a negative effect on legal trade volume.
(X) produced for illegal trade (S) and the amount for legal trade (L) for the smuggling firm increase. The (DPR) will rise and total exports will increase, which will increase total tax revenues. The effect on (S-S'), however, is ambiguous. This contrasts with the results found in the paper by Martin and Panagariya. In their paper, the real resource cost is independent of the terms of trade; and thus, an increase in the terms of trade has no effect on the real resource cost. The above discussion establishes proposition 4.

VI. The Effect of Joint Product Smuggling on Total Exports and Trade Tax Revenues.

Expanding on the work by Johnson (1974), Pitt (1981), Deardorff and Stolper (1990), and Fausti (1992), this section will examine the effect of joint product smuggling on export production and tax revenue collection. In this section an additional assumption is made: that the exported good (X) is a pure export.  

The first issue addressed in this section is the impact of the introduction of joint-product smuggling on total exports, legal exports and tax revenue collection:

PROPOSITION 5. The introduction of joint-product smuggling will: 1) increase total export production; and 2) have an ambiguous effect on legal exports. Thus, the introduction of joint product smuggling has an ambiguous effect on tax revenue collection.

---

14 A pure export good implies that there is no domestic consumption of that good.
To establish proposition 5, the analysis begins with the question of how the introduction of joint product smuggling affects total export production, denoted \( X \). The small country assumption made earlier implies that the demand for exports is perfectly elastic. The level of export production is therefore determined by supply. Assuming that the level of total export production, after the export tax is levied but before smuggling is introduced, is equal to \( L_1 \),

\[
X_1 = L_1. \tag{14}
\]

After smuggling is introduced, the level of total export production is equal to the sum of legal and illegal exports,

\[
X_2 = L_2 + S_2. \tag{15}
\]

Assuming the supply of total exports has a positive relationship with the price of exports, one would expect that whenever \( P* \geq P^l \), then \( X_2 \geq X_1 \). Given that \( S_2 \geq 0 \), then \( L_2 + S_2 \geq X_1 \), or equivalently, \( S_2 \geq X_1 - L_2 \). Using (14), this implies condition (16),

\[
S_2 \geq L_1 - L_2. \tag{16}
\]

Equation (16) demonstrates that the production of exports destined to be marketed via illegal channels is greater than the change in the production of exports destined to be marketed via legal channels. However, as in Pitt's paper, it can not be determined if the amount of \( X \) marketed via legal trade channels increases or decreases. Therefore, the affect of smuggling on
export tax revenues is ambiguous. The above discussion establishes proposition 5.

Changing the assumptions again, assume that the export tax is set to maximize revenues collected before joint product smuggling begins. Since it is assumed that country produces a pure export good, domestic consumption can be ignored. Domestic production and thus export supply is assumed to be solely dependent on the exogenous world price for the exported good. Any ad valorem tax levied on exports must be fully absorbed by domestic producers. Given this set of circumstances, it is assumed that legal export supply is actually a function of the export tax \( L(t) \), ceteris paribus. The government's total revenue function is defined as,

\[
TR = t \cdot P^f \cdot L(t), \quad L'<0, \quad L'=dL/dt. \quad (17)
\]

The sign for \( L' \) is taken from appendix B. Total tax revenue is defined as revenue collected on exports evaluated at world prices. To determine the revenue maximizing tax rate, the first

---

15 Fausti (1992) used this approach to discuss the impact of smuggling on total exports. Pitt (1981), and Deardorff and Stolper (1990) also derive ambiguous results for the effect of the introduction of smuggling on legal trade. They indicate that the introduction of smuggling could actually increase tax revenue collected. In the paper by Bhagwati and Srinivasan (1973), the introduction of strictly clandestine smuggling has a strictly negative effect on revenue collected from a given tariff rate.

