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by Joseph Santos

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Can Futures Markets Quell Money Market Volatility? A Look At US Money Markets Before and Since Commodities Futures Contracts

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Abstract

This paper offers the introduction of futures markets, and the resulting substitution away from consignment contracts around 1874, as the reason why early US money markets are relatively more volatile, and far less seasonal, than their post-1874 counterparts. Until 1874, movements in interest rates were erratic and financial instabilities imparted relatively large shocks to money markets, particularly in the autumn months. After 1874, the effects of financial instabilities on interest rates diminished and the regularization of seasonal movements was attained. The paper demonstrates the plausibility of this claim using the standard mean-variance framework of the spot price volatility literature, where producers and speculators are assumed risk averse and risk neutral, respectively. Results indicate that the ability to hedge in the futures markets increases the price sensitivities of aggregate supply and aggregate demand, thereby diminishing the variability of both the price level and the interest rate in the presence of supply and/or demand shocks.

Key Words: interest rate volatility, seasonal cycles, futures contracts, financial innovations, antebellum money markets.
1. Introduction

The effect of financial innovations on the behaviors of macroeconomic time series initiated much research in both economics and finance in the last two decades. Two such cases included the Federal Reserve’s role in the cessation of transitory fluctuations in interest rates, and the stabilizing effects of commodity futures contracts on spot price variability. And although examined recently, the origins of both topics date back to the very nadir of macroeconomic studies. Indeed, early students of transitory seasonal fluctuations debated whether such movements were relevant to the study of other, seemingly more important, business cycles [See Jevons (1884), Kemmerer (1910), Kuznets (1933), Mitchell (1927), Pigou (1929), Burns and Mitchell (1947)]. Similarly, historians of the North American grain trade long suggested that commodity futures markets diminished spot price volatility [See Clark (1966), Chandler (1977), Irwin (1954) and Rothstein (1966)].

This paper unites these literatures by linking a change in the behavior of US short-term interest rates to the evolution of commodity futures trading. In particular, the introduction of futures markets, and the resulting substitution away from consignment contracts, is offered as the primary reason why early US money markets exhibit relatively less volatility, and far more seasonality, after 1874. The paper argues that, until 1874, movements in interest rates are erratic and financial instabilities impart relatively large shocks to money markets, particularly in the autumn months. While after 1874, the effects of financial instabilities on interest rates diminishes and the regularization of seasonal movements is attained, thanks in large part to the growth in commodity futures trading at that time.

The plausibility of this claim is demonstrated using the standard mean-variance framework of the spot price volatility literature. In particular, the paper models the optimizing behaviors of risk averse producers and risk neutral speculators in the absence and presence of futures contracts. Results indicate that the ability to hedge in futures markets increases the price sensitivities of aggregate supply and aggregate demand, thereby diminishing the variability of both the price level and the interest rate in the presence of supply and/or demand shocks.
2. The Traditional Interpretation of US Short-term Interest Rate Behavior

Most economic historians accept that US short-term interest rate fluctuations prior to the founding of the Fed are mean reverting and seasonal [See Friedman and Schwartz (1963), Grandy and LaCroix (1996) and Mankiw, Miron and Weil (1987)]. According to Kemmerer (1910), in a monograph prepared for the National Monetary Commission, annual transitory fluctuations in interest rates were fueled primarily by seasonal interregional cash transfers that financed the planting, harvesting and moving of the nation’s crops. Indeed, that economists consider such transitory fluctuations in early American money markets commonplace is evidenced by the vast (and only) current literature on this topic; namely, that which examines its abrupt disappearance around 1914. [See Angelini (1992), Barsky, et. al. (1988), Clark (1986), Fishe (1991), Fishe and Wohar (1990), Friedman and Schwartz, (1963), Holland and Toma (1991), Kool (1995), Mankiw and Miron (1986), Mankiw, Miron, and Weil (1987), Miron (1986), Miron (1988), and Toma (1993)]. The literature surrounding this conundrum is centered on the establishment of the Federal Reserve System. Indeed, the most popular explanation is that of Miron (1988), who argues that the Fed began to smooth short-term interest rates shortly after 1914, thus quelling all transitory fluctuations present in US money markets.

3. Transitory Fluctuations in Antebellum Money Markets

In this section, the time series properties of antebellum interest rates (in Boston, New York, Philadelphia and New Orleans) are examined and compared to those of their post-1875 counterparts in order to determine if US money markets behave in a manner relatively unchanged prior to 1914. All four series, presented in figure 1, are taken from Bodenhom (1992) and represent short-term discount rates on commercial paper and bills of exchange.

That both ante- and post-bellum US money market rates, prior to the establishment of the Federal Reserve in 1914, are mean reverting is well-established in the literature [See Bodenhom (1992) and Grandy and LaCroix (1993)]. Nonetheless, summary statistics presented in table 1 illustrate that antebellum money markets are relatively more volatile than either of the post-1875 series. Indeed, Boston, the northernmost money market, and New Orleans, the southernmost marketplace and ‘hub’ of the cotton trade, report standard deviations in excess of 400 basis points; in addition, ante-bellum rates report the highest means of any series.

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4 The data are from, Bodenhom, Howard, "Capital Mobility and Financial Integration in Antebellum America," pp. 603-608. See data appendix for details.
5 In addition to being stationary, both Bodenhom (1992) and Grandy and LaCroix (1993) identify the antebellum series as cointegrated.
Table 1: Summary statistics, antebellum and postbellum series, monthly data in levels.

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Adjusted R²</th>
<th>F Value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York, 1843:07 – 1859:12</td>
<td>6.82</td>
<td>2.70</td>
<td>0.01</td>
<td>1.13</td>
<td>0.3401</td>
</tr>
<tr>
<td>New York (a), 1875:01 – 1910:12</td>
<td>4.88</td>
<td>1.12</td>
<td>0.10</td>
<td>5.24</td>
<td>0.0001</td>
</tr>
<tr>
<td>New York (b), 1920:01 – 1933:12</td>
<td>3.71</td>
<td>1.97</td>
<td>0.02</td>
<td>1.41</td>
<td>0.1635</td>
</tr>
<tr>
<td>Boston, 1836:01 – 1859:12</td>
<td>9.28</td>
<td>4.73</td>
<td>0.00</td>
<td>0.89</td>
<td>0.5545</td>
</tr>
<tr>
<td>Philadelphia, 1839:02 – 1857:06</td>
<td>8.74</td>
<td>3.30</td>
<td>0.01</td>
<td>1.13</td>
<td>0.3395</td>
</tr>
<tr>
<td>New Orleans, 1839:11 – 1859:12</td>
<td>9.12</td>
<td>4.05</td>
<td>0.02</td>
<td>1.48</td>
<td>0.1390</td>
</tr>
</tbody>
</table>

Adjusted $R^2$ is based on the following regression specification:

$x_t = c + \delta_2(t2) + \delta_3(t3) + \cdots + \delta_{12}(t12) + e_t$. First differences of NY (b) are used due to nonstationarity of these data.

Moreover, the adjusted $R^2$s reported in table 1 suggest seasonal variations are responsible for relatively little of the transitory fluctuations in antebellum interest rates; for example, in New York the seasonal cycle explains 1% of the total variation in the rate between 1843-1859, 10% between 1875-1910 and 2% between 1920-1936.

Indeed, as with the post-Fed series, the hypothesis that the seasonal coefficients are jointly zero cannot be rejected for any of the antebellum samples.\(^7\) Hence, fluctuations in antebellum rates are transitory and large, but not seasonal in nature.

That a seasonal cycle is not always present in US interest rates prior to the founding of the Federal Reserve is seemingly puzzling. But, a closer inspection illustrates that antebellum rates exhibit seasonal patterns (figure 2.a.) like those of 1875-1910 (figure 2.b.); by contrast, the complete absence of a seasonal pattern occurs only in the post-1914 sample (figure 2.c.).\(^8\) Hence, an appropriate description of ante-bellum rate behavior is that occasional and large transitory fluctuations in the autumns of some years between 1836 and 1859 produce a seasonal pattern in interest rates that is not statistically significant due to the irregularity with which these fluctuations occur. Indeed, while autumn rates exceed their summer counterparts in 92% of the years between 1875 and 1910, such differentials are observed in only 54% of the antebellum years. Hence, while fluctuations in US interest rates are transitory up to 1914, antebellum interest rates are relatively more volatile and less seasonal than any other pre-Fed series.

4. An Institutional Explanation for the Behavior of Early US Money Markets

In general, two explanations for the relatively volatile and nonseasonal nature of US interest rates prior to the late 1800s exist. The first assumes that market volatility and the absence of seasonal strains are unrelated. That

\(^7\) Joint test of seasonal significance, $H_0: \delta_2 = \delta_3 = \cdots = \delta_{12} = 0$, where $\delta_i$ is the seasonal dummy in month $i$.

\(^8\) Autocorrelation functions of the first difference of each data series, examined at multiples of the seasonal span (12), confirm these results.

\(^4\) These sample periods are chosen simply for expositional purposes; an actual 'break date' is chosen in a later section of the paper.
is, either seasonal pressures, fueled by agriculture and popularized by Kemmerer (1910), did not exist prior to this
time or seasonal strains were present, but sterilized by some sort of monetary intervention (centrally planned or
otherwise). The second explanation links the presence of volatility with the absence of seasonality. That is,
seasonal strains were present, but had no discernable effect on money markets prior to this time due to interest rate
volatility.

