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THE EFFECT OF UNCERTAINTY ON A
JOINT PRODUCT MODEL OF SMUGGLING

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ABSTRACT

Extending the seminal work of Bhagwati and Hansen (1973) on smuggling, Pitt (1981) developed a joint export smuggling model to investigate the welfare effect of illegal transactions. This paper develops an extension of Pitt's original model which allows many of the interesting features of the Bhagwati and Hansen model to be re-examined within a joint product model of smuggling framework. The extension is made through the following modifications to Pitt's assumptions: 1) firms that export are free to engage in joint product smuggling or strictly legal trade; and 2) uncertainty is introduced into the model via active government enforcement.

The modifications enable the model to reexamine the ambiguous welfare results derived in the papers by Pitt, and Bhagwati and Hansen. The model explains why the ambiguous welfare results were derived and demonstrates that the welfare effect of smuggling can indeed be positive, even if smuggling incurs a real resource cost.
I. Introduction.

The paper by Pitt (1981) on illegal transactions in international trade questioned the results of the seminal paper on illegal transactions by Bhagwati and Hansen (1973). The purpose of this paper is to explore how Pitt's welfare results are affected when risk and the firm's attitude toward risk are introduced in conjunction with the firm's freedom to choose between strictly legal trade and joint product smuggling. The modified model of joint product smuggling developed in this paper permits many of the interesting features of the Bhagwati and Hansen model to be re-evaluated within a joint product framework. This is accomplished by allowing government enforcement to play an active role in the model without incurring a real resource cost to combat smuggling.\(^1\) Active enforcement generates a crime theoretic framework for the analysis of joint product smuggling in this paper.\(^2\)

In this essay Pitt's model of joint product export smuggling is modified to incorporate active government enforcement of the smuggling laws. Active enforcement is the assumption used to introduce uncertainty into the joint export smuggling model. The paper then develops a decision mechanism which evaluates the uncertainty and provides a set of decision rules for the firm to

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\(^1\) It is a common assumption in the smuggling literature that enforcement effort against smuggling incurs a zero real resource cost because the legal system of country is already in place. Thus, increased enforcement effort against smuggling requires only a reallocation of resources within the legal system.

\(^2\) Martin and Panagariya (1984) were the first to introduce the crime theoretic approach to the analysis of smuggling.
follow in making its decision to smuggle or engage in strictly legal trade. The firm's attitude toward risk affects this decision process. The model requires the smuggling firm to include the real resource costs of smuggling and expected punishment in its output price. These two factors affect production and output price if the firm smuggles. The firm's smuggling decision determines the long run equilibrium domestic price ratio. The results of the model indicate that: 1) it is the firm's attitude toward risk in conjunction with the real resource cost of smuggling that determines the welfare effect of smuggling; 2) if firms are risk neutral or risk averse and they decide to smuggle, then smuggling is welfare enhancing under certain conditions; and 3) the assumption of a significant real resource cost is only a partial explanation for the ambiguous welfare results found in the earlier smuggling literature.

II. Assumptions.

3 The concept of a firm decision mechanism which determines whether a firm smuggles or not was first developed in a paper by Fausti (1992).

4 Smuggling models which incorporate variable firm risk preference can be found in papers by Scholër (1989) and Fausti (1992).

5 In the papers by Bhagwati and Hansen (1973), Pitt (1981), Bhagwati (1981), and Martin and Panagariya (1984), the real resource cost of smuggling is the only factor responsible for the ambiguous welfare results. Sheikh (1989) argues that ambiguous welfare results can only occur if firms are assumed risk preferring. Scholër (1989) and Fausti (1992) argue that if the attitude toward risk varies over the traded goods industry, smuggling can be strictly welfare enhancing.
The basic assumptions of Pitt's model of smuggling are the starting point for this paper. Pitt assumes the small country case with the terms of trade fixed. The country produces two traded goods, an exportable (X) and an importable (M), employing primary factors purchased in competitive markets. Production and trade are carried out by identical firms. Legal and illegal trade in exports is carried out by the same firm. The law of one price holds in the domestic economy.