16 If one assumes that legal trade is a function of the domestic price of exports, \( L(P) \), and \( P = P^f \cdot (1-t) \), the results remain unaltered. The decision to make legal trade a function of the tax was done to simplify the mathematics presented in the paper.
derivative \((dTR/dt)\) is derived and set to zero in equation (18) and the revenue maximizing tax rate is given in equation (19),
\[
dTR/dt = P^f\cdot L + F^f\cdot t\cdot L' = 0, \quad (18)
\]
\[
t_0 = -(L/L') > 0. \quad (19)
\]

When joint product smuggling is introduced, the government's total revenue function is altered in the following manner,
\[
TR = t\cdot P^f\cdot L(t,S) + P^f\cdot t\cdot L'L + p\cdot F\cdot P^f\cdot G(L,S), \quad S' < 0, \quad S' = dS/dt. \quad (20)
\]
Notice that \(TR\) is now also a function of confiscation revenues. The legal export supply function \((L)\) is altered so that legal exports are a function of \(t\) and \(S\). The illegal export supply function is denoted \(G(L,S)\). The partial derivative \(\partial L/\partial S\) captures the affect of the introduction of smuggling on legal exports. From the discussion above its sign was determined to be ambiguous. The signs for \(L'\) and \(S'\) are assumed to be consistent with those derived in appendix B. Following the same procedure as above, the revenue maximizing tax rate is derived,
\[
dTR/dt = P^f\cdot L + P^f\cdot t\cdot [L' + \partial L/\partial S\cdot S'] +
\]
\[
P^f\cdot F\cdot [G_L\cdot L' + GL\cdot \partial L/\partial S\cdot S' + G_S\cdot S'] = 0, \quad (21)
\]
\[
t_1 = -{(L + P\cdot F\cdot [G_L\cdot L' + G_L\cdot \partial L/\partial S\cdot S' + G_S\cdot S'])} / 
\]
\[
(L' + \partial L/\partial S\cdot S') > 0. \quad (22)
\]
Equation (22) brings the discussion to proposition 6:

**PROPOSITION 6.** The introduction of joint-product smuggling will reduce the revenue maximizing tax rate if \(\partial L/\partial S < 0\), and have an ambiguous effect if \(\partial L/\partial S > 0\).

To establish proposition 6, the ratio of \(t_1/t_0\) is examined under the following assumptions: 1) \(\partial L/\partial S = 0\); 2) \(\partial L/\partial S < 0\); and 3)
If the ratio is less than one, then revenue maximizing tax rate is lower when there is smuggling activity. If the ratio is greater than one, then the revenue maximizing tax rate is higher when there is smuggling activity. In appendix (D) it is demonstrated that when the introduction of smuggling has no effect on legal exports i.e., \( \partial L/\partial S = 0 \), the ratio \( t_1/t_0 < 1 \). If it is assumed that the introduction of smuggling reduces legal exports (\( \partial L/\partial S < 0 \)), then the ratio is again less than one. If it is assumed that the introduction of smuggling increases legal exports (\( \partial L/\partial S > 0 \)), then the ratio is positive but its magnitude is ambiguous. However, if the contribution of confiscation revenues to total revenues (TR) is small, then \( t_1/t_0 > 1 \). The implication for this case is that the revenue maximizing tax rate increases in the presence of smuggling. The results derived in appendix (D) establish proposition 6.\(^\text{17}\) The policy implication for lesser developed countries which employ trade taxes as a revenue raising device is that they may over or under estimate the tax rate necessary for revenue maximization if smuggling and enforcement levels are not accounted for in their calculations.

VII. Smuggling, Enforcement and Taxes: The Welfare Implications.

The real resource cost of smuggling in this model is equal to \( P^f \cdot (S - S^*) \) and represents the total welfare loss associated

\(^{17}\) This set of results extends the discussion by Johnson (1974) and Bhagwati and Srinivasan (1973) on how the introduction of smuggling affects the revenue maximizing tax rate.
with smuggling. The negative welfare effect can be divided into two parts: 1) a negative effect on prices and production; and 2) a loss in government revenue. The negative price effect is internalized by the smuggling firm and is reflected in the firm's output price. The welfare loss due to a real resource cost not accounted for in the smuggling firm's output price is the value of lost government confiscation revenues that would have accrued if \( S - S^* = 0 \). The welfare loss not accounted for by the firm can be considered a dead weight loss to society (DWL) and is equal to: 

\[
\text{DWL} = (\rho \cdot F) \cdot P^f \cdot (S - S^*).
\]

The overall welfare effect of smuggling depends on whether the positive welfare effect of an improvement in domestic relative prices due to the introduction of smuggling outweighs the negative welfare effect of the dead weight loss.