The first explanation is not plausible. The existence of seasonal strains, imparted on US credit markets by
the agricultural cycle, is documented in the literature. Moreover, the smoothing of interest rates over a period of
four decades is highly unlikely in the absence of a central monetary authority. Finally, this explanation requires that
antebellum seasonal patterns mimic their post-1914 counterparts (figure 2.c.), which they clearly do not do. We
propose that the second explanation is most appropriate. Namely, the inability to detect a seasonal cycle prior to the
late 1800's is due to money market volatility. That is, although the annual process of planting, harvesting and
moving crops occurred at similar times throughout each year, market volatility obscured the seasonal cycle.
Moreover, we associate the quelling of money market volatility with the evolution of commodity futures contracts.
But, before expounding on this point, the date at which volatility diminished sufficiently, and a regular seasonal
component emerged, must be established.

5. A Regime Switch in the 19th Century

In this section, the most likely date at which a break in interest rate variability occurred is estimated. To
obtain a continuous data set, we restrict our analysis in this section to Macaulay's commercial paper rate (1836-
1933). To estimate the time at which a break occurred in short-term rate behavior, we use the maximum
likelihood technique developed by Goldfeld and Quandt (1973) and employed in Mankiw, et. al. (1987) to detect the
break in 1914. In particular, the short-term interest rate is modeled as the following:

\[
\begin{align*}
\text{r}_{t+1} &= a_0 + b_0 r_t + \beta_{0s} X_s + \nu_{t+1}, & t=1,2,\ldots,T-2, T-1 \\
\text{r}_{t+1} &= a_0 + b_0 r_t + \beta_{ns} X_s + \nu_{t+1}, & t=T,T+1,\ldots,T
\end{align*}
\]

where o and n denote old and new regimes, respectively; is the first period of the new regime. The error terms on
the two regression equations, namely \(\nu_{t+1}\), old and new, are assumed to be distributed \(N(0,\sigma_o^2)\) and \(N(0,\sigma_n^2)\)

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9 See Chandler (1977), Friedman and Schwartz (1963), Jevons (1884), and Kemmerer (1910).
10 Although timely, we believe the establishment of the National Banking System in 1863, and the concomitant extinction of state banks of issue
is irrelevant to this discussion. First, as Friedman and Schwartz (1963) argue, the growing importance of deposits in the 1860s, coupled with
relatively less austere state banking regulation, lead to a rapid expansion in the number and deposits of state banks by 1871. Indeed, the deposits
of state banks roughly equaled those of their national counterparts by this time.
respectively, while \((a_0, b_0, \beta_m)\) and \((a_n, b_n, \beta_m)\) are the regression coefficients calculated using OLS; specifically, \(a, b\) and \(\beta\) are the constant term, autoregressive term and seasonal dummy coefficients, respectively. Given these assumptions, the break date can be estimated by maximizing the likelihood function conditional on \(T_s\).¹²

In this model a break in short-term interest rates implies a change in either the regression coefficients specified in (4) or the population variance \(\sigma^2\). But, given that the autoregressive term included in (4) is both mean reverting and significant throughout the sample, and seasonal pressures existed throughout the entire nineteenth century, we identify a break in the series as a shift in its population variance, \(\sigma^2\).

Results indicate a change in variance occurred near November, 1873.¹³ That is, US short-term interest rates began to exhibit a statistically significant seasonal component around this time due to diminished volatility. Indeed, the variance of this series is larger before this date than after by a statistically significant factor of 7.1.¹⁴

Hence the findings thus far suggest that prior to 1874, movements in interest rates were erratic and financial instabilities imparted relatively large shocks to money markets, particularly in the autumn months. After 1874, the effects of financial instabilities on interest rates diminished and the regularization of seasonal movements was attained. We attribute this change in behavior of short-term rates in the nineteenth century to the introduction of futures markets and the resulting substitution away from consignment contracts in the agricultural trade shortly after 1874.

6. The Financing and Marketing of Grain and Cotton Before the 1870’s

Prior to the 1870’s, the US grain trade was financed through a network of producers, purchasing agents, commission houses, and produce dealers. Producers, including farmers, millers and local merchants, were situated in the western-most portion of the network while the dealers, who purchased grain, were situated primarily in the East. Purchasing agents and commission houses, located in Buffalo, New York and Liverpool, bridged western production and eastern consumption with the financial resources necessary to facilitate trade.¹⁵

¹¹ These data consist of antebellum Boston from 1836-1859 and the New York commercial Paper rate from 1860-1933.
¹² The break date can be estimated by maximizing the following likelihood function conditional on \(T_s\):

\[ L(r_{ll,T}) = \left( \frac{1}{2\pi} \right)^{n/2} \exp \left( \frac{-1}{2} \sum (r_i - x, x)^2 \right) \]

This is done by first calculating the maximum likelihood estimates for the parameters in the model and then choosing the \(T_s\) which has the greatest likelihood. While this methodology allows for heteroscedasticity across the two subsamples, it assumes that the innovation variance is constant within each subsample. That is, the model specifies constant heteroscedasticity. In addition, the error term is assumed pure white noise and hence autocorrelated errors are not considered. Based on an examination of residuals of the differenced data, these assumptions seem appropriate.

¹³ Indeed, the standard error of the estimate from (4) fell from 2.08 between 1836 and 1873 to only .57 between 1874 and 1910.
¹⁴ F-test 7.13, F-critical 1.69 with \(n_1=432, n_2=456\).
The most common method of financing the grain trade was where a producer offered his harvest to purchasing agents who in turn sold the grain to commission houses in the East. From there, the grain was sold to either a final purchaser or another commission house. In all cases, the final purchaser and the final purchase price was unknown at the point of production. For this reason, agents and commission houses at each stage of the process attempted to reduce their exposure to price risk during crop movements by operating on a consignment basis. That is, rather than purchase the grain outright from the farmer, and hence risk any price fluctuations until the grain could be sold, agents did not take ownership of the grain, effectively acting on behalf of the farmers. According to Rothstein (1966), this approach linked farmers, agents and commission houses such that the "entire procedure was attended by considerable risk and speculation, which was assumed by both the consignee and consignor."1

7. Changes in the Financing and Marketing of Grain and Cotton in the 1870’s

Storage and shipment technologies such as grain elevators and railroads became available in the 1850’s. Because these implements required that staples be stored and transported in bulk, shipments could no longer be tagged according to farmer or region. Moreover, due to the high volume of transports, purchasers were unable to inspect and choose their bundles upon delivery. This presented a problem in the East because produce agents often gauged the quality (and hence price) of a staple on the basis of such information and inspections. Hence, a nationally accepted system of grading and standardizing staples was required.

In the 1850’s, grain exchanges emerged, and their roles included weighing, inspecting and classifying each commodity shipment. Established in 1848, the Chicago Board of Trade began this practice in the late 1850’s. St. Louis and Buffalo exchanges adopted similar methods in 1854. Like the grain industry, cotton exchanges would grade, standardize and inspect cotton. The first US cotton exchange formed in New York one year after the Liverpool Cotton Brokers Association in 1869, while a ‘complete’ network of grading and standardizing was not in

16 Rothstein, M., “The International Market for Agricultural Commodities, 1850-1873,” p. 120.
17 Ibid., p. 120. The financing of the cotton trade was somewhat different. Although credit systems played a role in Southern agriculture since Colonial times, the network of intermediaries differed from that of the grain trade. In particular, factors were the principle lenders of funds for the purchase of agricultural inputs, and served as both purchasing agents and intermediaries for large Southern planters and Northern and European money-lenders. The method of credit extension from year to year was such that current credit was provided on the basis of future crop production. Liens were often placed on future harvests when current production proved insufficient to pay outstanding credit balances. However, after the Civil War, factors frequently lacked the funds to make advances to farmers and hence were forced to seek advances from commission houses in the North and in Europe. A system similar to that in the West evolved such that large commission houses dealt with correspondents in Liverpool, and factors became the local agents (receivers) within the hinterland [See Rothstein (1966)].
19 Chandler, A.D., The Visible Hand, p. 211.
20 Regarding the grain trade around 1848, a 1936 Chicago Board of Trade Bulletin writes, “A bushel of wheat was measured in a basket leveled off on top with a stick... Warehouses were of the flat type and of limited capacity... Future contracts as we know them today were unknown.”
place until 1874. The East, and the New York Produce Exchange in particular, accepted the methods of grading used in the West and South as a national standard at this time. This complete network made business communication easier and less subject to expensive arbitration. Speaking about the late 1840's, a Chicago Board of Trade bulletin adds,

"Without standard weights per bushel, there was opportunity for shady practices in measuring quantity, and because of lack of standard grades the matter of quality was often the basis for altercation and bitter dispute between buyer and seller. Lacking adequate storage facilities and with no contracts other than the warehouse receipt, there was much speculation in both cash grain and warehouse receipts. Violent price fluctuations were frequent as deliveries were either greater than the need for immediate shipment or less than was required to load the waiting ships for movement East."