The following additional assumptions are made so that a model of smuggling incorporating uncertainty can be developed: 1) firms that smuggle may not incur a significant real resource cost; 6) smugglers (firms) are natives and therefore their utility functions are embodied in the country's social welfare function; 3) export taxes are assumed to be non-prohibitive; 4) firms must bear the risk of illegal activity and they cannot insure against criminal penalties; 5) exporting firms have a choice between strictly legal trade or smuggling, with the choice based on profit maximization; and, 6) if the domestic exporting firm decides to smuggle, it will then produce a joint product, and legal trade will act as a cloak for the firm's illegal activity. The firm can use four methods to smuggle exports: a) under-invoicing of exports; b) falsely declared

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6 Cooper (1974) and Deardorff and Stolper (1990) argue that smuggling may not impose any significant real cost on society over legal trade.
exports; c) under-assessment of exports; and d) clandestine smuggling of unreported production.\textsuperscript{7}

III. A Joint Product Model of Smuggling.

In addition to the assumptions made in the previous section, it is assumed each firm can trade illegally according to a modified Pitt smuggling function,

\[ S^* = G(L,S). \]

The variable \( S^* \) is the quantity of good \( X \) made ready to be smuggled. The variable \( S^* \) in this model is defined as exports made ready for smuggling across the domestic border or, in other words, smuggling attempted. The variable \( L \) is the quantity of good \( X \) legally traded and \( S \) is the quantity of good \( X \) input into smuggling activity. The function \( G \) is strictly concave and a twice differentiable linear homogeneous function. The function \( G \) is assumed to have the following properties:

\[ G_L \geq 0, \quad G_{LL} \leq 0, \tag{2} \]
\[ 1 \geq G_s \geq 0, \quad G_{ss} \leq 0, \tag{3} \]
\[ G(0,S)=0, \quad G(L,0)=0 \tag{4} \]
\[ S-S^* \geq 0, \quad \partial(S-S^*)/\partial L < 0, \quad \partial(S-S^*)/\partial S > 0. \tag{5} \]

Assumption (2) states that the marginal smuggling product of legal trade used in smuggling is non-negative and is declining in \( L \). Assumption (3) states that a unit increase in the smuggling input \( S \) results in a positive but less than unit increase in

\textsuperscript{7} Deardorff and Stolper (1990) discuss the widespread use of smuggling method (d) in a number of African countries.
actual amount of the export made ready to be smuggled, and the marginal product of \( S \) is declining. Assumption (4) states that legal trade is a necessary input into smuggling or the probability of detection is one. Assumption (5) states that firms can choose to engage in legal trade only. Assumption (6) prohibits the real resource cost of smuggling from being negative. The real resource cost of smuggling \( S - S^* \) is the smuggler's selling cost in excess of its legal trade alternative selling cost (excluding taxes). It is assumed that the actual magnitude of smuggling's real resource cost is exogenous to the model. However, a change in one of the endogenous variables \( L \) or \( S \), affects the marginal resource cost of smuggling. A one-unit increase in \( L \), ceteris paribus, reduces the marginal real resource cost of smuggling. A one-unit increase in \( S \), ceteris paribus, increases the marginal real resource cost of smuggling.

In the literature, smuggling's ambiguous welfare effect is the direct result of how the real resource cost assumption is modeled.\(^8\) A negative welfare effect results from an excessive real resource cost incurred by smugglers, while an insignificant real resource cost produces a positive welfare effect. As an example, Pitt assumes that the cost of smuggling is composed of either penalties and confiscation or a mixture of a real resource cost and penalties and confiscation. His welfare result is ambiguous because the

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composition of the cost mix is unknown. We alter Pitt's assumption and assume that the difference between \((S)\) and \((S^*)\) is a real resource cost incurred from the use of cloaking tactics employed to evade detection.\(^9\)

Smuggling is assumed to incur a risk of detection \((\rho)\), \((1 \geq \rho \geq 0)\) such that \((\rho=1)\) if \((L=0)\). The expected value of illegal goods intercepted as they are moved over the border is \((\rho \cdot P^f \cdot S^*)\) or \((P^f \cdot S^*)\) if \((L=0)\). The variable \((P^f)\) is the world price of exports. The expected value of successful smuggling is \([(1-\rho)P^f \cdot S^*]\). The variable \((F)\) is a multiple of the value of intercepted illegal goods which is imposed as a fine, \((F \geq 1)\). The expected cost of interception to the smuggler is \((\rho \cdot F \cdot P^f \cdot S^*)\) and is at least \((P^f \cdot S^*)\) if \((L=0)\). Expected smuggling revenue net of interception cost is equal to \([(1-\rho \cdot F) \cdot P^f \cdot S^*]\) and is non-positive if \((L=0)\).