A comparison of the welfare level attained when "all firms smuggle" to the welfare level attained when "all firms engage in strictly legal trade" can be determined by answering two question: 1) what effect does smuggling have on the domestic price ratio? and 2) is the change in the total value of exported goods smuggled greater than the total value of those exports if they were shipped through legal channels instead?. The first question is answered by equations (11) through (13): smuggling will only occur if \( P^* \geq P^l \). The second question can be answered by first assuming \( P^* > P^l \); then, by rearranging equation (10) the following condition (23) results,

\[
P^f \cdot S^* \cdot (1 - \rho \cdot F) - P^f \cdot S \cdot (1 - t) > 0.
\] (23)
Condition (23) states that if \((P^* > P^k)\), then the smuggling firm receives a higher total value for its exports by engaging in illegal trade. The firm, however, does not consider the (DWL) to society generated by the real resource cost associated with smuggling. In order for smuggling to increase the total social value of exports (welfare) in comparison to the strictly legal trade alternative, condition (24) must be met,

\[
P^f \cdot S^* \cdot (1 - \rho \cdot F) - P^f \cdot S \cdot (1 - t) - (\rho \cdot F) \cdot P^f \cdot (S - S^*) > 0.
\]  

(24)

Condition (24) reduces to condition (25), which can be considered the income effect attributed to smuggling when smuggling incurs a real resource cost,

\[
(S^* + S) - (\rho \cdot F) > (1 - t).
\]  

(25)

In comparing condition (25) to equation (12), it is clear that (25) is the stronger condition. This implies it is possible for smuggling to cause a decline in the total social value of exports. The above analysis demonstrates that smuggling can reduce the social value of exports if condition (25) is not met. An analysis of the social welfare effect of smuggling, however, must also consider the effect smuggling has on the domestic price ratio. For this purpose an indirect utility function \((V)\) is introduced and it is assumed it can be used as a proxy for the social welfare function. Assume welfare is a function of the domestic price ratio (DPR) and income \((Y)\).^{18} Assume income is positively related to the total social value of exports. The

\[\hline\]

^{18} Under the assumption of risk neutrality, income is an appropriate proxy for welfare.
total social value of exports includes both private and public sector revenues generated by the export trade. Assume all other income sources are held constant and enforcement effort does not incur a real resource cost. Under these assumptions the indirect utility function is defined as $V(DPR, Y)$, and has the following properties: $\partial V/\partial DPR > 0$, $\partial V/\partial Y > 0$.\(^{19}\)

The discussion above leads to the next proposition in the paper:

**PROPOSITION 7.** The welfare effect of introducing joint-product smuggling can not be strictly negative or all smuggling will end.

To establish proposition 7, the condition given in equation (12) must first be satisfied. This implies that firms will smuggle. The welfare effect of smuggling will be positive if condition (25) is met. This is due to the fact that the change in domestic price ratio and the change in the total social value of exports are both positive. The welfare effect of smuggling, however, is ambiguous if condition (25) is not met. This ambiguous result is the consequence of the (DPR) increasing, while (Y) declines. The ambiguous result derived above provides a stronger argument in the favor of smuggling than the ambiguous welfare results obtained by Bhagwati and Hansen and Pitt. Their ambiguous welfare results are the consequence of smuggling either

\[^{19}\] The indirect utility function ($V$) has the following properties: 1) ($V$) is continuous at all $DPR > 0$, and $Y > 0$; 2) ($V$) is non-decreasing in $(DPR)$ and $(Y)$; and 3) ($V$) is homogenous of degree zero in $(DPR)$ and $(Y)$. It should be noted that an increase in the (DPR) implies an improvement in domestic relative prices. For a discussion of the properties of indirect utility functions see Varian (1984).
having a strictly positive or strictly negative effect on welfare depending on the magnitude of the real resource cost. Given that the magnitude of the real resource cost is unknown in their papers they conclude that the welfare effect is ambiguous. In this paper smuggling does not have a strictly negative effect on welfare. The welfare effect is strictly positive or ambiguous when the real resource cost is excessive. In the joint product model, smuggling activity will become indeterminate or end before the welfare effect of smuggling becomes strictly negative. Thus proposition 7 is established. Furthermore, unlike their analysis, this paper provides the mathematical conditions necessary for a positive or an ambiguous welfare result to occur.\(^\text{20}\)

The next issue to be addressed is the affect of a change in the level of enforcement on welfare:

**Proposition 8.** The welfare effect of increased enforcement is strictly negative when the welfare effect of smuggling is strictly positive.