In addition, a national system of grading standards allowed for the use of 'to arrive' or futures contracts. A futures contract stipulated the quality, amount, price and (a future) delivery date of a staple; the staple was purchased in cash upon delivery. High volume futures contracting required the standardization of staples because contracts were made before the deliverable was harvested. Hence, both parties to the contract had to agree on the quality of the deliverable before it was exchanged. Not until the 1870's was the language used to define such 'quality' accepted generally by all parties involved, including the conservative business community in the East. Indeed, Irwin (1954) notes that,

"After 1856 it appears that the use of time contracts in grain marketing increased at a moderate rate for several years. They then received a great stimulus from the activity in the grain trade which resulted from the Civil War. Their growth and development continued until they ripened into organized trading in grain futures, perhaps early in the 1870s."

Futures trading also required a technological infrastructure capable of communicating prices across markets, nationally and internationally, in a timely fashion. The telegraph provided this service. Perfected in 1837, the telegraph shaped the commodity exchanges some thirty to forty years later by allowing prices in the East to be communicated to the South and West, allowing grain and cotton to be purchased before harvest or while the goods were in transit. In summary, some form of futures trading existed prior to the Civil War, however, innovations in staple standardization, transportation and communications developed such that futures trading

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22 Chandler, A.D., The Visible Hand, p. 211.
24 The Chicago Board of Trade, “The Development of the Chicago Board of Trade” p. 12.
26 Irwin, Harold S. Evolution of Futures Trading, p. 77-8.
27 Nonetheless, futures trading took place before such technological infrastructures became available. Rothstein (1965) explains that regular mail service by "fast boats" enabled British grain importers in the 1840's to send ahead samples of a staple which was still in transit. Merchants on the floor of London's Baltic Exchange entered into buy and sell contracts on the "to arrive" staple based on inspection of these samples.
became fully operational around 1874. On the size of futures markets around the late 1870s, the Board of Trade noted,

For at least ten years after it had been transacted by the members of the Board of Trade, and during the [Civil] war, this business had grown to immense proportions, at times establishing a price on grain and provision throughout the civilized world. But until the adoption of these [1865 trading] rules, defining the rights of parties under contract for future delivery, the Board had no adequate machinery for adjustment of disputes which [sic] were constantly arising.

8. The Effect of Futures Markets on US Money Market Volatility: A Model

Futures markets quelled money market volatility via hedging, a risk-transferring scheme practiced by producers, shippers and speculators. Indeed, Chandler writes that traders began to hedge their portfolios immediately following the introduction of the modern (post 1874) futures contract. Moreover, on this matter the Chicago Board of Trade observed, “Merchants and transportation interests were using the futures market to insure delivery of grain when and where it was needed and at a known price which permitted safeguarding of handling charges, a practice today [1936] known as ‘hedging.’ The notion that hedging transfers price risk, or the volatility of an asset’s cash market price, from staple producers and shippers to speculators, thus immunizing the portfolio of the former from the volatility of cash market prices, has been suggested, both anecdotally and theoretically by several authors [See Clark (1966), Chandler (1977), Chari and Jagannathan (1990), Kawai (1983), Morgan, Rayner and Ennew (1994), Turnovsky (1983) and Turnovsky and Campbell (1985). However, the effect of futures markets on the variance of interest rates remains both unclear and relatively uninvestigated.

That futures markets quell rate volatility is shown below using the standard mean-variance framework of the spot price volatility literature [See Chari, Jagannathan, and Jones (1990), Kawai (1983), Turnovsky (1983), and Turnovsky and Campbell (1985)]. In similar fashion, this paper builds upon the optimizing behaviors of producers and speculators in the absence and presence of futures contracts in order to expound on the effect of commodity futures on interest rates, rather than prices. In general, results indicate that the ability to hedge in the futures markets increases the price sensitivities of aggregate supply and aggregate demand, thereby diminishing the variability of both the price level and the interest rate in the presence of supply and/or demand shocks.

To begin, we assume that output is produced by a perfectly competitive representative firm facing the profit

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31 The Chicago Board of Trade, “The Development of the Chicago Board of Trade” p.18.
32 See appendix 1 for derivation details.
33 In the parlance of modern finance, a ‘perfect’ or ‘textbook’ hedge was attained if the basis, defined as difference between cash and future prices, remained constant throughout the period for which the hedge was employed.
34 Chandler, A.D., The Visible Hand, p. 212.
function specified in (1), where costs are quadratic in the level of planned, rather than actual, output $\bar{Y}_t$. In the presence of a futures market for commodity $Y_t$, planters in period $t-1$ may choose to engage in a hedge, thereby committing themselves to deliver $Z_{t-1}$ units of output in period $t$, at a (futures) price $P_{t}^{f}$ specified in period $t-1$. The term $\phi$ in (1) is a binary variable denoting the presence ($\phi = 1$) or absence ($\phi = 0$) of a futures market. That costs are incurred on planned rather than actual output is appropriate since firms (planters) expend resources based upon ex ante growing objectives rather than ex post yields.

$$\pi_{t}^{P} = P_{t}^{f} \left[ Y_{t} - \phi Z_{t-1}^{f} \right] + P_{t-1}^{f} \phi Z_{t-1}^{f} - \frac{1}{2} c \bar{Y}_{t}^{2} \quad (1)$$

Actual output, given by (2), is equal to planned output plus a random disturbance term $\nu_t \sim N(0,1)$ representing natural (supply) shocks to each period's output.

$$Y_{t} = \bar{Y}_{t} + \nu_{t}, \quad E_{t-1}[\nu_{t}] = 0, \quad E_{t-1}[\nu_{t}^2] = \sigma_{\nu}^{2} \quad (2)$$

Substituting (2) into (1) yields the profit function specified in (3),

$$\pi_{t}^{P} = P_{t}^{f} \left[ \bar{Y}_{t} + \nu_{t} - \phi Z_{t-1}^{f} \right] + P_{t-1}^{f} \phi Z_{t-1}^{f} - \frac{1}{2} c \bar{Y}_{t}^{2} \quad (3)$$

Following Tumovsky (1983), the representative firm is assumed to be risk averse, concerned with both the level and variability of profits, and hence maximizes the following objective function (4).

$$\Lambda_{t}^{P} = E_{t-1}[\pi_{t}^{P}] - \frac{1}{2} \alpha \text{ var}_{t-1}[\pi_{t}^{P}] \quad (4)$$

The role played by the coefficient $\alpha$, a term that captures the relative risk aversion of the representative producer, will be discussed in detail below. In the absence of futures markets ($\phi = 0$), the representative firm maximizes $\Lambda_t$ with respect to $\bar{Y}_t$, yielding,

$$\bar{Y}_t = E_{t-1}[P_{i}] - \alpha \text{ cov}_{t-1}[P_{i}, \nu_{t}] \cdot \frac{c}{c + \alpha \text{ var}_{t-1}[P_{i}]} \quad (5)$$

While in the presence of futures markets ($\phi = 1$), the representative firm maximizes $\Lambda_t$, with respect to $\bar{Y}_t$ and $Z_{t-1}$.

Solving simultaneously yields,

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11. The Chicago Board of Trade, "The Development of the Chicago Board of Trade" p. 16.
In addition to producers, this model considers also the behavior of risk neutral speculators who store commodity $Y_i$ and hold long and short positions $X_{i-1}$ in the futures markets in anticipation of favorable spot and futures price movements, respectively. The profit function of the representative speculator (8) is comprised of: (i) storage revenues, where $I_{t-1}$ is the quantity of inventory held over from period $t-1$; (ii) futures market transaction revenues, where $X_{i-1}$ is the speculator's quantity of long ($X_{i-1} < 0$) or short ($X_{i-1} > 0$) positions in the futures markets; and (iii) quadratic storage costs.\footnote{Similar specifications can be found in Bond (1984), Kawai (1983), Turnovsky (1983) and Turnovsky and Campbell (1985).}

\begin{equation}
\bar{Y}_i = \frac{P_{t-1}^f}{c} \tag{6}
\end{equation}

\begin{equation}
Z_{i-1} = \frac{P_{t-1}^f - E_{t-1}[P_t] + \alpha \text{cov}_{t-1}[P_t, P_t]}{\alpha \text{var}_{t-1}[P_t]} + \frac{P_{t-1}^f}{c} \tag{7}
\end{equation}

Once again, following Turnovsky (1983), the representative speculator is risk neutral and maximizes the following objective function (9).\footnote{Turnovsky, Stephen J. “The Determination of Spot and Future Prices With Storable Commodities,” p.1363.} The coefficient beta captures the relative risk aversion of the representative speculator.

\begin{equation}
\pi_i = I_{t-1}(P_t - P_{t-1}) + \phi X_{t-1}(P_{t-1}^f - P_t) - \frac{1}{2} \frac{1}{d} \frac{1}{t-1} \frac{dX_{t-1}^2}{d}
\tag{8}
\end{equation}

In the absence of futures markets ($\phi = 0$), the representative speculator sets the expected marginal return on inventory storage $\frac{d\Lambda_i}{dI_{t-1}}$ equal to $R_{t-1}$, the return on a financial asset representing the opportunity costs of storing commodities [See Bond (1984)], yielding,

\begin{equation}
I_{t-1} = \frac{E_{t-1}[P_t] - P_{t-1} - R_{t-1}}{d + \beta \text{var}_{t-1}[P_t]} \tag{10}
\end{equation}