The expected value of output price per unit of smuggled good at the border for the smuggling firm is \(E[P^s] = (1-\rho \cdot F) \cdot P^f\), and is non-positive if \((L=0)\). The expected value of revenue per input unit of the smuggled good at the border for the smuggling firm is \(E[P^s] \cdot (S^*/S) = (1-\rho \cdot F) \cdot P^f \cdot (S^*/S)\), and is non-positive if \((L=0)\). The expected value for the output price per unit of legally exported goods is \(E[P_L] = P^f \cdot (1-t) = P_L\), and represents the legal tax distorted price for exports. The variable \((t)\) denotes the export tax.

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\(^9\) The real resource cost, for example, may take the form of: 1) special packing cost necessary to hide smuggled goods; and 2) the transport cost of shipping unreported production out of the country via clandestine ports.
tax. It is assumed the firm knows the values of these risk factors.

It is assumed each firm has a decision to make. The firm can engage in joint product smuggling or it can sell its output at the legal domestic export tax distorted price \(P^L\), as implied by the assumption \(G(L,0)=0\).\(^{10}\) If the firm decides to smuggle, it receives the weighted average price for its total output.

If the firm decides to become involved in joint product illegal trade, then its situation can be thought of as a lottery. The expected value of the lottery is dependent on variables \((\rho, Y^*, F)\). The probability of apprehension \((\rho)\) is determined by the government. It is assumed the firm's probability of being caught is \((\rho)\), if it engages in cloaking activities. If it does not cloak its illegal activity, then the probability of apprehension is equal to one. The variable \((F)\) determines the monetary equivalent of the punishment imposed on the firm by the government if it is caught in the illegal act of smuggling. Fines are considered a transfer to the government. As in Pitt's paper; profit maximization in production implies producing on the production possibility curve where the marginal rate of transformation equals domestic relative prices \((P^*)\). The variable \((Y^*)\) represents profits from joint product export trade.

Smugglers are assumed to be profit maximizers. Expected profit for the smuggling firm is given by equation (7),

\(^{10}\) Strictly legal trade profits are derived from equation (7) when it is assumed \(S=0\).
E(Y*) = P^f \cdot G(L,S) - (\rho) \cdot F \cdot P^f \cdot G(L,S) + P^f \cdot (1-t) \cdot L - P^* \cdot (L+S). \quad (7)

The term \([P^f \cdot G(L,S) - (\rho) \cdot F \cdot P^f \cdot G(L,S)]\) denotes expected smuggling revenues; \(P^f \cdot (1-t) \cdot L\) represents revenues for legal trade. As in Pitt's article, firms earn zero economic profit in the long run. Setting equation (7) to zero and solving for \(P^*\) generates an expression for the long run equilibrium domestic price ratio as a weighted average of prices received for goods legally exported in conjunction with goods illegally exported:

\[ P^* = [(1 - \rho \cdot F) \cdot P^f \cdot (S^*)]/(L+S) + [P^f \cdot (1-t) \cdot (L)]/(L+S). \quad (8) \]

The exporting firm's decision of whether to engage in strictly legal trade \((S=0)\) or engage in smuggling and produce a joint product \((L+S^*)\) will determine the long run equilibrium domestic price ratio \((DPR)\). If firms smuggle, then Pitt's "price disparity" result is generated \((P^* > P^L)\).

IV. The Role of Uncertainty in the Smuggling Decision.

The act of smuggling incurs a risk. The exporting firm's attitude toward the risk will effect its decision to smuggle or engage in strictly legal trade. This section applies the methodology developed by Arrow (1970) and Pratt (1964) to address the uncertainty issue in this paper.

Joint product illegal trade profit represents an uncertain prospect and legal trade profit represents a certain prospect. The term \((\pi)\) represents the difference between the expected value of illegal profit and legal profit; it follows that there exists a

\[ 11 \quad \text{The first order conditions can be found in appendix (A).} \]
such that the firm is indifferent between legal and illegal trade. The value of \( \pi^* \) depends on the firm's attitude toward risk. Applying Pratt's results we can define \( \pi^* \) as a risk premium and the functional form of \( \pi^* \) is given in equation (9),

\[
\pi_i^* = \frac{1}{2} \cdot \text{VAR}(Y) \cdot -\{U''(Y) + U'(Y)\}.
\]