To establish proposition 8, the affect of increased enforcement is examined below. Proposition 2 established that increasing either (\(\rho\)) or (\(F\)) causes a decline in L and S and the DPR. If smuggling is not eliminated, then the total social value of exports under the smuggling regime declines, as indicated by equation (25). This establishes proposition 8.

\(\text{\textsuperscript{20}}\) As in the paper by Deardorff and Stolper, the welfare effect of smuggling in this model is strictly positive, if smuggling does not incur a real resource cost.
The last issue to be addressed is the affect of an increase in the tax rate on welfare:

**PROPOSITION 9.** The welfare effect of an increase in the tax rate is strictly negative.

To establish proposition 9, the affect of an increase in the tax rate is examined below. Proposition 3 established that increasing the export tax causes Land S and the (DPR) to decline. Equation (25) indicates that an increase in the export tax will not alter smuggling's improvement of the total social value of exports over the strictly legal trade alternative. This result establishes proposition 9.

The final issue is the case of smuggling coexisting with strictly legal trade. This can occur only when \((P^* = P^L)\). If smuggling coexists with strictly legal trade, then the change in the domestic price ratio is zero and the change in the total social value of exports is negative as condition (25) would indicate for \((P^* = P^L)\). For the situation depicted by equation (13), the welfare effect is negative. However, in this case the existence of strictly legal trade and/or smuggling is indeterminate. This result mirrors the result attained in the paper by Bhagwati and Hansen when \((P^* = P^L)\), and the conclusion of this paper concurs with their conclusion of "the less smuggling the better" for this case. For this case, any change in an exogenous variable which causes a decline in the (DPR) will eliminate smuggling.
VIII. Summary.

The purpose of this essay is to extend the analysis of Bhagwati and Hansen and other economists who have made a contribution to a greater understanding of the economic consequences of illegal transactions in international trade. The focus of this essay is the effect of enforcement and taxes on smuggling, welfare, and tax revenue collection. The results of the model are as follows: 1) smuggling can have a strictly positive effect on welfare; 2) the level of enforcement, taxes, and the real resource cost affects the firm's decision to smuggle; 3) if smuggling is welfare enhancing, an increase in export taxes has a negative effect on joint export trade and welfare; 4) if the welfare effect of smuggling is positive, then increased government enforcement against smuggling has an negative effect on welfare and total exports; and if increased enforcement does not eliminate smuggling, then tax revenues will fall; 5) improvement in the terms of trade will increase the amount of legal and illegal goods the smuggling firm will export; and 6) the presence of smuggling reduces the revenue maximizing tax rate.

The general conclusion of the paper is that if a country tries to eliminate smuggling, it may reduce welfare and tax receipts. If one compares the policy implications derived in this paper with those found in the earlier literature, it is clear that there is a difference in the economic impact of policy changes on the Pitt type of joint product smuggling as compared
to the Bhagwati and Hansen type of clandestine smuggling. Before a government decides to implement a policy in reaction to smuggling activity, it should be aware of which type of smuggling is most prevalent in its economy.
IX. References


Appendix (A)

The profit maximization first order conditions for equation (7) are,

\[(1a) \quad \frac{\partial Y}{\partial L} = (1-p\cdot F) \cdot Pf \cdot GL + Pf \cdot (1-t) - P^* = 0,\]

\[(2a) \quad \frac{\partial Y}{\partial S} = (1-p\cdot F) \cdot Pf \cdot Gs - P^* = 0.\]

The term \((Pf)\), is the fixed international terms of trade and \((t)\) is the ad valorem export tax rate. First order conditions \((1a)\) and \((2a)\) state that the marginal cost of an additional unit of tradeable will just equal its revenue in trade, be it legal or illegal trade. An additional unit of legal trade will result in additional legal revenue \(Pf \cdot (1-t)\) and additional smuggling revenue \((1-p\cdot F) \cdot Pf \cdot Gs\).