Meanwhile, in the presence of futures markets ($\phi = 1$), the representative speculator totally differentiates $\Lambda_i$, with respect to $I_{t-1}$ and $X_{t-1}$. Once again, setting $\frac{d\Lambda_i}{dI_{t-1}} = R_i$ and $\frac{d\Lambda_i}{dX_{t-1}} = 0$, and solving simultaneously yields,

\begin{equation}
I_{t-1} = \frac{P_{t-1}^f - P_{t-1} - R_{t-1}}{d} \tag{11}
\end{equation}

\begin{equation}
X_{t-1} = \frac{P_{t-1}^f - P_{t-1} - R_{t-1}}{d} + \frac{P_{t-1}^f - E[P_t]}{\beta \text{var}_{t-1}[P_t]} \tag{12}
\end{equation}
Following Turnovsky and Campbell (1985), we assume that the current spot price, $P_s$, deviates from its expected value by the weighted sum of the aggregate demand and supply disturbances, respectively, yielding the following aggregate supply in the absence of futures markets,

$$Y_t^{AS} = bE_{t-1}[P_t] + u_t \quad (13)$$

where

$$b_t = \frac{1 - \varrho_t}{\alpha + \alpha \beta \varpi_{t-1}[P]}$$

Likewise, in the presence of futures markets, aggregate supply is simply

$$Y_t^{AS} = \frac{1}{c}P_t + u_t \quad (14)$$

Lastly, in both the absence and presence of futures markets, the following specifications are employed for consumption and money demand respectively.

$$C_t = \xi y_t, \quad \xi \in (0,1)$$

$$M^D = Ke^{\alpha t} P_t, \quad I < 0 \quad (10)$$

The money demand equation is transformed logarithmically (denoted by lower case), and a linear approximation of the expression $\ln P_t$ is taken around $P_o$, yielding, $m^D = k + (\ln P_0 - 1) + IR_t + \frac{1}{P_0}$. Hence, in the absence of futures markets, a demand-side equilibrium is established in the goods and money markets by solving (15) simultaneously, where real investment expenditure is given from (11); $u_t$ is a zero mean and finite variance demand shock.

$$c_t + i_t + u_t = y_t, \quad \text{goods market}$$

$$M^d = M^S, \quad \text{money market} \quad (15)$$

Demand-side equilibrium yields the following aggregate demand function,

$$AD: \quad Y_t = -\theta \frac{\gamma_t}{I} (m - k - p_{0t} + 1) + \theta r_t E_t[P_{t+1}] - \theta r_t \sigma_{t} + \theta u_t \quad (16)$$

where $\sigma = \left(1 - \frac{1}{P_0} \right), \quad \frac{dY_t}{dP_t} |_{m^D} < 0, \quad \frac{dY_t}{dm} > 0$

Finally, the aggregate equilibrium condition (17) is derived, where aggregate demand is set equal to current output

\[ As seen originally in Cagan (1956). \]
plus inventory holdings carried over from the prior period.

\[ AD_t = y_t^{AS} + i_{t-1} \quad (17) \]

where,

\[ y_t^{AS} = b_t E_{t-1} \{ P_t \} + \nu_t \]

From (17), the one period and asymptotic variances of interest rates follow as

\[ \sigma^2_R(1) = \frac{\sigma^2_\eta}{(P_0)^2 \left( d \left( \frac{1}{F_1 \alpha} \omega - \frac{F_2}{F_1} \lambda_1 \right) \right)^2}, \]

\[ \sigma^2_R^2 = \frac{\sigma^2_\eta}{(P_0)^2 \left( \left( 1 - \frac{1}{F_1} \right) \omega \frac{F_2}{F_1} \lambda_1 \right)^2}, \]

respectively.

Likewise, in the presence of futures markets, a demand-side equilibrium is established in the goods, money and futures markets simultaneously (18).

\[ c_t + i_t + u_t = y_t \quad \text{goods market} \]
\[ M^d = M^S \quad \text{money market} \]
\[ z_{t-1}^f + x_{t-1}^f = 0 \quad \text{futures market} \]

Demand-side equilibrium yields the following aggregate demand function,

\[ AD : Y_t = \theta \left( \frac{m - k - P_0 + 1}{\alpha} \right) \left( \frac{1}{d F_1} \right) - 1 + \theta \frac{F_2}{d F_1} E_t \{ P_{t+1} \} + \theta \left( \frac{1}{d F_1} \right) \omega \right) P_t + \theta \mu_t \]

where,

\[ F_{t_1} = \frac{1}{\text{var}_t \{ P_t \} \left( \frac{1}{\alpha} + \frac{1}{d} \right) + \frac{1}{c} + \frac{1}{d}}, \quad F_{t_2} = \frac{1}{\text{var}_t \{ P_t \} \left( \frac{1}{\alpha} + \frac{1}{d} \right) + \text{var}_t \{ \nu_t \}} \]

and \( \frac{dY_t}{dP_t} |_{\Delta m} < 0 \), \( \frac{dY_t}{dP_t} |_{\Delta m} > 0 \)

Once again, the aggregate equilibrium condition (19) is derived, where aggregate demand is set equal to current output plus inventory holdings carried over from the prior period.

\[ AD_t = y_t^{AS} + i_{t-1} \quad (19) \]

where,

\[ y_t^{AS} = \frac{1}{c} P_{t-1}^{f} + \nu_t \]

From (19), the one period and asymptotic variances of interest rates follow as

\[ \sigma^2_R(1) = \frac{\sigma^2_\eta}{(P_0)^2 \left( d \left( \frac{1}{F_1} \omega - \frac{F_2}{F_1} \lambda_1 \right) \right)^2}, \]

\[ \sigma^2_R^2 = \frac{\sigma^2_\eta}{(P_0)^2 \left( \left( 1 - \frac{1}{F_1} \right) \omega \frac{F_2}{F_1} \lambda_1 \right)^2}, \]

respectively.
Due to the model’s non-linear structure, an arithmetic comparison of variances in the presence, and absence, of futures markets is not tractable, hence a simulation approach is adopted to expound the effects of futures markets on money market volatility. While both systems of equations (w/ & w/o futures) require nine parameters: $\theta, M, \bar{k}, P_0, c, d, l, \alpha$, and $\beta$, only changes in the values of the latter five are germane to the hypothesis at hand; these are, $c, d, l, \alpha$ and $\beta$, or producer’s cost elasticity, speculator’s return (on inventory) elasticity, interest rate elasticity of money demand, producer’s relative risk aversion and speculator’s relative risk aversion, respectively.

The following two plates illustrate the effects of changes in $c, d, l$, and $\beta$, for an interest rate elasticity of money demand, $l$, equal to -.9, in the presence of supply and demand shocks, respectively. The results that follow were produced using a nonlinear systems-solving algorithm available through SAS.

Several simulation results are worth noting. Namely, futures markets quell interest rate volatility most effectively when: (1) producers and speculators are relatively risk-averse (relatively high $\alpha$ and $\beta$); (2) shocks emanate from the demand, as opposed to the supply, side of an economy; (3) an economy’s aggregate demand and supply curves are relatively inelastic (relatively small $c$ and large $d$); and (4) interest rate sensitivity of money demand is relatively large. Lastly, the effect of futures markets on the mean level of the interest rate is ambiguous.

In general, the intuition for why futures markets quell money market volatility is as follows: futures markets increase the relative sensitivities of producers and speculators to anticipated fluctuations in the price level, causing each to behave in a manner that offsets partially any potential movement in the price level. As movements in the price level are offset, so too are fluctuations in the real money supply, hence interest rate movements are offset as well. This can be seen for speculators by comparing investment decisions in the absence, and presence, of futures markets ((10) and (11), respectively). In the absence of futures markets a rise in the variance of the price level weakens a speculator’s investment sensitivity to anticipated price increases (by a multiple of the latter’s coefficient of relative risk aversion). For example, without the availability of a futures contract in which to engage, a speculator is less likely to sell inventories as prices rise, hence prices rise, and interest rates fall, by more than they otherwise would. On the contrary, price volatility plays no direct role in a speculator’s inventory decision in the presence of futures contracts. On this point, the Chicago Board of Trade notes,

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41 SAS’ Model procedure is used to solve these nonlinear systems.
42 While the variance of the interest rate is dependent upon a given value of $P_0$, the relative magnitudes of these variances, in the presence and absence of futures contracts are not.
43 Interest rate volatility is relatively less in the presence of futures contracts, the greater the absolute value of interest rate sensitivity of money.
The speculator was present to absorb, as a "long," offerings that could not otherwise be readily taken by the trade, or to supply contracts during times of high prices and small deliveries by going "short." ... Even at this early date [1850-1860] the inherent speculative risk in taking title to grain for merchandising purposes was being passed on to the speculator who desired to assume this risk in the hope of profit. 

Likewise, producers are more apt to increase production in anticipation of a price rise, when a futures contract is available, hence mitigating partially a price rise in the next period. Once again, a comparison of supply functions in the absence, and presence, of futures contracts ((5) and (6), respectively) illustrates this point.