The measure of absolute risk aversion is defined as

\[-\left\{ \frac{U''(Y)}{U'(Y)} \right\},\]

and is employed as the measure of the firm's attitude toward risk in this section. The following conditions arise from equation (9): 1) the value of \( \pi^* \) for the firm will be negative if \( (U''>0) \); 2) positive if \( (U''<0) \); and 3) zero if \( (U''=0) \). This implies, respectively, that the firm prefers, averts, or is neutral toward risk. In this paper the risk preferring case is not addressed. The variable \( \pi^* \) represents the insurance premium the firm would be willing to pay if it could insure itself against criminal penalties. Therefore, \( \pi^* \) represents the minimum value of risk premium necessary to make the firm indifferent to smuggling. This implies that, in equilibrium, at the margin, smugglers that are risk averse earn higher profits than in legal activities. Smugglers who are risk neutral earn the same amount of profits as in legal trade.\(^12\) Hence, \( \pi^* \) serves as proxy for Cooper's "threshold of law abidingness."\(^13\) Whenever \((\pi > \pi^*)\), then the firm will become involved in smuggling.

\(^{12}\) Becker (1968) used this approach to examine the effect of uncertainty on criminal behavior.

\(^{13}\) See Cooper (1974), for a discussion of the factors which influence a firm to smuggle or continue in legal trade, p.186.
In the paper by Sheikh (1989), a positive equilibrium level of economic profit is considered a "reward" to smugglers who are risk averse as compensation for the mental anguish incurred by participating in a risky venture. In this paper, the risk premium ($\pi^*$) represents the smuggler's compensation. As in Sheikh's paper, there is a unique perfectly competitive equilibrium in this model where the cost associated with the amount of mental anguish is exactly equal to the reward for risk ($\pi^*$).

Assume the exporting firm reacts to uncertainty as described above. Long run economic profit is then equal to zero for the risk neutral firm. It follows that economic profit is positive for the risk averse firm. This assumption modifies equation (7) and long run equilibrium expected profit for the smuggling firm is:

$$E(Y^*) = P^f \cdot G(L,S) - (\rho) \cdot F \cdot P^f \cdot G(L,S) + P^f \cdot (1-t) \cdot L - \pi^* \cdot (L+S) = v^*.$$  \hfill (10)

Solving equation (10) for $P^*$ generates a new expression for the (DPR):

$$P^* = \left[ (1-t) \cdot P^f \cdot (S^*) \right] / (L+S) + \left[ P^f \cdot (1-t) \cdot (L) \right] / (L+S) - \pi^* / (L+S). \hfill (11)$$

Long run equilibrium domestic relative price is now a function of the weighted average price of joint product smuggling, which includes the risk premium ($\pi^*$). The firm's decision to engage in joint product smuggling or strictly legal trade is determined by the firm's decision criteria condition:

$$\max[P^f \cdot (1-t), P^f \cdot (S^*+S) \cdot (1-\rho \cdot F) - \pi^* / S]. \hfill (12)$$
Condition (12) states that if the expected value of revenue per input unit of smuggled good, less the per unit risk premium, is greater than the per unit revenue that could be earned by selling (S) through legal channels, then all firms smuggle and DPR=P\(^*\). If not, then the DPR=P\(^L\). The following statements outline the firm's decision mechanism for engaging in joint product smuggling or the strictly legal trade alternative:

\[ P^f \cdot S^* \cdot (1-\rho \cdot F) - \pi^* = P^f \cdot S \cdot (1-t), \] (13)

if \( P^f \cdot (S^* / S) \cdot (1-\rho \cdot F) - \pi^* / S < P^f \cdot (1-t) \), then \( S=0 \), DPR is \( P^L \), (14)

if \( P^f \cdot (S^* / S) \cdot (1-\rho \cdot F) - \pi^* / S > P^f \cdot (1-t) \), then \( S>0 \), DPR is \( P^* \), (15)

if \( P^f \cdot (S^* / S) \cdot (1-\rho \cdot F) - \pi^* / S = P^f \cdot (1-t) \),

then the type of firm activity is indeterminate, \( P^L=P^* \). (16)

Statement (13) compares total revenue \( (P^f \cdot S^* \cdot (1-\rho \cdot F)) \) coming from illegal trade minus the risk premium to the total revenue \( (P^f \cdot S \cdot (1-t)) \) which would be earned by channeling \( S \) through legal channels. Statements (14-16) are derived from (13).\(^{15}\)

Under the assumption of risk neutrality (\( \pi^* = 0 \)) and a real cost associated with smuggling \( (S-S^*>0) \), the (DPR) is determined by the firm's decision to smuggle or engage in strictly legal trade. The firm's choice is based on the decision criteria found in statements

\(^{14}\) Condition (12) is derived from equation (11). Condition (12) makes a comparison of revenue coming from illegal trade \( (P^f \cdot S^* \cdot (1-t)) \) to the revenue which would be earned by channeling illegal goods through legal channels, \( (P^f \cdot S \cdot (1-t)) \).