The profit maximization second order conditions for equation (7) are,

\[(3a) \quad \frac{\partial^2 Y}{\partial L^2} = (1-p\cdot F) \cdot Pf \cdot GLL < 0,\]

\[(4a) \quad \frac{\partial^2 Y}{\partial L \partial S} = (1-p\cdot F) \cdot Pf \cdot GLS > 0,\]

\[(5a) \quad \frac{\partial^2 Y}{\partial S^2} = (1-p\cdot F) \cdot Pf \cdot GSS < 0,\]

\[(6a) \quad \frac{\partial^2 Y}{\partial S \partial L} = (1-p\cdot F) \cdot Pf \cdot GSL > 0.\]

The partial derivative \((G_s)\) is the marginal product of \((S)\) in the production of \((S')\) and is assumed to be positive. The partial derivative \((G_L)\) is the marginal product of legal trade in production of \((S')\) and is assumed to be positive. The second order partial derivatives \((G_{ss})\) and \((G_{LL})\) are assumed to be nega-
tive because of the concavity assumption imposed on \((G)\). The cross partial derivatives, \((G_{aL}, G_{La})\), are assumed positive and small. This implies that the marginal productivity of either input increases if the other input is increased. The second order conditions for profit maximization hold when it is assumed that the cross partial derivatives are positive and small.

\[
(7a) \quad A = P^f \begin{bmatrix}
(1-\rho \cdot F) \cdot G_{LL} & (1-\rho \cdot F) \cdot G_{LS} \\
(1-\rho \cdot F) \cdot G_{SL} & (1-\rho \cdot F) \cdot G_{SS}
\end{bmatrix}, \quad \text{DET} \ (A) > 0.
\]

Under the assumption that second order conditions given in equation (7a) hold, profit maximization is assured for the smuggling firm.
Appendix (B)

In order to analyze the effect of a change in the trade tax or a change in enforcement variables or the world price of exports on the smuggler's optimal level of legal exports (L) and illegal attempted exports (S), we have to rewrite the first order conditions in appendix (A) in the following style in order to perform a comparative static analysis,

\[(1b) \quad Z_1 = (L, S; P^f, \rho, F, t) = (1-\rho \cdot F) \cdot P^f \cdot G_L + P^f \cdot (1-t) - P^* = 0,\]

\[(2b) \quad Z_2 = (L, S; P^f, \rho, F, t) = (1-\rho \cdot F) \cdot P^f \cdot G_s - P^* = 0.\]

Assuming the second order conditions for a profit maximization are satisfied, we have the following pair of implicit functions: 1) \(L = L(P^f, \rho, F, t)\); 2) \(S = S(P^f, \rho, F, t)\). Taking the total differential of \((Z_1)\) and \((Z_2)\), with respect to the endogenous and exogenous variables, the following results are derived for \(dL\), \(dS\), \(dt\), \(d\rho\), \(dF\), \(dP^f\):

\[(3b) \quad \frac{dz_1}{dL} = [(1-\rho \cdot F) \cdot P^f \cdot G_{LL}] < 0,\]

\[(4b) \quad \frac{dz_1}{dS} = [(1-\rho \cdot F) \cdot P^f \cdot G_{LS}] > 0,\]

\[(5b) \quad \frac{dz_1}{dt} = [P^f] > 0,\]

\[(6b) \quad \frac{dz_1}{d\rho} = [F \cdot P^f \cdot G_{L}] > 0,\]

\[(7b) \quad \frac{dz_1}{dF} = [\rho \cdot P^f \cdot G_{L}] > 0,\]
\[ (8b) \frac{dz_1}{dp} = - [(1-p \cdot F) \cdot G_L + (1-t)] < 0, \]

\[ (9b) \frac{dz_2}{dS} = [(1-p \cdot F) \cdot P^f \cdot G_{SS}] < 0, \]

\[ (10b) \frac{dz_2}{dL} = [(1-p \cdot F) \cdot P^f \cdot G_{SL}] > 0, \]

\[ (11b) \frac{dz_2}{dt} = 0, \]

\[ (12b) \frac{dz_2}{dp} = [F \cdot P^f \cdot G_s] > 0, \]

\[ (13b) \frac{dz_2}{dF} = [\rho \cdot P^f \cdot G_s] > 0, \]

\[ (14b) \frac{dz_2}{dP^f} = - [(1-p \cdot F) \cdot G_s] < 0. \]