The intuition regarding the remaining simulation results (enumerated above) is as follows: first the greater the relative risk aversions of producers and speculators, the less their actions accommodate price movements in the absence of futures contracts, so futures markets are relatively more stabilizing; second, unlike the demand-side, the supply-side is able to employ both inventory (speculators) and production (producers) responses to a potential change in the price level, so demand shocks are relatively better accommodated with and without futures contracts; third, since futures markets increase price-sensitivities of both aggregate supply and demand schedules, the effect of this increased sensitivity is relatively more pronounced when these functions are relatively insensitive prior to the introduction of such contracts; lastly, the explanation for the role that the interest rate elasticity of money demand plays in quelling interest rate volatility is the same as that provided by the standard L.M relationship.

Hence, futures contracts insulated traders from the annual fluctuations in staple prices caused by variations in planting and harvesting conditions. As a result, when traders borrowed from the money market to purchase these staples, the amount of cash that they required (demanded) was also relatively insulated from staple price volatility. Hence, while real shocks continued to hit agricultural markets after 1874, the demand and supply for loanable funds remained relatively unaffected. Therefore, money markets were less inclined to react to real shocks and hence the volatility in the annual cost of borrowing decreased. Chandler notes that this financial innovation "lowered the cost of credit required to move crops" such that relatively low-interest short-term credit was easily obtained by shippers from local commercial banks as the risk associated with such loans diminished. As a result, interest rate volatility decreased, and a statistically significant seasonal pattern is discernable after 1874.

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\[ \text{To conserve space, only results for } l = -9 \text{ are reported here.} \]

\[ \text{The Chicago Board of Trade, "The Development of the Chicago Board of Trade" p.16. Interestingly, this publication also notes that "it is clearly evident that the stabilizing influence and price registration functions of a futures market were in use almost from the organization of the Board of Trade in 1848, and were not an outgrowth of the Civil War as is popularly believed (emphasis added)."} \]

\[ \text{Chandler, A.O. The Visible Hand, p. 212.} \]
Plate 1.a.: Simulation Results, $l = -0.9$, in the Presence of Supply Shocks

Parameter Sets

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<tr>
<th>Parameter Sets</th>
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<td>$\beta$</td>
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<td>0.100</td>
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<td>0.010</td>
<td>0.100</td>
<td>0.100</td>
<td>0.010</td>
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| $\sigma^2_{R_{no\text{ futures}}}$ | 3.966 | 3.727 | 3.541 | 3.360 | 0.010 | 0.010 | 0.010 | 0.010 |
| $\sigma^2_{R_{futures}}$            | 3.305 | 3.302 | 3.302 | 3.302 | 0.010 | 0.010 | 0.010 | 0.010 |
| $\frac{\sigma^2_{R_{futures}}}{\sigma^2_{R_{no\text{ futures}}}}$ | 0.833 | 0.890 | 0.933 | 0.983 | 1.000 | 0.993 | 1.001 | 1.000 |
| $\bar{R}_{no\text{ futures}}$       | 11.683 | 11.939 | 11.689 | 11.928 | 0.831 | 0.825 | 0.828 | 0.822 |
| $\bar{R}_{futures}$                 | 11.534 | 11.870 | 11.870 | 11.908 | 0.783 | 0.814 | 0.814 | 0.817 |
| $\frac{\bar{R}_{futures}}{\bar{R}_{no\text{ futures}}}$ | 0.987 | 0.994 | 1.016 | 0.998 | 0.942 | 0.987 | 0.983 | 0.994 |
| $CV_{futures}$                       | 0.158 | 0.153 | 0.153 | 0.153 | 0.131 | 0.125 | 0.125 | 0.125 |
| $CV_{no\text{ futures}}$             | 0.170 | 0.162 | 0.161 | 0.154 | 0.123 | 0.124 | 0.123 | 0.124 |
Plate 1.b.: Simulation Results, $\lambda = -.9$, in the Presence of Demand Shocks

Parameter Sets

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<td>$\sigma_{\bar{R}}^2_{no,futures}$</td>
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<td>7.430</td>
<td>7.429</td>
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<tr>
<td>$\sigma_{\bar{R}}^2_{futures}$</td>
<td>0.573</td>
<td>0.727</td>
<td>0.831</td>
<td>0.958</td>
<td>0.990</td>
<td>0.983</td>
<td>1.000</td>
<td>0.999</td>
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<td>$\sigma_{\bar{R}}^2_{no,futures}$</td>
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<td>$\bar{R}_{futures}$</td>
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<td>11.754</td>
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<td>$\bar{R}_{no,futures}$</td>
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<td>1.059</td>
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<td>$CV_{futures}$</td>
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<td>$CV_{no,futures}$</td>
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<td>0.269</td>
<td>0.269</td>
<td>0.234</td>
<td>0.186</td>
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However, that the futures contract did not eradicate seasonal patterns in interest rates must be noted. Borrowing increased during the harvest and crop moving seasons regardless of whether or not a futures contract was employed. For example, in the presence of futures contracts: wheat was purchased in September, wheat prices fluctuated in September, transactions were still settled in cash, and if the long position required a loan to purchase wheat, that loan was acquired in September. The only difference was that traders were certain about the price they would pay in the market for the staple. In the context of this example, the demand for money increased every September just as it did prior to the existence of futures markets. Likewise, if the September harvest was poor, staple prices rose accordingly. Hence, the annual movements in the marketing of wheat remained seasonal while the volatility of staple prices, and interest rates, did diminished as the transfer of price risk from traders to speculators
quelled money market volatility.\textsuperscript{46} This led to a decrease in the variance of interest rates and hence statistically significant seasonality was introduced into US money markets.

Lastly, an additional explanation for the decrease in money market volatility around 1874 is the substitution of futures for consignment contracts.\textsuperscript{47} Regarding the abandonment of consignment contracts (and adoption of futures contracts) after 1874, Chandler writes, "No longer did the financing of the movement of the crops require long and often risky negotiations between one commission merchant and another."\textsuperscript{48} Once futures markets were fully operational, the crowding out of the consignment contract was immediate.\textsuperscript{49}

Negotiations based on consignment were inherently risky because none of the parties involved purchased the produce from the farmers, millers and merchants at the point of production. Rather, the owners of the produce were compensated upon the execution of a final sale in the East. Prior to this sale, traders at every level of the marketing process were unaware of the price that they would pay or receive for the produce. Regardless of the motive, contracts made on consignment and financed with short-term credit tied money markets to the volatility of staple prices. This occurred because a sudden change in price would unexpectedly affect the financial positions of borrowers and lenders. This would lead to relatively large fluctuations in interest rates.

The design of a futures contract was opposite that of a consignment contract. By generally adopting the former in 1874, players in the agricultural trade could choose between business transactions of varying risk. For the conservative business-person wishing to purchase staples from the West, a futures contract and a hedging scheme was the ideal combination for guarding oneself against price risk. Likewise, for the speculator, schemes such as selling staples short in anticipation of a price decrease or taking a long position in anticipation of a price increase enabled quick gains (and losses) to be made.

In summary, the emergence of futures markets, in conjunction with hedging techniques and the eradication of consignment contracts, could explain the decrease in interest rate volatility observed after 1874 and the subsequent emergence of statistically significant seasonality. In addition, this explanation is consistent with the observed seasonal patterns in US short-term interest rates throughout the 19th and early 20th centuries.

\textsuperscript{46} Indeed, Chandler (1977) states that such price volatility diminished with the advent of futures contracts.
\textsuperscript{47} Chandler, A.D., \textit{The Visible Hand}, p. 211.
\textsuperscript{48} Ibid., p. 212.
\textsuperscript{49} Ibid., p. 212.
\textsuperscript{50} See Chandler (1977), Clark (1966), and Hammond (1897).
8. Conclusion

This paper linked the change in the behavior of US short-term interest rates around 1874 with the evolution of commodity futures contracts. In particular, the introduction of futures markets, and the resulting substitution away from consignment contracts around 1874, was used to explain why early US money markets are relatively more volatile, and far less seasonal, than their post-1874 counterparts. The paper argued that the inability to detect a seasonal cycle prior to the 1870's was most likely due to money market volatility. Until 1874, movements in interest rates were erratic and financial instabilities imparted relatively large shocks to money markets, particularly in the autumn months. After 1874, the effects of financial instabilities on interest rates diminished and the regularization of seasonal movements was attained.