\(^{15}\) The introduction of the firm's decision mechanism to the analysis of smuggling eliminates the possibility of obtaining the strictly ambiguous welfare results found in Bhagwati and Srinivasan (1974) or Pitt (1981).
Statement (16) reveals the necessary condition for the coexistence of legal trade only firms \((S=0)\) with firms that smuggle (joint product exports). This model, unlike models in the previous literature, requires the smuggling firm to account for the real resource cost incurred by smuggling in the firm's output price structure. However, if the firm decides to smuggle, welfare may not be enhanced due to the real resource cost.

The real resource cost of smuggling in this model is equal to \(p^f \cdot (S-S^*)\) and represents the total welfare loss associated with smuggling evaluated at world prices. The negative welfare effect can be divided into two parts: 1) a negative effect on prices and therefore production; and 2) a loss in government revenue. The negative price effect \((1-p \cdot F) \cdot p^f \cdot (S-S^*)\) is internalized by the smuggling firm and is reflected in the firm's output price. Therefore, the negative price effect which is generated by just a proportion of the real resource cost is taken into consideration when the firm makes its decision to smuggle or engage in strictly legal trade. The welfare loss due to a real resource cost not accounted for in the smuggling firm's output price is the value of lost government confiscation revenues that would have accrued if \((S-S^* \approx 0)\). The welfare loss not accounted for by the firm can be considered a dead weight loss to society (DWL) and it is equal to:

\[
\text{DWL} = (p \cdot F) \cdot p^f \cdot (S-S^*).
\]

The overall welfare effect of smuggling depends on whether additional revenues accruing to the firm from the act of smuggling
outweigh the negative welfare effect of the dead weight loss due to the real resource cost.

A comparison of the welfare level attained when all risk neutral firms smuggle to the welfare level achieved when all firms engage in strictly legal trade can be determined by answering two questions: 1) what effect does smuggling have on the domestic price ratio; and 2) is the total social value of exported goods smuggled ($S^*$) greater than the total social value of those exports if ($S$) were shipped through legal channels? The first question is answered by statements (14) through (16), smuggling will only occur if expected smuggling revenue is greater than or equal to legal trade revenue, which implies ($P^* \geq P^L$). The second question can be answered by first assuming ($P^* > P^L$), then by rearranging statement (13),

\[ P^f \cdot S^* \cdot (1 - \rho \cdot F) - \pi^* - P^f \cdot S \cdot (1 - t) > 0. \]

(18)

If ($P^* > P^L$), then (18) states that the smuggling firm receives a higher total value for its exports by engaging in illegal trade. The firm, however, does not consider the (DWL) to society generated by the real resource cost associated with smuggling. For smuggling to increase the total social value of exports in comparison to the strictly legal trade alternative, statement (19) must be true,

\[ P^f \cdot S^* \cdot (1 - \rho \cdot F) - \pi^* - P^f \cdot S \cdot (1 - t) - P^f \cdot (\rho \cdot F) \cdot (S - S^*) > 0. \]

(19)

If statement (19) is true, then the change in total revenues generated from smuggling over non-smuggling is greater than (DWL),

\[^{16}\text{Note, when it is assumed that firms are risk neutral then } (\pi^* = 0).\]
and the total social value of exports increases. Simplifying (19) we have (20),
\[ p^f (S^*/S) > p^f (1-t) + p^f (\rho F) + \pi^*/S. \]  

In comparing (20) to decision criteria statement (15), it is clear that (20) is the stronger condition. Which indicates that it is possible for firms to decide to engage in smuggling and have the act of smuggling reduce the total social value of exports. If however, the per unit revenue of smuggling input is greater than the combined per unit value of: 1) the legal trade revenue alternative for (S); and 2) expected punishment, then smuggling increases the social value of exports. We can now assert that statement (20) is a necessary and sufficient condition for risk neutral firms to engage in smuggling and increase the total social value of exports over the non-smuggling alternative.