Imposing the second order conditions on the Hessian matrix below, the determinant of the Hessian is positive. This is a reasonable assumption since the main diagonal matrix elements are negative and large -- large, that is, in comparison to the off diagonal elements of the matrix as described in appendix (A). This assumption is used to determine the signs in the comparative static analysis below,

\[ \frac{L}{dz_1/dL} = \frac{S}{dz_1/dS} = \frac{-}{+} > 0. \]

By applying Cramer's rule the following comparative static results can be derived: \( \partial L/\partial t, \partial L/\partial \rho, \partial L/\partial F, \partial L/\partial P^f, \partial S/\partial P^f, \partial S/\partial t, \partial S/\partial \rho, \partial S/\partial F, \)
\[
\begin{align*}
(16b) \quad \begin{vmatrix} L & S \\ - & + \\ + & - \end{vmatrix} & \quad \frac{\partial L}{\partial t} = \frac{dz_1}{dt}, \\
& \quad \frac{\partial S}{\partial t} = \frac{dz_2}{dt}, \\

(17b) \quad \begin{vmatrix} + & + \\ 0 & - \\ + & + \end{vmatrix} & < 0, \\
& \quad \begin{vmatrix} - & + \\ + & 0 \\ + & + \end{vmatrix} < 0, \\

(18b) \quad \begin{vmatrix} L & S \\ - & + \\ + & - \end{vmatrix} & \quad \frac{\partial L}{\partial \rho} = \frac{dz_1}{d\rho}, \\
& \quad \frac{\partial S}{\partial \rho} = \frac{dz_2}{d\rho}, \\

(19b) \quad \begin{vmatrix} + & + \\ + & - \\ + & + \end{vmatrix} & < 0, \\
& \quad \begin{vmatrix} - & + \\ + & + \\ + & + \end{vmatrix} < 0, \\

(20b) \quad \begin{vmatrix} L & S \\ - & + \\ + & - \end{vmatrix} & \quad \frac{\partial L}{\partial F} = \frac{dz_1}{dF}, \\
& \quad \frac{\partial S}{\partial F} = \frac{dz_2}{dF}, \\

(21b) \quad \begin{vmatrix} + & + \\ + & - \\ + & + \end{vmatrix} & < 0, \\
& \quad \begin{vmatrix} - & + \\ + & + \\ + & + \end{vmatrix} < 0,
\end{align*}
\]
\[ \begin{array}{ccc} L & S \\ - & + \\ + & - \end{array} \quad \begin{array}{ccc} \frac{\partial L}{\partial P^f} & = & \frac{dz_1}{dP^f} \\ \frac{\partial S}{\partial P^f} & = & \frac{dz_2}{dP^f} \end{array} \]

(22b)

\[
\begin{array}{ccc} \frac{\partial L}{\partial P^f} = - - + & > 0, \\ \frac{\partial S}{\partial P^f} = + - + & > 0. \end{array}
\]

(23b)
Appendix (C)

The equilibrium domestic price ratio, Equation (8), is given below in (1c). The effect of an exogenous variable change on \( P^* \) is provided below. The indirect affects due to changes in the exogenous variables on \( (L) \) and \( (S) \) are ambiguous. Therefore, I will assume the direct effect dominates.

\[
(1c) \quad P^* = \frac{[(1-p \cdot F) \cdot P^* \cdot (S^*)]}{(L+S)} + \frac{[P^f \cdot (1-t) \cdot (L)]}{(L+S)}.
\]

\[
(2c) \quad \frac{\partial P^*}{\partial p} = - \frac{(P^f \cdot F \cdot S^*)}{(L+S)} < 0.
\]

\[
(3c) \quad \frac{\partial P^*}{\partial F} = - \frac{(P^f \cdot p \cdot S^*)}{(L+S)} < 0.
\]

\[
(4c) \quad \frac{\partial P^*}{\partial t} = - \frac{(P^f \cdot L)}{(L+S)} < 0.
\]

\[
(5c) \quad \frac{\partial P^*}{\partial P^f} = \frac{[(1-p \cdot F) \cdot (S^*)]}{(L+S)} + \frac{[(1-t) \cdot (L)]}{(L+S)} > 0.
\]