That futures trading quelled money market volatility was demonstrated using the standard mean-variance framework of the spot price volatility literature. The paper built upon the optimizing behaviors of producers and speculators in the absence and presence of futures contracts. Results indicated that the ability to hedge in the futures markets increases the price sensitivities of aggregate supply and aggregate demand, thereby diminishing the variability of both the price level and the interest rate in the presence of supply and/or demand shocks.
Appendix 1. The Model

The Producer’s Problem

\[ \pi_p = P_1^f \left( Y_t - \phi Z_{t-1} \right) + P_1^f \phi Z_{t-1} - \frac{1}{2} c \bar{Y}_t \]  
(1)

\[ Y_t = \bar{Y}_t + \nu_t, \quad E_{t-1}[\nu_t] = 0, \quad E_{t-1}[\nu_t^2] = \sigma^2_v \]  
(2)

Substituting (2) into (1) yields (3),

\[ \pi_p = P_1^f \left( \bar{Y}_t + \nu_t - \phi Z_{t-1} \right) + P_1^f \phi Z_{t-1} - \frac{1}{2} c \bar{Y}_t \]  
(3)

Following Turnovsky (1983), the representative firm is assumed to be risk averse, concerned with both the level and variability of profits, and hence maximizes the following objective function (4).

\[ \Lambda_p = E_{t-1}\left[ \pi_p^2 \right] - \frac{1}{2} \alpha \text{var}_{t-1}\left[ \pi_p^2 \right] \]  
(4)

where,

\[ E_{t-1}\left[ \pi_p^2 \right] = E_{t-1}\left[ P_1^f \left( \bar{Y}_t - \phi Z_{t-1} \right) \right] + E_{t-1}\left[ P_1^f \phi Z_{t-1} \right] + P_1^f \phi Z_{t-1} - \frac{1}{2} c \bar{Y}_t \]

\[ \text{var}_{t-1}\left[ \pi_p^2 \right] = \text{var}_{t-1}\left[ P_1^f \left( \bar{Y}_t - \phi Z_{t-1} \right)^2 \right] + \text{var}_{t-1}\left[ P_1^f \phi Z_{t-1} \right] + 2(\bar{Y}_t - \phi Z_{t-1}) \text{cov}_{t-1}\left[ P_1^f, P_1^f \phi Z_{t-1} \right] \]

In the absence of futures markets (\( \phi = 0 \)), the representative firm maximizes \( \Lambda_p \) with respect to \( \bar{Y}_t \), yielding,

\[ \bar{Y}_t = \frac{E_{t-1}\left[ P_1^f \right] - \alpha \text{cov}_{t-1}\left[ P_1^f, P_1^f \phi Z_{t-1} \right]}{c + \alpha \text{var}_{t-1}\left[ P_1^f \right]} \]  
(5)

While in the presence of futures markets (\( \phi = 1 \)), the representative firm maximizes \( \Lambda_p \), with respect to \( \bar{Y}_t \) and \( Z_{t-1} \). Solving simultaneously yields,

\[ \bar{Y}_t = \frac{P_{1,1}^f}{c} \]  
(6)

\[ Z_{t-1} = \frac{P_{1,1}^f - E_{t-1}\left[ P_1^f \right] + \alpha \text{cov}_{t-1}\left[ P_1^f, P_1^f \phi Z_{t-1} \right]}{c} \]  
(7)

The Speculator’s Problem

\[ \pi_s = I_{t-1}\left( P_t - P_{t-1} \right) + \phi X_{t-1}\left( P_{t-1}^f - P_t \right) - \frac{1}{2} d t_{t-1}^2 \]  
(8)

Once again, following Turnovsky (1983), the representative speculator is risk neutral and maximizes the following objective function (9).

\[ \Lambda_s = E_{t-1}\left[ \pi_s^2 \right] - \frac{1}{2} \beta \text{var}_{t-1}\left[ \pi_s^2 \right] \]  
(9)

where,

\[ E_{t-1}\left[ \pi_s^2 \right] = I_{t-1}\left( E_{t-1}\left[ P_1^f \right] - P_{t-1} \right) + \phi X_{t-1}\left( P_{t-1}^f - E_{t-1}\left[ P_1^f \right] \right) - \frac{1}{2} d t_{t-1}^2 \]

\[ \text{var}_{t-1}\left[ \pi_s^2 \right] = \left( I_{t-1} - \phi X_{t-1} \right)^2 \text{var}_{t-1}\left[ P_1^f \right] \]
In the absence of futures markets (φ = 0), the representative speculator totally differentiates \( A_{t-1} \) with respect to \( I_{t-1} \), and, borrowing from Bond (1984), sets the expected marginal return on inventory storage \( \frac{dA_t}{dI_{t-1}} \) equal to \( R_{t-1} \), the return on a financial asset representing the opportunity costs of storing commodities, which yields,

\[
\frac{dA_t}{dI_{t-1}} = E_{t-1}[P_t] - P_{t-1} - (d + \beta \text{var}_{t-1}[P_t])I_{t-1}
\]

\[
I_{t-1} = \frac{E_{t-1}[P_t] - P_{t-1} - R_{t-1}}{d + \beta \text{var}_{t-1}[P_t]} \tag{10}
\]

While in the presence of futures markets (φ = 1), the representative speculator totally differentiates \( A_{t-1} \), with respect to \( I_{t-1} \) and \( X_{t-1} \). Once again, setting \( \frac{dA_t}{dI_{t-1}} = R_t, \frac{dA_t}{dX_{t-1}} = 0 \) and solving simultaneously yields,

\[
I_{t-1} = \frac{P_{f,t-1} - P_{t-1} - R_{t-1}}{d} \tag{11}
\]

\[
X_{t-1} = \frac{P_{f,t-1} - P_{t-1} - R_{t-1}}{d} + \frac{P_{f,t} - E[P_t]}{\beta \text{var}_{t-1}[P_t]} \tag{12}
\]

Following Turnovsky and Campbell (1985), we assume that the current spot price, \( P_t \), deviates from its expected value by the weighted sum of the aggregate demand and supply disturbances, respectively:

\[
P_t = E_{t-1}[P_t] + \rho_u u_t + \rho_v v_t \tag{13}
\]

where,

\[
E_{t-1}[u_t] = E_{t-1}[v_t] = 0
\]

\[
E_{t-1}[u_t^2] = \sigma_u^2; E_{t-1}[v_t^2] = \sigma_v^2
\]

and \( \rho_u, \rho_v \) represent the responses of the spot price to aggregate demand and aggregate supply disturbances, respectively.\(^5\) Hence,

\[
\text{cov}_{t-1}[P_t, P_{f,t}] = \rho_u \text{var}_{t-1}[v_t] E_{t-1}[P_t] \tag{14}
\]

Lastly, substituting (14) into (5) yields the following aggregate supply in the absence of futures markets,

\[
Y_{t}^{AS} = bE_{t-1}[P_t] + \nu_t \tag{15}
\]

where

\[
b_t = \frac{1 - \alpha \rho_v \text{var}_{t-1}[v_t]}{c + \alpha \text{var}_{t-1}[P_t]}
\]

Likewise, in the presence of futures markets, aggregate supply is simply

\[
Y_{t}^{AS} = \frac{1}{c} P_{f,t} + \nu_t \tag{16}
\]

---

Finally, in both the absence and presence of futures markets, the following specifications are employed for consumption and money demand respectively.

\[ C_t = \xi v_t, \quad \xi \in (0, 1) \]
\[ M^D_t = K e^{\Phi P_t}, \quad l < 0 \]

The money demand equation is transformed logarithmically (denoted by lower case), and a linear approximation of the expression \( \ln P_t \) is taken around \( P_0 \), yielding, \( m^D = k + (\ln P_0 - 1) + lR_t + \frac{1}{P_0} P_t \).

In the absence of futures markets, a demand-side equilibrium is established in the goods and money markets by solving (17) simultaneously, where real investment expenditure is given from (11); \( u_t \) is a zero mean and finite variance demand shock.

\[ C_t + I_t + u_t = Y_t \quad \text{goods market} \]
\[ M^D_t = M^S_t \quad \text{money market} \]

where

\[ C_t = \xi Y_t, \quad \xi \in (0, 1) \]
\[ I_{t-1} = \gamma_t \left[ E_{t-1} \left[ P_t \right] - P_{t-1} - R_{t-1} \right] \]
\[ \gamma_t = \frac{1}{d + \beta \text{var}_t[P_t]} \]
\[ m_t = k + (\ln P_0 - 1) - lR_t + \frac{1}{P_0} P_t \]
\[ M^S_t = \bar{M} \]
\[ E_{t-1} \left[ u_t \right] = 0, \quad E_{t-1} \left[ u_t^2 \right] = \sigma_u^2 \]

Demand-side equilibrium yields the following aggregate demand function,

\[ AD : \quad Y_t = -\theta Y_t \left( m - k - p_0 + 1 \right) + \theta \gamma_t \left[ E_{t-1} \left[ P_t \right] - \gamma_t \left[ P_t \right] - \gamma_t \right] + \theta u_t \]

where \( \theta = 1 - \frac{1}{P_0} \), \( \frac{dY_t}{dP_t} \bigg|_{AD} < 0 \), \( \frac{dY_t}{dm} > 0 \)

The aggregate equilibrium condition is derived by setting aggregate demand equal to current output plus inventory holdings carried over from the prior period.

\[ AD_t = Y_t^{AS} + I_{t-1} \]

where,

\[ Y_t^{AS} = \beta_t E_{t-1} \left[ P_t \right] + \nu_t \]

Solving (19) and rearranging yields,

\[ C + \theta \left( \frac{Y_t}{P_0} \right) P_t + \theta \gamma_t \left( E_{t-1} \left[ P_{t-1} \right] - P_t \right) = \gamma_t \left( E_{t-1} \left[ P_t \right] - P_{t-1} \right) + \beta_t \left( \frac{Y_t}{P_0} \right) P_{t-1} + (\nu_t - \theta u_t) \]
\[ C = \frac{\gamma(m - k - p_0 + 1)(1 - \theta)}{l} > 0 \]

If we define the long-run average price attained when expectations are realized by \( \bar{P} = \frac{C}{b + \frac{\gamma}{l}P_0 (1 - \theta)} \), and redefine the following variables in mean deviation form, \( p_t = P_t - \bar{P}, E_{t+j-1}[p_{t+j}] = E_{t+j-1}[p_{t+j}] - \bar{P} \), we get

\[ \theta \left( \frac{\gamma}{lP_0} \right) p_t + \theta \gamma (E_t[p_{t+j}] - p_t) = \gamma (E_t[p_t] - p_{t-1}) + b E_{t-1}[p_t] + \left( \frac{\gamma}{lP_0} \right) p_{t-1} + (\nu_t - \theta \nu_t) \quad (21) \]