An analysis of the social welfare effect of smuggling, however, must also consider the effect smuggling has on the (DPR). For this purpose an indirect utility function (V) is introduced. It is assumed (V) can be used as a proxy for social welfare. Assume welfare is a function of the (DPR) and income (Y). It is assumed that an improvement (increase) in (DPR) increases social welfare. It is assumed that income is positively related to the total social value of exports. The total social value of exports includes both private and public sector revenues generated by the export trade. Assume all other income sources are held constant and enforcement effort does not incur a real resource cost. Under these
assumptions the following indirect utility function is defined as
$V(DPR,Y)$, and has the following properties; $\frac{\partial V}{\partial DPR}>0$, $\frac{\partial V}{\partial Y}>0$.\(^{17}\)

If statement (15) is true then firms will smuggle. If condition (20) is also satisfied, then the welfare effect will be positive. This is due to the fact that the change in domestic price ratio and the change in the total social value of exports are both positive, and welfare improves via the social welfare function ($V$). The welfare effect of smuggling, however, is ambiguous if condition (20) is not met. This ambiguous result is the outcome of the (DPR) still increasing, but (Y) declining. This set of results establishes a stronger argument in favor of smuggling than the ambiguous welfare results obtained by Bhagwati and Hansen, and Pitt. Their ambiguous welfare results are the consequence of smuggling either having a strictly positive or strictly negative effect on welfare, depending on the magnitude of the real resource cost. Given that the magnitude of the real resource cost is unknown in their papers, they conclude that the welfare effect is ambiguous. In this paper, smuggling does not have a strictly negative effect on welfare. The welfare effect is strictly positive or ambiguous when the real resource cost exceeds the costs associated with legal trade. In the joint product model, smuggling activity will become indeterminate or end before the welfare effect

\(^{17}\) The indirect utility function ($V$) has the following properties: 1) ($V$) is continuous at all DPR>0, and Y>0; 2) ($V$) is non-decreasing in (DPR) and (Y); and 3) ($V$) is homogenous of degree zero in (DPR) and (Y). It should be noted that an increase in the (DPR) implies an improvement in domestic relative prices. For a discussion of the properties of the indirect utility function see Varian (1984).
of smuggling becomes strictly negative. Unlike their analysis, however, this paper provides the mathematical condition necessary for the ambiguous welfare result to occur, otherwise smuggling has an unambiguous positive welfare effect.

Smuggling coexisting with strictly legal trade can occur when \( (P^*=F') \). If smuggling is coexisting with strictly legal trade, then the change in the domestic price ratio is zero and the change in the total social value of exports is negative as (19) indicates since \( (P^f \cdot S^* \cdot (1-\rho \cdot F) - P^f \cdot S \cdot (1-t)=0) \). In this situation (16), smuggling either ends or the welfare effect is negative. However, in this case the existence of strictly legal trade and/or smuggling is indeterminate. This result mirrors that attained by Bhagwati and Hansen when \( (P^*=P^i) \), and their conclusion of "the less smuggling the better" holds.

If it is assumed that the real resource cost of smuggling is insignificant \( (S^*/S=1) \), with penalties and confiscation representing the significant cost to the smuggling firm, then Pitt's strictly positive welfare result is reproduced in this model.\(^{18}\) Statements (13) and (15) indicate that the firm will smuggle only if export revenue earned from smuggling is greater than export revenue from strictly legal trade, in this case the welfare effect is strictly positive. Statement (16) expresses that legal-trade-only firms \( (S=0) \) may coexist with firms that smuggle

\(^{18}\) This is a strong assumption for this model, and is made only to discuss Pitt's results under this assumption within the context of this model.
(joint product) only when the value of expected punishment equals the export tax, which implies the export price received from both types of trade are equal. If smuggling exists in this situation, then the welfare effect is neutral. This situation allows both types of firm activity to coexist in the Pitt framework.

The implications of these results are: 1) if the expected punishment associated with smuggling is less than the export tax, all firms smuggle; 2) without a significant real resource cost associated with smuggling, the welfare level for the "all firms smuggling" situation is greater than the non-smuggling alternative due to smuggling's relative price effect; and 3) the welfare effect of smuggling is dependent on the level of expected punishment \((\rho \cdot F)\).

In the previous example of a risk neutral firm not incurring a real resource cost, it was demonstrated that the firm was indifferent in the choice between strictly legal trade or smuggling when \((t = \rho \cdot F)\). For this specific case the actual profit differential \((\pi)\) is equal to zero, and the risk premium \((\pi')\) required by the risk neutral firm is equal to zero. The risk averse firm, however, requires \((\pi \geq \pi')\) to engage in smuggling. Suppose for example, that the actual risk premium being generated when a firm smuggles \((S^*)\) in lieu of exporting \((S)\) through legal channels is equal to:

\[ \pi \]

\[ \text{19} \]

Without a significant real resource cost to smuggling, the cost of smuggling to the firm represents just a transfer of revenue to the government. The aggregate rate of transformation in trade in this situation is the free trade terms of trade as in Pitt's paper.
\[ P^f \cdot S^* \cdot (1 - \rho \cdot F) - P^f \cdot S \cdot (1 - t) = \pi. \] 

(21)

For the risk averse firm \((\pi^*)\) is positive. If \((\pi)\) is greater than \((\pi^*)\), then the firm will smuggle as indicated by statement (15).

The effect of smuggling on social welfare depends on if the risk premium earned by smugglers is greater than the (D WL) associated with smuggling. In comparing (20) to decision criteria statement (15), it is clear that it is possible for firms to decide to engage in smuggling and have the act of smuggling reduce the total social value of exports. If, however, the per unit revenue of smuggling input is greater than the combined per unit value of: 1) the legal trade revenue alternative for \((S)\); 2) expected punishment; and 3) the risk premium, then smuggling increases the social value of exports. We can now assert that statement (20) is a necessary and sufficient condition for risk averse firms to engage in smuggling and increase the total social value of exports over the non-smuggling alternative.

If condition (20) holds, the welfare effect of smuggling by risk averse firms is positive. This result occurs because both the (DPR) and \((\gamma)\) increase. Income increases because the total social value of exports in the presence of smuggling is greater than the non-smuggling alternative. If (20) is not true, then the welfare effect of smuggling is ambiguous. The results of the risk averse case mirror the risk neutral case.

The next welfare issue to be addressed is when smuggling and strictly legal trade coexist. This situation is stated in (16). The coexistence of the two types of trade can only occur when if
\[ P^* = \frac{P^L + \pi^*}{(L+S)} \]. This implies that the law of one price breaks down and allows a type of parallel market structure to develop. The (DPR) is now composed of a weighted average of \((P^*)\) and \((P^L)\).\(^{20}\)

The effect on the total social value of exports for this case is again determined by (20). Condition (20) reveals that the total social value of exports, if firms smuggle, is less than the total social value of exports for the legal trade alternative. The welfare result for this case is ambiguous: The (DPR) increases and income declines and therefore the welfare effect of smuggling is ambiguous when smuggling coexists with strictly legal trade. This result implies the "less smuggling the better" result of Bhagwati and Hansen does not hold when firms are risk averse.

The next issue to be addressed is the effect of increased enforcement on smuggling and welfare. Starting with the assumptions that there is not a significant real resource cost associated with smuggling and firms are risk neutral, increased enforcement will have a negative effect on the (DPR) if \((\rho \cdot F < t)\), and eliminate smuggling when the level of expected punishment becomes greater than the export tax.\(^{21}\) Equation (8) and condition (12) verify the last statement: 1) if enforcement is increased,

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\(^{20}\) This type of parallel market structure is type of market structure which develops in many lesser developed countries when smuggling is present. The development of this type of market structure is discussed in the empirical paper by Cooper (1974) and the theoretical paper by Fausti (1992).

\(^{21}\) The following partial derivatives derived from equation 8: \(\frac{\partial P^*}{\partial \rho}, \frac{\partial P^*}{\partial F}\), are negative. However, the indirect affects due to changes in the enforcement variables on \(L\) and \(S\) are ambiguous. Therefore, I will assume the direct effect dominates.
then \( P' \) declines as equation (8) indicates; and 2) if the value of \( P' \) declines below \( P^b \), then condition (12) states that all smuggling will end. Under the "no real cost" assumption, increased enforcement has a negative impact on welfare due to its negative effect of the (DPR).