Following Turnovsky (1983), we take conditional expectations of (21) at time \( t-1 \) for an arbitrary period \( j = 0, 1, 2, \ldots t+j \), which yields,

\[ \theta \gamma E_{t-1}[p_{t+j}] - (\gamma (1 + \theta \omega) + b) E_{t-1}[p_{t+j}] + \gamma \omega p_{t+j-1} = 0 \quad (22) \]

This is a second order difference equation in the predictions \( E_{t-1}[p_{t+j}] \), the solution to which is

\[ E_{t-1}[p_{t+j}] = A_1 \lambda_1^j + A_2 \lambda_2^j \quad (23) \]

where \( \lambda_1, \lambda_2 \) are the roots of the quadratic equation,

\[ (\theta \lambda - 1)(\lambda - \omega) = \frac{b}{\gamma} \lambda \quad (24) \]

The roots of this equation are real and positive such that \( 0 < \lambda_1 < \frac{1}{\theta} < 1, \quad \lambda_2 > \omega > 2 \)

We take only the root that is less than one, as this is consistent with our assumption that the path of future price expectations is stationary. Hence, \( A_2 = 0 \), and the solution to (23) is

\[ E_{t-1}[p_{t+j-1}] = \lambda_1^j \eta_j \quad (25) \]

setting \( j=0 \), yields the following initial condition,

\[ E_{t-1}[p_{t+j-1}] = \lambda_1^i \eta_i \quad (26) \]

Substituting into (22) yields,

\[ \theta \gamma (\omega - \lambda_1) p_t = (\gamma (\omega - \lambda_1) - b \lambda_1) p_{t-1} + (\theta \nu_t - \nu_t) \quad (27) \]

simplifying yields,

\[ p_t = \lambda_1 p_{t-1} + \eta_t \left( \frac{\theta \nu_t}{\theta \gamma (\omega - \lambda_1)} \right), \quad \text{where} \quad \eta_t = \theta \nu_t - \nu_t \quad (28) \]

Hence, the one period variance in prices can be written as,

\[ \sigma_p^2(1) = \frac{\sigma_q^2}{(\theta \gamma (\omega - \lambda_1))^2} \quad (29) \]

with asymptotic variance.
\[ \sigma_p^2 = \frac{\sigma_\eta^2}{\left(1 - \frac{\lambda_1^2}{\text{Var}(\omega - \lambda_1)}\right)^2} \]  

Likewise, expressing the interest rate in mean deviation form,

\[ r_t = \frac{R_t - \bar{R}}{\bar{P}_t} \]  

yields the following one period variance in interest rates,

\[ \sigma_\eta^2(l) = \frac{\sigma_p^2(l)}{\left(\bar{P}_0\right)^2} \cdot \left(\frac{\sigma_\eta^2}{\text{Var}(\omega - \lambda_1)}\right)^2 \]  

with asymptotic variance,

\[ \sigma_R^2 = \frac{\sigma_p^2}{\left(\bar{P}_0\right)^2} \cdot \frac{\sigma_\eta^2}{\left(\text{Var}(\omega - \lambda_1)\right)^2} \]  

Moreover, taking expectations of (28) yields,

\[ \rho_u = \frac{1}{\gamma(\omega - \lambda_1)} \quad \rho_v = -\frac{1}{\gamma(\omega - \lambda_1)} \]

\[ b = \frac{1 + \frac{\alpha \sigma_\eta^2}{\gamma(\omega - \lambda_1)}}{c + \alpha \sigma_p^2(l)} \]

\[ \gamma = \frac{1}{d + \beta \sigma_p^2(l)} \]

Interest Rate Volatility in the Presence of Futures Markets.

In the presence of futures markets, a demand-side equilibrium is established in the goods, money and futures markets simultaneously (34).

\[ C_t + I_t + u_t = Y_t \quad \text{goods market} \]
\[ M^d_t = M^S_t \quad \text{money market} \]
\[ Z_{t-1}^f + X_{t-1}^f = 0 \quad \text{futures market} \]

where

\[ C_t = \xi Y_t, \quad \xi \in (0,1) \]
\[ I_{t-1} = \frac{P_{t-1}^f - P_{t-1}^f - R_{t-1}}{d} \]
\[ Z_{t-1}^f = \frac{P_{t-1}^f - E_{t-1}[P_t] + \alpha \text{cov}_{t-1}[P_t, P_{t+1}]}{\alpha \text{var}_{t-1}[P_t]} \cdot \frac{P_{t-1}^f}{c} \]
\[ X_{t-1} = \frac{P_{t-1} - P_r - R_{t-1}}{d} + \frac{p_{r-1} - E[p_r]}{\beta \text{var}_{t-1}[P_r]} \]

\[ m_t = (k + \ln P_0 - 1) - lR_t + \frac{1}{P_0} \]

\[ M^* = M \]

\[ E_{t-1}[u_t] = 0, \quad E_{t-1}[\sigma^2] \]

Demand-side equilibrium yields the following aggregate demand function,

\[ AD: \quad Y_t = \phi \left( \frac{m - k - P_0 + 1}{ld} \left( \frac{1}{F_1} d - 1 \right) + \frac{\theta}{F_1} \frac{F_2}{E[p_{t+1}]} + \frac{\theta}{d} \left( \frac{1}{F_1} d - 1 \right) \right) P_t + \partial u_t \tag{35} \]

where,

\[ F_1 = \frac{1}{\text{var}_{t-1}[P_r]} \left( \frac{1 + \alpha + \beta}{\alpha} + \frac{1}{c} \right) \quad F_2 = \frac{1}{\text{var}_{t-1}[P_r]} \left( \frac{1 + \beta - P_0 \text{var}_{t-1}[u_t]}{\alpha} \right) \]

and \( \frac{dY_t}{dP_t} |_{AD} < 0 \), \( \frac{dY_t}{dm} > 0 \)

Next, the aggregate equilibrium condition is derived, where aggregate demand is set equal to current output plus inventory holdings carried over from the prior period.

\[ AD_t = Y_t^{AS} + I_{t-1} \tag{36} \]

where,

\[ Y_t^{AS} = \frac{1}{c} P_{t-1} + u_t \]

Solving (36) and rearranging yields,

\[ C_f + \frac{\theta}{d} F_2 \frac{E[p_{t+1}]}{F_1} + \frac{\theta}{d} \left( \frac{1}{F_1 d} - 1 \right) \phi P_t = \left( \frac{1}{c} + \frac{1}{d} \right) \frac{F_2}{F_1} E_{t-1}[P_r] \]

\[ + \left( \frac{1}{c} + \frac{1}{d} \right) \frac{1}{F_1 d} \phi P_{t-1} + (\phi - \partial u_t) \tag{37} \]

where,

\[ C_f = \frac{(m - k - P_0 + 1)}{ld} \left( (\phi - 1) \left( \frac{1}{F_1 d} - 1 \right) - \frac{1}{F_1 c} \right) > 0 \]

As before, if we define the long-run average price attained when expectations are realized by

\[ \bar{P} = \frac{C_f}{\left( (1 - \phi) \left( \frac{1}{F_1 d} - 1 \right) + \frac{1}{F_1 c} \left( F_2 + \frac{\phi}{d} \right) - F_2 (\phi - 1) \right)} \]

and redefine the following variables in mean deviation form,
\( p_t = P_t - \overline{P}, E_{t+j-1}[p_{t+j}] = E_{t+j-1}[p_{t+j}] - \overline{P} \), we get

\[
\frac{\theta}{d} \frac{F_2}{F_1} E_t[p_{t+1}] + \theta \left( \frac{1}{F_1 d} - 1 \right) \omega p_{t+1} \left( \frac{1}{c} + \frac{1}{d} \right) F_2 E_t[p_t] - \left( \frac{1}{c} + \frac{1}{d} \right) F_1 d \omega p_{t+1} (\omega u_t - \nu_t) = 0
\]

Once again, following Turnovsky (1983), we take conditional expectations of (38) at time \( t-1 \) for an arbitrary period \( j = 0, 1, 2, \ldots t+j \), which yields,

\[
\frac{\theta}{d} \frac{F_2}{F_1} E_t[p_{t+j+1}] + \left( \frac{1}{F_1 d} - 1 \right) \omega \left( \frac{1}{c} + \frac{1}{d} \right) F_2 E_{t-1}[p_{t+j}] - \left( \frac{1}{c} + \frac{1}{d} \right) F_1 d \omega p_{t+j-1} = 0
\]

This is a second order difference equation in the predictions \( E_{t-1}[p_{t+j+1}] \), the solution to which is

\[
E_{t-1}[p_{t+j+1}] = A_1 \lambda_1^j + A_2 \lambda_2^j
\]

where \( \lambda_1, \lambda_2 \) are the roots of the quadratic equation,

\[
(\omega - \lambda) \left( \frac{F_1 - \frac{1}{c} - \frac{1}{d}}{F_2} \right) = \left( \frac{d + \theta \omega}{c} \right) \lambda
\]