Relaxing the "no real cost" assumption, the welfare effect of smuggling is shown to be either ambiguous or strictly positive. The welfare result is dependent on the real resource cost, the value of expected punishment and the risk premium \( \pi^*=0 \). The welfare effect of increased enforcement is ambiguous (negative) if the welfare effect of smuggling is ambiguous (positive). Relaxing the assumption of risk neutrality, for the risk averse case, when smuggling has a positive (ambiguous) impact on welfare, the welfare effect of increased enforcement is negative (ambiguous). The policy conclusion concerning increased enforcement to reduce smuggling in this paper contradicts the policy conclusion of the "the less smuggling the better" found in the earlier literature. The results derived in this paper cast doubt on the economic welfare argument for the eradication of smuggling which was the implied policy conclusion in the earlier literature.

The next issue to be discussed is the results contained in a paper by Sheikh (1989). Sheikh argues that the ambiguous welfare results derived in the earlier literature are the direct result of smugglers being risk preferring. This paper's model demonstrates that when the presence of smuggling results in an ambiguous welfare outcome, it is due to the assumption of a large real resource cost,
not to the assumption of risk preferring behavior by firms.\textsuperscript{22} Sheikh also asserts that incorporating risk by itself lowers welfare and thus all previous models over-predict the positive impact of smuggling on welfare. This assertion is only true when firms are risk averse. However, it is not the inclusion of risk, but the assumption of risk aversion that lowers welfare. This point is discussed next.

The final issue to be discussed is the long run equilibrium results of the model. This paper examined two firm risk preference states: 1) risk averse firms; and 2) risk neutral firms. The results of the model demonstrate that long run equilibrium profit and (DPR) are effected by the state of nature assumed about firm risk preference. In the risk neutral case, long run economic profit is equal to zero. The long run domestic price ratio is composed of a weighted average of \((P^s)\) and \((P^l)\). Under the assumption of firm risk aversion, long run economic profit is positive. However, it is assumed that \((\pi')\) represents compensation for the mental anguish suffered by firms due to the risk associated with smuggling. Thus, excessive profit in the non-competitive sense is not being earned. Comparing the two states, the long run equilibrium (DPR) is lower for the risk averse state. The risk averse state, therefore, reduces welfare when compared to the risk neutral state. However, under both states of nature it is possible for smuggling to have a strictly positive effect on welfare. The

\textsuperscript{22} It should be noted that an excessive real resource cost will cause all smuggling to end this paper's model.
state of nature effect in this model is consistent with the trade literature on uncertainty.\textsuperscript{23}

\textsuperscript{23} For a discussion of the effect of uncertainty on prices, output, and welfare see Batra (1975).
V. Summary.

A general equilibrium joint product model of smuggling incorporating features found in the papers by Bhagwati and Hansen, and Pitt was presented in this paper. The results of the paper indicate that: 1) a significant real resource cost will preclude smuggling activity from occurring; 2) if smuggling begins, it can have a strictly positive welfare effect when compared to the legal trade alternative; 3) firms that smuggle can coexist with firms that engage in strictly legal trade and if firms are risk averse the welfare effect of smuggling is ambiguous; 4) increased enforcement against smuggling can have a negative welfare effect; and 5) the real resource cost, expected punishment, and firm risk preference all play a role in determining the welfare effect of smuggling.

In conclusion, the presence of smuggling is just a manifestation of the economic reality that a tariff is a second best commercial policy instrument and should be avoided when formulating commercial policy goals. The answer to the smuggling problem is not increased enforcement, but the replacement of the tariff with a more suitable commercial policy instrument.

VI. Appendix (A).

The profit maximization first order conditions for equation (7) are,

\[
\frac{\partial Y}{\partial L} = (1 - \rho \cdot F) \cdot P^f \cdot G_L + P^f \cdot (1 - t) - P^* = 0, \tag{1a}
\]

\[
\frac{\partial Y}{\partial S} = (1 - \rho \cdot F) \cdot P^f \cdot G_S - P^* = 0. \tag{2a}
\]
The term \( P^f \), is the fixed international terms of trade and \( t \) is the ad valorem export tax rate. First order conditions (1a) and (2a) state that the marginal cost of an additional unit of tradeable will just equal its revenue in trade, be it legal or illegal trade. An additional unit of legal trade will result in additional legal revenue \( P^f \cdot (1-t) \) and additional smuggling revenue \( (1-\rho \cdot F) \cdot P^f \cdot G_L \).
VII. References.