The roots of this equation are real and positive such that \( 0 < \lambda_1 < 1 < \lambda_2 > \frac{F_1 - \frac{1}{c} - \frac{1}{d}}{F_2} > 1 \)

Once again, we take only the root that is less than one, as this is consistent with our assumption that the path of future price expectations is stationary. Hence, \( A_2 = 0 \), and the solution to (40) is

\[
E_{t-1}[p_{t+j-1}] = \lambda_1^j p_{t-1}
\]

setting \( j=0 \), yields the following initial condition,

\[
E_{t-1}[p_{t+j-1}] = \lambda_1^j p_{t-1}
\]

Substituting into (39) yields,

\[
\frac{\theta}{d} \left( \frac{1 + \frac{1}{c} + \frac{1}{d} F_2 \lambda_1}{F_1} \right) p_t = \left( \frac{\omega}{d} \left( \frac{1}{c} + \frac{1}{d} \right) F_1 \lambda_1 + \frac{\omega}{F_1 d} \right) p_{t-1} + (\omega u_t - \nu_t)
\]

simplifying yields,
\[ p_t = \lambda_t p_{t-1} + \frac{\eta_t}{\theta \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1} \]  

where \( \eta_t = \delta u_t - v_t \)  

(44)

Hence, the one period variance in prices, in the presence of futures markets, can be written as,

\[ \sigma_{p_t}^2(1) = \frac{\sigma_{\eta}^2}{\left( \frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1 \right)^2} \]  

(45)

with asymptotic variance,

\[ \sigma_{p_t}^2 = \frac{\sigma_{\eta}^2}{\left( 1 - \lambda_1^2 \right) \left( \frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1 \right)^2} \]  

(46)

Similarly, the one period variance in interest rates can be expressed as

\[ \sigma_{r_t}^2(1) = \frac{\sigma_{p_t}^2(1)}{(\Pi_0)^2} = \frac{\sigma_{\eta}^2}{\left( \frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1 \right)^2} \]  

(47)

with asymptotic variance,

\[ \sigma_{r_t}^2 = \frac{\sigma_{p_t}^2}{(\Pi_0)^2 \left( 1 - \lambda_1^2 \right) \left( \frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1 \right)^2} \]  

(48)

Moreover, taking expectations of (44) yields,

\[ \rho_u = \frac{1}{\frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1} \]  

\[ \rho_v = -\frac{1}{\frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1} \]  

\[ F_2 = \frac{1}{\sigma_{p_t}^2(1)} \left[ \frac{1}{\alpha} + \frac{1}{\beta} + \frac{\sigma_{\eta}^2}{\frac{\theta}{d} \left( 1 - \frac{1}{F_1 d} \right) \omega - F_2 \lambda_1} \right] \]  

A Review of Solutions in the Absence and Presence of Futures Contracts

Absence:

\[ \bar{p} = \frac{C}{b + \frac{\gamma}{\Pi_0} (1 - \theta)} \]
\[
\sigma^2_\mu(1) = \frac{\sigma^2_\eta}{(\theta_f(\omega - \lambda_1))^2}
\]
\[
\sigma^2_\nu = \frac{\sigma^2_\eta}{(1 - \lambda_1^2)(\theta_f(\omega - \lambda_1))^2}
\]
\[
\sigma^2_\kappa(1) = \frac{\sigma^2_\eta}{(IP_\nu)^2} = \frac{\sigma^2_\eta}{(IP_\nu)^2(\theta_f(\omega - \lambda_1))^2}
\]
\[
\sigma^2_\kappa = \frac{\sigma^2_\eta}{(IP_\nu)^2} = \frac{\sigma^2_\eta}{(IP_\nu)^2(1 - \lambda_1^2)(\theta_f(\omega - \lambda_1))^2}
\]
\[
\rho_\nu = \frac{1}{\gamma(\omega - \lambda_1)} - \frac{1}{\theta_f(\omega - \lambda_1)}
\]
\[
b = \frac{1 + \alpha a^2}{\theta_f(\omega - \lambda_1)}
\]
\[
\gamma = \frac{1}{d + \beta \sigma^2_\nu(1)}
\]
\[
(\theta - 1)(\lambda - \omega) = \frac{b}{\gamma}
\]

Presence:
\[
\bar{p} = \frac{-C_f}{\left(\left(\theta - 1\right)\left(\frac{1}{F_1d} - 1\right) - \frac{1}{F_1c} \left(F_2 + \frac{\gamma}{d}\right) - F_2(\theta - 1)\right)}
\]
\[
\sigma^2_\nu(1) = \frac{\sigma^2_\eta}{\left(\theta f \left(1 - \frac{1}{F_1d}\right) \omega - F_2 \lambda_1\right)^2}
\]
\[
\sigma^2_\nu = \frac{\sigma^2_\eta}{\left(1 - \lambda^2_1\right)\left(\theta f \left(1 - \frac{1}{F_1d}\right) \omega - F_2 \lambda_1\right)^2}
\]
\[
\sigma^2_\kappa(1) = \frac{\sigma^2_\eta}{(IP_\nu)^2} = \frac{\sigma^2_\eta}{\left(\theta f \left(1 - \frac{1}{F_1d}\right) \omega - F_2 \lambda_1\right)^2}
\]
\[
\sigma^2_\kappa = \frac{\sigma^2_\eta}{(IP_\nu)^2} = \frac{\sigma^2_\eta}{\left(1 - \lambda^2_1\right)\left(\theta f \left(1 - \frac{1}{F_1d}\right) \omega - F_2 \lambda_1\right)^2}
\]
\[
\rho_\nu = \frac{1}{d\left(1 - \frac{1}{F_1d}\right) \omega - F_2 \lambda_1}, \quad \rho_\nu = \frac{1}{d\left(1 - \frac{1}{F_1d}\right) \omega - F_2 \lambda_1}
\]
\[ F_1 = \frac{1}{\sigma^2(\beta)} \left( \frac{1 + \frac{1}{\alpha}}{c + \frac{1}{d}} \right), \quad F_2 = \frac{1}{\sigma^2(\beta)} \left( \frac{1 + \frac{1}{\alpha}}{c + \frac{1}{d}} + \frac{\sigma^2}{\beta} \right) \]

\[
(\partial \lambda - 1) \left( \frac{F_1 - \frac{1 - \frac{1}{d}}{c + \frac{1}{d}}}{F_2} \right) = \left( \frac{d + \frac{\sigma^2}{\beta}}{c} \right) \lambda
\]
Appendix 2. The Data

United States Short-term interest rates:

Macaulay's call money rates at the New York Stock Exchange, 1861:01 - 1936:12,

Macaulay's commercial paper rate, 1836:01 - 1936:12,

1836:01 - 1860:12:
PP. A248-250. Boston. Rates are averages of Bigelow's reported "beginning," "middle," and "end" of month. Bigelow describes these rates as "street rates on first class-paper in Boston...at the beginning, middle, and end of the month."\(^{51}\)

1861:01 - 1936:12:
PP. A142-161. Col (3). New York. From 1860 to 1923:12 'choice 60-90 day two name paper'; from 1924:01 to 1936:12 '4 to 6 month prime double and single name paper'.

Macaulay's 3-month time money rate in New York City, 1890:01 - 1936:12,

MM&W's 3-month time money rate in New York City, 1890:01 - 1936:12,
Source: Mankiw, N. Gregory and Jeffrey A. Miron. 1985. "The Changing Behavior of the Term Structure of Interest Rates," *NBER Working Paper #1669*. "Three Month Rate." Mankiw and Mankiw and Miron (1985) state that: "These data are time rates available at New York banks from 1890 to 1958; they are interest rates banks charged for loans of fixed maturity. In 1910, the National Monetary Commission compiled these data from 1890 to 1909 by tabulating them from the *Financial Review*, a periodical that analyzed current financial market developments. We updated this series to 1958 using the *Review* and the *Commercial and Financial Chronicle*, which took over the *Review* in 1921." Data are from the first week of each month.\(^{52}\)

MM&W's 3-month time money rate in New York City, 1890:01 - 1968:05

Regional antebellum interest rates on short-term bills of exchange,
  Boston, 1836:01-1859:12,
  Charleston, 1838:01-1859:12,
  New Orleans, 1839:11-1859:12,
  Philadelphia, 1839:02-1857:06.
Source:

\(^{51}\) For the period 1836:01-1859:12 these data are also found in Bodenhorn (1992)
\(^{52}\) See footnote 9 of Miron (1988).
BIBLIOGRAPHY


Chicago Board of Trade (bulletin). 1936 "The Development of the Chicago Board of Trade."


Stein, Jerome, L. 1984. "Real Effects of Futures Speculation: Rational Expectations and Diverse
Opinions,” working paper no. 88, Center for the Study of Futures Markets, Columbia University.


Figure 1: Short-term Antebellum Interest Rates, 1836-1860
Source: Bodenhorn (1992)
Figure 2.a Seasonal Patterns and Standard Errors, Macaulay's Paper Rate, 1836.01-1874.12
Figure 2.b. Seasonal Patterns and Standard Errors, Macaulay's Paper Rate, 1875.01-1910.12
Figure 2.c. Seasonal Patterns and Standard Errors, Macaulay's Paper Rate, 1920.01-1933.12

Seasonal variations (%)